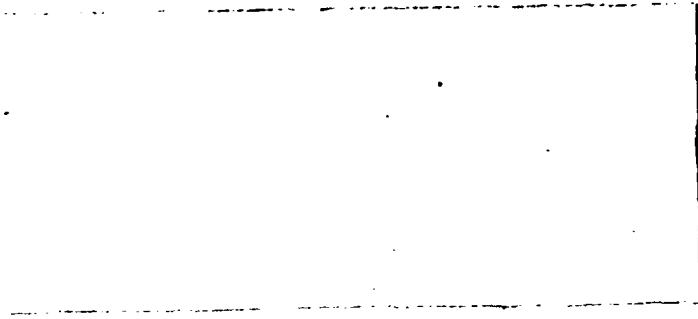


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**APPROXIMATING MULTIVARIATE NORMAL
ORTHANT PROBABILITIES USING THE
CLARK ALGORITHM**

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University of Illinois
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Biometric Lab Report #87-1**

July, 1987

**Departments of Psychiatry and Biometry
University of Illinois at Chicago**

and the

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Supported by Office of Naval Research Contract # N00014-85-K-0586.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified		1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution unlimited	
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE		5. MONITORING ORGANIZATION REPORT NUMBER(S)	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Biometric Lab Report #87-1		7a. NAME OF MONITORING ORGANIZATION Personnel & Training Research Programs Office of Naval Research (Code 1142PT)	
6a. NAME OF PERFORMING ORGANIZATION University of Illinois	6b. OFFICE SYMBOL (if applicable)	7b. ADDRESS (City, State, and ZIP Code) 800 N. Quincy Street Arlington, Virginia 22217-5000	
6c. ADDRESS (City, State, and ZIP Code) Illinois State Psychiatric Institute 529W 1601 West Taylor Street Chicago, Illinois 60612		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N 00014-85-K-0586	
8a. NAME OF FUNDING / SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (if applicable)	10. SOURCE OF FUNDING NUMBERS	
8c. ADDRESS (City, State, and ZIP Code)		PROGRAM ELEMENT NO 61153N	PROJECT NO. RR04206
		TASK NO. RR04204-01	WORK UNIT ACCESSION NO. NR 150-538
11. TITLE (Include Security Classification) Approximating multivariate Normal Orthant Probabilities Using the Clark Algorithm			
12. PERSONAL AUTHOR(S) Robert D. Gibbons, R. Darrell Bock and Donald Hedeker			
13a. TYPE OF REPORT Technical	13b. TIME COVERED FROM 8/01/85 TO 1/01/87	14. DATE OF REPORT (Year, Month, Day) July 15, 1987	15. PAGE COUNT 31
16. SUPPLEMENTARY NOTATION To be submitted for publication in the computational section of Psychometrika			
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	
05	09		
19. ABSTRACT (Continue on reverse if necessary and identify by block number)			
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20. DISTRIBUTION / AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Charles Davis		22b. TELEPHONE (Include Area Code) 202/696-4046	22c. OFFICE SYMBOL ONR 1142PT

**Approximating Multivariate Normal Orthant Probabilities
using The Clark Algorithm**

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ABSTRACT

The probability of m correlated random variables drawn from a multivariate normal distribution being non-negative is:

$$\int_0^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} \Phi_n(x_1, x_2, \dots, x_n) \partial x_1, \partial x_2, \dots, \partial x_n.$$

Exact results for this probability integral are unavailable for $m > 3$. Approximations for higher dimensional problems have generally yielded poor results except for *special cases*, such as compound symmetry, which is of limited value in practice. The purpose of this paper is to present a general approximation of this probability integral. The algorithm developed here is computationally tractable for $m = 50$ and accurate for very general correlational structures. The performance of this algorithm is compared to results based on Clark's (1961) original approximation, Gaussian quadrature formulae, and Monte Carlo simulation methods. Application of this approximation to problems of conditional dependence in IRT estimation problems is discussed.

1 INTRODUCTION

With the renewed interest in marginal maximum likelihood methods (MML) in item response theory (Bock and Aitken, 1981), the necessity for approximating orthant probabilities of the multivariate normal distribution has arisen. In the MML estimation of item parameters, for example, all $s \leq 2^n$ orthant probabilities must be evaluated in order to obtain the likelihood of the parameters of a given IRT model. Similarly, the same multivariate normal orthant probabilities are required to obtain Bayesian estimates of ability assuming dichotomous scoring.

Because IRT models assume conditional independence among the items, that is, all association among the n items is completely explained by their joint association with the latent variables, m -dimensional Gaussian quadrature formulae have been used to provide the necessary orthant probability estimates. In this case, the simple "product formulae" for numerical integration can be applied, because the residuals of the model have the simple uncorrelated multivariate normal distribution $e \sim (0, \sigma_j^2 I)$, where σ_j^2 is the so-called item "uniqueness", that is, $1 - \alpha_j^2$, and α_j is the item factor loading. However, these numerical integrations become extremely expensive as the dimensionality of the problem increases. To the extent that the items have a simple factor structure, these approximations are accurate to any practical degree, however their behavior with more general correlational structures is predictably poor and the extent of bias introduced into parameter estimates is unknown.

In the general case, the probability that m correlated random variables drawn from a multivariate normal distribution are non-negative is:

$$\int_0^\infty \int_0^\infty \cdots \int_0^\infty \Phi_m(x_1, x_2, \dots, x_m) \partial x_1, \partial x_2, \dots, \partial x_m.$$

Exact results for the bivariate normal distribution have been obtained from the work of Sheppard (see Kendall and Stuart, 1973). David (1953) has shown that trivariate normal probabilities are a direct extension of Sheppard's earlier result. However, exact results for m -variate normal probabilities for $m > 3$ are unknown.

Kendall (1941), McFadden (1960) and Moran (1948) have developed infinite expansions of the quadrivariate normal integral, but these results are computationally cumbersome. More tractable representations have been proposed by Abrahamson (1964), Dutt (1973), Dutt and Lin (1975) and Childs (1967). Even in the quadrivariate case, however, these approximations are intractable for general correlational structures and have, therefore,

only been applied under restrictions of compound symmetry or band matrices in which $\rho = .5$ just off the main diagonal.

In a previous paper, we have developed an IRT-type model for estimating trend in correlated proportions (Gibbons and Bock, 1987). In this case the binary items are not components of the same test, but rather, dichotomous classifications ordered in time. Since the association between temporally proximal responses will typically be greater than the association between temporally distal responses, the assumption of conditional independence is untenable and residual autocorrelation must be assumed. At the suggestion of James Heckman (personal communication) we adapted the so-called Clark Algorithm (Clark, 1961) to the problem of evaluating the likelihood of this model. Our modified Clark Algorithm combines both Gaussian quadrature formulae with Clark's original approximation which is extremely fast. The purpose of this paper is to present details of this modified approximation, illustrate its computation, and determine its accuracy in a few relevant examples.

2 THE CLARK ALGORITHM

Designating positive directions 1 and negative directions 0, we may represent the probability of the positive orthant of an m -variate distribution by $P(1, 1, \dots, 1)$, that of the negative orthant by $P(0, 0, \dots, 0)$, and that of any one of the other $2^m - 2$ orthants by inserting the appropriate pattern of 1's and 0's. The Clark algorithm provides a computing approximate for any orthant of a multivariate normal distribution with arbitrary vector mean and covariance matrix. Clark (1961) derives the following formulas.

Let any three successive components from an m -variate vector, y_1 , be distributed:

$$\begin{bmatrix} y_i \\ y_{i+1} \\ y_{i+2} \end{bmatrix} \sim N \left(\begin{bmatrix} \mu_i \\ \mu_{i+1} \\ \mu_{i+2} \end{bmatrix}, \begin{bmatrix} \sigma_i^2 & & \\ \sigma_i \sigma_{i+1} \rho_{i,i+1} & \sigma_{i+1}^2 & \\ \sigma_i \sigma_{i+2} \rho_{i,i+2} & \sigma_{i+1} \sigma_{i+2} \rho_{i+1,i+2} & \sigma_{i+2}^2 \end{bmatrix} \right)$$

Let $\tilde{y}_i = \max(y_i) = y_i$, and compute the probability that $y_{i+1} > \tilde{y}_i$ as follows:

set $z_{i+1} = (\mu_i - \mu_{i+1})/\zeta_{i+1}$,

where $\zeta_{i+1}^2 = \sigma_i^2 + \sigma_{i+1}^2 - 2\sigma_i\sigma_{i+1}\rho_{i,i+1}$.

Then $P(y_{i+1} > \bar{y}) = P(y_{i+1} - \bar{y} > 0)$
 $= \Phi(-z_{i+1})$

the value of the univariate normal distribution function at the standard deviate $-z_{i+1}$.

Now let $\tilde{y}_{i+1} = \max(y_i, y_{i+1})$ and assume (as an approximation) that $(y_{i+2}, \tilde{y}_{i+1})$ is bivariate normal with means,

$$\begin{aligned} \mu(y_{i+2}) &= \mathcal{E}(y_{i+2}) = \mu_{i+2} \\ \mu(\tilde{y}_{i+1}) &= \mathcal{E}(\tilde{y}_{i+1}) = \mu_i\Phi(z_{i+1}) + \mu_{i+1}\Phi(-z_{i+1}) + \zeta_{i+1}\phi(z_{i+1}), \end{aligned}$$

variances

$$\begin{aligned} \sigma^2(y_{i+2}) &= \mathcal{E}(y_{i+2}^2) - \mathcal{E}^2(y_{i+2}) = \sigma_{i+2}^2, \\ \sigma^2(\tilde{y}_{i+1}) &= \mathcal{E}(\tilde{y}_{i+1}^2) - \mathcal{E}^2(\tilde{y}_{i+1}), \end{aligned}$$

where

$$\mathcal{E}(\tilde{y}_{i+1}^2) = (\mu_i^2 + \sigma_i^2)\Phi(z_{i+1}) + (\mu_{i+1}^2 + \sigma_{i+1}^2)\Phi(-z_{i+1}) + (\mu_i + \mu_{i+1})\zeta_{i+1}\phi(z_{i+1}),$$

and correlation

$$\rho(\tilde{y}_{i+1}, y_{i+2}) = \frac{\sigma_i\rho_{i,i+2}\Phi(z_{i+1}) + \sigma_{i+1}\rho_{i+1,i+2}\Phi(-z_{i+1})}{\sigma(\tilde{y}_{i+1})}$$

Then,

$$P(y_{i+2} = \max(y_i, y_{i+1}, y_{i+2})) = P((y_{i+2} - y_{i+1} > 0) \cap (y_{i+2} - y_i > 0))$$

is approximated by

$$\begin{aligned} P(y_{i+2} > \tilde{y}_{i+1}) &= P(y_{i+2} - \tilde{y}_{i+1} > 0) \\ &= \Phi\left(\frac{\mu_{i+2} - \mu(\tilde{y}_{i+1})}{\sqrt{\sigma_{i+2}^2 + \sigma^2(\tilde{y}_{i+1}) - 2\sigma_{i+2}\sigma(\tilde{y}_{i+1})\rho(\tilde{y}_{i+1}, y_{i+2})}}\right) \end{aligned}$$

Assuming as a working approximation that \tilde{y}_{i+1} is normally distributed with the above mean and variance, we may therefore proceed, recursively from $i = 1$ to $i = m - 1$, where y_{m+1} is an independent dummy variate with mean zero and variance zero (i.e. $y_{m+1} = 0$).

Then

$$\begin{aligned} P(y_{m+1} = \max(y_1, y_2, \dots, y_m)) \\ &= P((y_{m+1} - y_1 > 0) \cap (y_{m+1} - y_2 > 0) \cap \dots \cap (y_{m+1} - y_m > 0)) \\ &= P((-y_1 > 0) \cap (-y_2 > 0) \cap \dots \cap (-y_m > 0)) \end{aligned}$$

approximates the probability of the negative orthant of the specified multivariate normal distribution. The probability of any other orthant can be obtained by reversing the signs of the variates corresponding to 1's in the orthant pattern.

3 COMPUTATIONAL EXAMPLE

For computational purposes, it is convenient to set $\Phi(-z) = 1 - \Phi(z)$ and rewrite Clark's equations as:

$$\mu(\tilde{y}_{i+1}) = \mu_{i+1} + (\mu_i - \mu_{i+1})\Phi(z_{i+1}) + \zeta_{i+1}\phi(z_{i+1}),$$

$$\begin{aligned} \mathcal{E}(\tilde{y}_{i+1}^2) &= \mu_{i+1}^2 + \sigma_{i+1}^2 + (\mu_i^2 + \sigma_i^2 - \mu_{i+1}^2 - \sigma_{i+1}^2)\Phi(z_{i+1}) \\ &\quad + (\mu_i + \mu_{i+1})\zeta_{i+1}\phi(z_{i+1}), \end{aligned}$$

$$\sigma^2(\tilde{y}_{i+1}) = \mathcal{E}(\tilde{y}_{i+1}^2) - \mathcal{E}^2(\tilde{y}_{i+1}),$$

$$\begin{aligned} \sigma^2(\tilde{y}_{i+1}, y_{i+2}) &= \sigma^2(y_{i+1}, y_{i+2}) + [\sigma^2(\tilde{y}_i, y_{i+2}) \\ &\quad - \sigma^2(y_{i+1}, y_{i+2})]\Phi(z_{i+1}) \end{aligned}$$

Given this transformation, the i th step now only requires a single evaluation of $\Phi(\cdot)$ and $\phi(\cdot)$. For example, suppose that

$$\underline{\mu} = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} 3 & & \\ 1 & 2 & \\ 1 & 0 & 2 \end{bmatrix}$$

Given these values for $\underline{\mu}$ and Σ , the probability of the negative orthant (- - -) can be obtained as follows.

First find $P(y_3 > \tilde{y}_2)$ as:

$$\begin{aligned}\zeta_2 &= \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}^2} \\ &= \sqrt{3 + 2 - 2(1)} = 1.732\end{aligned}$$

$$\begin{aligned}z_2 &= (\mu_1 - \mu_2)/\zeta_2 \\ &= (3 - 2)/1.732 = .577\end{aligned}$$

$$\begin{aligned}\mathcal{E}(\tilde{y}_2) &= \mu_2 + (\mu_1 - \mu_2)\Phi(z_2) + \zeta_2\phi(z_2) \\ &= 2 + (3 - 2)\Phi(.577) + 1.732\phi(.577) = 3.30\end{aligned}$$

$$\begin{aligned}\mathcal{E}(\tilde{y}_2^2) &= \mu_2^2 + \sigma_2^2 + (\mu_1^2 + \sigma_1^2 - \mu_2^2 - \sigma_2^2)\Phi(z_2) \\ &\quad + (\mu_1 + \mu_2)\zeta_2\phi(z_2) \\ &= 6 + 6\Phi(.577) + 5(1.732)\phi(.577) = 13.23\end{aligned}$$

$$\begin{aligned}\sigma^2(\tilde{y}_2) &= \mathcal{E}(\tilde{y}_2^2) - \mathcal{E}^2(\tilde{y}_2) \\ &= 13.23 - 3.30^2 = 2.34\end{aligned}$$

$$\begin{aligned}\sigma^2(\tilde{y}_2, y_3) &= \sigma_{23}^2 + (\sigma_{13}^2 - \sigma_{23}^2)\Phi(z_2) \\ &= \Phi(.577) = .72\end{aligned}$$

Therefore;

$$\begin{aligned}P(y_3 > \tilde{y}_2) &= \Phi\left(\frac{\mu_3 - \tilde{\mu}_2}{\sqrt{\sigma_3^2 + \tilde{\sigma}_2^2 - 2\tilde{\sigma}_{23}^2}}\right) \\ &= \Phi\left(\frac{2 - 3.3}{\sqrt{2 + 2.34 - 2(.72)}}\right) = .222\end{aligned}$$

To determine $P(\tilde{y}_3 > 0)$ we set $y_4 = 0$ and $\sigma_4^2 = 0$, and compute:

$$\begin{aligned}\zeta_3 &= \sqrt{\bar{\sigma}_2^2 + \sigma_3^2 - 2\bar{\sigma}_{23}^2} \\ &= \sqrt{2.34 + 2 - 2(.72)} = 1.703\end{aligned}$$

$$\begin{aligned}z_3 &= (\bar{\mu}_2 - \mu_3)/\zeta_3 \\ &= (3.3 - 2)/1.703 = .763\end{aligned}$$

$$\begin{aligned}\mathcal{E}(\tilde{y}_3) &= \mu_3 + (\bar{\mu}_2 - \mu_3)\Phi(z_3) + \zeta_3\phi(z_3) \\ &= 2 + (3.3 - 2)\Phi(.763) + 1.703\phi(.763) = 3.518\end{aligned}$$

$$\begin{aligned}\mathcal{E}(\tilde{y}_3^2) &= \mu_3^2 + \sigma_3^2 + (\bar{\mu}_2^2 + \bar{\sigma}_2^2 - \mu_3^2 - \sigma_3^2)\Phi(z_3) \\ &\quad + (\bar{\mu}_2 + \mu_3)\zeta_3\phi(z_3) \\ &= 6 + 7.23\Phi(.763) + 5.3(1.703)\phi(.763) = 14.307\end{aligned}$$

$$\begin{aligned}\sigma^2(\tilde{y}_3) &= \mathcal{E}(\tilde{y}_3^2) - \mathcal{E}^2(\tilde{y}_3) \\ &= 14.307 - 3.518^2 = 1.931\end{aligned}$$

$$\begin{aligned}\sigma^2(\tilde{y}_3, y_4) &= \sigma_{34}^2 + (\sigma_{14}^2 - \sigma_{34}^2)\Phi(z_3) \\ &= 0\end{aligned}$$

Therefore;

$$\begin{aligned}P(y_4 > \tilde{y}_3) &= \Phi\left(\frac{\mu_4 - \bar{\mu}_3}{\sqrt{\sigma_4^2 + \bar{\sigma}_3^2 - 2\bar{\sigma}_{34}^2}}\right) \\ &= \Phi\left(\frac{0 - 3.518}{\sqrt{0 + 1.931 - 2(0)}}\right) = \Phi(-2.53)\end{aligned}$$

Hence, $P(0 > \tilde{y}_3) = \Phi(-2.53) = .006$, which is the probability of orthant (- -).

4 THE MODIFIED CLARK ALGORITHM #1

In our previous paper (Gibbons and Bock, 1987), we noted that the accuracy of the Clark approximation diminishes with increasing magnitude of

the correlations. If we apply the Clark approximation directly to estimates of inter-item correlations, it will generally yield biased results due to the size of correlations. This is true regardless of whether the correlation matrix exhibits the property of conditional independence. Alternatively, if we examine the residual inter-item correlation matrix at fixed points on the ability scale, we will observe the identity matrix for conditionally independent solutions or small residual correlations for those item pairs that are conditionally dependent. In light of this, we evaluate the response function at several fixed points on the ability scale using Gauss-Hermite quadrature, and correct these estimates using the Clark algorithm. These corrections depend only on the residual inter-item correlations, which in practice should be quite small. The first modified Clark algorithm proceeds as follows.

Step 1 Obtain a factor solution of dimension m , using full information factor analysis for binary data (Bock and Aitken, 1981, and Bock, Gibbons and Muraki, 1986) if the correlations are unknown or using principal factor analysis if the item-correlations are known, as in the following simulations.

Step 2 Using the estimated factor loading matrix $\lambda_{n \times k}$ compute the estimated residual correlation matrix $R'_{n \times n}$ as $\rho'_{ij} = \sum_{t=1}^m \lambda_{it} \lambda_{jt}$ for $i \neq j$ else $\rho'_{ij} = 1$, where t is the dimensionality of the space we are conditioning on plus 1. For example, in the unidimensional case, $t = 2$. This correlation matrix represents the degree of residual conditional dependence.

Step 3 Given the previous values of item thresholds γ_j and item factor loadings λ_{ij} for the $t - 1$ principal factors, compute the invariant item parameters a_j (slope) and b_j (intercept). For example, in the unidimensional case,

$$a_j = \lambda_j / \sqrt{1 - \lambda_j^2}$$

and

$$b_j = -\gamma_j / \sqrt{1 - \lambda_j^2}$$

Step 5 At each point on the ability dimension (i.e. at each quadrature node X_k) compute the value of the response function for each item as:

$$z_{jk} = c_j + a_j X_k$$

where $c_j = -a_j b_j$ and X_k are the nodes of the Gauss-Hermite polynomial (see Stroud and Secrest, 1966).

Step 6 At each quadrature point, substitute the values of $x_{j,k}$ for the mean vector $\underline{\mu}$ and R' for the covariance matrix Σ and compute the Clark approximated probability $C_i(X_k)$. Accumulating these probabilities over all quadrature nodes for a given response pattern (\underline{x}_i) yields the desired marginal probability estimate

$$\begin{aligned} h(\underline{x}_i) &= \int_{-\infty}^{\infty} C_i(\theta)\phi(\theta)\partial(\theta) \\ &= \sum_{k=1}^q C_i(X_k)A(X_k) \end{aligned}$$

where $A(X_k)$ is the corresponding weight at quadrature node X_k .

We note that in practice, the effect of assuming normality of the maximum of two jointly normal variables, produces probability overestimates in the tails of the distribution. As such, we apply an empirically based correction factor to these probability estimates which involves raising the Clark adjusted probability estimate to the power 1.3. This correction factor appears to provide the necessary adjustment across the entire quadrature space.

5 THE MODIFIED CLARK ALGORITHM #2

Algorithm #1 is computationally expensive, because the Clark approximation must be evaluated for each response pattern at every point in the quadrature space. In a problem with 1000 unique response patterns and two principal factors, each with 10 quadrature points, the Clark approximation must be invoked 100,000 times per iteration. An alternate approach is to simply use the Clark approximated probability as a correction term, applied directly to the usual probability estimate obtained from the quadrature solution; that is:

$$\begin{aligned} h(\underline{x}_i) &= C_i \int_{-\infty}^{\infty} P_i(\theta)\phi(\theta)\partial(\theta) \\ &= C_i \sum_{k=1}^q P_i(X_k)A(X_k) \end{aligned}$$

where

$$\begin{aligned}
P_1 &= P(x_1, \dots, x_n) \\
&= \int_{-\infty}^{\infty} \left[\prod_{j=1}^n [p_j(\theta)]^{x_j} [1 - p_j(\theta)]^{1-x_j} \right] \phi(\theta) d(\theta) \\
&= \sum_{k=1}^q \left[\prod_{j=1}^n p_j(X_k)^{x_j} [1 - p_j(X_k)]^{1-x_j} \right] A(X_k)
\end{aligned}$$

Returning to the previous example of 1000 unique response patterns and 10 quadrature points in each of two dimensions (i.e. 100 points in total), the Clark algorithm need only be invoked 1000 times per iteration; that is, once for each response pattern in contrast to 100,000 times for algorithm #1.

The probabilities obtained using algorithm #2 differ from those obtained from algorithm #1 in that they will not sum to unity even if all response patterns are realized in the sample. When the number of items is small, say ten or less, these probabilities can be normalized to yield the appropriate metric; however, in larger problems (i.e. with eleven or more items) normalization is not possible, because all patterns generally are not realized in even large samples. We note, however, that maximum likelihood estimation does not require normalized probabilities as long as their relative magnitudes are invariant to transformation of scale. This condition does hold for algorithm #2.

6 ILLUSTRATION

To examine the accuracy of the modified Clark Algorithm, we designed the following limited simulation study. First, we simulated one million five-variate normal deviates for each of the following conditions:

1) compound symmetric matrices with $\rho = 0.2$ through $\rho = 0.8$. For example,

$$R = \begin{bmatrix} 1.0 & & & & \\ 0.5 & 1.0 & & & \\ 0.5 & 0.5 & 1.0 & & \\ 0.5 & 0.5 & 0.5 & 1.0 & \\ 0.5 & 0.5 & 0.5 & 0.5 & 1.0 \end{bmatrix}$$

2) Autocorrelated matrices with $\rho = 0.2$ through $\rho = 0.8$. For example,

$$R = \begin{bmatrix} 1.0 & & & & \\ 0.5 & 1.0 & & & \\ 0.25 & 0.5 & 1.0 & & \\ 0.125 & 0.25 & 0.5 & 1.0 & \\ 0.0625 & 0.125 & 0.25 & 0.5 & 1.0 \end{bmatrix}$$

9) Conditionally dependent correlation matrices with principal factor loadings of $\lambda_{11} = \lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{15}$ ranging from 0.5 to 0.7 and two method related factors, the first with $\lambda_{21} = \lambda_{22} = \lambda_{23}$ ranging from 0.2 through 0.7 and $\lambda_{24} = \lambda_{25} = 0.0$, and the second with $\lambda_{31} = \lambda_{32} = \lambda_{33} = 0.0$ and $\lambda_{34} = \lambda_{35}$ ranging from 0.2 through 0.7.

For example, the correlation matrix corresponding to factor pattern matrix:

$$\lambda = \begin{bmatrix} 0.7 & 0.3 & 0.0 \\ 0.7 & 0.3 & 0.0 \\ 0.7 & 0.3 & 0.0 \\ 0.7 & 0.0 & 0.3 \\ 0.7 & 0.0 & 0.3 \end{bmatrix}$$

is

$$R = \begin{bmatrix} 1.0 & & & & \\ 0.58 & 1.0 & & & \\ 0.58 & 0.58 & 1.0 & & \\ 0.49 & 0.49 & 0.49 & 1.0 & \\ 0.49 & 0.49 & 0.49 & 0.58 & 1.0 \end{bmatrix}$$

In each of the above simulated conditions, the mean vectors were zero, therefore, the binary response patterns were obtained by dichotomizing the simulated normal deviates at zero. The dichotomized response patterns were then sorted and unique patterns and their respective frequencies were accumulated. When divided by one million, these frequencies yield the so called Monte Carlo probability estimates which should be exact to at least four decimal places; that is, the standard error of the simulated probabilities is:

$$\left(\frac{p_j(1-p_j)}{N} \right)^{\frac{1}{2}}$$

which has a maximum value of:

$$\left(\frac{.5(1-.5)}{1000000}\right)^{\frac{1}{2}} = .0005$$

Accuracy of the estimated probabilities was determined by computing the average absolute deviation for each method (i.e. the average absolute difference between Monte Carlo and Clark estimates over all response patterns).

In an effort to examine the properties of these approximations in larger sets of items, that are more typical in practice, we produced two 10-item simulations. The first, consisted of ten million multivariate normal deviates for the compound symmetric case ($\rho = 0.5$), and the second for the conditionally dependent example with principal factor loadings of $\lambda_{1j} = 0.6$ and method related factor loadings of $\lambda_{2,1} \dots \lambda_{2,10} = 0.4$ and $\lambda_{3,11} \dots \lambda_{3,20} = 0.4$.

7 RESULTS

Results of the simulations are displayed in Tables 1-5. Inspection of Table 1, which displays results for the five-item compound symmetric case, reveals that on average, both the quadrature solution and the modified Clark algorithm #2, recover the Monte Carlo estimates to four decimal places (i.e. average difference $\rho = 0.5$, .0002) whereas the modified Clark algorithm #1 is slightly less accurate (i.e. average difference $\rho = 0.5$, .0009). In contrast, the original Clark algorithm is inferior to the other methods and this inferiority increases with larger correlations ($\rho = 0.8$, average difference = .0163). These results are exactly as expected for a conditionally independent correlational structure. In contrast, inspection of Table 2 reveals that autocorrelation produces inferior estimates for the quadrature solution (eg. $\rho = 0.5$, average difference = .0070) whereas the modified Clark algorithms produced more consistent estimates across the entire range (eg. $\rho = 0.5$, average difference for algorithm #1 = .0041 and for algorithm #2 = .0036). Algorithm #2 was in general, slightly better than algorithm #1. The original Clark algorithm produced reasonable estimates through $\rho = 0.5$, but deteriorated quickly for values of $\rho > 0.5$. Since the elements of the correlation matrix become more homogeneous for extreme values of ρ (eg. $\rho = 0.8$) it is not surprising that the performance of the quadrature solution stabilized for values of $\rho > 0.6$.

In terms of five-item conditionally dependent correlational structures (see Table 3), the modified Clark algorithms performed similarly whereas the original Clark algorithm and the quadrature solution were consistently

inferior, for even moderate dependence (λ_{2j} or $\lambda_{3j} > 0.3$). The modified Clark algorithm #2 produced average differences that were slightly smaller than algorithm #1 and as little as one-sixth the size of the standard quadrature solution. Overall, performance was better for solutions in which the principal factor loadings were smaller (eg. $\lambda_{11} = 0.5, \lambda_{21} = 0.5, \lambda_{31} = 0.0$, average difference for algorithm #1 = .0034, average difference for algorithm #2 = .0031, average difference for original Clark = .0041 and average difference for quadrature = .0084 versus $\lambda_{11} = 0.7, \lambda_{21} = 0.5, \lambda_{31} = 0.0$, algorithm #1 = .0068, algorithm #2 = .0048, original Clark = .0091, and quadrature = .0099).

Results for the larger item-sets are presented in Table 4 for the compound symmetric case and Table 5 for the conditional dependent case. In general, results for the larger item-sets parallel those of the smaller item-sets. For the compound symmetric case ($\rho = 0.5$) the average differences were .000065 for algorithm #1, .000007 for algorithm #2, .000007 for the quadrature solution. For the conditionally dependent case ($\lambda_{1j} = 0.6, \lambda_{2j} = 0.4$ or 0.0 and $\lambda_{3j} = 0.4$ or 0.0), the average differences were .000218 for algorithm #1, .000242 for algorithm #2, .000329 for the quadrature solution.

8 DISCUSSION

The results of this study clearly demonstrate that when the assumption of conditional independence is violated, bias in the standard IRT probability estimates are produced. Both modified Clark algorithms presented here minimize this bias; however, algorithm #2 produces slightly more accurate results than algorithm #1 for small numbers of items, at a remarkable savings in computation. Algorithm #2 decreases the required computation by a factor of q^m , where m is the number of underlying dimensions and q is the number of quadrature points in each dimension. When compared to the standard quadrature solution, algorithm #2 requires an additional s evaluations of the Clark algorithm, where s is the number of uniquely observed response patterns. Conversely, when method related factors do exist, multiple-factor solutions will greatly increase the computational complexity of the standard IRT approach, whereas no increase in computation is required for algorithm #2.

It is important to point out that the use of these approximations should not replace multiple factor solutions where the additional factors contribute to our understanding of individual differences. Indeed, the methods described here treat these potentially meaningful effects as errors of measure-

ment. By allowing for more general types of measurement variability, these methods can mask important characteristics of item-person interactions. There are, however, situations where small method related effects confound our ability to accurately characterize the dominant dimension of interest, because the inter-item association is not strictly a function of a single underlying trait. In these cases, the ability to segment these more complex measurement errors from our estimates of the central trait or aptitude of interest is a highly desirable goal of measurement.

Our future research in the application of the Clark algorithm for likelihood evaluation of IRT models, will focus on the estimation of ability. When method related factors violate the assumption of conditional independence, current methods for estimating ability (Bock and Aitken, 1981) will produce incorrect results. Some preliminary work in this area suggests that the variance in Bayes "expected *a posteriori*" estimates (EAP) for fixed levels of ability, increases by a factor of 2 to 3 in the presence of even moderate dependence. Substituting Clark estimates for the conditional probabilities that are usually employed in calculating EAP estimates should account for this "extra" normal variability, and therefore, provide more accurate ability estimates.

In addition, we will also further explore the difference between our two modified Clark algorithms, by focusing on their behavior at individual points in the quadrature space. In this way, we hope to even further improve the performance of algorithm #1 relative to algorithm #2, and perhaps, develop a third procedure which is an improvement over both.

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TABLE 1
COMPOUND SYMMETRY

r	Average Difference			
	Algorithm #1	Algorithm #2	Quadrature	Clark
.2	0.0007	0.0003	0.0003	0.0022
.3	0.0011	0.0003	0.0003	0.0034
.4	0.0019	0.0002	0.0003	0.0049
.5	0.0009	0.0002	0.0003	0.0068
.6	0.0016	0.0002	0.0002	0.0091
.7	0.0022	0.0002	0.0002	0.0122
.8	0.0006	0.0006	0.0007	0.0163

TABLE 2
AUTOCORRELATION

ρ	Average Difference			
	Algorithm #1	Algorithm #2	Quadrature	Clark
.2	0.0013	0.0013	0.0037	0.0017
.3	0.0022	0.0021	0.0051	0.0026
.4	0.0031	0.0028	0.0062	0.0037
.5	0.0040	0.0036	0.0070	0.0050
.6	0.0050	0.0044	0.0075	0.0067
.7	0.0054	0.0051	0.0075	0.0090
.8	0.0060	0.0054	0.0071	0.0121

Table 3
CONDITIONAL DEPENDENCE

λ_1	CD	Average Difference			
		Algorithm #1	Algorithm #2	Quadrature	Clark
.5	.2	0.0011	0.0005	0.0014	0.0023
	.3	0.0016	0.0011	0.0031	0.0024
	.4	0.0022	0.0020	0.0054	0.0031
	.5	0.0034	0.0031	0.0084	0.0041
	.6	0.0047	0.0045	0.0121	0.0057
	.7	0.0075	0.0065	0.0173	0.0083
	.6	.2	0.0018	0.0006	0.0015
.3		0.0025	0.0014	0.0032	0.0037
.4		0.0031	0.0025	0.0058	0.0046
.5		0.0047	0.0039	0.0089	0.0060
.6		0.0070	0.0058	0.0130	0.0081
.7		0.0123	0.0083	0.0204	0.0120
.7		.2	0.0015	0.0008	0.0016
	.3	0.0026	0.0018	0.0036	0.0057
	.4	0.0041	0.0031	0.0063	0.0070
	.5	0.0068	0.0048	0.0099	0.0091
	.6	0.0117	0.0073	0.0159	0.0125
	.7	0.0242	0.0170	0.0305	0.0229

Table 4
 COMPOUND SYMMETRY
 $\rho = 0.5$
 25 PATTERNS FOR 10 ITEMS

Pattern	Algorithm #1	Algorithm #2	Quadrature	Monte Carlo
1111111111	.097932	.091086	.090469	.090958
1111111110	.009606	.009115	.009096	.009055
1111111101	.009229	.009119	.009096	.009049
1111111100	.002018	.002018	.002018	.002011
1111111011	.008887	.009122	.009096	.009094
1111111010	.001965	.002016	.002018	.002024
1111111001	.001980	.002017	.002018	.002008
1111111000	.000793	.000758	.000761	.000758
1111110111	.008578	.009125	.009096	.009077
1111110110	.001917	.002017	.002018	.002021
1111110101	.001930	.002018	.002018	.002013
1111110100	.000780	.000758	.000761	.000759
1111110011	.001948	.002019	.002018	.002018
1111110010	.000779	.000758	.000761	.000747
1111110001	.000781	.000758	.000761	.000769
1111110000	.000478	.000432	.000434	.000436
1111101111	.008302	.009126	.009096	.009126
1111101110	.001874	.002017	.002018	.002050
1111101101	.001885	.002018	.002018	.002015
1111101100	.000768	.000758	.000761	.000741
1111101011	.001901	.002019	.002018	.002040
1111101010	.000767	.000758	.000761	.000753
1111101001	.000768	.000759	.000761	.000750
1111101000	.000473	.000432	.000434	.000433
1111100111	.001924	.002019	.002018	.002037
REMAINING PATTERNS	.831716	.836967	.837527	.837249
AVERAGE DIFFERENCE	.000065	.000007	.000007	

Table 5
 CONDITIONAL DEPENDENCE
 $\lambda_{1j} = 0.6$
 λ_{2j} and $\lambda_{3j} = 0.4$ or 0.0
 25 PATTERNS FOR 10 ITEMS

Pattern	Algorithm #1	Algorithm #2	Quadrature	Monte Carlo
1111111111	.058630	.099093	.049837	.071364
1111111110	.008145	.010562	.007744	.008396
1111111101	.007586	.010753	.007744	.008384
1111111100	.002818	.002365	.002115	.002726
1111111011	.007119	.010809	.007744	.008387
1111111010	.002676	.002399	.002115	.002707
1111111001	.002645	.002432	.002115	.002743
1111111000	.001620	.000993	.000889	.001767
1111110111	.006694	.010750	.007744	.008432
1111110110	.002594	.002414	.002115	.002732
1111110101	.002525	.002429	.002115	.002723
1111110100	.001582	.001006	.000889	.001743
1111110011	.002482	.002438	.002115	.002749
1111110010	.001572	.001012	.000889	.001762
1111110001	.001547	.001016	.000889	.001751
1111110000	.001434	.000732	.000539	.002087
1111101111	.006241	.010513	.007744	.008455
1111101110	.002570	.002417	.002115	.002767
1111101101	.002458	.002409	.002115	.002741
1111101100	.001592	.001025	.000889	.001752
1111101011	.002370	.002395	.002115	.002723
1111101010	.001561	.001021	.000889	.001760
1111101001	.001518	.001015	.000889	.001744
1111101000	.001444	.000748	.000539	.002076
1111100111	.002300	.002362	.002115	.002720
REMAINING PATTERNS	.866271	.814880	.882800	.842793
AVERAGE DIFFERENCE	.000218	.000242	.000329	

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