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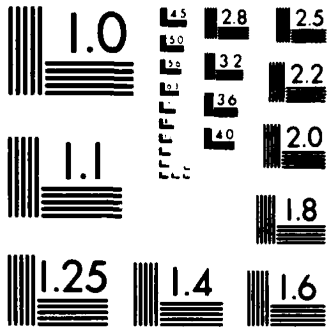

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Path-integral treatment of intensity behavior for rays in a sound channel

by

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ABSTRACT

The intensity coherence function (ICF) of the acoustic wave function from a point source is derived by the path-integral technique for transmission through internal waves in the presence of a sound channel. Separations in time are emphasized, although separations in transverse horizontal position, vertical position, and acoustic frequency are discussed. Approximate intensity coherence times, lengths, and bandwidths due to internal-wave fluctuations are derived. Analytic approximations suitable for computer coding are presented for the micropath focussing parameter γ, which controls the deviation of higher intensity moments from the Rayleigh-distribution values.

INTRODUCTION

The intensity coherence function (ICF) contains important statistical information about the acoustic field that has traversed a medium filled with random fluctuations. We show in this paper how to derive a number of results about the ICF which were indicated in our earlier works on the subject.[1-3] In particular, we show how to derive the ICF from the path-integral technique for transmission in the presence of a deterministic sound channel. The ICF is a fourth moment of the field. This paper follows logically after our paper[4] on the mutual coherence function (MCF), which is a second moment. We also present some analytic approximations to the micropath focussing parameter γ, which controls the deviation of the higher moments of intensity from the Rayleigh-distribution values. These approximations are suitable for computer coding.

We derive a path-integral expression for the ICF of time, and then give explicit rules for calculating it in the special case of internal-wave medium fluctuations. We are able to explicitly evaluate the case of internal waves because the accepted spectrum of internal waves implies a nearly quadratic structure function and solutions are known for path integrals with quadratic actions. In our results, the intensity coherence time, coherent bandwidth, and coherence lengths are determined by weighted averages of the medium fluctuation spectra. We also indicate already-known general results on the ICF of space and frequency, and we derive a new result for the relation between the scales of the ICF of time and MCF of frequency for an internal-wave medium. A companion paper presents comparisons of these theoretical results with experiment.[5]

I. PATH-INTEGRAL EXPRESSION FOR THE ACOUSTIC WAVEFUNCTION

We begin with the wave equation for the pressure as a function of space and time in the presence of a spatially varying wavespeed. We follow the notation of the review article by Flatté.[3] The sound speed can be expressed as

$$C(\underset{\sim}{x}, t) = C_o[1 + U_o(z) + \mu(\underset{\sim}{x}, t)] \tag{1}$$

where C_o is a reference sound speed, $U_o(z)$ is a dimensionless function of the depth z representing the deterministic sound channel, and $\mu(\underset{\sim}{x}, t)$ is a random, zero-mean function of position representing the effect of medium fluctuations such as internal waves. The wave equation for an acoustic wave is unaffected by the time dependence of μ because μ has only components with very low frequency.

The acoustic pressure ϕ obeys the wave equation

$$\partial_{tt}\phi - C^2 \nabla^2\phi = 0 \tag{2}$$

The parabolic approximation consists of considering solutions in which waves are travelling only at small angles to a particular direction; in the ocean this direction is in the horizontal, labelled by x. Thus we try

$$\phi = \exp[i(qx - \sigma t)]\psi(\underset{\sim}{x}, t) \tag{3}$$

where q and σ are the wavenumber and frequency of an acoustic wave travelling along the x axis at speed C_o: that is, $q = \sigma / C_o$. The "reduced" wavefunction ψ is slowly varying in space and time compared with q and σ, and satisfies a parabolic equation:[6]

$$2iq\partial_x\psi = \{-\partial_{yy} - \partial_{zz} + 2q^2[U_o + \mu]\}\ \psi \tag{4}$$

Equation (4) is a Schrodinger equation, and thus its solution can be directly expressed in terms of a Feynman path integral[7]

$$\psi = N \int D\underset{\sim}{z}\ \exp\{iqS_o(\underset{\sim}{z}) - iq \int_0^R \mu\left[x, \underset{\sim}{z}(x), t\right] dx\} \tag{5}$$

where the path integration (indicated by D) is over all paths $\underset{\sim}{z}(x) = [y(x), z(x)]$ connecting the source to the receiver. The phase associated with the path in the absence of fluctuations is

$$qS_0 = q \int_0^R \left[\frac{1}{2}(\partial_x y)^2 + \frac{1}{2}(\partial_x z)^2 - U_o(z)\right] dx \tag{6}$$

and N is a normalization factor chosen by convention so that $\psi \equiv 1$ for $\mu = 0$.

II. PATH-INTEGRAL EXPRESSIONS FOR THE ICF

The ICF is $<I(B)I(A)>$ where the angle brackets indicate averaging over the ensemble of random μ functions. The ICF measures the coherence between the intensities at two different points, labelled by A and B. These points may be separated in space, frequency, time, or a combination. Consider spatial separations that lie in the (y,z) plane. The ICF is a fourth moment of the field; we designate it by:

$$G_4(B,A) \equiv <\psi^*(B)\psi(B)\psi^*(A)\psi(A)> = <|N|^4 \int Dz_1\, Dz_2\, Dz_3\, Dz_4$$

$$\exp\left[iqS_o(1)-iqS_o(2)+iqS_o(3)-iqS_o(4)-iq\int_0^R \{\mu(z_1)-\mu(z_2)+\mu(z_3)-\mu(z_4)\}\,dx\right]> \quad (7)$$

where paths 1 and 2 arrive at point A and paths 3 and 4 at point B. (All paths begin at one point: the source.) The ensemble average applies only to the μ's, so we may write

$$G_4(B,A)>=|N|^4 \int Dz_1\, Dz_2\, Dz_3\, Dz_4$$

$$\exp\left[iqS_o(1)-iqS_o(2)+iqS_o(3)-iqS_o(4)-\frac{1}{2}M\right] \quad (8)$$

where

$$M=q^2<\left(\int_0^R [\mu(z_1)-\mu(z_2)+\mu(z_3)-\mu(z_4)]dx\right)^2> \quad (9)$$

and we have used the fact that

$$<\exp(i\alpha)>=\exp(-\frac{1}{2}<\alpha^2>) \quad (10)$$

if α is a zero-mean, Gaussian random variable, such as any combination of μ's is assumed to be. Even if α is not a Gaussian random variable, (10) can still be true if the higher moments of α are small.

The problem of finding a useful result for the ICF now reduces to evaluating the quadruple path integral in (8). We first expand the deterministic sound channel function to second order in the displacement of the paths away from the equilibrium ray. That is, we define $z_r(x)$ as the function that satisfies the ray equation (in the parabolic approximation):

$$\partial_{xx} z_r + U_o'(z_r) = 0 \tag{11}$$

with appropriate boundary conditions determined by the initial and final points. We expand $U_o(z_i)$ around the point $z_r(x)$. Let $v_i(x) \equiv z_i(x) - z_r(x)$; then

$$U_o(z_i) \approx U_o(z_r) + U_o' v_i + \frac{1}{2} U_o'' v_i^2 \tag{12}$$

where it is understood that U_o' and U_o'' are evaluated at $z_r(x)$. This expression will be valid as long as the effective bundle of acoustic energy stays well confined around the unperturbed ray, $z_r(x)$. If more than one solution of (11) exists (deterministic multipath), then we treat one unperturbed ray at a time. Addition of the results depends on the coherence between deterministic rays, which can often be well estimated. At caustics our method breaks down.

The quantities $\underset{\sim}{v}_i(x)$ can be thought of as four new path variables defined as deviations from the equilibrium ray.

We can make a transformation of the four path variables to a new set given by[8]

$$\begin{aligned}
2\,\underset{\sim}{\alpha} &= \underset{\sim}{v}_1 + \underset{\sim}{v}_2 + \underset{\sim}{v}_3 + \underset{\sim}{v}_4 \\
2\,\underset{\sim}{\beta} &= \underset{\sim}{v}_1 + \underset{\sim}{v}_2 - \underset{\sim}{v}_3 - \underset{\sim}{v}_4 \\
2\,\underset{\sim}{\gamma} &= \underset{\sim}{v}_1 - \underset{\sim}{v}_2 - \underset{\sim}{v}_3 + \underset{\sim}{v}_4 \\
2\,\underset{\sim}{\delta} &= \underset{\sim}{v}_1 - \underset{\sim}{v}_2 + \underset{\sim}{v}_3 - \underset{\sim}{v}_4
\end{aligned} \tag{13}$$

The $\underset{\sim}{\alpha}$ path-variable integration can be carried out by noting that M is assumed to be independent of the centroid of the four paths. The $\underset{\sim}{\delta}$ variable is then forced to be zero for the case of intensity correlations.

We are left with a double path integral over $\underset{\sim}{\beta}$ and $\underset{\sim}{\gamma}$ for the intensity correlation at two nearby points a distance R from a point source. The two points A and B are separated transversely by $\underset{\sim}{z}_s$ and in time by t_s:

$$G_4(\underset{\sim}{z}_s, t_s) = |N|^2 \int D\,\underset{\sim}{\beta}\;\; D\,\underset{\sim}{\gamma}$$

$$\exp\left\{-iq\int_O^R\left[(\partial_x \underset{\sim}{\beta})\cdot(\partial_x \underset{\sim}{\gamma}) - U_o''\beta_z\gamma_z\right]dx - \frac{1}{2}M\right\} \tag{14}$$

and

$$M=\int_O^R\left[\,2d(\underset{\sim}{\gamma},0)+2\,d(\underset{\sim}{\beta},t)-d(\underset{\sim}{\beta}+\underset{\sim}{\gamma},t)-d(\underset{\sim}{\beta}-\underset{\sim}{\gamma},t)\right]dx \tag{15}$$

where $d(\underset{\sim}{z},t)$ is the phase structure function density defined in Esswein and Flatté.[9] The dummy path variables $\underset{\sim}{\beta}$ and $\underset{\sim}{\gamma}$ are constrained by the above treatment to have boundary conditions given by $\underset{\sim}{\beta} = \underset{\sim}{\gamma} = 0$ at $x=0$ and $\underset{\sim}{\beta} = \underset{\sim}{z}_s$, $\underset{\sim}{\gamma} = 0$ at $x=R$. The β_z and γ_z are the vertical components of $\underset{\sim}{\beta}$ and $\underset{\sim}{\gamma}$.

At this point it is worth noting that a number of endpoint terms that resulted from the integrations by parts have been subsumed in the normalization, which is required to give unity at zero separation of the two receivers.

III. EVALUATION OF THE MONOCHROMATIC ICF FOR INTERNAL WAVES

Under the Markov approximation, the phase structure function density can be expressed as

$$d(y,\,z,\,t) = 2q^2 \langle\mu^2\rangle L_P f(y,\,z,\,t) \tag{16}$$

where $\langle\mu^2\rangle$ and L_P have been defined previously[2,3] and $f(y,z,t)$ is the phase correlation function (PCF) defined by Esswein and Flatté.[9] The PCF has been evaluated for internal waves by Esswein and Flatté,[9] using a combination of analytical and numerical techniques. Since y and z are both functions of range, x, (16) must in general be evaluated by a numerical integration code.

At small separations, approximations to the PCF are possible. For example,

$$f(0,\,0,\,t) = \frac{1}{2}\{\omega^2\}\,t^2 \tag{17}$$

$$f(0,\,z,\,0) = \frac{1}{2}\{k_V^2\}\,z^2\ln(z_o/z) \tag{18}$$

where $\{\omega^2\}$ is an average internal-wave frequency that is dependent on the local depth and position of the ray $z_r(x)$, and likewise the quantities $\{k_V^2\}$ and z_o. All three quantities are evaluated in Esswein and Flatté.[9] Because (18) is nearly quadratic the logarithm may rather accurately be replaced by a constant. One of our problems will be to establish the best value of that constant. It will be useful to remember that two important quantities defining the type of fluctuations are Φ and Λ where

$$\Phi^2 = q^2 \int dx <\mu^2> L_P \quad . \tag{19}$$

and

$$\Lambda\Phi^2 = q \int dx <\mu^2> L_P \; \{k_V^2\} \; |g(x, x)| \tag{20}$$

where $g(x,x)$ is a Green's function defined in Flatté.[3] We also define $L_V \equiv \{k_V^2\}^{-\frac{1}{2}}$. Evaluations of Φ and Λ for some particular examples are given by Esswein and Flatté.[10]

We will now use (16), (17) and (18) to evaluate G_4. We need to note that the horizontal (y) dependence of $f(y, z, t)$ is very weak; therefore we will approximate the PCF as $f(z, t)$ with y set to zero. Then (14) becomes:

$$G_4(z_a, t) = |N|^2 \int D\beta\, D\gamma \exp\left\{ - iq \int_O^R \left[\partial_x\beta\, \partial_x\gamma - U_o''\beta\gamma\right] dx - \frac{1}{2} M \right\} \tag{21}$$

and

$$M = q^2 \int_O^R <\mu^2> L_P \left[2f(\gamma, o) + 2f(\beta, t) - f(\beta+\gamma, t) - f(\beta-\gamma, t)\right] dx \tag{22}$$

with endpoint conditions given by $\gamma(0) = \gamma(R) = 0$ and $\beta(0) = 0, \beta(R) = z_a$.

IV. EVALUATION OF THE ICF OF TIME

Consider the intensity coherence function $<I(t)I(0)>$; that is, set $z_a = 0$ in (21) and (22). It is not difficult to show that the main contributions to the double path integral come from two regions, one where $\beta \lesssim L_V/\Phi$ and the other where $\beta \lesssim L_V/\Phi$. The two regions are independent, so first consider the region where γ is small. In that case, expansion of the last two f's in M yields

$$M \approx 2q^2 \int_O^R <\mu^2> L_P \; f(\gamma,0)dx \tag{23}$$

which is now independent of t. Since we normalize to the answer when $t=0$, this term gives unity. Now the term where β is small yields

$$M \approx 2 \int_O^R <\mu^2> L_P \left[f(\gamma,o)+f(\beta,t)-f(\gamma,\, t)\right]dx \tag{24}$$

which is separable in β and γ. To evaluate (21) we must either do the β or γ path integral first. It is of interest that in the phase-screen case, one does the γ integral first, but here we find it necessary to do the β integral first.[8] That is

$$<\mathrm{I}(t)\mathrm{I}(0)>=1+N \int D\gamma \exp\left\{q^2 \int_O^R <\mu^2> L_P \left[f(\gamma,0)-f(\gamma,t)\right]dx\right\}$$

$$\cdot \int D\beta \exp\left\{-iq \int_O^R \left[\partial_x\beta\,\partial_x\gamma - U_o''\beta\,\gamma\right]dx - q^2 \int_O^R <\mu^2> L_P f(\beta,t)dx\right\} \tag{25}$$

We know from the phase-screen example that the β integral is cut off by the $f(\beta,t)$ term in the exponential, that is when

$$q^2 \int_O^R <\mu^2> L_P f(\beta,t)dx \approx 1 \tag{26}$$

but this is approximately

$$\Phi^2 \left(\frac{\beta}{L_V}\right)^2 \ln(L_V/\beta)=1 \tag{27}$$

Thus the important contributions come when $\beta \approx L_V/\Phi$ as expected, and the logarithm may be replaced by ln Φ. Now the β integral may be done since it is a quadratic path integral. Let

$$H = q^2 <\mu^2> \frac{L_P \ln\Phi}{L_V^2} \tag{28}$$

Completing the square and doing the β integral, which is now subsumed in the normalization, we are left with the γ integral:

$$<\mathrm{I}(t)\mathrm{I}(0)> = 1 + N \int D\gamma \exp\left\{ -\int_O^R \frac{q^2}{4H} \left[\partial_{zz}\gamma + U_o''\gamma\right]^2 dx \right.$$

$$\left. -\frac{1}{2} q^2 \int_O^R <\mu^2> \frac{L_P}{L_V^2} \{\omega^2\}\ t^2 \gamma^2 \ln\left(\frac{L_V}{\gamma}\right) dx \right\} \tag{29}$$

and again we must ask what value of γ to use in approximating the logarithm by a constant. Again from the phase-screen example we know that the γ integral is cut off by the geometric term (the first term) in the exponent. Appendix A shows that the appropriate value of γ is $L_V\ [2\Lambda\Phi(\ \ln\Phi)^{1/2}]$ in order to make the geometric term of order unity. We define

$$\beta_o \equiv \frac{1}{2\Lambda\Phi(\ln\ \Phi)^{1/2}} \tag{30}$$

and we note that we are going to replace $\ln\ (L_V/\gamma)$ by $\ln\ \beta_o$.

With that approximation, the γ integral is a quadratic path integral. Although quadratic path integrals can be done, the form of (28) is not convenient because of the appearance of the function H as a coefficient of $\partial_{zz}\gamma$, and because of the appearance of $\left[\partial_{zz}\gamma\right]^2$. Well-known standard techniques can be employed most easily if we return to the original double path integral (21), with our new understanding that M can be expressed in quadratic form. Remembering that we are setting $z_a = 0$, we can express the ICF in terms of a matrix form within the double path integral:

$$<\mathrm{I}(t)\mathrm{I}(0)> = <I^2> \left[1 + K(t)\right] \tag{31}$$

$$K(t) = N \int D\beta D\gamma \exp\left\{ -\frac{i}{2} q \int_O^R (\gamma,\beta) \left[(\partial_{zz} + U_o'') \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right.\right.$$

$$\left.\left. -\frac{2i}{q} \begin{pmatrix} H & O \\ O & J \end{pmatrix} \right] \begin{pmatrix} \gamma \\ \beta \end{pmatrix} dx \right\} \tag{32}$$

where

$$J = \frac{1}{2} q^2 <\mu^2> \frac{L_P}{L_V^2} \{\omega^2\}\ t^2 \ln\beta_o \tag{33}$$

where N is determined by the requirement that $K(t)\rightarrow 1$ as $t \rightarrow 0$. The general solution to this type of path integral is given by solving a set of coupled ordinary differential equations for a matrix M:

$$O\ M = 0 \tag{34}$$

where the differential matrix operator O is given by the expression inside the square brackets in (31), the initial conditions on M are $M(x=0) = 0, M'(x=0) = I$, and the solution is

$$K(t) = \left\{ \frac{\det M_o(R)}{\det M(R)} \right\}^{\frac{1}{2}} \tag{35}$$

where M_o is the solution with $t = 0$.

One can show that the result for $K(t)$ is independent of multiplying the H-J matrix by any diagonal matrix. In particular, if we multiply by

$$\begin{pmatrix} c & 0 \\ 0 & \frac{1}{c} \end{pmatrix} \tag{36}$$

with

$$c = -i\omega_i t \left\{ \frac{\ln\beta_o}{2\ln\Phi} \right\}^{\frac{1}{2}} \tag{38}$$

we find our ordinary differential equation is

$$\left[(\partial_{xx} + U_o'') \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + q <\mu^2> \frac{L_P}{L_V^2} \alpha \begin{pmatrix} 0 & 1 \\ 1 & \frac{\{\omega^2\}}{\omega_i^2} \end{pmatrix} \right] M = 0 \tag{38}$$

with

$$\alpha = \omega_i\ t(\ln \Phi \ln\beta_o)^{\frac{1}{2}} \tag{40}$$

This is the equation that is used in evaluating $K(t)$.

An important approximation may be made if $\{\omega^2\}$ is not a function of x, which would happen for a short, horizontal ray, or if the weighting function $<\mu^2> L_P/L_V^2$ were strongly peaked around upper turning points so that only $\{\omega^2\}$ values near horizontal enter in a significant way. In that case the two equations become identical to each other, and related to the equation for the micropath bandwidth function Q derived in Dashen et al.[4] As a result, one can calculate K from the equation

$$K(t) = |Q(\Delta\sigma)|^2 \quad (40)$$

where the pseudofrequency $\Delta\sigma$ is given by

$$\frac{\Delta\sigma}{\sigma} = \frac{\alpha}{\ln\Phi}\left[\frac{2\{\omega^2\}}{\omega_i^2}\right]^{1/2} = \left[\frac{2\{\omega^2\}\ln\beta_o}{\ln\Phi}\right]^{1/2} t \quad (41)$$

and σ is the acoustic frequency.

V. SMALL-TIME APPROXIMATION OF THE ICF

Calculation of $K(t)$ is cumbersome and lengthy. It is useful to have a simpler approximation based on the limit as $t \to 0$. It can be shown that $K(t)$ approaches unity in a quadratic manner; that is, there exists a constant ν' such that

$$K(t) \to 1 - \nu'^2 t^2 \quad (42)$$

We can obtain an expression for ν' immediately in the case that (40) is valid, (that is, constant $\{\omega^2\}$) since we have an expression for $Q(\Delta\sigma)$ as $\Delta\sigma \to 0$. From (41) and Eq. (65) of Dashen et al[4] we find

$$\frac{\nu'^2}{\tau_o^2} = 4\sigma^2 \{\omega^2\} \frac{\ln\beta_o}{\ln\Phi} \quad (43)$$

If $\{\omega^2\}$ is not constant we must treat (38) when $\alpha \to 0$. The same technique that led to Eq. (67) of Dashen et al,[4] only in matrix form, leads to:

$$\nu'^2 = q^2 \ln\beta_o \ln\Phi \int_O^R dx \, \langle\mu^2\rangle L_P \{k_V^2\} \, \{\omega^2\}$$

$$\int_O^R dx' \, \langle\mu^2\rangle L_P \{k_V^2\} \left[g(x,x')\right]^2 \quad (44)$$

where it is understood that the first $\langle\mu^2\rangle L_P\{k_V\}^2$ is evaluated at x and the second at x'. In actual numerical evaluation $g(x,x')$ can be broken into two factors, one a function of x and the other a function of x' (with different functions for the two cases $x \gtrless x'$); this can speed up the calculation substantially.

VI. SEPARATIONS IN FREQUENCY AND SPACE

We collect here results presented in other papers for the intensity correlations between receivers separated in space or in frequency:

$$<I(\underset{\sim}{z})I(0)> = \left[1 + \exp\left\{-D(\underset{\sim}{z})\right\}\right] <I^2> \tag{45}$$

$$<I(\Delta\sigma)I(0)> = \left[1 + |Q(\Delta\sigma)|^2\right] <I^2> \tag{46}$$

where $D(\underset{\sim}{z})$ is the phase structure function for transverse separation $\underset{\sim}{z}$. The use of $<I^2>$ as a normalizing factor avoids problems with the first-order correction discussed in Flatté, Bernstein, and Dashen.[11] The above results imply that the intensity coherence lengths and bandwidths are simply $\sqrt{2}$ smaller than the acoustic-field coherence scales evaluated in Dashen et al.[4]

VII. HIGHER ORDER MOMENT CORRECTIONS; THE MICRORAY FOCUSSING PARAMETER γ

We present here some analytical approximations we have used to calculate γ, where it is remembered that higher intensity moments are approximately given by[11]

$$<I^N> \approx N! <I>^N \left[1 + \frac{N(N-1)}{2}\gamma\right] \tag{47}$$

In terms of internal waves with the spectrum assumed in previous references, the expression for γ is:[11]

$$\gamma = 2q^2 \int_O^R dx <\mu^2> L_P \{P(j,x)\} \tag{48}$$

where

$$P(j,x) \equiv \left\{1 - \cos\left[j^2 \frac{k_v^2}{q} g(x,x)\right]\right\}$$

$$\cdot \exp\left\{-2q^2 \int_O^R dx' <\mu^2> L_P M_J \sum_{J=1}^{\infty} \frac{1 - \cos\left[\frac{k_v k_v'}{q} jJg(x,x')\right]}{J(J^2+j_*^2)}\right\} \tag{49}$$

$$\{P(j,x)\} \equiv M_j \sum_{j=1}^{\infty} \frac{P(j,x)}{j(j^2+j_*^2)} \tag{50}$$

where $k_v = \pi n(z)/n_o B$, $k_v' = \pi n(z')/n_o B$, and M_j and $j*$ are constants defined in Flatte.[3]

In the case of no sound channel, where the rays are straight lines, the contributions to the x integral are either relatively uniform from 0 to R or are peaked near the end points. However, for curved rays the contribution may peak at different places along the ray. To avoid doing a double-integral, double-sum problem, we need approximations.

First, the summation over J is similar to the summation necessary to calculate the vertical phase-structure-function density, and yields

$$\sum_J \frac{1-\left[\frac{k_v k_v'}{q} jJg(x,x')\right]}{J(J^2+j_*^2)} \approx \frac{1}{2}\ln\Phi \left[\frac{k_v k_v'}{q} jg(x,x')\right]^2 \tag{51}$$

Then we may express $P(j,x)$ much more simply in terms of two functions of x, called α and β:

$$P(j,x) = (1-\cos\beta j^2)\exp(-\alpha j^2) \tag{52}$$

and

$$\alpha = M_j \ln\Phi \, k_v^2 \int_O^R dx' <\mu^2> L_P k_v'^2 \left[g(x,x')\right]^2 \tag{53}$$

$$\beta = \frac{k_v^2}{q} g(x,x) \tag{55}$$

The integral expression for α is the same that appears as the inner integral in r_o^2 and ν'^2.[4]

We still have two integrals and a sum. The sum can be approximated by an integral and can be done analytically, if α is small:

$$\{P(j,x)\} \approx \frac{M_j}{2j_*^2}\left[\frac{1}{2}\ln\left(1+\frac{\beta^2}{\alpha^2}\right) + e^{\alpha j_*^2} Ei(-\alpha j_*^2)\right.$$

$$\left. - \mathrm{Re}\left\{e^{(\alpha+i\beta)j_*^2} Ei\left[-(\alpha+i\beta)j_*^2\right]\right\}\right] \tag{55}$$

We have found the following empirical rules for evaluating $\{P(j,x)\}$: First, for very small $\alpha < 10^{-3}$ we may do asymptotic expansions to obtain:

$$\{P(j,x)\} = \frac{1}{4}\frac{\beta^2}{\alpha} \tag{56}$$

Second for $10^{-3} < \alpha < 10^{-1}$ we use (55) with appropriate expansions of the Ei(x) function for small argument. We retain terms to x^6.

Third, for $\alpha > 10^{-1}$ we use the direct sum (50) with a decreasing number of terms as α becomes large. (The cutoffs we have used for 1 through 6 terms are at values of α equal to 5., 0.64, 0.33, 0.2, 0.13, and 0.1.)

Thus we have reduced our problem to a double integral, with the inner integral necessary for the calculations of several interesting quantities: τ_o, the width of the micropath bandwidth function Q; ν', the time scale of intensity decorrelation; and now γ, the micropath focussing parameter that controls the higher moments of intensity.

VIII. SUMMARY AND CONCLUSIONS

The derivation of the intensity coherence function (ICF) of time by the path-integral technique has been given. Allowance for a deterministic sound channel and the presence of reasonable inhomogeneity and anisotropy in the fluctuation field has been included. The ICF has been evaluated for fluctuations dominated by internal waves, which have a vertical structure function that is nearly quadratic. Reasonably simple expressions in terms of environmental measurements for acoustic intensity coherence times and coherent bandwidths have been given.

All of the parameters necessary to approximate the various ICF's can be evaluated on a mini/micro computer in a few minutes for a typical ocean-acoustic experiment.

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Appendix A

In order to estimate the appropriate value of γ to insert in the logarithm in (29), we need to find the typical order of magnitude that γ takes on within the path integral. We argue that this typical value is that which makes the first integral in the exponential of order unity. (Larger γ would make negligible contribution to the integral because of the exponential cutoff; smaller γ has smaller phase space. As long as it is small enough, the second term in the exponential is less important; the crossover time where the second term will dominate occurs at $\Lambda\Phi^2$ times the intensity correlation time.)

We can express the requirement on γ as

$$\frac{1}{4}\, q^2 \int \left[\mathrm{O}\phi\right]^2 dx \approx 1 \tag{A.1}$$

where

$$\mathrm{O} \equiv \mathrm{H}^{-\frac{1}{2}} (\partial_{xx} + \mathrm{U}_o'')\, \mathrm{H}^{-\frac{1}{2}} \tag{A.2}$$

$$\phi = \mathrm{H}^{\frac{1}{2}} \gamma \tag{A.3}$$

Thus O is an operator and we can imagine its eigenvalues and eigenfunctions, with

$$\mathrm{tr}\ \mathrm{O}^{-1} = \int \mathrm{H}\,(x)\, g\,(x,x)\ dx = \sum_{j=o}^{\infty} \frac{1}{\epsilon_j} \approx \frac{1}{\epsilon_o} \tag{A.4}$$

where $g(x,x)$ is the Green's function for the operator $\partial_{xx} + \mathrm{U}_o''$ defined in Flatté,[3] and the lowest eigenvalue is ϵ_o.

The definitions[3] of the diffraction parameter Λ and the strength parameter Φ can be used to estimate ϵ_o knowing H from (28):

$$\epsilon_o^{-1} \approx q\, \Lambda \Phi^2 \ln \Phi \tag{A.5}$$

Let us define γ_o as our estimate of the order of magnitude of γ. We can make such an estimate in many different ways; we choose the following:

$$\gamma_o^2 = \frac{\int H(x)\gamma^2 dx}{\int H(x) dx} \tag{A.6}$$

which is an estimate with weighting along the ray determined by H(x). Noting that

$$O\phi \approx \epsilon_o \phi \tag{A.7}$$

we can express (A.1) as

$$\int H(x)\, \gamma^2 dx \approx \frac{4}{q^2}\, \epsilon_o^{-2} \tag{A.8}$$

and we can also use the definition of the strength parameter Φ to show that

$$\int H(x) dx \approx \frac{\ln\Phi}{L_v^2}\, \Phi^2 \tag{A.9}$$

It follows that

$$\gamma_o \approx 2L_v \Lambda \Phi\, (\ln\Phi)^{1/2} \tag{A.10}$$

which provides the appropriate expression for the definition of β_o in (30).

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