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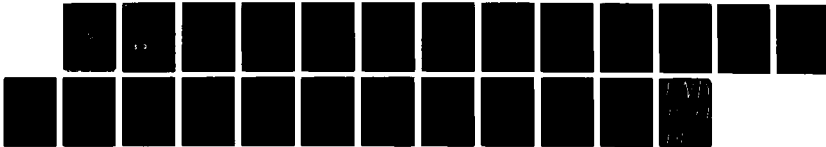
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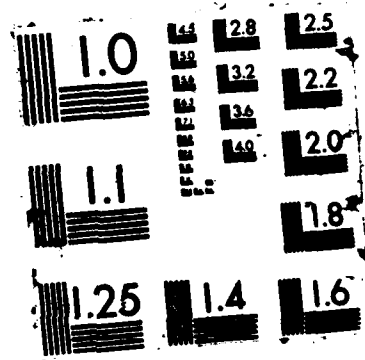
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ASYMPTOTIC NORMALITY OF POLY-T DENSITIES
WITH BAYESIAN APPLICATIONS

George Y. Wong

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New York, New York 10021

October, 1987

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ASYMPTOTIC NORMALITY OF POLY-T DENSITIES
WITH BAYESIAN APPLICATIONS

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ABSTRACT

A poly-t density is a density which is proportional to a product of at least two t-like factors, each of which is of the form $\{1 + (\underline{x} - \underline{\mu})^T \underline{M} (\underline{x} - \underline{\mu})\}^{-d/2}$ where d is a positive number, $\underline{\mu}$ is an arbitrary location vector and \underline{M} is a symmetric semi-positive definite scale matrix. In general, \underline{M} is a function of d . Such a density arises, for example, in the Bayesian analysis of a linear model with a normal error term, independent normal priors on the linear parameters and inverted-gamma priors on the variance components. A theorem about the asymptotic normality of the density as a subset of the individual d 's tend to infinity is proved under very general conditions. A corollary specifically related to the Bayesian linear model is also given. Detailed results are illustrated in the familiar Bayesian multiple linear regression model with two variance components. The Tiao-Zellner expansion for approximating the particular poly-t form involving two proper multivariate t factors is extended to the case of two arbitrary t-like factors.

1. INTRODUCTION

A p -dimensional random vector \underline{x} is said to have a poly- t distribution if its density is proportional to a product of $L \geq 2$ "t-like" factors, or

$$f(\underline{x} | d_1, \dots, d_L) \propto \prod_{k=1}^L \{1 + (\underline{x} - \underline{\mu}_k)^T \underline{M}_k(d_k) (\underline{x} - \underline{\mu}_k)\}^{-d_k/2} \quad (1)$$

where $d_k > 0$, $\underline{\mu}_k$ is a $p \times 1$ location vector and $\underline{M}_k(d_k)$ is a $p \times p$ symmetric semi-positive definite scale matrix, $k=1, \dots, L$. To ensure that the right-hand side of (1), which we denote by $g(\underline{x} | d_1, \dots, d_L)$, is normalizable, we must require that $d_1 + \dots + d_L > p$ and $\underline{M}_1(d_1) + \dots + \underline{M}_L(d_L)$ be positive definite (Dickey 1986). The normalizing constant $\int g(\underline{x} | d_1, \dots, d_L) d\underline{x}$, however, cannot be expressed in a simple closed form. When L is smaller than p , Dickey showed that the normalizing constant can be expressed in an $(L-1)$ -fold integral.

A useful form of density (1) is obtained by letting

$$d_k = \nu_k + m_k, \quad \underline{M}_k(d_k) = \underline{M}_k / \nu_k \quad (2)$$

for some $\nu_k > 0$, m_k such that $\nu_k + m_k > 0$, and \underline{M}_k symmetric semi-positive definite. When $m_k = p$ and \underline{M}_k is positive definite, then the k th t-like factor is proportional to a proper multivariate t density with ν_k degrees of freedom. Such a poly- t density form plays an important role in the Bayesian analysis of a general linear model with a normal error term. The Bayesian approach considered here is described as follows: (1) A vague prior is imposed on a subset of the linear parameters; (2) The rest of the parameters are partitioned into different subsets, and an exchangeable normal prior with mean 0 and an unknown variance component is imposed on each subset independently; and (3) The prior variances and the error variance are given independent inverted gamma distributions. It can be shown that the posterior density of the linear parameters is of the above poly- t density form. Details about poly- t posterior

distributions can be found in Dickey (1974), Drèze (1977), Rajagopalan and Broemeling (1983), Broemeling and Abdullah (1984), and Broemeling (1985).

In this article, we are concerned with the asymptotic normality of a poly-t density as some or all of d_1, \dots, d_L become large. The asymptotic normal density can be used to approximate the poly-t density directly. Also, it can serve as an important sampling distribution for the Monte Carlo evaluation of any probability statement about \underline{X} , or any moment of \underline{X} . Despite its usefulness in Bayesian linear modeling, the asymptotic normality of a poly-t density has never been formally established. In Section 2, we prove the normality result for the poly-t density (1) under fairly general conditions. As will be seen there, the proof is much harder than that in the case of a single multivariate t density; the difficulty is mainly due to the fact that the normalizing constant of a poly-t density is in terms of an intergral expression.

In Section 3, we illustrate the asymptotic results on a Bayesian linear model with two variance components. There we show that the appropriate asymptotic normal density may be made the leading term in a generalization of the Tiao-Zellner expansion (1964) to approximate the posterior density of the linear parameters.

2. MAIN RESULTS

We show that under some fairly general conditions, the poly-t density (1) converges to a proper multivariate normal density. It follows from a result of Scheffe (1947) that the poly-t variable actually converges in distribution to the multivariate normal variable with the limiting normal density. We also give a corollary on a special form of (1) which is particularly useful in Bayesian linear modeling. For convenience, we write $\underline{A} > 0$ and $\underline{A} > 0$ to mean that the symmetric matrix \underline{A} is semi-positive definite and positive definite, respectively. Also, we denote the density of a multivariate normal density

with the mean $\underline{\mu}$ and covariance matrix $\underline{\Sigma}$ by $f_N(\cdot | \underline{\mu}, \underline{\Sigma})$.

Theorem For the general poly-t density (1), assume that

(1) $d_k M_k(d_k)$ converges to $\underline{M}_k > 0$ as $d_k \rightarrow \infty$, $k=1, \dots, L$, and $\underline{M} = \underline{M}_1 + \dots + \underline{M}_L > 0$, and

(2) there exist semi-positive definite matrices $\underline{B}_1, \dots, \underline{B}_L$ such that

$\underline{B}_1 + \dots + \underline{B}_L > 0$ and $d_k M_k(d_k) - \underline{B}_k > 0$ for all d_k sufficiently large.

Then as d_1, \dots, d_L all tend to infinity, $f(\underline{x} | d_1, \dots, d_L)$ will tend to the limiting density $f_N(\underline{x} | \underline{\mu}, \underline{M}^{-1})$, where $\underline{\mu} = \underline{M}^{-1}(\underline{M}_1 \underline{\mu}_1 + \dots + \underline{M}_L \underline{\mu}_L)$.

Proof. Without any loss of generality, we prove the theorem for the case $L=2$. To show that $f(\underline{x} | d_1, d_2)$ converges to $f_N(\underline{x} | \underline{\mu}, \underline{M}^{-1})$ as $d_1, d_2 \rightarrow \infty$, it suffices to show that (i) $g(\underline{x} | d_1, d_2)$ tends to a limit $g(\underline{x})$ proportional to $f_N(\underline{x} | \underline{\mu}, \underline{M}^{-1})$, and (ii) $\int g(\underline{x} | d_1, d_2) d\underline{x} \rightarrow \int g(\underline{x}) d\underline{x}$, as $d_1, d_2 \rightarrow \infty$.

To prove (i), we make use of assumption (1) and obtain

$$g(\underline{x} | d_1, d_2) = \prod_{k=1}^2 \{1 + Q_k/d_k + o(1/d_k)\}^{-d_k/2},$$

where $Q_k = (\underline{x} - \underline{\mu}_k)^T \underline{M}_k (\underline{x} - \underline{\mu}_k)$, $k = 1, 2$. It follows that $g(\underline{x} | d_1, d_2)$ tends to

$$g(\underline{x}) = \exp \{ - (\underline{\mu}_1 - \underline{\mu}_2)^T \underline{M}_1^{-1} \underline{M}_2 (\underline{\mu}_1 - \underline{\mu}_2) / 2 \} \times \exp \{ - (\underline{x} - \underline{\mu})^T \underline{M} (\underline{x} - \underline{\mu}) / 2 \},$$

which is proportional to $f_N(\underline{x} | \underline{\mu}, \underline{M}^{-1})$.

To prove (ii), we show that for d_1 and d_2 sufficiently large, $g(\underline{x} | d_1, d_2)$ is dominated by an integrable function $h(\underline{x} | d_1, d_2)$, which monotonically decreases to an integrable function $h(\underline{x})$; moreover, $\int h(\underline{x} | d_1, d_2) d\underline{x}$ also decreases monotonically to $\int h(\underline{x}) d\underline{x}$. Then using a generalization of the Lebesgue Dominated Convergence Theorem (see, for example, Royden, p.89), we conclude that the limit of $\int g(\underline{x} | d_1, d_2) d\underline{x}$ is $\int g(\underline{x}) d\underline{x}$.

We now make use of assumption (2) to prove the above claims. First, it is an immediate consequence of the assumption that for sufficiently large d_1 and d_2 ,

$$g(\underline{x} | d_1, d_2) \leq h(\underline{x} | d_1, d_2) = \prod_{k=1}^2 \{1 + (\underline{x} - \underline{\mu}_k)^T \underline{B}_k (\underline{x} - \underline{\mu}_k) / d_k\}^{-d_k/2}.$$

Also, it can be directly verified that as $d_1, d_2 \rightarrow \infty$, $h(\underline{x}|d_1, d_2)$ monotonically decreases to an integrable function

$$h(\underline{x}) = \exp \left\{ - (\underline{\mu}_1 - \underline{\mu}_2)^T \underline{B}_1 (\underline{B}_1 + \underline{B}_2)^{-1} \underline{B}_2 (\underline{\mu}_1 - \underline{\mu}_2) / 2 \right\} \\ \times \exp \left\{ - (\underline{x} - \underline{\nu})^T (\underline{B}_1 + \underline{B}_2) (\underline{x} - \underline{\nu}) / 2 \right\},$$

where $\underline{\nu} = (\underline{B}_1 + \underline{B}_2)^{-1} (\underline{B}_1 \underline{\mu}_1 + \underline{B}_2 \underline{\mu}_2)$. To establish the integrability of $h(\underline{x}|d_1, d_2)$, let $m = \min(d_1, d_2)$. Then the monotonicity of the function implies that

$$h(\underline{x}|d_1, d_2) \leq \prod_{k=1}^2 \left\{ 1 + (\underline{x} - \underline{\mu}_k)^T \underline{B}_k (\underline{x} - \underline{\mu}_k) / m \right\}^{-m/2} \\ \leq \left\{ 1 + (\underline{x} - \underline{\nu})^T (\underline{B}_1 + \underline{B}_2) (\underline{x} - \underline{\nu}) / m \right\}^{-m/2}.$$

For sufficiently large d_1 and d_2 , the t-like factor on the right-hand side, denoted by $h_m(\underline{x})$, is proportional to a multivariate t density with $m-p$ degrees of freedom, mean $\underline{\nu}$ and scale matrix $\underline{B}_1 + \underline{B}_2$; therefore, $h(\underline{x}|d_1, d_2)$ is integrable. Let m be fixed. Define $\phi(\underline{x}|d_1, d_2) = h_m(\underline{x}) - h(\underline{x}|d_1, d_2)$, for $d_1, d_2 > m$. Applying the Monotone Convergence Theorem to this sequence of functions together with the fact that $h_m(\underline{x})$ is integrable, we conclude that $\int h(\underline{x}|d_1, d_2) d\underline{x}$ tends to $\int h(\underline{x}) d\underline{x}$. This completes the proof.

We point out that if only the elements of a subset of d_1, \dots, d_L tend to infinity, then the theorem will apply to the product of the corresponding t-like factors while the remaining factors will stay put.

We now consider a special form of the poly-t density (1) with scale matrix

$$\underline{M}_k(d_k) = w_k(d_k) \underline{N}_k \quad (3)$$

for some positive function w_k and semi-positive definite matrix \underline{N}_k . The following corollary concerns the asymptotic behavior of such a poly-t density.

Corollary For the particular poly-t density with $\underline{M}_k(d_k)$ of the form (3), $k=1, \dots, L$, assume that $d_k w_k(d_k) \rightarrow c_k > 0$ as $d_k \rightarrow \infty$, $k=1, \dots, L$ and $\underline{M} = \underline{M}_1 + \dots + \underline{M}_L > 0$,

where $\underline{M}_k = c_k \underline{N}_k$ is the limit of $d_k \underline{M}_k(d_k)$. Then $f(\underline{x}|d_1, \dots, d_L) \rightarrow f_N(\underline{x}|\underline{\mu}, \underline{M}^{-1})$ as $d_1, \dots, d_L \rightarrow \infty$.

Proof We need to check that the two assumptions of the theorem hold for this particular form of poly-t density. Assumption (1) is obviously satisfied. For assumption (2), we take $\underline{B}_k = \epsilon_k \underline{M}_k$ for some appropriately chosen positive number ϵ_k . Clearly, $\underline{B}_1 + \dots + \underline{B}_L = \epsilon_1 \underline{M}_1 + \dots + \epsilon_L \underline{M}_L > 0$. We must choose ϵ_k such that $d_k \underline{M}_k(d_k) - \underline{B}_k = \{d_k w_k(d_k) - \epsilon_k c_k\} \underline{N}_k > 0$ for d_k sufficiently large. Since $d_k w_k(d_k) \rightarrow c_k$ as $d_k \rightarrow \infty$, it follows that for d_k large enough, $d_k w_k(d_k) > c_k - n_k > 0$ where n_k is an arbitrary positive number. Therefore, we can choose $\epsilon_k = (c_k - n_k)/c_k$. This completes the proof.

An example of a poly-t density with scale matrices of the specific form (3) is when d_k and $\underline{M}_k(d_k)$ are given by (2). Here we may choose $w_k(d_k) = 1/(d_k - m_k)$, and $\underline{N}_k = \underline{M}_k$. Note that the \underline{M}_k matrix in (2) is the limit of $d_k \underline{M}_k(d_k)$ since $c_k = 1$.

3. APPLICATION TO BAYESIAN LINEAR MODELING

The results of the previous section are now applied to a multiple regression model with an exchangeable normal prior on the regression coefficients and independent inverted gamma priors on the variance components. The sampling model is represented by

$$\underline{y} = \underline{X} \underline{\beta} + \underline{\epsilon},$$

$$\underline{\epsilon} \sim N(\underline{0}, \sigma^2 \underline{I}_n),$$

where \underline{y} is an $n \times 1$ vector of observations, \underline{X} is an $n \times p$ fixed design matrix in correlation form, and \underline{I}_n is the $n \times n$ identity matrix. The prior specification is given by

$$\underline{\beta} \sim N(\mu \underline{1}_p, \sigma_{\beta}^2 \underline{I}_p),$$

$$v\lambda/\sigma^2 \sim \chi_{\nu}^2, \quad v_{\beta}\lambda_{\beta}/\sigma_{\beta}^2 \sim \chi_{\nu_{\beta}}^2,$$

where $\underline{1}_p$ is a $p \times 1$ vector of ones, and the λ 's and ν 's are hyperparameters that determine the prior means and variances of the variance components. For example $E(\sigma^2) = v\lambda / (v-2)$, and $V(\sigma^2) = 2v^2\lambda^2 / (v-2)^2 (v-4)$. Therefore, a large value of λ reflects a strong prior certainty the λ is a correct guess of σ^2 . Additionally, we assume that prior knowledge about μ is vague.

In the special case of the one-way random effects model with a block diagonal matrix $\underline{X} = \text{diag}(\underline{1}_{n_1}, \dots, \underline{1}_{n_p})$, where n_i is the sample size of the i th group, Hill (1965, 1977, 1980) presented exact and approximate posterior inference of σ_{β}^2 and σ^2 . His results can be used to obtain the posterior moments of the group means $\underline{\beta}$. Lindley and Smith (1972) estimated the parameters of the above general model using a computationally simple posterior joint modal approach. None of these authors, however, discussed the approximation of the posterior density of $\underline{\beta}$, which we denote by $f(\underline{\beta}|\underline{y})$. In the following, we show how the results of Section 2 may be used to approximate this density.

From Lindley and Smith, the posterior density of $\underline{\beta}$ is

$$f(\underline{\beta}|\underline{y}) \propto (1 + \underline{\beta}^T \underline{H} \underline{\beta} / v_{\beta} \lambda_{\beta})^{-(v_{\beta} + p - 1)/2} \times \{1 + (\underline{\beta} - \hat{\underline{\beta}})^T \underline{X}^T \underline{X} (\underline{\beta} - \hat{\underline{\beta}}) / (v\lambda + ns^2)\}^{-(v+n)/2},$$

where $\underline{H} = \underline{I}_p - (1/p)\underline{1}_p \underline{1}_p^T$, $\hat{\underline{\beta}}$ is a least squares estimate of $\underline{\beta}$ and s^2 is the residual mean square. The posterior density of $\underline{\beta}$ is a poly-t density of the special form (2) with

$$\nu_1 = \nu_{\beta}, \quad \nu_2 = v + ns^2/\lambda,$$

$$m_1 = p-1, \quad m_2 = n - ns^2/\lambda,$$

$$\underline{M}_1 = \underline{H}/\lambda_{\beta}, \quad \underline{M}_2 = \underline{X}^T \underline{X} / \lambda.$$

When there is prior near certainty concerning σ_{β}^2 and σ^2 so that both ν_{β} and ν are large, a simple application of the corollary shows that $f(\underline{\beta}|\underline{y})$ is approximately normal with mean $\underline{\mu}$ and covariance matrix $\underline{\Sigma}$ given by

$$\underline{\mu} = (\underline{H}/\lambda_{\beta} + \underline{X}^T \underline{X}/\lambda)^{-1} \underline{X}^T \underline{y},$$

$$\underline{\Sigma} = (\underline{H}/\lambda_{\beta} + \underline{X}^T \underline{X}/\lambda)^{-1}.$$

When there is a considerable amount of uncertainty concerning the prior value λ_{β} (or λ) so that ν_{β} (or ν) is not assigned a large value, the normal approximation may be improved upon by using a generalization of the Tiao-Zellner expansion. An alternative was proposed by Dickey (1967) using an expansion based on Appell's polynomials (1880) in the univariate case ($p=1$); however, the multivariate version has not been studied. The original Tiao-Zellner expansion is designed for a special case of a poly-t density with $L=2$ proper multivariate t factors with mean $\underline{\mu}_k$, scale matrix \underline{M}_k and ν_k degrees of freedom, $k=1,2$. In the expansion, the poly-t density is approximated by a density in the form of $f_N(\underline{x}|\underline{\mu}, \underline{M}^{-1})$, where $\underline{\mu} = \underline{M}^{-1}(\underline{M}_1 \underline{\mu}_1 + \underline{M}_2 \underline{\mu}_2)$ and $\underline{M} = \underline{M}_1 + \underline{M}_2$, multiplied by a double infinite series in the inverse powers of ν_1 and ν_2 . When most of the probability mass is concentrated in the intersection of two ellipsoids $\{\underline{x} | (\underline{x} - \underline{\mu}_k)^T \underline{M}_k^{-1} (\underline{x} - \underline{\mu}_k) \leq \nu_k\}$, $k=1,2$, the expansion will yield reasonable approximation with only a few terms in the series expansion.

The Tiao-Zellner expansion may be generalized to the case of a poly-t density of the form (2). Lindley (1971) mentioned the use of this expansion in the case of the one-way model. However, he did not specify the limiting normal density. Also, the degrees of freedom ν_1 and ν_2 were not correctly stated. We now sketch the essential steps in the generalization using $f(\underline{\beta}|\underline{y})$ in (4) as an example.

Let ν_k and m_k be as defined in (5), $Q_1 = \underline{\beta}^T \underline{H} \underline{\beta} / \lambda_{\beta}$, and $Q_2 = (\underline{\beta} - \hat{\underline{\beta}})^T \underline{X}^T \underline{X} (\underline{\beta} - \hat{\underline{\beta}}) / \lambda$. Following Tiao and Zellner, we obtain for the first factor

$$(1+Q_1/v_1)^{-(v_1+m_1)/2} = \exp(-Q_1/2) \exp[Q_1/2 - \{(v_1+m_1)/2\} \log(1+Q_1/v_1)].$$

This is of the same form as that obtained in (3.2) of Tiao and Zellner except that we replace p there by m_1 . Therefore, the functional forms of the polynomials $p_j = p_j(Q_1)$ in the expansion

$$(1+Q_1/v_1)^{-(v_1+m_1)/2} = \exp(-Q_1/2) \sum_{j=0}^{\infty} p_j v_1^{-j}$$

are of the same forms as those given in Tiao and Zellner except that the constant p is replaced by m_1 (for instance, $p_1 = (Q_1 - m_1 Q_1)/4$). Similarly, for the second factor,

$$(1+Q_2/v_2)^{-(v_2+m_2)/2} = \exp(-Q_2/2) \sum_{j=0}^{\infty} q_j v_2^{-j}$$

where the constant p which appears in the polynomials $q_j = q_j(Q_2)$ in Tiao and Zellner is now replaced by m_2 .

Following the rest of the derivation of Tiao and Zellner, we obtain the generalized expansion

$$f(\underline{\beta}|\underline{y}) = f_N(\underline{\beta}|\underline{y}, \underline{M}^{-1}) \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} d_{ij} v_1^{-i} v_2^{-j},$$

where the leading normal term is the one guaranteed by the corollary, and the d_{ij} 's are polynomials in p_i and q_j given in (3.9) of Tiao and Zellner. The coefficients which appear in the polynomial d_{ij} , namely $b_{rs} = E\{p_r(Q_1)q_s(Q_2)\}$ for $r \leq i, s \leq j$, also have to be modified accordingly. The modification essentially consists of the following steps: (1) Change the constant p in $p_r(Q_1)$ given in Tiao and Zellner to m_1 ; (2) Change the constant p in $q_s(Q_2)$ given in Tiao and Zellner to m_2 , and (3) Use the bivariate moment-cumulant inversion formulas as given by Cook (1951) to express b_{rs} in terms of the mixed cumulants of Q_1 and Q_2 . Expressions for these cumulants are derived in Tiao and Zellner. Following these four steps, the coefficients b_{rs} can be evaluated

in a straightforward manner. We omit the details here.

In the case of a product of two t-like factors of the form (2), it is also feasible to approximate the density by direct one-dimensional numerical integration. Dickey (1968) showed that integration can be facilitated if the scale matrices are simultaneously diagonalized first (more details are presented in Box and Tiao (1973, chapter 9)). The generalized Tiao-Zellner expansion offers a simple approximation alternative in terms of familiar multivariate normal calculations. When there are many such t-like factors, such as in the posterior analysis of a general mixed linear model with many variance components, it will be virtually impossible to approximate the density by high-dimensional numerical integration. The generalized expansion, however, can be extended to the case of many t-like factors in a straightforward manner. Obviously, the computational complexity will increase rapidly as the number of t-like factors increases.

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