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TECHNICAL REPORT BRL-TR-2852

SMALL SAMPLE COMPARISONS OF
EXPONENTIAL RELIABILITY ESTIMATES
FOR TYPE I CENSORING

LINDA L. CRAWFORD MOSS
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OCTOBER 1987

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US ARMY BALLISTIC RESEARCH LABORATORY
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19 ABSTRACT (Continue on reverse if necessary and identify by block number) A simulation study was used to evaluate several Chi Square approximations for the lower confidence limit of the exponential parameter used in reliability for Type I censoring. The study showed that the approximation using χ^2 with $2c+1$ degrees of freedom (where c is the number of failures) gave closer estimates to the exact estimate than χ^2_{2c} or χ^2_{2c+2} . The exact estimates of the exponential parameter are based on an algorithm from D.L. Bartholomew (1965). However, it was discovered that not every estimate of the exponential parameter, $\hat{\theta}$, could be found in Bartholomew's equation. Estimates near the maximum and minimum values of $\hat{\theta}$ were nonsensical.			
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I. INTRODUCTION

The problem of estimating reliability using censored data and determining a lower confidence limit on reliability is discussed in this paper. Censoring occurs when testing is terminated before all items have failed. There are two basic methods of censoring: (1) Type I censoring, where testing is terminated at some predetermined time; and (2) Type II censoring, where testing is terminated after a preassigned number of failures. When failures occur, either an item is replaced or it is not replaced. For this paper, we are interested in Type I censoring without replacement for small sample sizes (less than 25) with one or more failures. Suppose we have n items on test until time T (censored time). Each item is tested until it fails or the c failures occur at times, t_1, t_2, \dots, t_c , where $t_i < t_{i+1} < T$.

One could simply ignore the failure times and just consider that c failures occurred out of n items, then use the binomial distribution to put a lower confidence limit (l.c.l.) on reliability. However, too much information is ignored and the confidence bound is very conservative.

A better approach would be to utilize both number of failures and failure times. If the failure times (t_i) are assumed to be exponentially distributed, the probability density function (pdf) for t is $f(t) = \frac{1}{\theta} e^{-\frac{t}{\theta}}$, and the reliability is $R(t) = e^{-\frac{t}{\theta}}$, $t > 0, \theta > 0$. In a 1960 paper, Epstein¹ gave estimation procedures for both point and confidence limit estimates for θ . For Type I censoring without replacement of the failed items, the exponential parameter θ can be estimated by its maximum likelihood estimate, (m.l.e.), which is

$$\hat{\theta} = A/c \quad (1)$$

where

$$A = \sum_{i=1}^c t_i + (n - c) T = \text{Total amount of test time}$$

$$\sum_{i=1}^c t_i = \text{sum of the failure times for the } c \text{ items that failed}$$

n = total number of items on test

c = total number of failures

T = censored time

$$\begin{aligned} \text{(This is a biased estimate: } E(\hat{\theta}) &= 1/c \left[\sum_{i=1}^c E(t_i) + (n - c) T \right] \\ &= 1/c [c\theta + (n - c) T] = \theta + \frac{(n - c) T}{c} \end{aligned}$$

¹ Epstein, Benjamin, "Estimation from Life Test Data," *Technometrics*, Vol. 2, No. 4, Nov 1960

Then the lower 100 (1 - α) percent confidence interval for θ is estimated by

$$\hat{\theta}_{2c+2} = 2\Lambda/\chi^2_{2c+2}(\alpha) \quad (2)$$

where $\chi^2_{2c+2}(\alpha)$ is the upper α percentage point of a χ^2 distribution with $2c + 2$ degrees of freedom. Therefore, the lower limit on reliability using this l.c.l. for θ would

be $\hat{R}_{2c+2}(t) = e^{-\frac{t}{\hat{\theta}_{2c+2}}}$. Epstein's approximate procedure is the most commonly used and is cited in text books such as Bazovsky²; Mann, Shafer, and Singpurwalla³; Hahn and Shapiro⁴; and others.

In lieu of the many approximations that exist for the l.c.l. of θ , the exact distribution of θ can be found. This exact distribution of θ was derived by D.J. Bartholomew⁵ for small sample sizes where at least one failure occurs, i.e., $c > 0$.

Bartholomew's exact method of calculating the l.c.l., $\hat{\theta}_{\text{exact}}$, for the exponential parameter, θ , is the solution to

$$\begin{aligned} \Pr(\theta_0 \geq \hat{\theta}) &= \frac{1}{1 - e^{-nT/\hat{\theta}_{\text{exact}}}} \sum_{c=1}^n \binom{n}{c} \sum_{i=0}^c \binom{c}{i} (-1)^i B \\ &= \alpha \end{aligned} \quad (3)$$

where

$$B = \exp \left\{ -\frac{T}{\hat{\theta}_{\text{exact}}} (n - c + i) \right\} \int_x^{\infty} p(\chi^2_{2c}) d\chi^2_{2c}$$

and

$$x = \frac{2c}{\hat{\theta}_{\text{exact}}} \left\langle \hat{\theta} - \frac{T}{c}(n - c + i) \right\rangle .$$

The symbol $\langle - \rangle$ means that the expression is to be taken as zero if the contents are negative. Subscripts on θ 's indicate l.c.l. (except for θ_0) and subscripts on χ indicate degrees of freedom.

² Bazovsky, Igor. "Reliability Theory and Practice." Prentice-Hall, Inc., 1961.

³ Mann, N.R., Shafer, R.E., and Singpurwalla, N.D. "Methods for Statistical Analysis of Reliability and Life Data." Wiley, 1974.

⁴ Hahn and Shapiro. "Statistical Models in Engineering." Wiley, 1967.

⁵ Bartholomew, D.J. "The sampling Distribution of an Estimate Arising in Life Testing." Technometrics, Vol. 5, No. 3, August 1963.

This equation is a weighted sum of χ^2_{2c} integrals. This is quite cumbersome without the aid of a computer, especially when $n > 2$. It is obvious that even for a sample size of two, Bartholomew's exact method is quite complicated. Complications and complexities using distribution theory leads, in most cases, to the use of asymptotic arguments for inference about parameters. However, because we are only interested in small samples, asymptotic results are not applicable. Therefore, we decided to compare Bartholomew's exact method with Epstein's method and variations of Epstein's method.

II. SIMULATION

A simulation study of the exponential distribution was performed with $\theta = 1$ and $T^\dagger = 1$. We first chose $\alpha = 0.05$ and then chose $\alpha = 0.10$ while letting the sample size, n , range from 2 to 20. For each sample size, 2000 simulations were run.

Uniform random numbers were used to generate random exponentially distributed times. After each exponential time was generated it was compared to $T = 1$. If the exponential time was less than $T = 1$, then it was considered to be a failure occurring at t_i . After n of these times were generated with c failures, A and $\hat{\theta}$ were calculated. Lower confidence limits for θ were then calculated using Bartholomew's exact method, $\hat{\theta}_{\text{exact}}$, and using Epstein's method with modifications in the degrees of freedom for χ^2 . The subscript on $\hat{\theta}$ indicates the degrees of freedom for χ^2 . Thus lower confidence limits $\hat{\theta}_{2c+2}$, $\hat{\theta}_{2c+1}$, and $\hat{\theta}_{2c}$ were calculated.

The lower limit on reliability was calculated using the computed lower confidence limit for θ in the following equation

$$R_L(T) = e^{-\frac{T}{\hat{\theta}_L}} \quad (4)$$

where

T = Censored time

$\hat{\theta}_L$ = Computed lower confidence limit.

III. DISCUSSION OF RESULTS

For those simulations where $\alpha = .05$, we would expect $\hat{\theta}_L$ to exceed θ only 5% of the time, i.e., $P(\hat{\theta}_L > \theta) = .05$. Since there were 2000 cases for each sample size, we would expect 100 of our $\hat{\theta}_L$ to exceed $\theta = 1$. **Table 1** shows the number exceeding 1.

† One nicety is that T , θ , and $\hat{\theta}_L$ scale proportionally, so if $T = 60$ hours, $\theta = 30.0$, then $\hat{\theta}_L = 18.33901$. If T and θ are scaled by dividing by 60, $T = 1$ and $\theta = 0.50$, then $\hat{\theta}_L = .30565$, which is $18.33901 \div 60$. Thus we chose to keep $T = 1$ for our examples and simulations.

TABLE 1. Simulation Study with $\alpha = 0.05$
 Observed Cases where $\hat{\theta}_L > \theta$
 Expected Number = 100

n	$\hat{\theta}_{\text{exact}}$	$\hat{\theta}_{2c}$	$\hat{\theta}_{2c+1}$	$\hat{\theta}_{2c+2}$
2	76	0	0	0
3	95	0	0	0
4	87	261	6	0
5	102	123	118	13
6	87	162	57	47
7	104	136	108	35
8	105	156	80	69
9	111	155	110	37
10	98	134	82	54
15	98	121	88	63
20	89	106	83	63

The results obtained for $n = 4, 5, 6, 7, 8, 9, 10, 15$ and 20 for the $\hat{\theta}_{\text{exact}}$ are typical of what one expects. However, we see that for $n = 2$, the exact is conservative. By conservative we mean that the lower limit tends to be too low, since in less than 5% of the simulated runs, $\hat{\theta}_{\text{exact}} > \theta$ but $\hat{\theta}_{2c}$, $\hat{\theta}_{2c+1}$, and $\hat{\theta}_{2c+2}$ are extremes, i.e., for $n = 2$ and 3 there are no $\hat{\theta}_L$ greater than $\theta = 1$. We can see that $\hat{\theta}_{2c+2}$ is very conservative, with $\hat{\theta}_{2c+1}$ being better. For $\hat{\theta}_{2c}$, we get more than 100 lower limits exceeding the true value, except for $n = 2$ and 3 .

Similarly, simulations were run with $\alpha = .10$. Here we would expect 200 cases of the $\hat{\theta}_L$ to exceed θ . **Table 2** gives the actual number of cases for each sample size. Again we see that $\hat{\theta}_{2c}$ overestimates the number of lower confidence limits of $\hat{\theta}_L$ which exceeds the true value of θ which equals 1 for sample sizes except $n = 2$. $\hat{\theta}_{2c+2}$ is very conservative and $\hat{\theta}_{2c+1}$ is less conservative (except for $n = 2$ and 3).

Another observation made from the simulations for computed values of $\hat{\theta}$ was that for each value of $\hat{\theta}$, the lower limit on θ tended to follow the pattern;

$$\hat{\theta}_{2c+2} < \hat{\theta}_{2c+1} < \hat{\theta}_{\text{exact}} < \hat{\theta}_{2c} \quad (5)$$

Examples of this can be observed for $n = 10$ and 20 by examining Figures 1 and 2. This indicates again that $\hat{\theta}_{2c+2}$ and $\hat{\theta}_{2c+1}$ are conservative, i.e., they underestimate $\hat{\theta}_L$. On the other hand, $\hat{\theta}_{2c}$ overestimates $\hat{\theta}_L$ slightly.

The $\hat{\theta}_L$'s were used to establish a lower limit on reliability, \hat{R}_L , at time T . Selected values of $\hat{\theta}$ were plotted against the \hat{R}_L 's for the four methods under study. The plots appear in Figures 3 and 4 for $n = 10$ and $n = 20$, respectively. We can see that for small values of $\hat{\theta}$, the four methods give approximately the same lower limit on

TABLE 2. Simulation Study with $\alpha = 0.10$
 Observed Cases where $\hat{\theta}_L > \theta$
 Expected Number = 200

n	$\hat{\theta}_{\text{exact}}$	$\hat{\theta}_{2c}$	$\hat{\theta}_{2c+1}$	$\hat{\theta}_{2c+2}$
2	174	0	0	0
3	190	274	0	0
4	181	252	193	2
5	192	280	126	114
6	180	263	180	67
7	188	258	152	123
8	201	279	198	110
9	193	254	163	130
10	183	225	171	112
15	191	232	167	125
20	170	203	158	117

reliability. For larger values of θ , we can see that \hat{R}_{2c} lies above \hat{R}_{exact} everywhere, with \hat{R}_{2c+1} and \hat{R}_{2c+2} lying generally below \hat{R}_{exact} . We also can see that \hat{R}_{2c+1} is much closer to \hat{R}_{exact} than the other two, especially for $n = 20$.

For each sample size, the maximum absolute difference in lower reliability estimates were computed, i.e., $|\hat{R}_{\text{exact}} - \hat{R}_{2c}|$, $|\hat{R}_{\text{exact}} - \hat{R}_{2c+1}|$ and $|\hat{R}_{\text{exact}} - \hat{R}_{2c+2}|$. These differences are plotted versus sample size in Figure 5. We can see that the differences were larger for small sample sizes ($n = 2$ and $n = 5$) and generally \hat{R}_{2c+2} differed the most from the \hat{R}_{exact} . As the sample size got larger ($n = 10, 15$ and 20), all the reliability estimates converged i.e., the differences converged to zero.

Earlier in the report it was mentioned that one alternative to this confidence limit estimate problem would be to ignore the failure times and simply use the number of failures to put a binomial lower confidence limit on reliability. To show how conservative these binomial limits are, we used the minimum and maximum lower reliability estimates obtained from Bartholomew's exact method (2000 simulations; $n = 10$) and compared them to the binomial lower confidence estimates. These comparisons are shown in Figure 6 for the various number of failures observed. We can see that the binomial limits are always at or below the minimum calculated using Bartholomew's method. Thus by ignoring the failure time information, we are lowering our estimate of reliability.

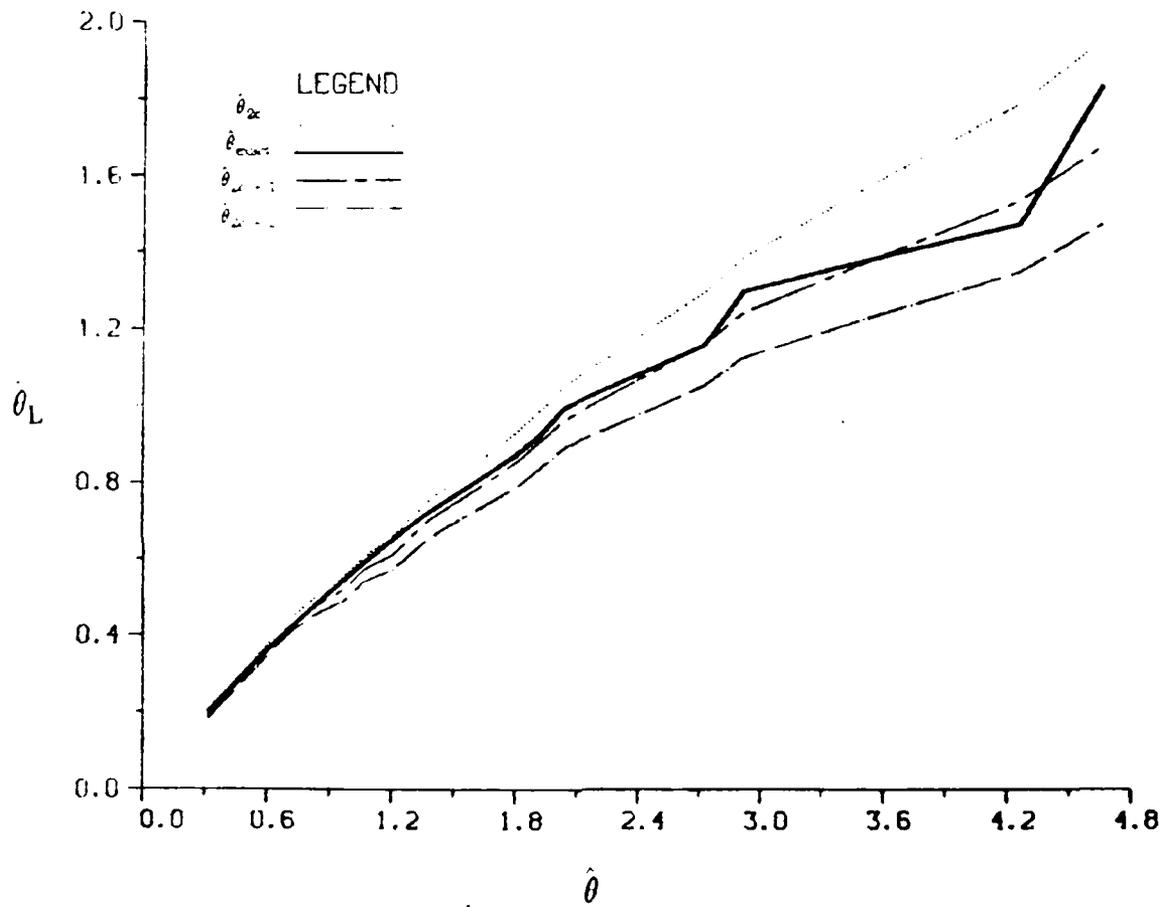


Figure 1. Lower 95% Confidence Limits on θ . Exact vs Estimates ($n = 10$).

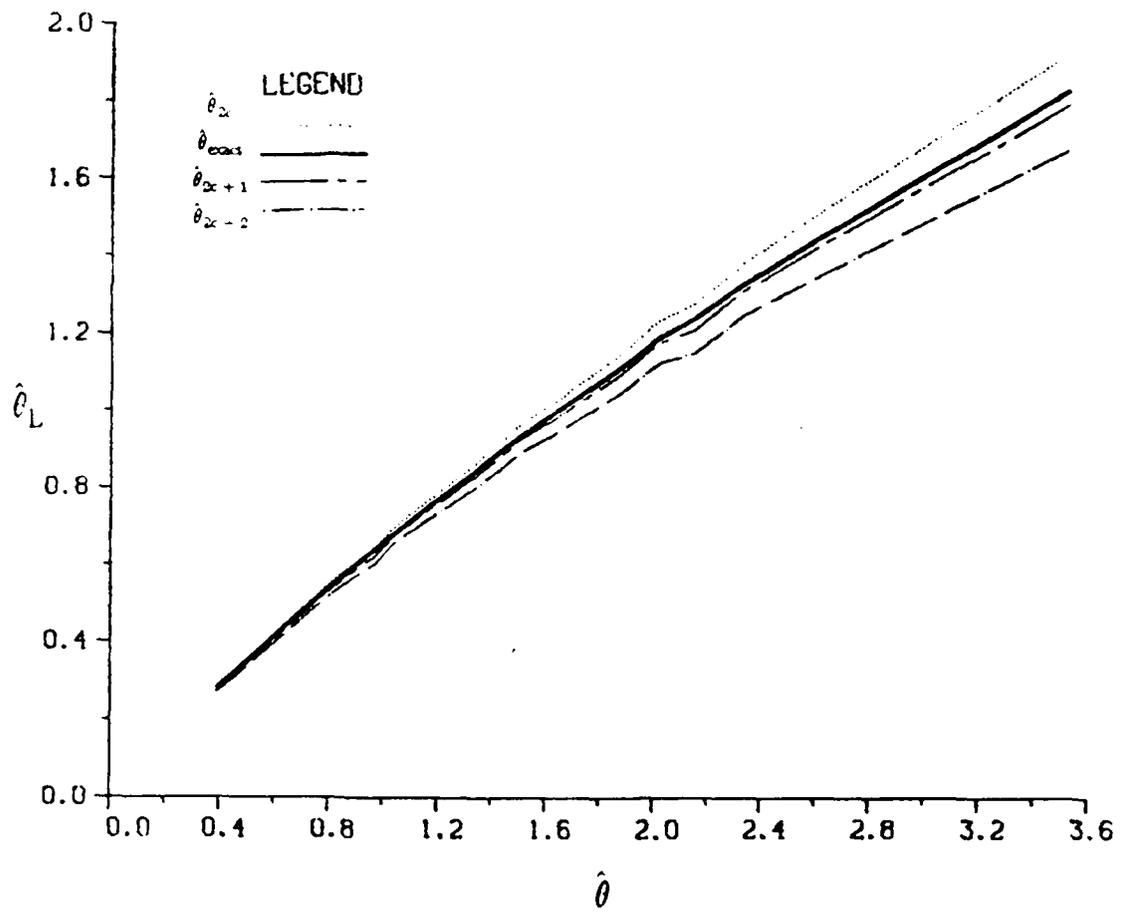


Figure 2. Lower 95% Confidence Limits on θ . Exact vs Estimates ($n = 20$).

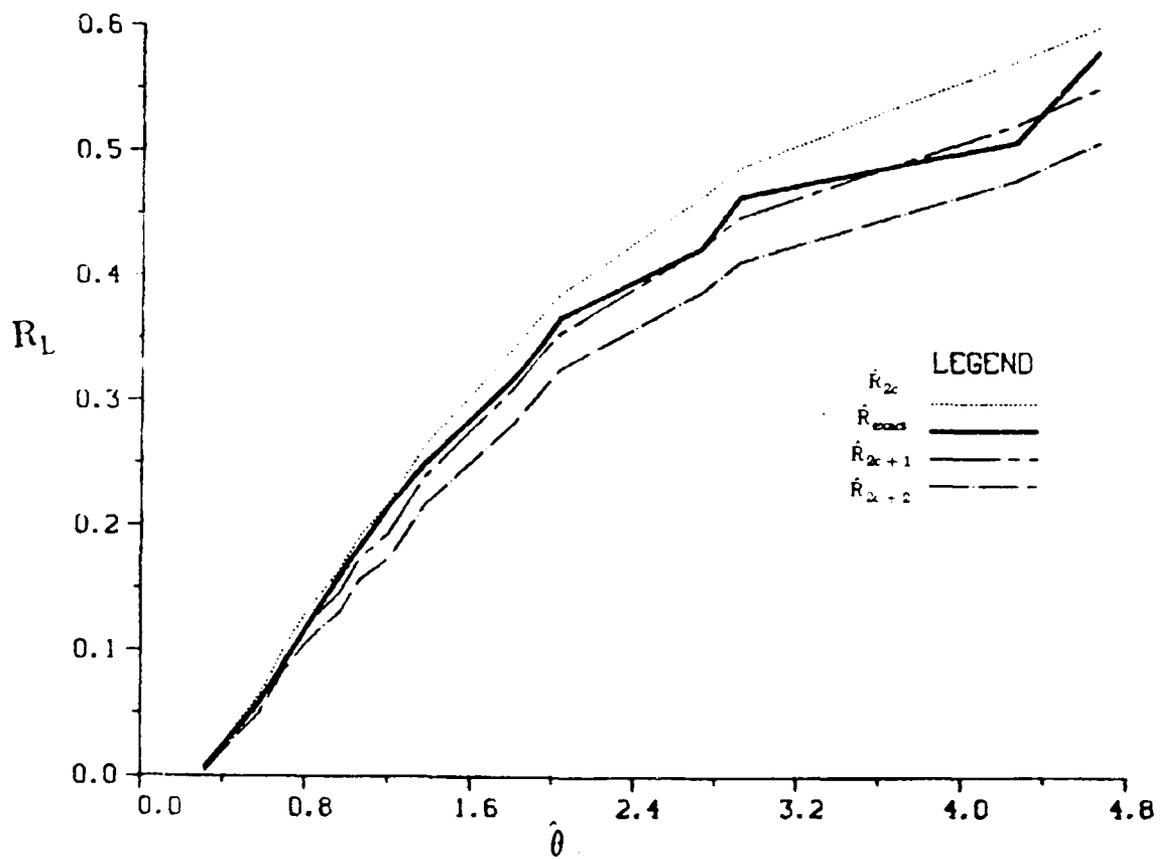


Figure 3. Reliability Lower 95% Confidence Limits. Exact vs Estimates ($n = 10$).

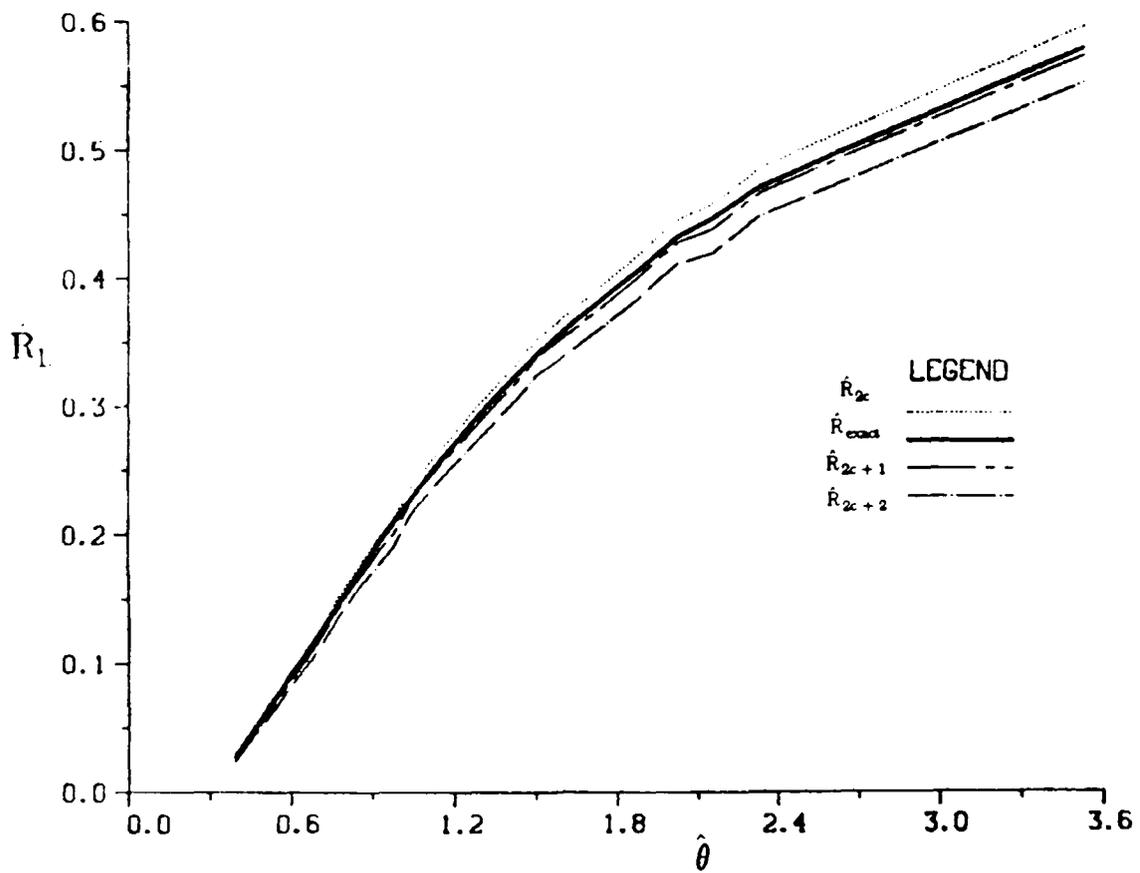


Figure 4. Reliability Lower 95% Confidence Limits. Exact vs Estimates ($n = 20$).

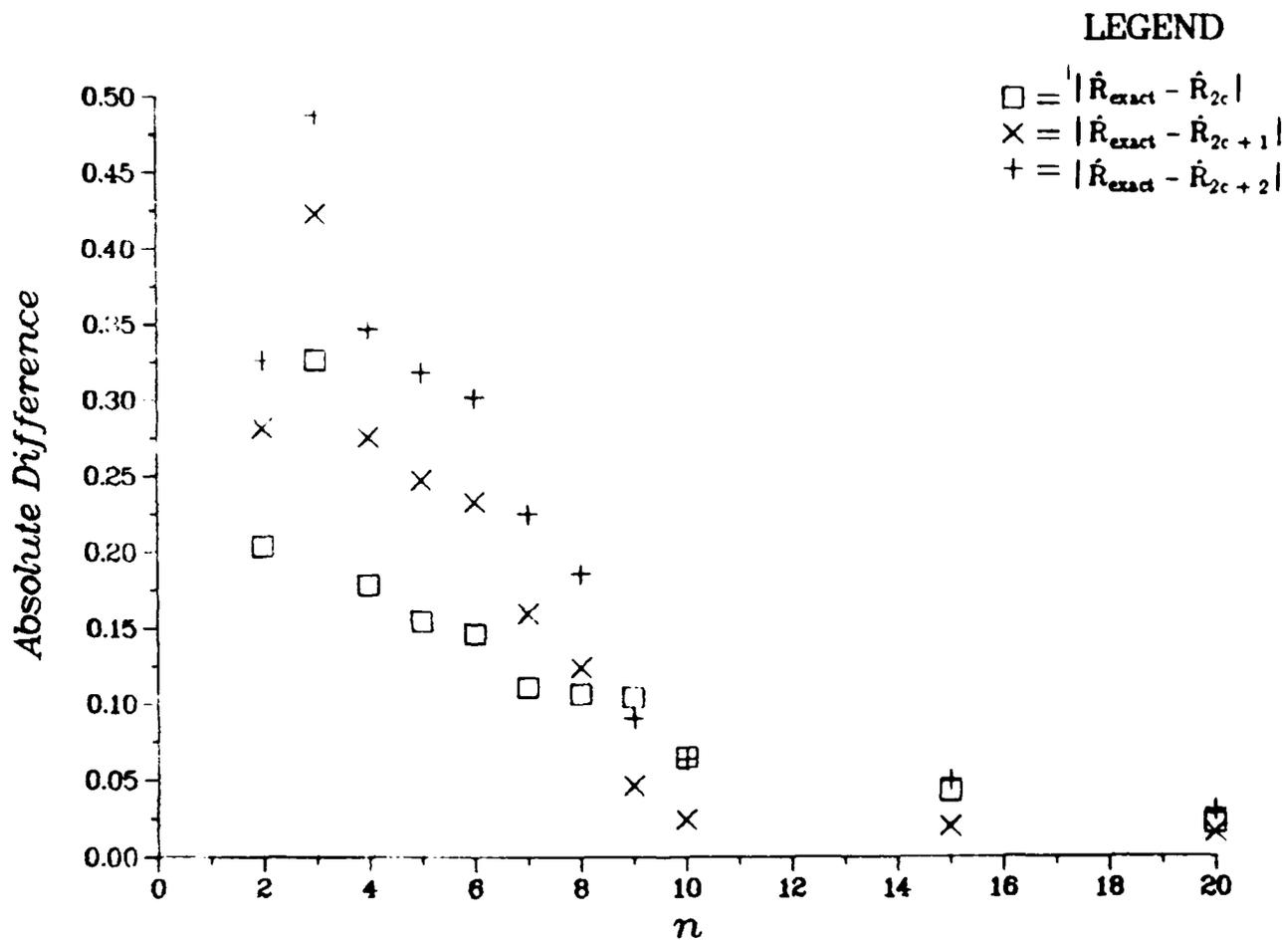


Figure 5. Maximum Absolute Difference in Reliability Lower Confidence Limits for $\alpha = .05$.

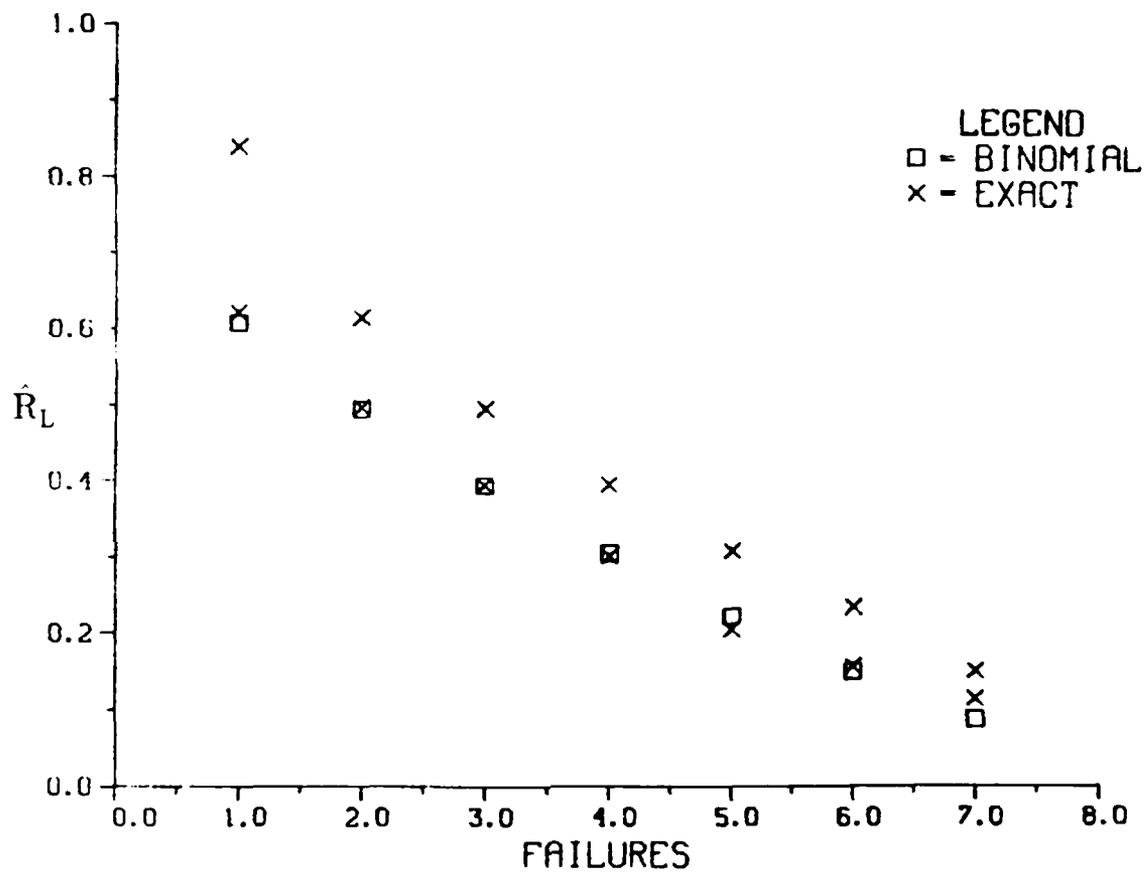


Figure 6. Comparison of Binomial with Maximum and Minimal Exact Reliability Lower 95% Confidence Limit.

IV. UNRESOLVED FINDING

During the simulation it was noted that on occasion we were obtaining estimates of $\hat{\theta}_{\text{exact}}$ that were greater than $\hat{\theta}$. Some $\hat{\theta}_{\text{exact}}$'s were several magnitudes greater than $\hat{\theta}$. Then there were a few instances where computer error messages were obtained instead of an estimate of $\hat{\theta}_{\text{exact}}$. Looking into the problem we noted these anomalies were occurring when $\hat{\theta}$ approached its maximum value.

Take for instance the case when $n = 2$, $c = 1$, $\theta = 1$, $T = 1$ and $\alpha = .05$. We can obtain a closed form for $P\{\theta_0 \geq \hat{\theta} | \theta\} = \alpha$. However, this is not a continuous function: there are three ranges:

$$\text{For } [0 < \hat{\theta} \leq .5], \quad \alpha = \frac{1}{1 - e^{-2/\theta}} \left[-e^{-2/\theta} + \left(2\frac{\hat{\theta}}{\theta} + 1\right) e^{-2\hat{\theta}/\theta} \right] \quad (6)$$

$$\text{For } [.5 < \hat{\theta} < 1], \quad \alpha = \frac{1}{1 - e^{-2/\theta}} \left[2e^{-1/\theta} - e^{-2/\theta} + \left(\frac{2\hat{\theta} + 2}{\theta} - 1\right) e^{-2\hat{\theta}/\theta} \right] \quad (7)$$

$$\text{For } [1 < \hat{\theta} < 2], \quad \alpha = \frac{2}{1 - e^{-2/\theta}} \left[e^{-\hat{\theta}/\theta} - e^{-2/\theta} \right]. \quad (8)$$

The discontinuities at 0, 1 and 2 can easily be seen by looking at the maximum and minimum values that $\hat{\theta}$ can take on. Recall that

$$\begin{aligned} \hat{\theta} &= \frac{A}{c} \\ &= \frac{\sum_{i=1}^c t_i + (n - c) T}{c} \end{aligned} \quad (9)$$

Let ϵ be a very small time, $T = 1$, and $n = 2$. Then for one failure ($c = 1$) near T , $\hat{\theta} = [(T - \epsilon) + (n - 1)T]/1 = 2 - \epsilon$ is the maximum value that $\hat{\theta}$ can assume. For two failures near T ,

$$\begin{aligned} \hat{\theta} &= (t_1 + t_2)/2 \\ &= 2(T - \epsilon)/2 \\ &= 1 - \epsilon. \end{aligned} \quad (10)$$

Assume one failure occurs near time 0, $\hat{\theta} = [(0 + \epsilon) + T]/1 = 1 + \epsilon$; for two failures near 0, $\hat{\theta} = [(0 + 2\epsilon)/2] = 0 + \epsilon$. The maximum value of $\hat{\theta}$ is $2 - \epsilon$. This indicates that the failure occurred only ϵ time before the censoring time T .

To study the problem, we selected $\hat{\theta} = 1.95$ and started at $\hat{\theta}_{\text{exact}} = 0.2$ and solved for α . Then $\hat{\theta}_{\text{exact}}$ was increased by increments of $\Delta = .2$ until $\hat{\theta}_{\text{exact}}$ reached 2.0, the theoretical maximum. The largest α obtained while solving for α at each increment was less than 0.03 (See Figure 7). To get an $\alpha = 0.05$ for $\hat{\theta} = 1.95$, it was determined that

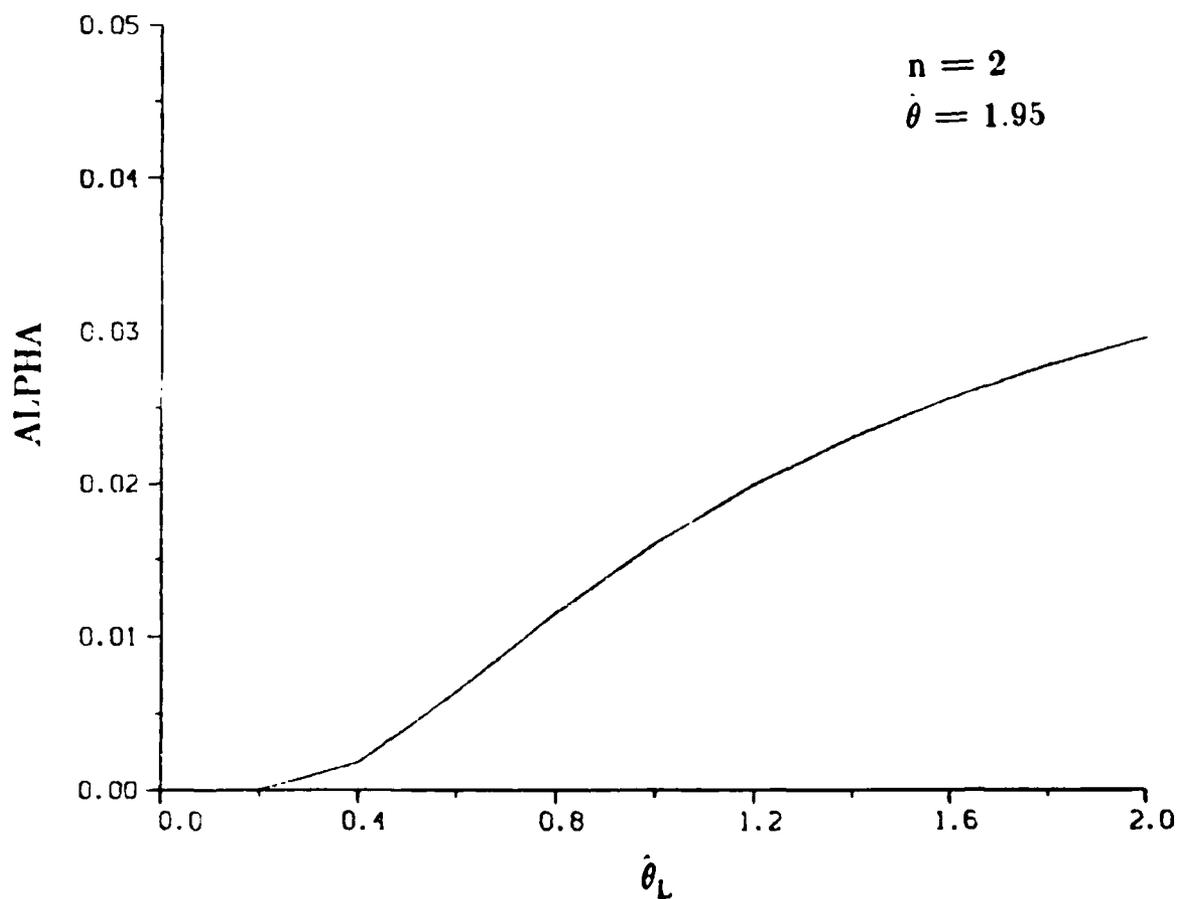


Figure 7. Alpha Level versus $\hat{\theta}_L$ ($\hat{\theta} = 1.95$).

$\hat{\theta}_{\text{exact}}$ would have to be greater than 500. This means that $\hat{\theta}_{\text{exact}}$ solved in the equation would be much larger than the set value. Similarly, when $n = 3, 4,$ and 5 there were a few cases where $\hat{\theta}_{\text{exact}} > \theta$. This phenomena did not occur when $n = 6, 7, 8, 9, 10, 15$ and 20 . At $\alpha = 0.10$, this also occurred for $n = 2$ thru 7 . Similarly we selected $\theta = 1.98$ and solved for α . The largest α obtained was less than 0.015 (See Figure 8). Yet we have not been able to explain this anomaly.

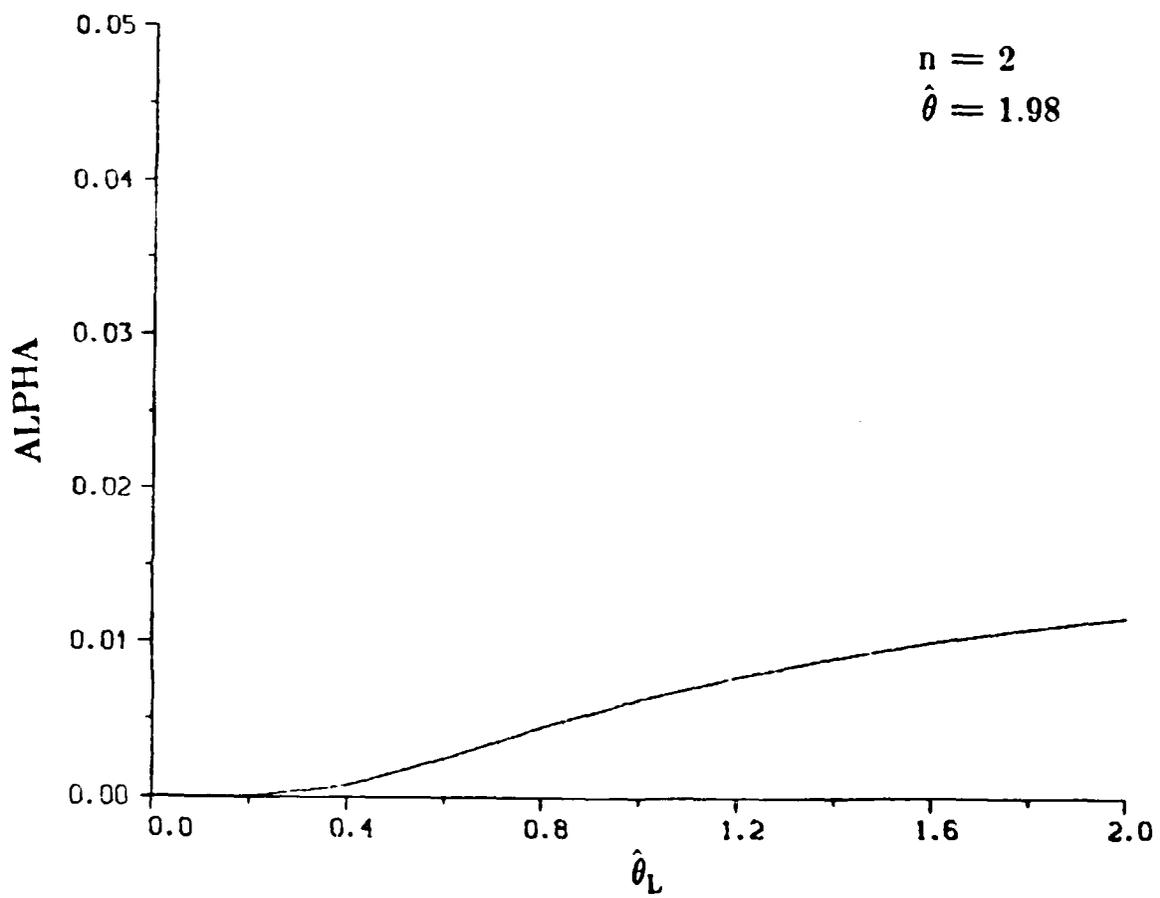


Figure 8. Alpha Level versus $\hat{\theta}_L$ ($\hat{\theta} = 1.98$).

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5. Bartholomew, D.J., "The Sampling Distribution of an Estimate Arising in Life Testing," *Technometrics*, Vol. 5, No. 3, August 1963.

NOMENCLATURE

- m.l.e. - maximum likelihood estimate
- l.c.l. - lower confidence limit
- θ - exponential parameter
- $\hat{\theta}$ - m.l.e. estimate on θ
- $\hat{\theta}_L$ - l.c.l. on θ
- $\hat{\theta}_{\text{exact}}$ - l.c.l. on θ using Bartholomew's exact method
- $\hat{\theta}_{2c+2}$ - l.c.l. on θ using Epstein's method - subscript indicates d.f. for χ^2
- $\hat{\theta}_{2c+1}$ - l.c.l. on θ using Epstein's method (modified) - subscript indicates d.f. for χ^2
- $\hat{\theta}_{2c}$ - l.c.l. on θ using Epstein's method (modified) - subscript indicates d.f. for χ^2
- $\hat{R}_L(t)$ - computed l.c.l. on reliability using $\hat{\theta}_L$
- \hat{R}_{exact} - l.c.l. on reliability using $\hat{\theta}_{\text{exact}}$
- \hat{R}_{2c} - l.c.l. on reliability using $\hat{\theta}_{2c}$
- \hat{R}_{2c+1} - l.c.l. on reliability using $\hat{\theta}_{2c+1}$
- \hat{R}_{2c+2} - l.c.l. on reliability using $\hat{\theta}_{2c+2}$

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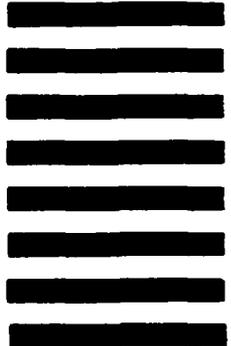
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