AD-A190 207 HIGH POWER HIGH FREQUENCY RADIATION FROM BEAM-PLASMA INTERACTIONS(U) CALIFORNIA UNIV IRVINE DEPT OF PHYSICS G BENFORD 14 JUN 87 AFOSR-TR-87-1029 AFOSR-82-0233 1/1

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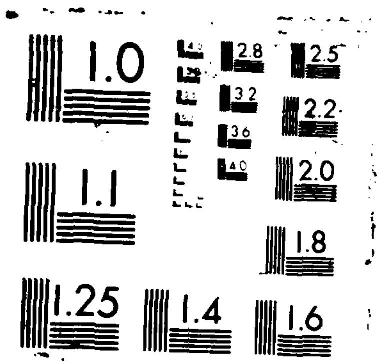
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REPORT DOCUMENTATION PAGE

1b. RESTRICTIVE MARKINGS	DTIC FILE COPY
3. DISTRIBUTION/AVAILABILITY OF REPORT	Approved for public release, distribution unlimited
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 87-1829
6a. NAME OF PERFORMING ORGANIZATION: Unievrsity of California, Irvine	6b. OFFICE SYMBOL (If applicable)
7a. NAME OF MONITORING ORGANIZATION	AFOSR
6c. ADDRESS (City, State, and ZIP Code): Department of Physics, University of California, Irvine 92717	7b. ADDRESS (City, State, and ZIP Code): Bldg 410, Bolling AFB, DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR	8b. OFFICE SYMBOL (If applicable): NP
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	AFOSR-82-0233
8c. ADDRESS (City, State, and ZIP Code): same as 7b.	10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO
61102F	2301	A8	

11. TITLE (Include Security Classification)

"HIGH POWER, HIGH FREQUENCY RADIATION FROM BEAM-PLASMA INTERACTIONS"

12. PERSONAL AUTHOR(S)
Dr. Gregory Benford

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Annual	FROM 82/6/15 TO 87/6/14	--------	43

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Experiment, Turbulence, Electron-Beams

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A theory has been developed for the probility distribution of the electric field in a highly turbulent plasma environment, Expermental optical studies have been carried out to quantify the distribution function for electric fields less than 10 kV/cm. Preliminary studeis have been conducted of the anisotrophy of microwave emission from beam-plasma systems.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☒ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION: Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL: DR. ROBERT J. BARKER	22b. TELEPHONE (Include Area Code): (202)767-5011 — 22c. OFFICE SYMBOL: NP

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

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AFOSR-TR- 87-1829

FINAL REPORT

AFOSR Grant #82-0233

Gregory Benford, Principal Investigator

6/15/82 - 6/14/87

A. <u>Theory for Strong Electric Correlations</u>. We have developed a theory for the probability distribution function of the electric field in a highly turbulent environment. This makes contact with our previous experiments, in which we found a characteristic distribution function, $\exp(-E^2)$ in a range of different beam-plasma strengths. The theory, as detailed in Appendix A, extends previous work which took into account collective effects in the plasma surrounding the ion. Our work takes into account the possibility of strong wave correlations overlengths exceeding the Debye length, λ_D. We find that to explain the 100kV/cm fields we have observed we must invoke correlations which are quite strong over dimensions of at least $10\lambda_D$. This is a new result with possible powerful consequences. It makes connection with the general body of soliton theory, which envisions the final stage of plasma turbulence as compressed packets of electric field on the scale of 10-100 Debye lengths, and containing field strengths comparable to the local thermal pressure. The fact that we have found this distribution in a very general way implies that it may be a common feature of strong turbulence. Further, it confirms our previous assertion that the specific form $\exp(-E^2)$ implies a one dimensional nature of the electric field spectrum. If the electric fields were spherically symmetric, for example, the distribution function would be $E^2\exp(-E^2)$. This implies a picture of one-dimensional beam-plasma turbulence which persists for the entire duration of our beam pulse, at least a microsecond.

B. Optical Study of electric fields less than 10kV/cm. We have carried out some of the studies we promised in our last proposal, of the distribution function for relatively weak electric fields. This is treated in detail in Appendix B and shows that we have successfully developed the diagnostics for this sensitive regime. We find that the distribution function remains the same at lower electric field strengths. We are now undertaking to measure the electric field distribution as a function of distance along the propagation axis, radius out from that axis, and time during the pulse(z, r and t). This will form the principal thesis topic of a graduate student, Ami Dovrat.

C. The anisotropic nature of turbulent microwave emission.

We have carried out preliminary studies of the anisoptropy of microwave emission. This shows that there is unexpected and quite possibly meaningful dependence of the polarization of radiation on the angle of emission. We hope to use this to probe the nature of the emitting entities deep within the plasma. Since we suspect these regions are quite small, on the range of 100 Debye lengths or less, radiation remains the easiest diagnostic of such features. We suspect that collapsed or "caviton" entities produce the radiation, and are generally oriented along the magnetic axis, which is the same as the beam axis. However, the polarization pattern does not correspond to such a simple one dimensional picture. We shall try to interpret the data in terms of a statistical distribution of dipoles within the plasma. This is being undertaken now both experimentally and theoretically.

D. In the last year we have produced several papers:

1. Scattering of Relativistic Electron Beams in a Plasma by Beam Induced Waves and Magnetic Field Errors, J. Plasma Phys. 35(1), 165 (1986).

2. Electric Field Spectra Beyond the Strong Turbulence Regime of Relativistic Beam-Plasma Interactions (with D. Levron and D. Tzach), Phys. Rev. Letts. 58 13 (1987).

3. Collisional Statistical Model for Super-Strong Plasma Turbulence, Phys. Fluids, 30 pp. 2579-2582 (1987).

4. Spectroscopic Measurements of Electric Field Intensities and of the Spectrum of Electric Field Fluctuations in Intense Relativistic Electron Beam-Driven Turbulent Plasma (D. Levron, W.T. Main, A. Fisher, G. Benford and K. Kato), p. 420, Beams '86 Proc. of the 6th International Conference on High-Power Particle Beams

5. Fundamental Studies of Microwave Emission from Relativistic Beam-Plasma Turbulence, (G. Benford, W. Main, A. Ben-Amar Baranga and K. Kato) p. 543, Beams '86, Proc. of the 6th International Conference on High-Power Particle Beams

Appendix A

Electric Microfield Distribution In Plasma With Long-Range Correlation

Gregory Benford and Xiaoling Zhai

October 2, 1987

Collective and individual particle correlations affect the probability distribution $W(E)$ of the electric microfield in a stationary, turbulent plasma. We extented previous work to include long-range correlations over lengths $\gg \lambda_D$, the Debye length. The characteristic distribution $W(E) \propto E^{d-1} \exp(-\frac{E^2}{<E^2>})$ emerges, with d the dimensionality of the electric field. The mean $<E^2>$ is proportional to the square of the correlated particle density. To describe a recent strongly turbulent experiment with $< E^2 >^{1/2} \approx 85kV/cm$ requires correlation over scales $\geq 10\lambda_D$. Comparison with observed microwave emission implies that more than one percent of the plasma volume experiences strong field regions

1. Introduction

Many experiments have measured the electric field distribution inside turbulent plasma. (Klein and Kunze 1973 ; Gallagher and Lavine 1971 ; Hamada 1970 ; Antonov *et al* 1970 ; Matt and Scott 1972). A recent measurement (Levron, Benford and Tzach, 1987) found a probability distribution of the electric Langmuir field in a strongly driven beam-plasma environment,

$$W(\mathbf{E}) \propto \exp(-\frac{E^2}{< E^2 >})$$

where $< E^2 >$ is about $(85\text{kV/cm})^2$.

Theoretical investigation (Ecker and Spatschek 1973; Hooper 1967; Ecker 1971) of a stationary equilibrium, homogeneous plasma model gave some physical explanation of the electric field distribution in terms of screening. All yield an exponential distribution. G. H. Ecker *et al* (1973) give

$$W(\mathbf{E}) \propto \exp(-\frac{E^2}{E_H^2})$$

where E_H is the Holtsmark field (Holtsmark 1919). It is about 0.16kV/cm in the experiment of D. Levron *et al*, about 10^{-3} times smaller than observed. Plainly collective effects and possibly long-range correlation must come into play to explain this striking difference. Generaly for turbulent electric field of dimensionality d, $W(E) \propto E^{d-1}\exp(-E^2)$. The Levron *et al* experiment implies d=1, with such strong antisotropy apparently arising from the beam-plasma instablity.

Here we develop a theory for $W(\mathbf{E})$ which decomposes the electric microfield distribution into three parts: individual particle, collective and long-range correlations. We give each an equal footing, following the strong wave field approach of Ecker and Spatschek with correlations added.

2. Basic Equations

Let us start from the Maxwell equations, with no net current $\mathbf{j} = 0$, $\mathbf{B} = 0$, and a homogeneous time-independent plasma.

$$\nabla \cdot \mathbf{E} = 4e\pi \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \qquad \nabla \times \mathbf{E} = 0 \tag{1}$$

$$\nabla \times \mathbf{B} = 0 \qquad \nabla \cdot \mathbf{B} = 0 \tag{2}$$

We can define a vector potential $\mathbf{A}$ with Fourier components $\tilde{A}_{\mathbf{k}}$

$$\mathbf{A} = c\sqrt{\frac{4\pi}{V}} \sum_{\mathbf{k}} \tilde{A}_k i\mathbf{k}_0 \, e^{i\mathbf{k}\,\mathbf{r}} \tag{3}$$

With $\mathbf{k}_0 = \frac{\mathbf{k}}{k}$, $\quad k = |\mathbf{k}|$, and since $\nabla \times \mathbf{E} = 0$, $\mathbf{E} = \nabla\psi$, with

$$\psi = -i\sqrt{\frac{4\pi}{V}} \sum_{\mathbf{k}} \frac{\tilde{E}_k}{k} e^{i\mathbf{k}\,\mathbf{r}} \tag{4}$$

Using equation (4) in equation (1),we can get:

$$\tilde{E}_{\mathbf{k}} = \frac{ie}{k}\sqrt{\frac{4\pi}{V}} \sum_i e^{-i\mathbf{k}\,\mathbf{r}_i} \tag{5}$$

The Hamiltonian of our system in the random phase approximation is

$$\begin{aligned} H &= \sum_i [\mathbf{p}_i + e/c\mathbf{A}(\mathbf{r}_i)]^2 \frac{1}{2m} + \int_V \frac{\mathbf{E}^2(\mathbf{r})}{8\pi} dV \\ &= \sum_i \frac{\mathbf{p}_i^2}{2m} + \frac{2e^2\pi}{V} \sum_i \sum_j \sum_{\mathbf{k}} \frac{1}{k^2} e^{i\mathbf{k}\,(\mathbf{r}_i - \mathbf{r}_j)} \end{aligned} \tag{6}$$

We are free to choose $\tilde{A}_{\mathbf{k}}$, so we choose $\tilde{A}_{\mathbf{k}} = 0$ at the second step of equation (6). The Hamiltonian has the canonical conjugate variables, $\mathbf{r}_i$, $\mathbf{p}_i$; $\tilde{A}_{\mathbf{k}}$, $\tilde{E}_{\mathbf{k}}$. The Hamiltonian equations are:

$$\dot{\tilde{A}}_{\mathbf{k}} = \frac{\partial H}{\partial \tilde{E}_{\mathbf{k}}}, \qquad \dot{\tilde{E}}_{\mathbf{k}} = -\frac{\partial H}{\partial \tilde{A}_{\mathbf{k}}} \tag{7}$$

Consider the ensemble distribution function:

$$F_\Gamma = \prod_{i=1}^{i=N} \delta(\mathbf{r}^i - \mathbf{r}_i(t))\delta(\mathbf{p}^i - \mathbf{p}_i(t)) \prod_{\mathbf{k}} \delta(A^{\mathbf{k}} - \tilde{A}_{\mathbf{k}})\delta(E^{\mathbf{k}} - \tilde{E}_{\mathbf{k}}(t)) \tag{8}$$

Where $\mathbf{r}^i$, $\mathbf{p}^i$, $A^{\mathbf{k}}$, $E^{\mathbf{k}}$ are the coordinates in Γ space, and $\mathbf{r}_i$, $\mathbf{P}_i$, $\tilde{A}_{\mathbf{k}}$, $\tilde{E}_{\mathbf{k}}$ the coordinates in μ space. We define

$$F_\mu = \sum_i \delta(\mathbf{r} - \mathbf{r}_i(t))\delta(\mathbf{p} - \mathbf{p}_i(t)) \prod_{\mathbf{k}} (\delta A^{\mathbf{k}} - \tilde{A}_{\mathbf{k}})\delta(E^{\mathbf{k}} - \tilde{E}_{\mathbf{k}}) \tag{9}$$

Here the sum is over all particles. Now consider the time derivative of $F_{i\mu}$

$$\begin{aligned}
(\frac{\partial F_{i\mu}}{\partial t})_{\mathbf{r},\mathbf{p},(A^{\mathbf{k}},E^{\mathbf{k}})} &= (\frac{\partial F_{i\mu}}{\partial \mathbf{r}_i})_{\mathbf{r},\mathbf{p},\mathbf{p}_i,(A^{\mathbf{k}},E^{\mathbf{k}}),(\tilde{A}_{\mathbf{k}},\tilde{E}_{\mathbf{k}})}\dot{\mathbf{r}}_i \\
&+ (\frac{\partial F_{i\mu}}{\partial \mathbf{p}_{\mathbf{i}}})_{\mathbf{r},\mathbf{p},\mathbf{r}_i,(A^{\mathbf{k}},E^{\mathbf{k}}),(\tilde{A}_{\mathbf{k}},\tilde{E}_{\mathbf{k}})}\dot{\mathbf{p}}_i \\
&+ \sum_{\mathbf{l}}(\frac{\partial F_{i\mu}}{\partial \tilde{A}_{\mathbf{l}}})_{\mathbf{r},\mathbf{p},\mathbf{r}_i,\mathbf{p}_i,(A^{\mathbf{k}},E^{\mathbf{k}}),(\tilde{A}_{\mathbf{k}},\tilde{E}_{\mathbf{k}})}\dot{\tilde{A}}_{\mathbf{k}} \\
&+ \sum_{\mathbf{k}}(\frac{\partial F_{i\mu}}{\partial \tilde{E}_{\mathbf{l}}})_{\mathbf{r},\mathbf{p},\mathbf{r}_i,\mathbf{p}_i,(A^{\mathbf{k}},E^{\mathbf{k}})}\dot{\tilde{E}}_{\mathbf{l}}
\end{aligned} \tag{10}$$

Replace $\dot{\tilde{A}}_{\mathbf{k}}$ by $\frac{\partial H}{\partial \tilde{E}_{\mathbf{k}}}$, $\dot{\tilde{E}}_{\mathbf{k}}$ by $-\frac{\partial H}{\partial \tilde{A}_{\mathbf{k}}}$, $\dot{\mathbf{r}}_i$ by $\frac{\partial H}{\partial \mathbf{p}_i}$, $\dot{\mathbf{p}}_i$ by, $-\frac{\partial H}{\partial \mathbf{r}_i}$. H as we define it in equation (6) then yields

$$(\frac{\partial F_{i\mu}}{\partial t}) = \frac{\mathbf{p}_i}{m}(\frac{\partial F_{i\mu}}{\partial \mathbf{r}_i}) - ie/m\sqrt{\frac{4\pi}{V}}\sum_{\mathbf{k}} \mathbf{k}_0 \cdot \mathbf{p}_i\, \mathbf{k}\, \tilde{A}_{\mathbf{k}}\, e^{i\mathbf{k}\,\mathbf{r}_i}$$

$$
\begin{aligned}
&+ \quad i\frac{2\pi e^2}{mV}\sum_{\mathbf{k}} \mathbf{l}(\mathbf{k}_0\cdot\mathbf{l}_0)\tilde{A}_{\mathbf{k}}\tilde{A}_{\mathbf{l}}(\mathbf{k}+\mathbf{l})\, e^{i(\mathbf{k}+\mathbf{l})\cdot\mathbf{r}_i}(\frac{\partial F_{i\mu}}{\partial \mathbf{p}_i}) \\
&- \quad \sum_{\mathbf{l}}(\frac{\partial F_{i\mu}}{\partial \tilde{A}_{\mathbf{l}}})\tilde{E}_{-\mathbf{l}} - \sum_{\mathbf{l}}[\frac{e}{m}\sqrt{\frac{4\pi}{V}}\sum_i(\mathbf{l}_0\cdot\mathbf{p}_i)\, e^{i\mathbf{k}\cdot\mathbf{r}_i} \\
&+ \quad \frac{2\pi e^2}{mV}\sum_i\sum_{\mathbf{k}}(\mathbf{k}_0\cdot\mathbf{l}_0)\tilde{A}_{\mathbf{k}}\, e^{i(\mathbf{k}+\mathbf{l})\cdot\mathbf{r}_i}](\frac{\partial F_{i\mu}}{\partial \tilde{E}_{\mathbf{l}}}) \qquad (11)
\end{aligned}
$$

Following Ecker *et al* (1973) we use the collective coordinate

$$
Q_{\mathbf{k}} = \sqrt{\frac{4\pi e^2}{k^2 V}}\sum_i e^{-i\mathbf{k}\cdot\mathbf{r}_i}
$$

Instead of putting all electron motions into the collective coordinates, we choose a value k_c of k such that the collective modes only enter with wave vector less than k_c. Clearly $k_c > r_0^{-1}$, and for undamped modes we require $k > k_c$, with k_c lying between r_0^{-1} and λ_D^{-1}. Previous calculations took $k_c\lambda_D \approx 1$, with λ_D the Debye length. Here we explicitly expand the formula to include correlations for $k < \lambda_D^{-1}$ (Fig 1). Define

$$
S = \exp(-\frac{1}{\hbar}\sum_{i,\mathbf{k},k>k_c}\sqrt{\frac{4\pi e^2}{Vk^2}}\tilde{A}_{\mathbf{k}}\, e^{i\mathbf{k}\cdot\mathbf{r}_i}) \qquad (12)
$$

Make the transformation:

$$
\begin{aligned}
\mathbf{p}_i' &= S^{-1}\mathbf{p}_i S \\
\tilde{A}_{\mathbf{k}}' &= S^{-1}\tilde{A}_{\mathbf{k}} S \qquad (13)
\end{aligned}
$$

$$
\mathbf{p}_i = \mathbf{p}_i' - \sqrt{\frac{4\pi e^2}{V}}\sum_{\mathbf{k},k\geq k_c}\mathbf{k}_0\tilde{A}_{\mathbf{k}}\, e^{i\mathbf{k}\cdot\mathbf{r}_i} \qquad (14)
$$

$$
for \quad k \geq k_c, \qquad \tilde{E}_{\mathbf{k}} = \tilde{E}_{\mathbf{k}}' + i\sqrt{\frac{4\pi e^2}{Vk^2}}\sum_i e^{-i\mathbf{k}\cdot\mathbf{r}_i}; \qquad (15)
$$

$$for \quad k < k_c, \qquad \tilde{E}_{\mathbf{k}} = \dot{\tilde{E}}_{\mathbf{k}} \tag{16}$$

3. The Distribution Function

We break the Hamiltonian equation (6), into contributions from the two salient regions of k space:

$$\begin{aligned} H &= \sum_i \frac{\mathbf{p}_i^2}{2m} + \frac{2e^2\pi}{V}\sum_i \sum_{j,i\geq j} \sum_{\mathbf{k},k>k_c} \frac{1}{k^2} e^{i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_j)} \\ &+ 1/2 \sum_{\mathbf{k},k<k_c} (E_{\mathbf{k}}E_{-\mathbf{k}} + \frac{\omega_k^3}{V}A_{\mathbf{k}}A_{-\mathbf{k}}) \end{aligned} \tag{17}$$

The plasma wave dispersion relation is

$$\sum_i \frac{\omega_k^2}{\omega_k^2 - (\frac{\mathbf{k}\cdot\mathbf{p}_i}{m})^2} e^{i\mathbf{k}\cdot\mathbf{r}_i} = 0 \tag{18}$$

$$\frac{4e^2\pi}{mV}\sum_i(\omega_k - \frac{\mathbf{k}\cdot\mathbf{p}_i}{m})^{-2} = 1 \tag{19}$$

In what follows we frequently assume a cold plasma, $\langle(k\mathbf{p}_i^2/m\omega)^2\rangle \ll 1$. Now we approximate the complex terms of equation (17). Since

$$\frac{2e^2\pi}{V}\sum_i\sum_j\sum_{\mathbf{k},k<k_c} \frac{1}{k^2} e^{i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_j)} \approx \frac{1}{2V}\sum_i\sum_{j,j\geq i} \frac{e^2 e^{-k_c|\mathbf{r}_i-\mathbf{r}_j|}}{|\mathbf{r}_i-\mathbf{r}_j|} \tag{20}$$

Using this to equation (11), through some algebra we get.

$$\begin{aligned} &\frac{\partial F_{i\mu}}{\partial t} + \frac{\mathbf{p}_i}{m}\cdot(\frac{\partial F_{i\mu}}{\partial \mathbf{r}_i}) - \sum_{\mathbf{k},k<k_c} \frac{\omega_k^2}{V}A_{-\mathbf{k}}\frac{\partial F_{i\mu}}{\partial E_{\mathbf{k}}} + \sum_{\mathbf{k}}\frac{\partial F_{i\mu}}{\partial A_{\mathbf{k}}} \\ &-e(\frac{\partial}{\partial \mathbf{p}})\int_V\int_{\mathbf{P}}\int_{\mathbf{A}} d\dot{\mathbf{r}}d\dot{\mathbf{p}}\prod_\lambda d\dot{A}^\lambda d\dot{E}_\lambda \frac{\partial}{\partial \mathbf{r}}\frac{e^{-k_c|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}F_{i\mu}\sum_j eF_{j\mu} = 0 \end{aligned} \tag{21}$$

We can illuminate this complicated result by using the Γ space density P_D, the first order distribution function f_{ic}, and the second order distribution function f_{ijc}

$$P_D = P_D(\mathbf{r}_1, \mathbf{r}_2, \ldots\ldots, \mathbf{r}_N, \mathbf{p}_1, \mathbf{p}_2, \ldots\ldots, \mathbf{p}_N, A_{\mathbf{k}_1}, A_{\mathbf{k}_2}, \ldots\ldots E_{\mathbf{k}_1}, E_{\mathbf{k}_2}, \ldots\ldots t) \quad (22)$$

$$f_{ic} = \int F_{i\mu} P_D \prod_{j=1}^{j=N} d\mathbf{r}_j \, d\mathbf{p}_j \prod_{\mathbf{k}} dA_{\mathbf{k}} dE_{\mathbf{k}} \quad (23)$$

$$f_{ijc} = \int F_{i\mu} F'_{j\mu} P_D \prod_{\nu=1}^{\nu=N} d\mathbf{r}_\nu d\mathbf{p}_\nu \prod dA_{\mathbf{k}} dE_{\mathbf{k}} \quad (24)$$

where N is the number of electrons in the system.

Mulitplying equation (21) by P_D , then integrating over all $\mathbf{r}_\nu$, $\mathbf{p}_\nu$, $A_{\mathbf{k}}$, $E_{\mathbf{k}}$, and using equation (23) and (24), yields

$$\frac{\partial f_{ic}}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f_{ic}}{\partial \mathbf{r}} - e\int_V \int_{\mathbf{P}} \int_{A_{\mathbf{k}}} \int_{E_{\mathbf{k}}} d\acute{\mathbf{r}} d\acute{\mathbf{p}} \prod_{\lambda} d\acute{A}_\lambda d\acute{E}_\lambda \frac{\partial}{\partial \mathbf{r}} \frac{e^{-k_c|\mathbf{r}-\acute{\mathbf{r}}|}}{|\mathbf{r}-\acute{\mathbf{r}}|} \frac{\partial}{\partial \mathbf{p}} \sum_j f_{ijc}$$
$$- \sum_{\mathbf{k}, k<k_c} \frac{\omega_k^2}{V} A_{\mathbf{k}} \frac{\partial f_{ic}}{\partial E_{\mathbf{k}}} + \sum_{\mathbf{k}} E_{\mathbf{k}} \frac{\partial f_{ic}}{\partial A_{\mathbf{k}}} = 0 \quad (25)$$

We define R as the probability function of collective electric field, with f_i the probability function for the individual particle electric fields, *i.e.* $f_{ic} = f_i R$, $f_{ijc} = f_{ij} R\acute{R}$. Since f_{ic} and f_{ijc} are separable, we can separate equation (25) into two equations:

$$\frac{\partial f_i}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f_i}{\partial \mathbf{r}} - e\int d\acute{\mathbf{r}} d\acute{\mathbf{p}} \frac{\partial}{\partial \mathbf{r}} \frac{e^{-k_c|\mathbf{r}-\acute{\mathbf{r}}|}}{|\mathbf{r}-\acute{\mathbf{r}}|} \frac{\partial}{\partial \mathbf{p}} (\sum_j f_{ij}) = 0 \quad (26)$$

$$\frac{\partial R}{\partial t} - \sum_{\mathbf{k}} \frac{\omega_k^2}{V} A_{\mathbf{k}} \frac{\partial R}{\partial E_{\mathbf{k}}} + \sum_{\mathbf{k}, k<k_c} E_{\mathbf{k}} \frac{\partial R}{\partial A_{\mathbf{k}}} = 0 \quad (27)$$

Equation (27) can be formally solved as

$$R = \prod_{\mathbf{k}, k<k_c} f_k (A_{\mathbf{k}} A_{-\mathbf{k}} \frac{\omega_k^2}{V} + E_{\mathbf{k}} E_{-\mathbf{k}}) \quad (28)$$

We can get $\mathbf{E}(\mathbf{r})$ from equation (5),

$$\mathbf{E}(\mathbf{r}) = \frac{i4\pi}{V} \sum_i \sum_{\mathbf{k}} \frac{\mathbf{k}_0}{k} e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_i)} \tag{29}$$

We can separate $\mathbf{E}(\mathbf{r})$ into single-particle and collective parts, with the first term counting individual particles:

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \mathbf{E}_{ind} + \mathbf{E}_{coll} \\ &= i\frac{4e\pi}{V} \sum_i \sum_{\mathbf{k},k>k_c} \frac{\mathbf{k}_0}{k} e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_i)} + \sqrt{\frac{4\pi}{V}} \sum_{\mathbf{k},k<k_c} \tilde{E}_{\mathbf{k}} \mathbf{k}_0 \, e^{i\mathbf{k}\cdot\mathbf{r}} \end{aligned} \tag{30}$$

Split the electric field $\mathbf{E}_{coll}$ into two parts:

$$\mathbf{E}_{coll} = \mathbf{E}'_{coll} + \mathbf{E}_{corr} = \sqrt{\frac{4\pi}{V}} \sum_{\mathbf{k},k<k_c} \tilde{E}_k \mathbf{k}_0 \, e^{i\mathbf{k}\cdot\mathbf{r}} \tag{31}$$

$\mathbf{E}'_{coll}$ represents collective screening while $\mathbf{E}_{corr}$ describes correlation over long range, $k < k_c$. Now we define $W(\mathbf{E})$ as a probability distribution function of the electric field, with P_{ind} and R_{coll} as distribution functions of $\mathbf{E}_{ind}$ and $\mathbf{E}_{coll}$ /

$$W(\mathbf{E}) = \int \delta(\mathbf{E} - \mathbf{E}_{ind} - \mathbf{E}_{coll}) R_{coll} R_{ind} d[\ldots\ldots]_{ind} d[\ldots\ldots]_{coll} \tag{32}$$

Let $W(\mathbf{q})$ be the Fourier transformtion of $W(\mathbf{E})$

$$\begin{aligned} W(\mathbf{q}) &= \int W(\mathbf{E}) e^{-i\mathbf{q}\cdot\mathbf{E}} d\mathbf{E} \\ &= \int e^{-i\mathbf{q}\cdot(\mathbf{E}_{ind}+\mathbf{E}_{coll})} R_{coll} R_{ind} d[\ldots]_{ind} d[\ldots]_{coll} \end{aligned} \tag{33}$$

Since $R_{ind} d[\ldots]_{ind}$ and $R_{coll} d[\ldots]_{coll}$ by assumption vary independently, we can write:

$$W(\mathbf{q}) = \int_{ind} e^{-i\mathbf{q}\cdot\mathbf{E}_{ind}} R_{ind} d[\ldots]_{ind} \int_{coll} e^{-i\mathbf{q}\cdot\mathbf{E}_{coll}} R_{coll} d[\ldots]_{coll} = W_{ind}(\mathbf{q}) W_{coll}(\mathbf{q})$$

We can take $\mathbf{r} = 0$ without changing the electric field distribution. Now consider $\tilde{E}_{\mathbf{k}}$ in two parts, $\tilde{E}_{\mathbf{k}} = \tilde{E}_{\mathbf{k}}^{corr} + \tilde{E}_{\mathbf{k}}^{coll}$, or explicitly,

$$\tilde{E}_{\mathbf{k}} = \frac{ie}{k}\sqrt{\frac{4\pi}{V}} \sum_{i=1}^{i=N_c} e^{-i\mathbf{k}\cdot\mathbf{r}_i} y(\frac{\mathbf{r}_i}{L}) + \frac{ie}{k}\sqrt{\frac{4\pi}{V}} \sum_{i=1}^{i=N_{nc}} e^{-i\mathbf{k}\cdot\mathbf{r}_i}$$

Here N_c represents the number of correlated particles and N_{nc} represents the number of uncorrelated particles. The spatial dependence of correlation is $y(r_i/L)$ which we shall choose to yield finite $< E^2 >$ in a correlation scale length L. For correlations we require $\mathbf{k}\cdot\mathbf{r}_i = 2m\pi$, where $m = 0; \pm1; \pm2; \pm3; \ldots\ldots\ldots\ldots$, so

$$\tilde{E}_{\mathbf{k}}^{corr} = \frac{ie}{k}\sqrt{\frac{4\pi}{V}} \sum_{i=1}^{i=N_c} y(\frac{\mathbf{r}_i}{L}) \approx \frac{4\pi ie}{k}\sqrt{\frac{4\pi}{V}} \int_{R_0}^{R_\infty} n_c y(r/L) r^2 dr$$

Here $R_\infty = \frac{3V}{4\pi}^{1/3}$, R_0 is a very small but finite length, and $n_c = \frac{N_c}{V}$ is the density of correlated particles.

We can write E_k^{corr} as,

$$\tilde{E}_{\mathbf{k}}^{corr} = \frac{ie}{k}\sqrt{\frac{4\pi}{V}} n_c G(k)$$

$$G(k) = 4\pi \int_{R_0}^{R_\infty} y(\frac{r}{L}) r^2 dr \tag{34}$$

Then

$$\mathbf{E}(\mathbf{r}) = \frac{4\pi}{V} n_c ie \sum_{\mathbf{k}, k<k_c} \frac{\mathbf{k}_0}{k} G(k) + \sqrt{\frac{4\pi}{V}} \sum_{\mathbf{k}, k<k_c} \tilde{E}_{\mathbf{k}} \mathbf{k}^0$$

It is easy to see that $\tilde{E}_{-\mathbf{k}} = -\tilde{E}_{\mathbf{k}}^*$ and

$$\mathbf{E}_{\mathbf{k}}^{coll+corr} = \sqrt{\frac{4\pi}{V}} \sum_{\mathbf{k}, k<k_c} \tilde{E}_{\mathbf{k}} \mathbf{k}_0 = \sqrt{\frac{4\pi}{V}} \sum_{\mathbf{k}, k<k_c} 2R_e(\tilde{E}_{\mathbf{k}}) \tag{35}$$

Where $\tilde{E}_{\mathbf{k}, k<k_c}$ includes $\tilde{E}_{\mathbf{k}}^{coll}$ and $\tilde{E}_{\mathbf{k}}^{corr}$, now use

$$R_{coll+corr} = \prod_{\mathbf{k}, k<k_c} f_{\mathbf{k}}(A^{\mathbf{k}} A^{-\mathbf{k}} \frac{\omega_k^3}{V} + E^{\mathbf{k}} E^{-\mathbf{k}}) \tag{36}$$

We have

$$W_{coll+corr}(\mathbf{r}) = \int e^{-i\mathbf{q}\cdot\mathbf{E}_{coll+corr}} R_{coll+corr}\, d[.....]_{coll} d[.....]_{corr}$$
$$= \int e^{-\mathbf{q}\cdot\sqrt{\frac{16\pi}{V}}\sum_{\mathbf{k},k<k_c} 2R_e(\tilde{E}_{\mathbf{k}})\mathbf{k}_0} \prod_{\mathbf{k},k<k_c} f_k[(R_e(\tilde{E})^2(I_m\tilde{E})^2 + \frac{\omega_k^3}{V}(R_eA_{\mathbf{k}})^2$$
$$+ (I_mA_{\mathbf{k}})^2]d[R_e\tilde{E}_{\mathbf{k}}]d[I_m\tilde{E}_{\mathbf{k}}d[R_eA_{\mathbf{k}}]d[I_mA_{\mathbf{k}}] \qquad (37)$$

Define $Z = R_eE_{\mathbf{k}}$, $\tilde{q}_{\mathbf{k}} = \mathbf{q}\cdot\mathbf{k}_0\sqrt{\frac{16\pi}{V}}$, and

$$< f_{\mathbf{k}}(Z^2) >= \int f_k d[R_eA_{\mathbf{k}}]d[I_mA_{\mathbf{k}}]d[I_mE_{\mathbf{k}}]$$

This enables us to write compactly

$$W_{coll+corr}(\mathbf{q}) = \prod_{\mathbf{k},k<k_c} \int e^{-i\tilde{q}_{\mathbf{k}}Z} < f_k(Z^2) > dZ$$

$$W_{coll+corr}(\mathbf{q}) = \prod_{\mathbf{k},k<k_c} < f_k(\tilde{\tilde{q}}^2) >$$

where $< f_k(\tilde{\tilde{q}}^2) >$ represents the Fourier transform of $< f_k(Z^2) >$, and

$$W_{coll+corr}(\mathbf{q}) = e^{\ln\prod_{\mathbf{k},k<k_c} <f_k(\tilde{\tilde{q}}^2)>} = e^{\sum_{\mathbf{k},k<k_c} <f_k(\tilde{\tilde{q}}^2)>}$$

Changing summation over $\mathbf{k}$ into integration

$$W_{coll+corr}(\mathbf{q}) = e^{\frac{V}{2\pi^3}\int_{\mathbf{k}<\mathbf{k_c}} d\mathbf{k}\ln <f_k(\tilde{\tilde{q}}^2)>}$$

Let us expand $\ln < f_k(\tilde{\tilde{q}}^2) >$ at $\tilde{q} = 0$.

$$\frac{1}{2\pi^3}\ln < f_K(\tilde{\tilde{q}}^2) > = C_{00} - C_{0\mathbf{k}}(\mathbf{k}_0\cdot\mathbf{q})\sqrt{\frac{16\pi}{V}} - C_{1\mathbf{k}}\frac{(\mathbf{k}^0\cdot\mathbf{q})^2 16\pi}{V}.......$$

C_{00} *and* $C_{0\mathbf{k}}$ must be zero, for otherwise the expression diverges! All terms beyond the third term vanish as $V \longrightarrow \infty$. Only the third term survives:

$$W_{coll+corr}(\mathbf{q}) = e^{-\int_{\mathbf{k},k<k_c} C_{1\mathbf{k}} 16\pi(\mathbf{k}_0\cdot\mathbf{q})^2 d\mathbf{k}} = e^{-\tilde{q}^2\frac{16\pi}{V}\int C_{1\mathbf{k}} d\mathbf{k}}$$

$W_{coll+corr}(\mathbf{q})$ is the Fourier component of $W_{coll+corr}(\mathbf{E})$, so

$$W_{coll+corr}(\mathbf{E}) = \int e^{-\mathbf{q}\cdot\mathbf{E}} W_{coll+corr}(\mathbf{q}) d\mathbf{q}$$

$$W_{coll+corr}(\mathbf{E}) = \frac{3\sqrt{3}}{512\pi^3(\int C_{1\mathbf{k}} d\mathbf{k})^{3/2}} exp(-\frac{E^2}{\frac{64\pi}{3}\int C_{1\mathbf{k}} d\mathbf{k}})$$

Take $< E^2 >= 32\pi \int C_{1\mathbf{k}} d\mathbf{k}$, then

$$W_{coll+corr}(\mathbf{E}) = \sqrt{27/8}\frac{1}{\pi^{3/2} < \mathbf{E}^2 >^{3/2}} e^{\frac{-E^2}{2/3 < E^2 >}}$$

We can see the distribution of electric field is proportional to $e^{\frac{-3E^2}{2 < E^2 >}}$, where $\mathbf{E}$ includes $\mathbf{E}_{coll}$ and $\mathbf{E}_{corr}$

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_{corr} + \mathbf{E}_{coll} = \frac{4\pi}{V} n_c e \sum_{\mathbf{k},k<k_c} \frac{\mathbf{k}_0}{k} G(\mathbf{k}) + \sqrt{\frac{4\pi}{V}} \sum_{\mathbf{k},k<k_c} \tilde{E}_{\mathbf{k}} \mathbf{k}^0 \qquad (38)$$

From equation (38), we can derive

$$\mathbf{E}_{coll} \cdot \mathbf{E}_{corr} = (\frac{4\pi}{V})^{3/2} n_c e \sum_{\mathbf{k},k<k_c} G(\mathbf{k})/k \tilde{E}_{\mathbf{k}}$$

and

$$\tilde{E}_{\mathbf{k}} = \frac{ie}{k}\sqrt{\frac{4\pi}{V}} \sum_{i=1}^{i=N_{nc}} e^{-i\mathbf{k}\,\mathbf{r}_i} \qquad (39)$$

Since

$$E^2_{coll} = \frac{4\pi}{V} \sum_{\mathbf{k},k<k_c} \sum_{i=1}^{i=N_{nc}} \sum_{j=1}^{j=N_{nc}} e^{i(\mathbf{k}\,\mathbf{r}_i - \mathbf{l}\,\mathbf{r}_j)} (\mathbf{k}_0 \cdot \mathbf{l}_0)$$

N_{nc} is the number of uncorrelated particles, a large number $\sum_{i=1}^{i=N_{nc}} e^{-i\mathbf{q}\,\mathbf{r}_i}$ vanshes, because $i\mathbf{k}\cdot\mathbf{r}_i$ is a random phase. If i=j and $\mathbf{k} = -\mathbf{l}$, then we have $e^{i(\mathbf{k}\,\mathbf{r}_i + \mathbf{l}\,\mathbf{r}_j)} = 1$, so

$$\mathbf{E}^2_{coll} = \frac{16\pi^2 e^2}{V} N_{nc}$$

$$\mathbf{E}_{corr}^2 = -\frac{16\pi^2 e^2}{V} n_c^2 \sum_{\mathbf{k},k<k_c} \sum_{\mathbf{l},l<k_c} \frac{\mathbf{k}}{k} \cdot \frac{\mathbf{l}}{l} G(k)G(l)$$

Later we will see that $< E_{corr}^2 >$ is a large number, so we can neglect $\mathbf{E}_{coll} \cdot \mathbf{E}_{corr}$ and find $\mathbf{E}_{coll+corr}^2 \approx \mathbf{E}_{coll}^2 + \mathbf{E}_{corr}^2$ Now we have, for a one-dimensional field,

$$W_{coll+corr}(\mathbf{E}) = \sqrt{\frac{27}{8}} \frac{1}{\pi^{3/2} < E^2 >^{3/2}} e^{-\frac{\mathbf{E}_{coll}^2 + \mathbf{E}_{corr}^2}{2/3 < E^2 >}}$$

$$< E^2 > = \frac{1}{2\pi^2} \int_{\mathbf{k},k<k_c} \tilde{E}_{\mathbf{k}} \tilde{E}_{-\mathbf{k}} d\mathbf{k} = < \mathbf{E}_{coll}^2 > + < \mathbf{E}_{corr}^2 > + U$$

Here the residue is

$$U = -\frac{1}{2\pi^2} \frac{4\pi}{V} n_c \int_{\mathbf{k},\mathbf{k}<\mathbf{k_c}} G(k) \sum_{i=1}^{i=N_{nc}} (e^{-i\mathbf{k}\,\mathbf{r}_i} + e^{i\mathbf{k}\,\mathbf{r}_i}) d\mathbf{k} \qquad (40)$$

Because of the random phase, both $\tilde{E}_{\mathbf{k}}^{coll} \tilde{E}_{-\mathbf{k}}^{corr}$ and $\tilde{E}_{\mathbf{k}}^{corr} \tilde{E}_{-\mathbf{k}}^{coll}$ become negligible compared with $< E_{coll}^2 >$ and $< E_{corr}^2 >$, so U=0 The distribution function of electric field is generally.

$$W_{coll+corr}(\mathbf{E}) = \sqrt{\frac{27}{8}} \frac{1}{\pi^{3/2}(< E_{coll}^2 > + < E_{corr}^2 >)} e^{-\frac{E_{coll}^2 + E_{corr}^2}{< E_{coll}^2 > + < E_{corr}^2 >}}$$

We are interested in the case where $\langle E_{corr}^2 \rangle \gg \langle E_{coll}^2 \rangle$, so $W_{coll+corr}(\mathbf{E}) \approx W_{corr}$. If $\mathbf{E}_{corr}$ is much stronger then $\mathbf{E}_{coll}$, the distribution funtion is nearly the same as the correlation electric field distribution. This is the distribution function for one dimension. For d dimensions, we should have

$$W \propto E^{d-1} e^{-(\frac{E^2}{< E^2 >})}$$

For three dimensions the distribution function becomes $W \propto E^2 e^{-(\frac{E^2}{< E^2 >})}$, a form often seen in weakly turbulent environments (Klein and Kunze, 1973).

Examining $< E^2_{coll+corr} >$,

$$< E^2_{coll} > \;=\; -\frac{4\pi\, e^2}{8\pi^3 V}\int_{\mathbf{k}<\mathbf{k}_c} N_{nc}\frac{1}{k^2}d\mathbf{k}$$

$$= -\frac{4\pi\, e^2}{8\pi^3 V}N_{nc}\frac{4\pi}{3}k_c \tag{41}$$

$$< E^2_{corr} > \;=\; \frac{n_c^2\, e^2}{2\pi^2 V}\int_{k<k_c}\frac{1}{k^2}G(k)^2 d\mathbf{k} \tag{42}$$

The spatial dependence of correlations, $y(\mathbf{r})$, allows finite $< E^2_{corr} >$ only if $y(r) = (L/r)^{3/2}$. This is an artifact of our method; in general, $y(r)$ must be rapidly declining. We find

$$G(k) \;=\; 4\pi\int_{R_0}^{R_\infty}(L/r)^{3/2}r^2dr = 6\pi L^{3/2}(V^{1/2} - V_0^{1/2}) \tag{43}$$

$$< E^2_{corr} > \;=\; \frac{n_c^2\, e^2\, 4\pi L^3 (4\pi)^2\, 4}{2\pi^2\, 9}k_c(1 + \frac{V_0}{V} - 2\frac{V_0^{1/2}}{V^{1/2}}) \tag{44}$$

Here $V_0 = R_0^3$; in the limit $V \longrightarrow \infty$

$$< E^2_{corr} > = \frac{128}{9}n_c^2 e^2 L^3 k_c \tag{45}$$

Since $< E^2_{coll} > = 16\pi^2 n_{nc} e^2$, we find

$$< E^2_{coll+corr} > = \frac{128}{9}n_c^2\, e^2 L^3 k_c + 6\, e^2 n_{nc} k_c \tag{46}$$

Here $n_{nc} = N_{nc}/V$ is the density of particles which are uncorrelated.

4. Comparison With Experiment

To compare with observations we use the the data of the D. Levron *et al* (1987). Their density of particles is about 10^{13}cm^{-3}, mean electron temperature is about 10eV, and the Debye length $\lambda_D \approx 7.4\times10^{-4}$cm, so $k_D \approx 8.4\times10^{3}\text{cm}^{-1}$.

The interparticle separation r_0 is about 1.3×10^{-5}cm. so $k_0 = \frac{2\pi}{r_0} \approx 57 k_D$. The Holtsmark field is $E_H = \frac{e}{r_0^2} = 0.16$kV/cm. We can write, with $\tilde{k} = \frac{k_c}{2\pi} r_0$.

$$< E^2 > = \frac{64}{3} E_H^2 (\frac{n_c}{n})(n_c L^3)\tilde{k} \tag{47}$$

Levron *et al* found $< E^2 >_{obs} \approx (85\text{kV/cm})^2$, about 3×10^5 times greater than E_H^2. To fit experiment, we need $\frac{n_c}{n}(n_c L^3)\tilde{k} \approx 1.3 \times 10^4$. Let us see what this implies. Define N_D as the number of particles inside the Debye sphere ($N_D \approx 1.5 \times 10^5$). $N_c = \frac{4\pi}{3} n_c L^3$ is the number of particles inside the correlated sphere. Then

$$\tilde{k}(\frac{n_c L^3}{N_D})\frac{n_c}{n} \approx 0.1 \, ; \qquad \frac{L}{\lambda_D} \approx \frac{0.25}{[\tilde{k}(\frac{n_c}{n})^2]^{1/3}} \tag{48}$$

If we use $k_c = k_D$, $\tilde{k}$ is about 0.02. Even if we choose $\frac{n_c}{n} \approx 1$, for complete conhence in the region, L is about λ_D, mor possibly taking $\frac{n_c}{n} = 0.1$, L is about $5\lambda_D$. Thus long-range correlations are necessary.

We can use further constraints set by the observed microwave power from the same experiment. Using the electric field distribution, we can calculate the power of the microwave radiation of the plasma

$$P_{total} = N_s \int\int W(E) dE \frac{dP}{d\Omega} d\Omega$$

Where $W(E) = \frac{2}{u\sqrt{\pi}} e^{\frac{E^2}{u^2}}$, is the electric field distribution (one dimension), with $u^2 = < E^2 >$. N_s is the number of cavitons. We take the power emitted by a single soliton correlated over a scale L. There are N_s such emitters, which we represent as simple dipoles oscillators of dipole length L. Then

$$\frac{dP}{d\Omega} = 54\pi^2 (sin\theta)^2 \frac{E^2}{8\pi} \sigma_T c (n_c L^3)^2$$

$$P_{total} = 576\pi^2 N_s \sigma_T c E_H{}^2 (n_c L^3)^3 (\frac{n_c}{n}) \tilde{k}$$

The experimentally observed result (Barauga, Benford, Tzach, and Kato, 1985) is about $P_{obs} = 10^5 P_5$ Watt. Since $E_H^2 \sigma_T c \approx 5 \times 10^{-23}$ Watt, we found

$$(\frac{n_c}{n})^4 (nL^3)^3 N_s \tilde{k} \approx 3 \times 10^{25} P_5 \tag{49}$$

Take $nL^3 \approx 5 \times 10^4 (\frac{L}{\lambda_D})^3$, plus the observed $u^2 = 64/3 E_H^2 \frac{n_c}{n}(n_c L^3)\tilde{k} = (85kV/cm)^2$, so that $\frac{n_c}{n}(n_c L^3)\tilde{k} \approx 10^4$. Therefore

$$(\frac{n_c}{n})^2 = \frac{10^4}{(nL^3)\tilde{k}}$$

Using equation (49)

$$\frac{10^8}{(nL^3)^2}(nL^3)^3 N_s \frac{1}{\tilde{k}} \approx 3 \times 10^{25} P_5 = 2 \times 10^{14} \frac{\tilde{k} P_5}{(\frac{L}{\lambda_D})^3}$$

Define the packing fraction of the turbulence, with the typical volume $V_3 = V_{expt}/10^3 cm^3$ of beam-plasma interaction,

$$f = \frac{N_s}{N_{max}} = \frac{4(\frac{L}{\lambda_D})^3}{10^{15} V_3}$$

The maxium number of emitting dipoles which could fit in V_{expt}, N_{max} is,

$$N_{max} = \frac{V_{expt}}{4/3\pi L^3} = \frac{10^{15} V_3}{4(\frac{L}{\lambda_D})^3}$$

The actual number of emitters is, using equation (49),

$$N_s = f N_{max} = f \frac{10^{15} V_3}{4(\frac{L}{\lambda_D})^3} \approx 2 \times 10^{14} \frac{\tilde{k} P_5}{(\frac{L}{\lambda_D})^3}$$

The packing fraction is,

$$f \approx 0.8 \frac{\tilde{k} P_5}{V_3} \tag{50}$$

So our result is free of the details of the emitting geometry and partiularly of $(L/lambda_D)$, as long as the individual volumes scale as L^3. Luckily, f is independent of L. Previous work showed that P_5 is in the range 0.1 to 1 (Barauga *et al* 1985). Further, we can take $\tilde{k}^{\cdot} = 0.1$ to 1.0, depending on the cutoff in k-space of individual particle effects. Most work has taken $k_c = k_D$, so $\tilde{k} \approx 0.1$(Ecker and Fisher, 1971). The result agrees with $f > 0.05$ from Levron *et al* (1988). They found their result by detailed consideration of observed optical lines. Such agreement is gratifying, considering the great difference between the two methods.

5. Conclusion

We see clearly that long-range correlations induced by large amplitude waves can account for the observed $\langle E^2 \rangle \gg E_H^2$. Normally, atoms see a Stark shift from interaction within r_0, and plasma effects extend the interaction to the scale λ_D , which for the Levron *et al* data is about $10r_0$. But this will yield only an increase of 10 to 100 in $< E^2 >^{1/2}$, not enough to explain the 10^3 increase above E_H seen. such enhancement requires a large volume $L^3 \approx 10^3\lambda_D^3$ of high amplitude oscillation ($\frac{n_c}{n} \approx 1$), or even greater interacting volume if $\frac{n_c}{n} \ll 1$

This result suggests a caviton-like phenomenon, with large density depressims ordered over scales $\geq 10\lambda_D$ by strong electric fields ($E^2/4\pi nkT \approx 1$) which produce ponderomotive pressure. Within our static model, no specific detailed interactions are required to yield the W(E) and $< E^2 >$ observed. There may be statistical considerations which give a $\exp(-E^2)$ distribution a more general meaning, however. Benford (1987) suggests that collisions and energy exchang among cavitons can produce an $\exp(-E^2)$ form as well.

We can see from equation (50) that the packing fraction f derived from the microwave power gives good agreement with the experimentally observed packing fraction of strong turbulence. Since the microwave power caculation is based on the $W(E)$ distribution, this gives support to our long-range correlation model.

Note that we have assumed a correlation existing over a three dimensional field, even though $\mathbf{E}$ may be a one or two dimensional field. We picture particles correlated by a group of waves which can be strongly anisotropic, so the correlation integral of equation (34) yields an L^3 form. If this were not so, the integral would be proportional to $\lambda_D^2 L$ for a one dimensional $\mathbf{E}$, and $\lambda_D L^2$ for a two dimensional one. Which form is appropriate for a given experiment must be decided by details of the case. Clearly, if only one dimensional correlations occur, the product $\lambda_D^2 L n_c$ will still be the same and L will necessarily be much larger, then we have estimated here.

This work was supported by the Air Force Office of Scientific Research and the Office of Naval Research.

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Appendix B

Fluctuating Electric Fields in an Intense Relativistic Electron Beam-Driven Turbulent Plasma

Electrostatic plasma oscillation in plasmas which contain atoms should give rise to satellite lines in the atomic spectrum. The satellite lines appear near the forbidden transition.[1]

If the plasma fields were time independent (or with rather low frequency) so that we can use quasi-static theory, they would produce a forbidden line through the static Stark effect in the well-known manner, i.e., the two satellites coalesce into one forbidden line. Using the ratio of the intensity of the forbidden plus satellite lines, to the allowed line intensity, we could derive the R.M.S. field as the combined field of oscillation at various frequencies.

The level system in Fig. B1, according to [1], has usually different levels for the upper states of both lines. Kawasaki et al., [2] compare forbidden to allowed lines which have the same initial state.

The two characteristics shown in Fig. B2 are:

1) We need not make any assumption (or measurement!) of the population density of the forbidden level, since the lines are emitted from the same level.

2) The population density of the upper level, i, can be enhanced by a resonance absorption from the level ℓ . For making

the resonance absorption effectively, the metastable state of HeI is suitable for the level ℓ. (Fig. B3)

The application of tunable dye lasers to this experiment offers several advantages:[3]

1) Spectroscopic observations integrate along the line of sight. Satellite intensities are always measured relative to the intensity of the allowed transition. This will cause serious errors, especially if the allowed transition is emitted from the total plasma along the line of sight. The plasma satellites, however, stem from only the turbulent region, i.e., most likely from the current-carrying part of the plasma. The laser-fluorescence technique enables us to gain spatial resolution. (See Fig. B4)

2) The laser is focused into the plasma and resonantly tuned to the allowed transition. This improves the signal-to-noise ratio, as well as the time resolution of the measurement. Possible impurity lines as well as lines from molecules can be discriminated against by pumping the allowed transition with a narrow-band laser beam.

In this way we have succeeded making measurements of the electric field and find $\langle E^2\rangle^{1/2} \sim 10\ \frac{kV}{cm}$ with a resolution of about $(mm)^3$.

We intend to go on measuring the E-field in different regions in the plasma -- on-axis and off-axis, close to the anode, and downstream. Also, we shall measure the fields in different times relative to the electron-beam pulse. This will yield E(r,z,t). We expect to see growth and dispersion of

$E(r,z,t)$, which will yield understanding of nonlnear turbulence characteristics.

A sensitive probing of this low-E region can make contact with conventional "strong" turbulence theory ($0.1 \lesssim W \lesssim 1$). Similarly, beam-induced fluorescence will yield about 1000 times more photons at high-E Stark shifts, and extend our measurement of N(E) to ≈ 300 kV/cm, with the same statistical reliability as our present highest measurement, 120 kV/cm. This will provide check for any break in the $\exp(-E^2)$ distribution (Appendices A and B), and thus for theoretical models of high-E turbulence.

Rapid switching of beam current and voltage will allow study of high-E lifetimes in the plasma. Our preliminary work (described under "Turbulence Lifetime Measurements" in our former proposal) shows that turbulence persists long after usual theory predicts its dissipation. We plan extensive experiments switching beam current on to explore how rapidly high-E regions appear, and similar switch-off experiments to measure lifetimes.

We will develop theoretical models to explain the wide range of experiments we have already done or will soon perform. This will include a careful consideration of microwave emission data, and comparison with turbulent electromagnetic emission models. Coupling of the microwave measures with direct N(E) data will provide a valuable constraint on theory.

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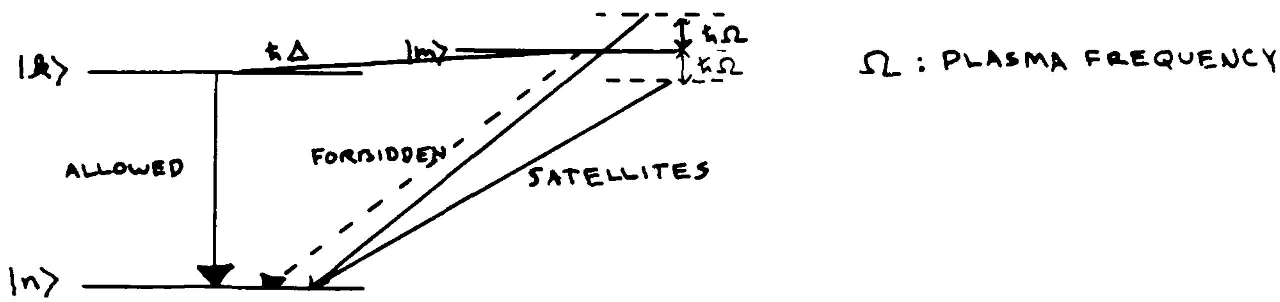

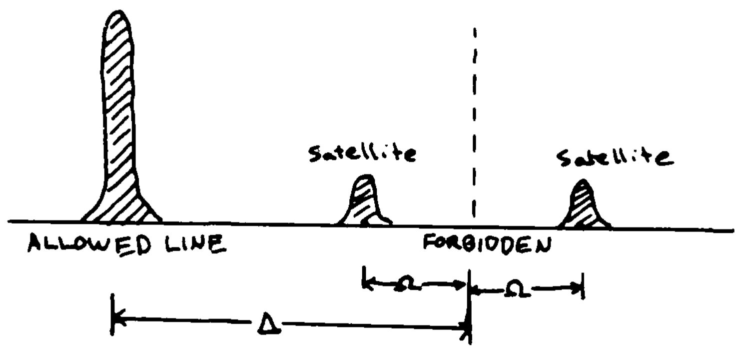

Figure B1: Level system which produces satellites to the dipole forbidden transition

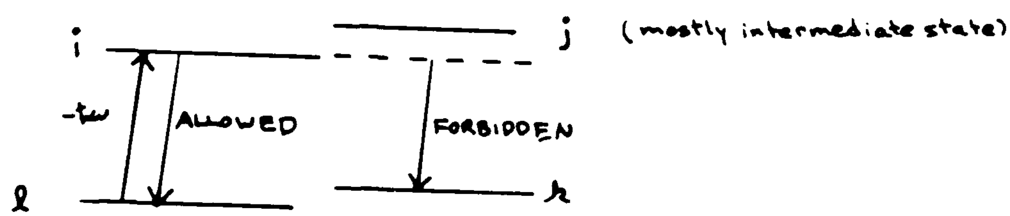

Figure B2: The forbidden line is caused by the level mixing of i with j due to electric fields, while the allowed line is due to the transition from i to l. An increase in population density of i enhances both of the lines.

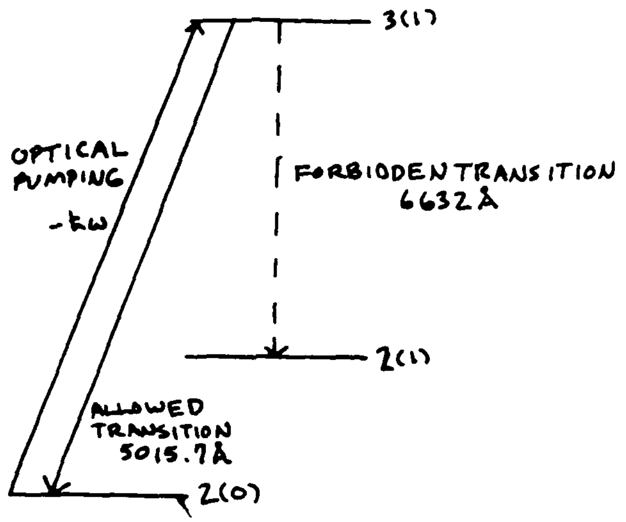

Figure B3: Level Diagram for our Experiment

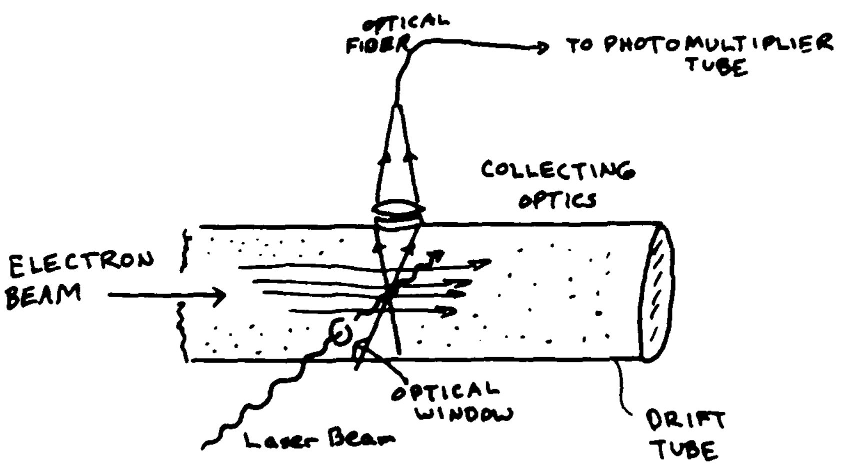

Figure B4: The experiment scheme. Subtracting measurements without the laser from measurements with the laser gives the contribution from the volume element on which the lens is focused.

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