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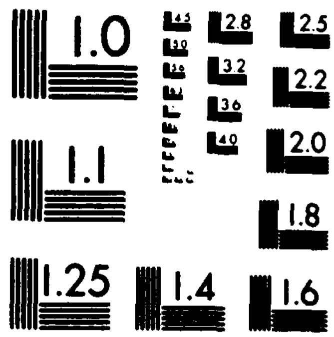

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Evolution of Mean Square Magnetic Vector Potential in Compressible, Two-Dimensional, Magnetofluid Turbulence

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Laboratory for Computational Physics and Fluid Dynamic

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The mean square magnetic vector potential is not invariant in ideal, two dimensional, compressible, magnetohydrodynamic turbulence. A new, related invariant for the compressible case is given. The results are demonstrated by numerical simulation. In the compressible case large amplitude fluctuations in the mean square magnetic vector potential are observed.

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CONTENTS

EVOLUTION OF MEAN SQUARE MAGNETIC VECTOR POTENTIAL IN COMPRESSIBLE, TWO-DIMENSIONAL, MAGNETOFLUID TURBULENCE

Introduction

Incompressible, two-dimensional magnetohydrodynamic (MHD) turbulence has been investigated under a wide range of conditions[1 2 3 4 5 6 7]. The starting point of these investigations is the identification of the quantities which remain constant in the absence of physical dissipation. A striking difference in compressible, two-dimensional MHD is the absence of the mean square magnetic vector potential as an ideal invariant. In this report we discuss this absence, and suggest a related invariant for the compressible case. The supporting results of numerical simulations are then given.

The nonlinear partial differential equations which govern the behaviour of an ideal, two-dimensional, compressible magnetofluid, written in a dimensionless form, are:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}), \qquad (1a)$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot (\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + \frac{1}{2}(p + B^2)\mathbf{I}), \qquad (1b)$$

$$\frac{\partial A}{\partial t} = \mathbf{v} \times \mathbf{B}, \qquad (1c)$$

Manuscript approved March 6, 1987

$$\frac{\partial E}{\partial t} = -\nabla \cdot ((E+p)\mathbf{v} + (|\mathbf{B}|^2 \mathbf{I} - 2\mathbf{B}\mathbf{B}) \cdot \mathbf{v}), \tag{1d}$$

where $\rho(x,y,t) \equiv$ mass density, $\mathbf{v}(x,y,t) \equiv$ flow velocity, $p(x,y,t) \equiv$ mechanical pressure, $\mathbf{I} \equiv$ unit dyad, $A(x,y,t) \equiv$ magnetic vector potential, $\mathbf{B}(x,y,t) \equiv$ magnetic induction field, and $E(x,y,t) \equiv$ total energy density. In addition, we utilize and ideal gas equation of state, *viz.*, $p(x,y,t) = (\gamma - 1)U(x,y,t)$, where $U(x,y,t)$ is the internal energy density, and γ is he ratio of specific heats (assumed equal to $\frac{5}{3}$). The normalization[8] is such that $E_0 = p_0 = B_0^2/8\pi = \rho V_A^2/2$, where V_A^2 is the square of the Alfvén speed. The effects of thermal conduction are not considered.

The ideal conservation properties of the mean square magnetic vector potential are determined by equation $1c$[1]. Upon multiplying this equation by A, we have:

$$\frac{\partial A^2}{\partial t} = -\nabla \cdot (\mathbf{v}A^2) + A^2 \nabla \cdot \mathbf{v} \cdot \tag{2}$$

Integrating this over a periodic box, or one with perfectly conducting boundary conditions, we have, after some algebra:

$$\frac{\partial}{\partial t} \int A^2 d^2x = \int (A^2 \nabla \cdot \mathbf{v}) d^2x \,. \tag{3}$$

For an incompressible magnetofluid, $\nabla \cdot \mathbf{v} = 0$, and the right hand side of equation 3 equals zero. In the compressible case, however, the right hand side of this equation can be finite, as can easily be shown by example. Thus, the magnetofluid's compressibility both removes the constraint that the mean square magnetic vector potential be an ideal invariant of the system, and provides a possible source or sink for this quantity.

A new, related, ideal invariant for the compressible, 2-d magnetofluid can be found by utilizing the mass density equation ($1a$). First, multiply equation 2 by the mass density ρ. After substituting from equation $1a$, this gives:

$$\frac{\partial \rho A^2}{\partial t} = -\nabla \cdot (\mathbf{v} \rho A^2) \cdot \tag{5}$$

When equation 5 is integrated over a periodic box, the right hand side will equal zero, implying that

$$\int \rho A^2 d^2 x \tag{6}$$

is an ideal constant of the motion for the compressible, two-dimensional magnetofluid. Note that equation 6 reduces to the mean square magnetic vector potential as $\rho \to 1$. This is the situation in compressible magnetofluids which exhibit negligible variation in mass density.

We note that $\int \rho A^n d^2 x$ is constant for $n = 0, 1, 2, 3, ..$ Hence the question naturally arises as to why we have singled out the $n = 2$ case for consideration. We believe that this is the significant value of n because of it's close analogy with the kinetic energy, $\int \rho \mid \mathbf{v} \mid^2 d^2 x$. Both of these quantities exhibit smooth transitions to the incompressible limit, *e.g.*, as $\rho \to 1$.

Results of numerical simulations

We have written a computer code which solves the physically dissipative version of equations $1a - 1d$. For the discretization, a dealiased Fourier pseudospectral method is used which will be described in greater detail elsewhere. For the run described here, 32×32 Fourier modes are used, and $\Delta t = 1/250$. A static, uniform, equilibrium state is considered. To initiate a nontrivial evolution in the magnetofluid, we initialize the magnetic field with random noise. With no physical dissipation, the code should exhibit conservation of the ideal invariants.

Figure 1 shows the evolution of the mean square magnetic vector potential as a function of time. Low-frequency, large amplitude oscillations in this quantity are apparent, suggesting that the compressible term serves alternatively as a source or sink of this quantity. The mean square magnetic vector potential varies by as much as approximately 25 % from its initial value. Figure 2 shows the integral given in equation 6 as a function of time for the same run. Note the difference in scaling between the two figures. The maximum variation of this quantity from its initial value is approximately 0.3 % over the course of the run. The conservation is especially good for the first three Alfvén transit times.

We have repeated these calculations for other randomly initialized cases, with similar results being obtained.

Discussion

Statistical description has proven to be the most useful method for characterizing turbulent magnetofluids. The first step in such descriptions is to identify those quantities which remain invariant for the ideal equations of motion. These invariants serve to define hypersurfaces in the phase space of Fourier coefficients of the independent variables of the system. If such a statistical theory can be formed for compressible, 2-d MHD, then it will differ from the incompressible case from the start because the ideal invariants are not the same.

Absence of conservation of mean square magnetic vector potential in a 2-d compressible, turbulent magnetofluid implies several things. First, an inverse cascade of mean square magnetic vector potential is no longer to be expected, since this is not a conserved quantity. Second, there exists in this case the possibility of a 2-d compressible turbulent dynamo. A related matter is the following: the magnetic energy need no longer decay selectively with respect to the mean square magnetic vector potential[4], since the decay of this latter quantity can be hastened or retarded by compressibility effects.

The new invariant which we have given reduces to the mean square magnetic vector potential in the limit of weak compressibility, where the variation of the mass density will be insignificant. Perhaps for this reason the mean square magnetic vector potential has proven a useful quantity in compressible magnetofluids like the solar wind, in which there is little variation in the mass density. In other magnetofluids, *e.g.*, the upper solar atmosphere, variation of the mass density cannot be ignored. The new invariant (equation 6) takes the mass density variation of the magnetofluid into account. We conjecture that the new invariant replaces the mean square magnetic vector potential in compressible, 2-d, magnetofluid turbulence. Further calculations will be required to support this conjecture. The rather difficult question of identifying the regions of Fourier space where the Fourier transformed version of equation 6 remains positive definite must also be addressed (a problem which must also be faced with respect to the kinetic energy). We also note here that the results of this paper can be extended in a straightforward way to the conservation of enstrophy in a compressible, 2-d, neutral fluid which is either isentropic or barotropic.

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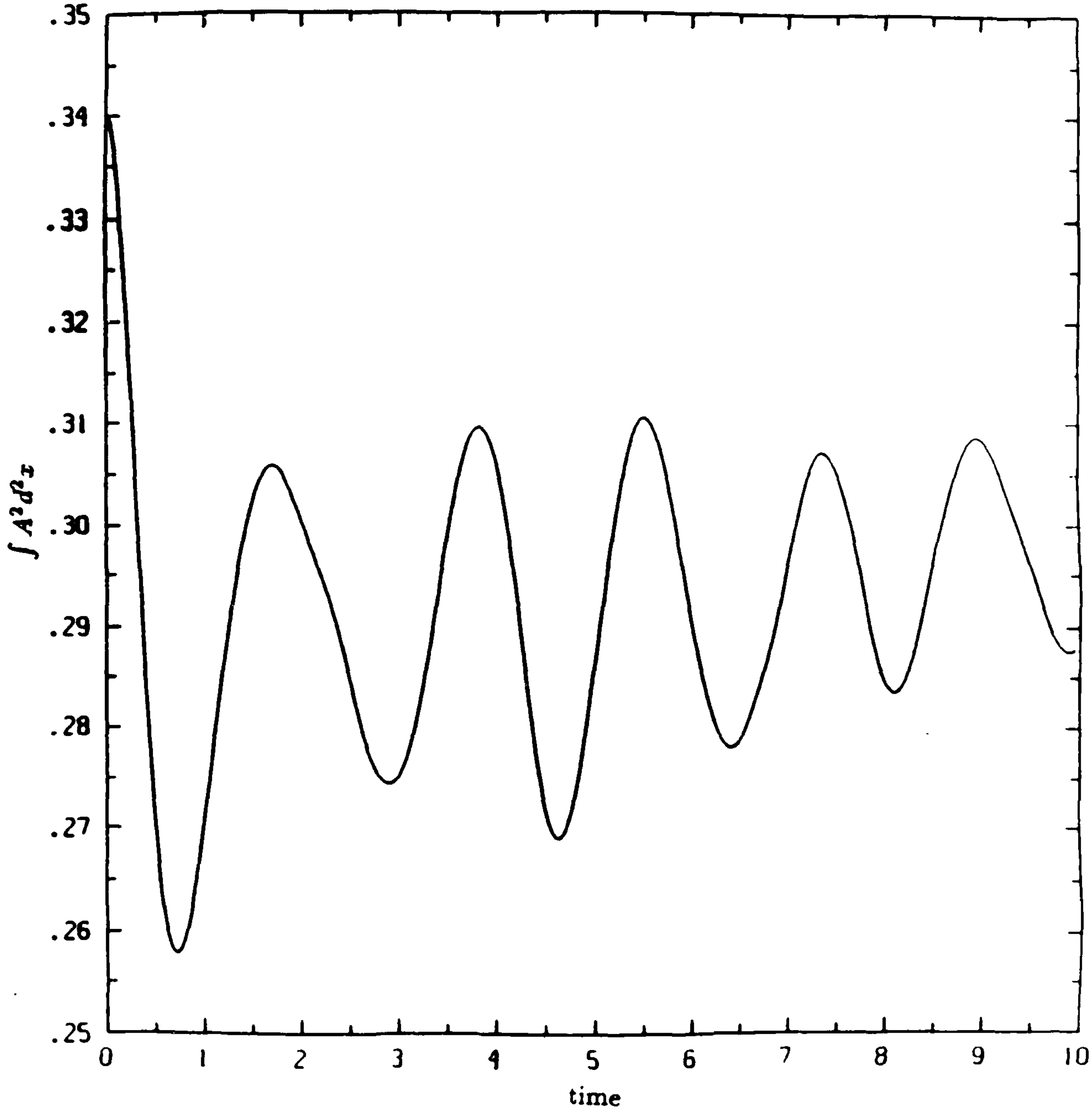

Figure 1. Plot of mean square magnetic vector potential, $\int A^2 d^2 x$, vs time.

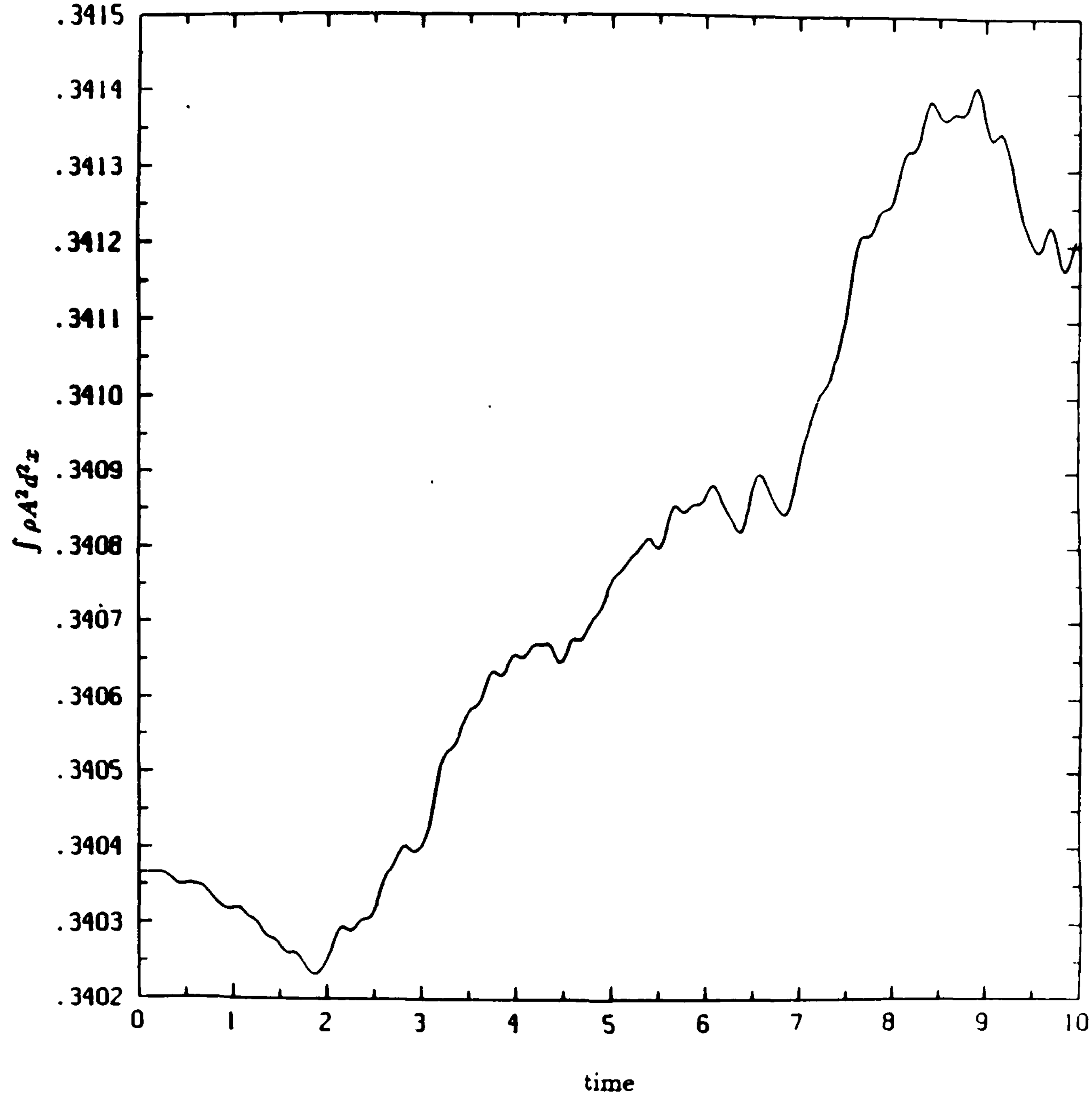

Figure 2. Plot of $\int \rho A^2 d^2 x$ vs time.

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