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We complete the discussion of the periodic fixed points of Bäcklund transformations for the Korteweg-de Vries equation. It will be shown that the systems of equations defined by the KdV periodic fixed points are equivalent to the periodic Kac-Van Moerbeke systems. As a consequence, for even order fixed points, the KdV systems are equivalent to the periodic Toda lattice.

The periodic fixed points of the Bäcklund transformation for the Boussinesq equation are found to have a Hamiltonian structure. The integrals of these systems are found.

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BÄCKLUND TRANSFORMATION AND THE SCHWARZIAN DERIVATIVE

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1 Bäcklund transformation and the Schwarzian derivative

1.1 Introduction

This report is on work supported by the AFOSR for six months during the period from January 1986 to June 1987. During this time we have developed a method for systematically deriving the connection between finite and infinite dimensional dynamical systems. That is, the periodic fixed points of Bäcklund transformations are finite dimensional invariant manifolds of the infinite dimensional system. When the system is integrable the invariant flow is a finite dimensional, integrable system. In earlier work supported by the AFOSR we have shown how do find Bäcklund transformations through use of the Painlevé property. The current set of results demonstrate that Bäcklund transformations can obtain nearly complete information about the system. For instance, the relationships between systems is made evident. On the invariant finite dimensional manifold an infinite dimensional system with a hamiltonian structure is described by commuting hamiltonian flows in the space and time variables. It is important to note that the reduction takes place in the original (phase space) variables and not in the inverse scattering variables that are commonly used to study integrable systems. In our opinion this allows a more intuitive and direct approach to the finite dimensional reductions and their stability to perturbation. Also, the method applies to systems with any number of independent variables which possess a Bäcklund transformation. The system need not be integrable. In this case, the reduction by fixed points need not be integrable either. In general, the study of periodic fixed points makes the use of Bäcklund transformations an effective procedure with important advantages over the use of inverse scattering in classifying the behavior of integrable systems. The analysis is local (not tied to boundary data), algebraic (almost algorithic) , direct and unifying.

1.2 Specific results

In reference [1] we introduced the method of periodic fixed points and applied this to the odd order fixed points of the Korteweg-de Vries equation. The *dual-hamiltonian* formulation of the invariant manifold is found and the integrals are obtained by an explicit construction. The construction applies a linear operator to the *casimir* integral to find a complete set of integrals in involution.

Reference [2] contains the results of the work supported by this grant. This paper will appear in the **Journal of Mathematical Physics** in September or October of 1987. Therein, we complete the discussion of the Korteweg-de Vries system by showing that the even order fixed points are a completely integrable system. We find that the KdV fixed points (modulo some technical stuff) are equivalent to the Kac-Van Moerbeke systems and for even order contain the periodic Toda lattice systems. The periodic Toda lattices are known to determine the finite zone potentials of the AKNS hierarchy of equations. Thus, the periodic fixed points unify the KdV and AKNS hierarchies of equations.

For the periodic fixed points of the Boussinesq system we find hamiltonian invariant manifolds that are similar but inequivalent to the KdV systems. For instance the integrals are found by the operator method but the dual-hamiltonian formulation does not follow in the same manner as the KdV case. The systems are intrinsically more complicated and exhibit for different orders of fixed point an interplay between constraints and casimir integrals which determines the six possible subsequences of hamiltonian systems. These are a reflection of the even-odd parity of the casimir integrals and the triplet structure of the constraints. We have found a complete set of integrals for these systems.

By inspection of the KdV and Boussinesq systems we formulate a generic form of hamiltonian system that is conjectured to be completely integrable. A subset of the integrals is found and it is shown how the structure through the various hierarchies of equations becomes increasingly complex. This complexity is caused by the presence of an increasing number of casimirs and constraints with each new sequence.

1.3 Examples

We illustrate some of the above remarks with an example. The periodic fixed points of degree seven define the following systems.

- The Korteweg-de Vries system.

$$\xi_{j,x} + \xi_{j+1,x} = \xi_j - \xi_{j+1}$$

- The Boussinesq system.

$$\xi_{j,x} + \xi_{j+1,x} + \xi_{j+2,x} = \xi_j - \xi_{j+2}$$

- The next system.

$$\xi_{j,x} + \xi_{j+1,x} + \xi_{j+2,x} + \xi_{j+3,x} = \xi_j - \xi_{j+3}$$

- $j = 1, 2, 3, \ldots, \pmod 7$

All three systems have a casimir integral of degree seven

$$H_7 = \prod_{j=1}^{7} \xi_j$$

and define hamiltonian systems of three degrees of freedom.

The KdV system has additional integrals of degree five, three and one. The Boussinesq system has integrals of degree four, two and one. The last system has integrals of degree three, two and one. These independent integrals are in involution and the systems are completely integrable [2,3].

Within the systems of this form the above represent the unique, nonequivalent forms. Some orbits of these are shown in the appendix.

2 References

1. *J. Weiss*, **Periodic fixed points of Bäcklund transformations and the Korteweg-de Vries equation**, *Journal of Mathematical Physics*, *2647*, **27**, *(1986)*

2. *J. Weiss*, **Periodic fixed points of Bäcklund transformations** , *Journal of Mathematical Physics*, **28**, to appear *September (1987)*

3. *J. Weiss*, **Work in progress.**

3 Appendix: Numerical orbits

Typical orbits for

1. Korteweg-de Vries system.
2. Boussinesq system.
3. Next system.

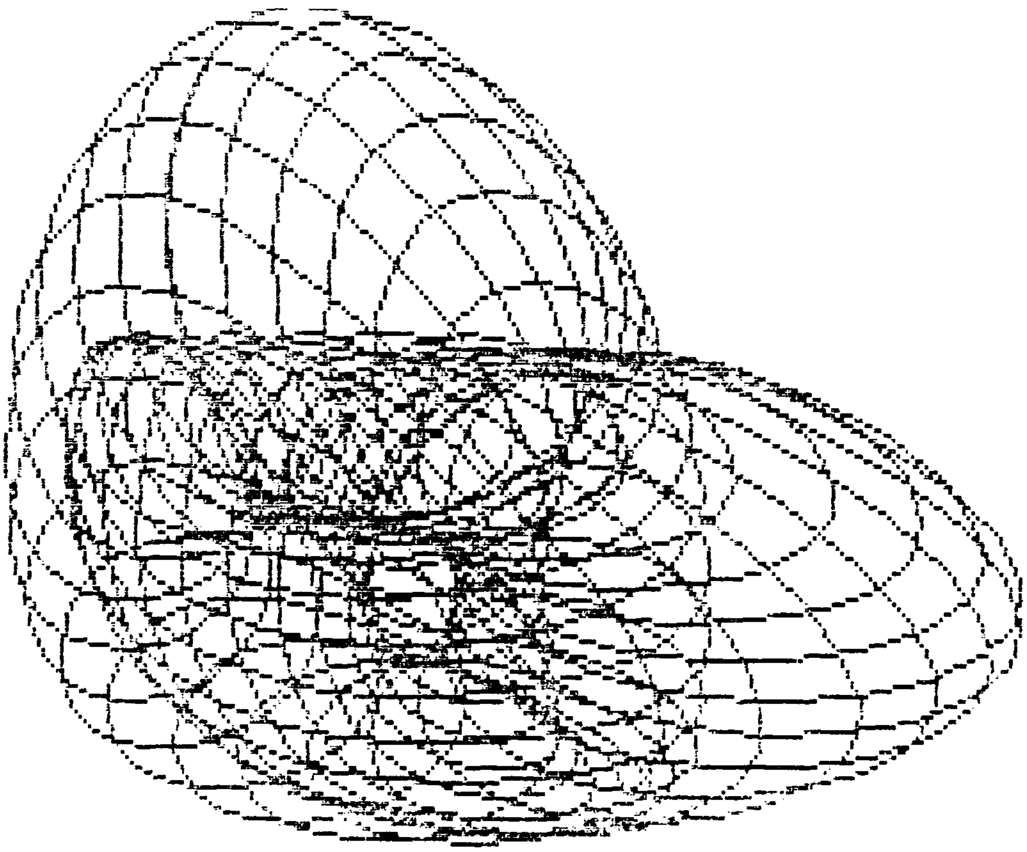
The phase portraits show ξ_1, ξ_2 with the initial conditions

$$\xi_1 = 1, \xi_2 = 2, \xi_j = 1, j = 3, 4, 5, 6, 7$$

plotted for $x \in (0, 100)$.

3.1 The Korteweg-de Vries orbit

3.2 The Boussinesq orbit



3.3 Orbit of the next equation in sequence