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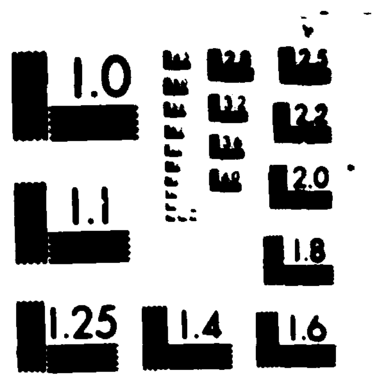

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DECISIONS THAT AFFECT OUTCOMES IN THE DISTANT FUTURE

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DECISION ANALYSIS PROGRAM

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The question that this research addresses is how decisions involving many citizens should be made when those decisions affect outcomes in the distant future. "Distant" means beyond the lifetimes of individuals alive now. The decision maker might be either a private company or a public agency. The contribution of this research is a comprehensive methodology

(continued)

for decision making in this situation, including a mathematical theory and techniques for assessing the required information.

The approach that is taken is a synthesis of concepts from economics with techniques for handling time preferences and uncertain outcomes from the theory of decision analysis. The result is a methodology for deciding whether to accept or reject individual projects with uncertain outcomes on future generations. The fundamental basis for decision making is the amount that current citizens are willing to pay for outcomes accruing to other individuals as well as outcomes affecting their own consumption. One important product of this research is a set of equations for approximating the amounts individuals would pay for a project The expressions include a wide range of realistic cases, such as non-expected-value preferences, uncertainty in individuals' lifetimes, and outcomes accruing to "others" in the same generation as well as others in the future.

It is shown that, under certain circumstances, an individual's willingness to pay is equal to his consumer surplus. Thus, cost-benefit analysis is a special case of the results derived in this research. However, there is an important philosophical difference between our approach and traditional analysis. The methodology proposed in this research assumes that only the preferences of current citizens enter the decision-making process. The future counts only to the extent that current individuals decide to value it.

If we accept this assumption, then the "social discounting" technique used in cost-benefit analysis is not an appropriate way to make decisions affecting outcomes on future generations. This is shown using an example of the government decision to store helium underground. In order for cost-benefit analysis to value projects in a manner consistent with current individuals' preferences, the discount rate would have to vary with the distribution of outcomes among people in each generation, how much current individuals value the future, and what it is that is valued about the future. Although we could force the cost-benefit approach to give a consistent answer by using a complicated discount rate, it is more reasonable to base the decision directly on individuals' preferences and the amounts they are willing to pay.

ABSTRACT

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Chapter 1

INTRODUCTION

1.1 A Statement of the Problem to be Solved

The question that this research addresses is how decisions affecting many citizens should be made when those decisions influence outcomes in the distant future. "Distant" means beyond the lifetimes of individuals alive now. The decision maker might be either a private company or a public agency. The product of this research is a comprehensive methodology for making such decisions, including a theory and a technique for assessing the required information.

The methodology that is developed calculates the net value of individual projects that have effects on both the current and future generations. An example of a project to which the methodology would apply is the underground storage of helium for future use. At the current time, helium is being vented to the atmosphere as a byproduct of natural gas production because the potential supply exceeds the current demand. It is anticipated that, after the year 2000, the demand for helium in advanced electric-power applications will rise dramatically. However, the helium supply at current prices will diminish as natural gas is depleted, necessitating the production of helium from air at much higher prices to meet the increased demand. Helium could be purchased now and stored until the future, thus making it available at lower cost to a future generation. The net value of a helium storage project, including the effect on both the current and future generations, could be calculated using the proposed methodology.

The methodology can also be applied to projects that make future individuals worse off while providing benefits to the current population. Projects that could fall in this category include increased development of mineral resources, extinction of animal species that have an immediate economic benefit, and the development of technologies that produce radioactive waste. In each case, the net value of the project, including the effect on current and future generations, could be calculated using the proposed methodology.

Decisions of this type have often been analyzed using cost-benefit analysis. In this case, the comparison of current and future outcomes is formulated as a question of social discounting. That is, at what rate should society discount future outcomes in order to compare them with current outcomes. The connection between the method proposed in this research and discount rates is explained in Chapter 4 using the decision of helium storage as an example.

1.2 A Summary of the General Approach for Solving the Problem

Some of the assumptions made in this research are similar to those made in cost-benefit analysis. For example, the method provides a way of accepting or rejecting individual projects with future outcomes. It may be used to decide how many projects to accept or how large a project should be, but the emphasis is always on tactical decisions and not on overall planning.

It is also assumed that the effect of the project on individuals is small enough that they are willing to delegate the decision-making function. In this research, the change in an individual's well-being as a result of a project is measured in terms of current dollars that he is willing to pay. Thus, each individual's willingness to pay for a project must be small compared to the present value of his lifetime income. If his willingness to pay is on the order of tens of thousands of dollars, for example, we would not want to apply the proposed procedure without more detailed analysis of the individual's preferences.

One important difference from traditional analysis is the assumption that only the preferences of current citizens enter the decision-making process. This does not mean that the future does not matter, but it counts only to the extent that current individuals decide to value it. In cost-benefit analysis, it is customary to think of society as trading between the preferences of current citizens and future citizens at the social rate of discount. However, since it is current citizens who must choose the discount rate, it is almost axiomatic that only current preferences enter the decision [22]. In this research, we explicitly recognize this fact and include only the preferences of current citizens.

This research also differs from traditional analysis in the assumption that all of the costs and benefits of a project are privately borne. That is, all outcomes are disaggregated and assigned to the individuals who ultimately are affected by them. For example, the project costs that are paid out of taxes are ultimately paid by taxpayers and thus are included as outcomes to individuals. This means that the government does not exist as an ultimate recipient of costs or benefits. Also, since outcomes are uncertain, this means that the risks of a project are ultimately borne by individuals. (Of course, the government may redistribute the risk through insurance, for example.) This assumption differs from authors such as Arrow and Lind [3], who analyze public decision making using the assumption that some or all of the outcomes of public investment are borne by the government.

Obviously, this research approaches the question of decision making when there are future outcomes from the point-of-view of the individual citizens who are affected by the decision. The important question becomes how much each citizen would be willing to pay for the outcomes of the decision. To each individual, the most important outcome of the project is probably the change in his own consumption. He would also be willing to pay something for the outcomes that accrue to others, both his neighbors in his own generation and individuals living in the future. Thus, the outcomes of a project for each individual are (1) changes in his own consumption during his lifetime and (2) changes in a vector of attributes describing outcomes to other individuals (such as the standard of living) in both his own and future generations. Of course, all outcomes are uncertain.

In this research, information on individuals' preferences and the uncertain state variables is combined to calculate the current dollars each individual would be willing to pay for the changes that result from the project (Fig. 1.1). It is shown that everyone can be made better off by adopting the project if and only if the total willingness to pay, summed over individuals, is positive. Aggregate willingness to pay, or the profit that could be made by undertaking the project, is developed as the criterion for decision making. If aggregate willingness to pay is positive, then all individuals could be induced to accept the project and it should be undertaken.

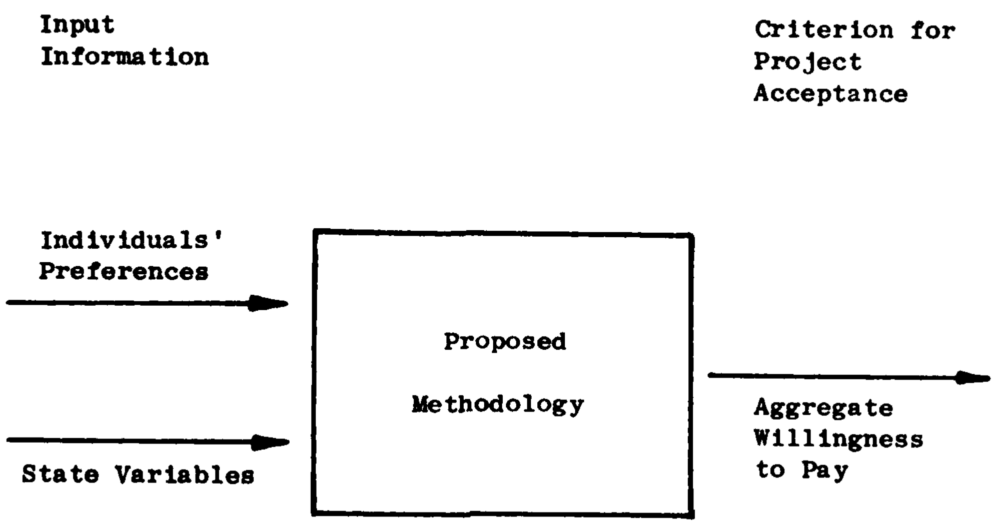

Fig. 1.1. AN OVERVIEW OF THE METHODOLOGY PROPOSED IN THIS RESEARCH.

1.3 Conclusions and Contributions

This research provides a comprehensive framework and methodology for making decisions when those decisions affect uncertain outcomes on future generations. It is a synthesis of some concepts of cost-benefit analysis with techniques for handling time preferences and uncertainty from the theory of decision analysis. This synthesis provides a practical solution to a problem that has been debated for many years.

The decision criterion proposed in this research depends on the amounts that individuals are willing to pay for the outcomes of a project. In Chapter 2, approximate expressions for individual willingness to pay are derived from a simple example involving an expected-value decision maker. The amount that an individual is willing to pay depends on the tradeoffs he would make between his own consumption at different times in his life and tradeoffs between his own consumption and that of others. In Chapter 2, a new numeraire is defined that describes how an individual feels about the consumption he receives during his own lifetime. This numeraire is a function of the prices and income an individual faces, allowing the corresponding terms of the expression for willingness to pay to be calculated from price changes in the market.

Then, additional interpretations of willingness to pay are made under special circumstances. For example, it is shown that, when all of the outcomes that an individual receives from a project consist of price changes within his lifetime, and all outcomes are certain, an individual's willingness to pay is equal to his consumer surplus. Thus, willingness to pay is the extension of the cost-benefit concept of surplus to include uncertainty and outcomes on future generations. Alternatively, cost-benefit analysis is a special case of the results derived in this research.

In Chapter 3, the assumptions underlying willingness to pay are extended to include a wider range of realistic cases. For example, the individual's lifetime is assumed to consist of more than one period and multiple attributes are considered describing outcomes to "others," both other individuals in the same generation and other individuals in the future. Expressions are given for willingness to pay that incorporate these extensions for both expected-value and non-expected-value individuals. Then, additional conclusions are drawn about willingness to pay. For example, it is shown that, for a particular representation of the individual's ordinal value function, his willingness to pay for a project is the change in his certain equivalent as a result of the project. In the final section of Chapter 3, the computation of willingness to pay when the individual's lifetime is uncertain is shown to depend on the probabilities of the individual being alive in each future year.

In Chapter 4, the decision about storing helium underground is used as an example to illustrate the connection between the methodology proposed in this research and "social discounting." First, a social discount rate is defined. Then, the rate that would be implied if the proposed method were used to make a decision about storing helium is calculated. It is shown that the social discount rate depends on not only individuals' preferences, but also time, and the units in which outcomes to others are measured. Thus, if one accepts the assumptions made in this research, discounting is not an acceptable way to make a decision.

In Chapter 5, techniques are presented for assessing the information describing state variables and individuals' preferences that is required to apply this research to actual decisions. Consistency checks are listed for insuring that the individual's answers are consistent with each other and a flow chart is given for the overall assessment procedure. In the final section of Chapter 5, results of a small experimental program to test the assessment procedure are discussed. Suggestions are made for overcoming the biases that interviewees exhibit during assessment.

In Chapter 6, suggestions are made for further research.

1.4 Related Research

The notion that future outcomes should be "discounted" because future goods are less desirable than present goods is part of classical economics. Irving Fisher developed the first comprehensive theory of interest or discount rates and their relation to individual preferences and investment opportunities [9]. His research showed how individuals make the optimal decisions by equating their rate of time preference to the rate of return on investment.

With the development of welfare economics, the question arose of how decisions with future outcomes should be made for a group of citizens. One answer came from the area of welfare economics called social cost-benefit analysis. There, the problem of outcomes at different times is formulated in terms of social discount rates. At what rate should society discount future outcomes in order to compare them with current outcomes?

A simple model can be used to explain most of the results in the social-discounting literature. Let time be divided into two periods and let there be a single good delivered in each period, defined simply as present and future consumption. At the end of the present period, the current generation dies and the future generation is born. Assume there exists a social welfare function depending on consumption in the two periods. Then, Pareto optimality results from maximizing social welfare subject to a constraint on the transformation between present and future consumption. At an equilibrium, the marginal rate of substitution for the social welfare function is equal to the marginal rate of transformation for the constraint, and they imply a discount rate r:

$$MRS = MRT = 1/(1 + r) \tag{1.1}$$

Most of the results in the social-discounting literature are expressions of the social discount rate in terms of either preferences (MRS) or transformation possibilities (MRT) for consumption in the two periods. Equation (1.1) is expressed and rearranged in various ways, but the underlying model of maximizing social welfare subject to an intertemporal constraint is the same.

For example, the "opportunity cost" approach concentrates on the intertemporal transformation and considers what else could be done with the money [7,8,20,21]. Relationships are derived between the social discount rate and interest rates for other investment opportunities such as long-term government bonds. The difficulty with this approach is that there are no markets for very distant outcomes from which to estimate prices, or equivalently interest rates, for the transformation frontier.

Even if there were markets, individuals would not need to have marginal rates of substitution between their own current dollars and dollars ten years from now equal to the corresponding interest rates. This result occurs because the future lotteries are not resolved and their outcomes known until the future. As Pollard pointed out, this means there is no fixed relationship between the marginal rate of substitution and the market interest rate [28]. The marginal rate of substitution that results for an individual depends on the outcomes that actually occur in the future and the form of his preferences.

A second approach in the social-discounting literature is to calculate discount rates from "social" time preferences [5,22,23,30,31]. This method models the tradeoffs that society is willing to make between aggregate consumption at various times, as expressed in the social welfare function. However, here, we end up with the classic welfare economics problem of finding a social welfare function. Sen has even expressed Arrow's impossibility theorem in terms of discount rates, showing that it is impossible to find a social discount rate that is consistent in a particular sense with individual time preferences [29].

Besides the vast literature on social cost-benefit analysis, there are at least two other solutions to the problem which this research addresses. One solution is to view social investment as a problem in determining the optimal growth path of the economy [2,19]. Rather than analyzing individual projects, as we are proposing, this approach models the relation between the total capital or resource stock, total investment, and total consumption for the economy over time. Then, an optimal growth path and the level of investment required to achieve it are determined by maximizing a social welfare function. Again, the problem is to determine the social welfare function.

A final solution is to treat decisions with future outcomes as questions of intergenerational equity [26]. This type of analysis gives votes to future citizens in the sense of asking "What distribution of resources between generations would seem fair if you did not know in what generation you would be?". This analysis has the problem that we, the current citizens, must decide to use the approach and how to apply it. Since, in the end, we must decide what is "fair" with regard to the future, why not base the decisions directly on our preferences for future outcomes?

The approach we are suggesting in this research synthesizes ideas from social cost-benefit analysis with techniques for handling individuals' preferences and uncertainty from decision analysis. The handling of individuals' preferences for consumption during their lifetime extends the multiperiod consumption model discussed first by Pollard [28] and then by Barrager [4]. The new numeraire for an individual's lifetime consumption, expressed in terms of prices, is a variation of the income compensation function defined by Hurwicz and Uzawa [15]. The relationship between the definition of willingness to pay in this research and consumer surplus is an extension of ideas found in Willig [34].

In Chapter 5, techniques are given for assessing the information about state variables and individuals' preferences that is necessary to apply this research to actual decisions. The method of assessing covariances uses an approximation given by Owen [27], and the method for assessing the derivatives of individuals' value functions is an extension of ideas from Keelin [18].

1.5 The Welfare Implications of this Research

It is clear that any methodology for making decisions with outcomes on future generations implies a social welfare function. Thus, we could ask what assumptions about social welfare are made in the approach that we are suggesting. The use of aggregate willingness to pay as a criterion for decision making in this research is based on the same assumption that underlies cost-benefit analysis, that is, the Kaldor compensation principle.

The Kaldor principle states that, in order to recommend a project, "it is quite sufficient...to show that even _if_ all those who suffer as a result are fully compensated for their loss, the rest of the community will still be better off than before. Whether [those who lose]...should in fact be given compensation or not, is a political question on which the economist, _qua_ economist, could hardly pronounce an opinion" [16]. The shortcomings of this criterion when compensation is not actually made are well documented [25].

If the methodology in this research is applied by a government that has the coercive power to collect tax and use it to undertake projects without the unanimous agreement of the citizens, then the welfare basis of the approach is only as strong as the Kaldor criterion. It is our opinion, however, that compensation should be accomplished. Otherwise, one cannot guarantee that everyone will be made better off by the project.

The social brokerage firm discussed in Chapter 2 is a private company that insures that social decisions are made by unanimity among current citizens. Compensation for the projects that the firm undertakes is accomplished because individuals are guaranteed to be made better off if they contract with the company. The social brokerage firm is both a construct for understanding decision making with future outcomes and a suggestion for actually implementing the approach in this research.

Chapter 2

A SIMPLE APPROXIMATION TO WILLINGNESS TO PAY FOR RISK-NEUTRAL INDIVIDUALS

2.1 Introduction and Summary of the Chapter

Assume that the current generation consists of one individual, who we will call CG. In this example, CG lives for one fixed period and, when he dies, one future individual is born. The decision that CG faces is whether to pay for a project that will benefit the future individual. In this chapter, we will develop a criterion which CG can use to make this decision.

The most important outcomes of the project from CG's point of view are the changes in his own consumption. The goods he purchases will change if his income decreases or the prices he pays increase in order to cover the cost of the project. He also cares about the standard of living of the future individual, and this will be measured by an additional attribute entering CG's preferences. The net benefit of the project to him is the current dollars he would sacrifice in addition to the change in his own consumption and the change in the well-being of the future individual that result from the project, in order to have it undertaken. For example, let x be the vector of CG's own consumption goods and z the income of the future individual. Also, denote outcomes with the project by the superscript $'$ and outcomes without the project by ^{o}. Then, the net benefit to CG, or what we will call "willingness to pay," can be represented diagramatically by the following:

$$(x^o, z^o) \xrightarrow[\text{What would CG pay?}]{} (x', z')$$

Of course, x and z are random variables both with and without the project.

The first step in calculating CG's willingness to pay is to develop a numeraire for his own consumption vector. Since it is often easier to measure prices in the market than changes in consumption, the numeraire will be expressed in terms of the prices and income he faces when he purchases goods. Then, this numeraire is combined with the attribute measuring the future individual's income to form an overall value function for CG. It is assumed that all outcomes are uncertain, but his overall value function is deterministic. Since CG has expected-value preferences, lotteries are ranked by their expected value.

The definition of willingness to pay in terms of expected values is expanded in a Taylor's series. The result is the approximate amount CG would pay to move from an initial lottery on his own consumption and the future individual's income to a new lottery when the project is undertaken. This quantity measures the net benefit to CG as a result of the project and is the appropriate criterion for his decision.

In the final section of this chapter, we discuss decision making for a group of current citizens who are in a situation similar to CG's. That is, they each live for the same fixed period and care only about their own consumption and the well-being of the future generation. The concept of a "social brokerage firm" is discussed as a framework for decision making in this setting. The Kaldor condition [16] is used to derive aggregate willingness to pay, summed over individuals, as the criterion for decision making.

2.2 The Numeraire for CG's Own Consumption

The assumption is made that the vector of goods that CG consumes is preferentially independent of the future individual's income, which CG also values. This requires that the tradeoffs that he would make between goods such as his own housing and clothing do not vary with the level of the future individual's income. As Keelin showed [18], this condition implies that a numeraire exists for CG's consumption and that his ordinal utility function, or value function, depends on his consumption only through this numeraire. Thus, his value function is of the following form:

$$V(h,z)$$

where

V = CG's overall value function

h = numeraire for CG's consumption

z = future individual's income

We want to express the value of CG's consumption, the numeraire h, in terms of the prices and income that he faces. To do this, we start with his fundamental or direct value function on consumption:

$\underline{v}(x_1, \ldots, x_n)$ is the direct value function defined on the vector of consumptions x_i of good i

In this research, we assume that the direct value function satisfies the set of assumptions made by Barrager [4]. This means that $\underline{v}$ is a sum of terms which are all either exponential, posynomial, or hyperbolic in the x_i.

Since $\underline{v}$ is expressed in terms of consumption, we substitute the ordinary Marshallian demand functions $d_i(p,m)$ into the direct value function to produce an "indirect" value function in terms of the vector of prices, p, and the income, m:

$$v(p,m) = \underline{v}\left[d_1(p,m),\ \ldots,\ d_n(p,m)\right]$$

where

$v(p,m)$ = indirect value CG receives at prices p and income m when he maximizes $\underline{v}$ subject to $\sum_i p_i d_i = m$

$d_i(p,m)$ = amount of good i that CG demands at prices p and income m

The same preference ordering can be represented by any monotonic increasing transformation of v. We will choose as a numeraire a particular transformation denoted by $h(p,m|p^b)$. It was called the "income compensation function" by Hurwicz and Uzawa [15], and they proved its existence under general conditions on the direct value function $\underline{v}$.

Put most simply, $h(p,m|p^b)$ is the income CG would need at fixed base prices p^b to be indifferent to prices p and income m. Each indifference curve has a constant value v so, mathematically, h is the solution to the equality:

$$v\left[p^b, h(p,m|p^b)\right] = v(p,m) \tag{2.1}$$

A graphical representation for the case of two goods is given in Fig. 2.1. If x_1 has a price of one, then the income corresponding to any budget constraint is just the horizontal intercept of the constraint. In this case, if CG had income h and price p^b for x_2, he could achieve the same indifference curve as he would have at price p and income m. Thus, h is the numeraire or value of price p and income m.

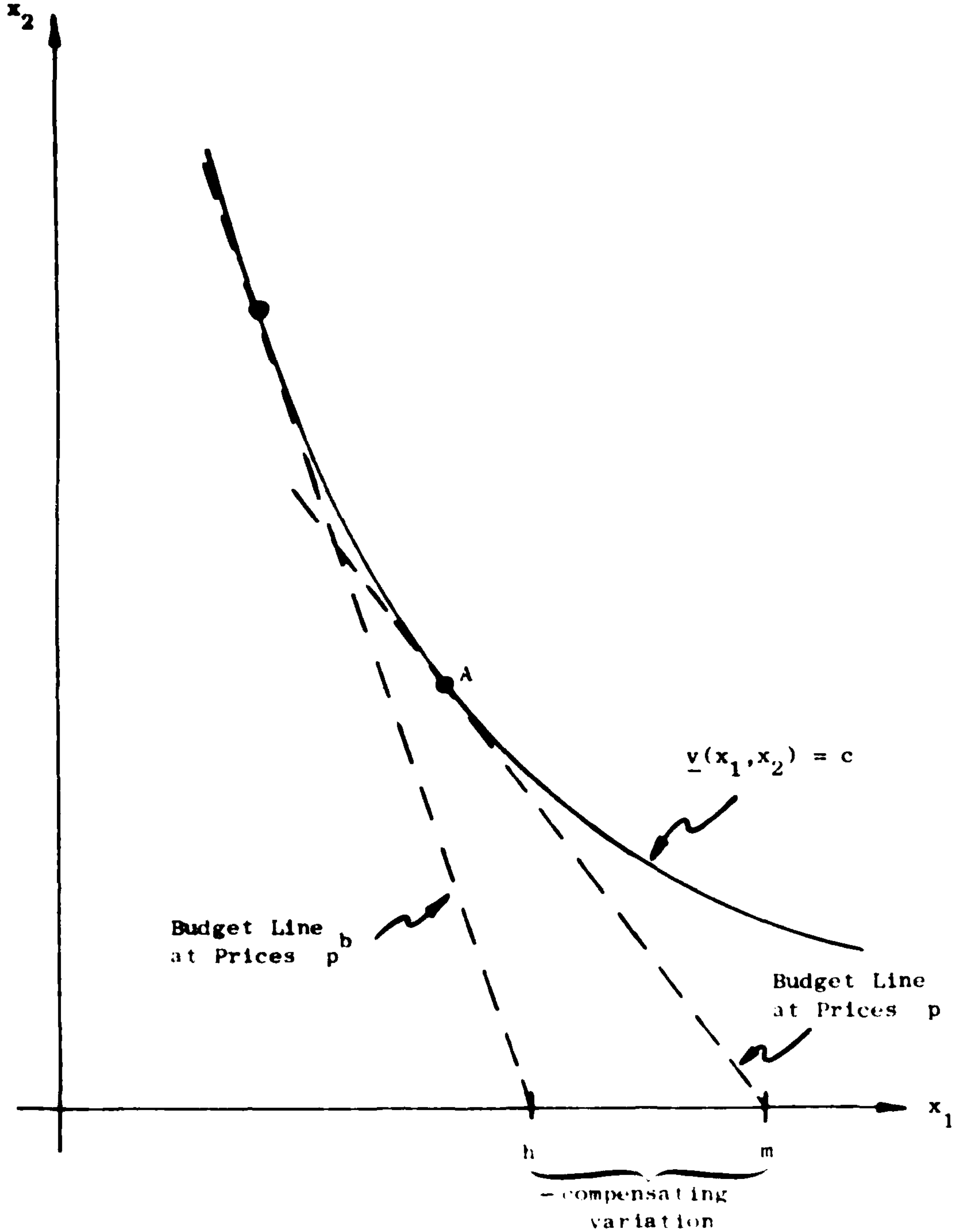

Fig. 2.1. INTERPRETATION OF THE NUMERAIRE h IN A TWO GOODS PROBLEM.

2.3 The Properties of the Numeraire

The numeraire h provides an ordering of price and income combinations which an individual faces in choosing his own consumption. The ordering is independent of the base price vector p^b and is consistent with the fundamental preferences on consumption. One of the reasons that we chose h as the numeraire is the simplicity of its derivatives when they are evaluated at the base prices. For example, the first derivative of h with respect to p_i is the i^{th} demand function, d_i. The derivatives through the second order are derived in Appendix B.

Another reason for choosing h is its interpretation in terms of compensating variations that are used in cost-benefit analysis. According to a theorem by Willig [34], h is the sum of m and the compensating variation:

$$h(p,m|p^b) = m + CV_{p \rightarrow p^b}|m$$

The quantity $CV_{p \rightarrow p^b}|m$ is the amount an individual would have to be compensated to change prices from p to base values p^b if he had income m. We can see this by looking back at Fig. 2.1. If CG had an income m and prices p, he would be indifferent if his income were reduced to h and prices were changed to p^b. The income change $h-m$ is the compensating variation.

According to Willig, the compensating variation is the appropriate measure of what economists call consumer surplus. Thus, the numeraire we are using for an individual's own consumption is his income plus his consumer surplus to change prices to base values.

The general shape of h is shown in Figs. 2.2, 2.3, and 2.4 which correspond to the case of two goods and hyperbolic consumption

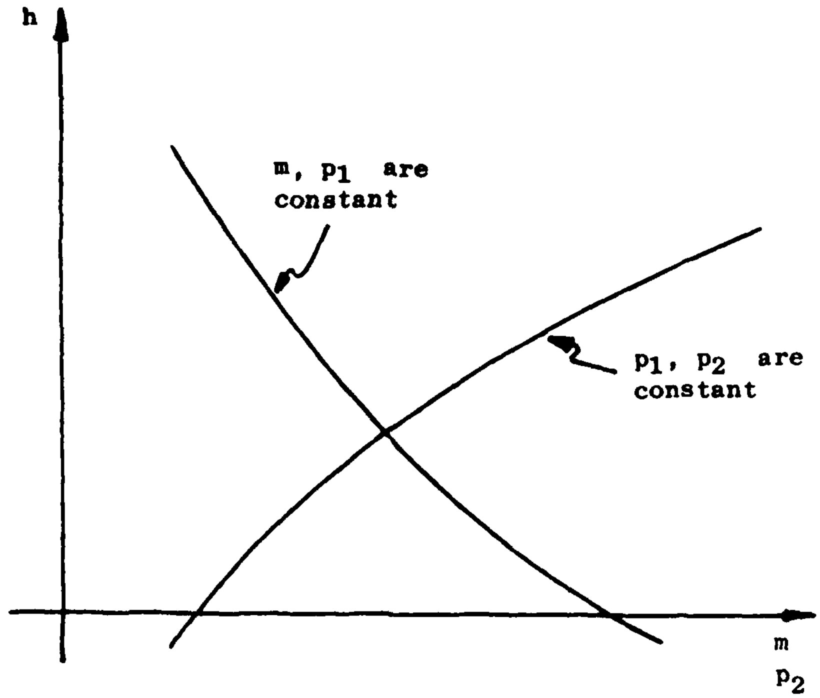

Fig. 2.2. THE NUMERAIRE h AS A FUNCTION OF PRICE AND INCOME.

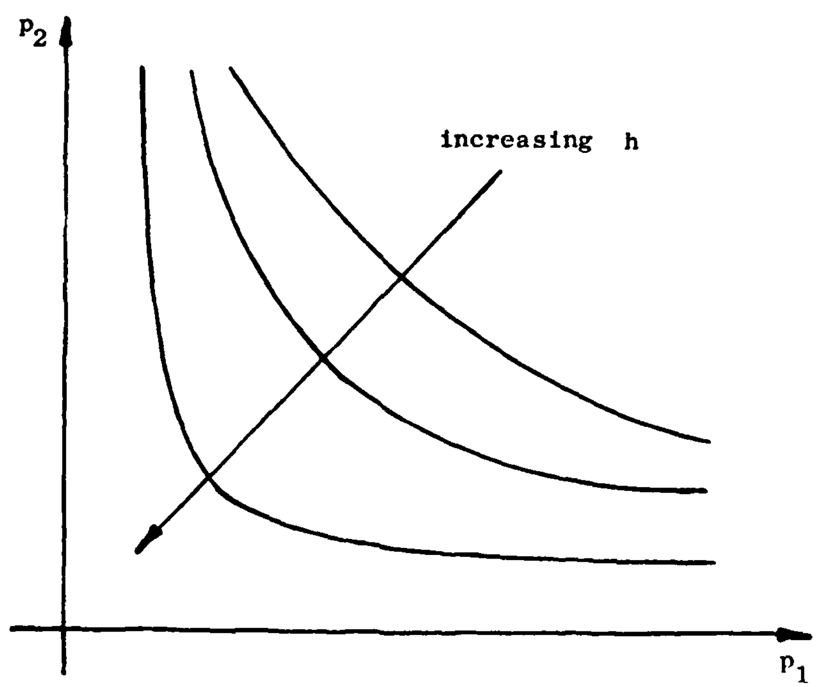

Fig. 2.3. INDIFFERENCE CURVES BETWEEN PRICES FOR THE NUMERAIRE h.

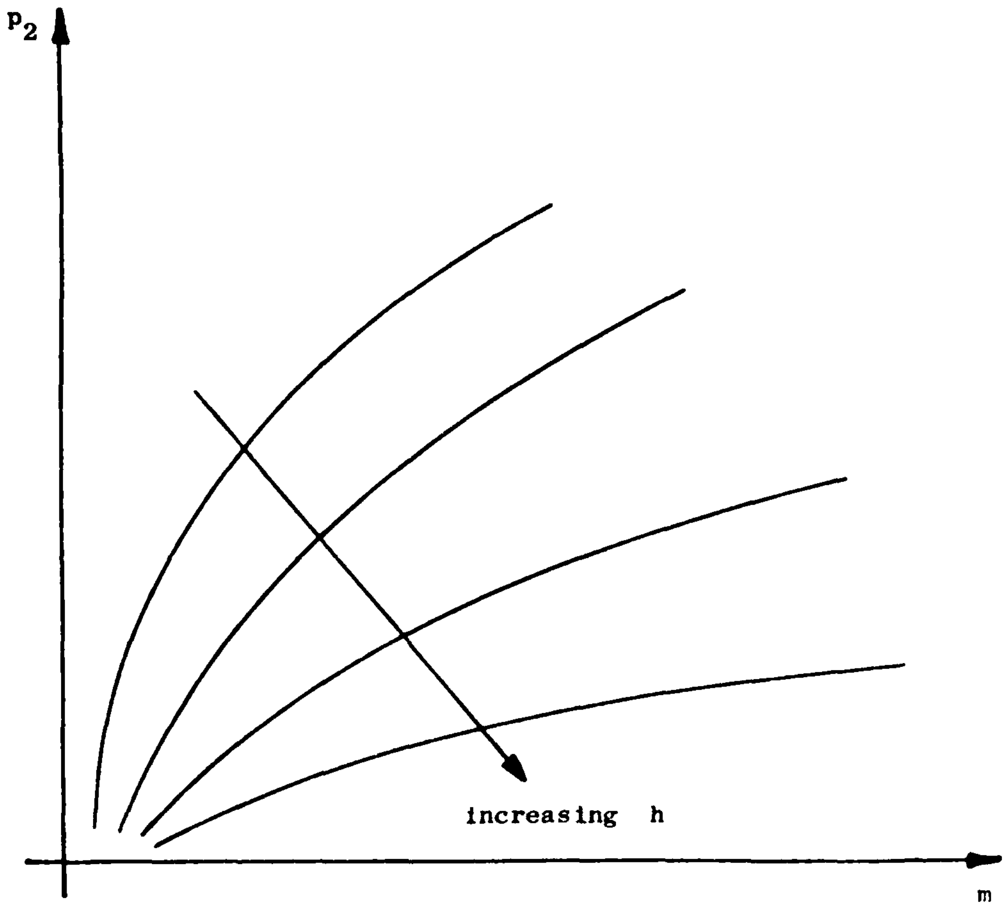

Fig. 2.4. INDIFFERENCE CURVES BETWEEN PRICES AND INCOME FOR THE NUMERAIRE h.

indifference curves of the form $\underline{v}(x_1,x_2) = x_1x_2$. For these preferences, h is given by the following,

$$h(p,m|p^b) = m\sqrt{\frac{p_2^b p_1^b}{p_1 p_2}}$$

where p_i is the component of p corresponding to the i^{th} good. The function h increases and the individual is better off with increasing income, and he is worse off when any price increases. The indifference curves between two prices are convex, that is, as one price increases, it takes less of a decrease in any other price to compensate him. The indifference curves between prices and income are concave. Expressions for h corresponding to the general hyperbolic, exponential, and posynomial preferences are given in Appendix A.

2.4 Willingness to Pay when CG is Risk Neutral

The numeraire h is combined with the future individual's income in CG's overall value function V(h,z). This value function is used to calculate his willingness to pay for changes in the compound lottery on h and z. Changes in his own consumption result from changes in the prices and income that are arguments of h. Thus, we ask what he would be willing to pay to change from the set of prices, his own income, and the future individual's income without the project to the values with the project:

$$(p^o, m^o, z^o) \xrightarrow{\hspace{3cm}} (p', m', z')$$

What would CG pay?

The variables p, m, and z are random variables both with and without the project.

The numeraire h is measured in dollars. Thus, if CG is risk neutral, willingness to pay in dollars is the amount that can be subtracted from the value of h with the project and still leave him with the same expected value as he would have without the project. Denoting willingness to pay by w, then w satisfies the following equation:

$$\left\langle V\left[h(p^o,m^o|p^b),z^o\right]\right\rangle = \left\langle V\left[h(p',m'|p^b) - w,z'\right]\right\rangle \tag{2.2}$$

This relation implicitly defines willingness to pay as a function of the uncertain state variables with and without the project:

$$w = w(p^o,p',m^o,m',z^o,z')$$

To calculate w exactly requires knowledge of CG's preferences V(h,z) and the joint probability distribution on prices, his own income, and the future individual's income, with and without the project. Instead, we will approximate w by expanding the value function in a Taylor's series about the vector of mean values without the project $(\overline{p^o},\overline{m^o},\overline{z^o})$. Keeping only first-order terms, we obtain:

$$\left\langle V\left[h(p,m|p^b),z\right]\right\rangle \doteq V\left[h\left(\overline{p^o},\overline{m^o}|p^b\right),\overline{z^o}\right] + \partial V/\partial h \sum_i \partial h/\partial p_i\left(\bar{p}_i - \overline{p_i^o}\right)$$

$$+ (\partial V/\partial h)\ \partial h/\partial m\left(\bar{m} - \overline{m^o}\right) + \partial V/\partial z\left(\bar{z} - \overline{z^o}\right) \tag{2.3}$$

where the horizontal bar over a variable denotes expectation. Using this expansion, the left-hand side of Eq. (2.2) is simply $V[h(\overline{p^o},\overline{m^o}|p^b),\overline{z^o}]$ and the right-hand side is equal to the following:

$$\left\langle V\left[h(p',m'|p^b) - w, z'\right]\right\rangle \doteq V\left[h\left(\overline{p^o},\overline{m^o}|p^b\right) - w, \overline{z^o}\right] + \partial V/\partial h \sum_i \partial h/\partial p_i\left(\overline{p_i'} - \overline{p_i^o}\right)$$

$$+ (\partial V/\partial h)\ \partial h/\partial m\left(\overline{m'} - \overline{m^o}\right) + \partial V/\partial z\left(\overline{z'} - \overline{z^o}\right)$$

Taking $\overline{p^o}$ as the base price vector in the definition of h, we can use the derivatives of h to simplify this expression. We also note that

$$V\left[h\left(\overline{p^o},\overline{m^o}|p^b\right) - w,\overline{z^o}\right] \doteq V\left[h\left(\overline{p^o},\overline{m^o}|p^b\right),\overline{z^o}\right] + (\partial V/\partial h)(-w)$$

Substituting all of these results into Eq. (2.2) produces the following first-order approximation for CG's willingness to pay:

$$w \doteq \left(\overline{m'} - \overline{m^o}\right) + \left[\sum_i d_i\Big|_{\overline{p^o},\overline{m^o},\overline{z^o}}\left(\overline{p_i^o} - \overline{p_i'}\right)\right] + \frac{\partial V/\partial z}{\partial V/\partial h}\Big|_{\overline{p^o},\overline{m^o},\overline{z^o}}\left(\overline{z'} - \overline{z^o}\right) \tag{2.4}$$

The quantity $\bar{m}$ is the expected value of CG's income. The quantity d_i is his demand for the i^{th} consumption good, so $d_i\bar{p}_i$ is the expected cost of the good i. The term $(\partial V/\partial z)/(\partial V/\partial h)$ is the marginal rate at which CG would trade his own income for the income of the future individual. Thus, the amount CG would be willing to pay for the project is approximately equal to the sum of

- the change in the expected value of his income
- the change in the expected cost of the goods he consumed before the project
- the change in the expected income of the future individual, evaluated in current dollars

Note that, in evaluating Eq. (2.4) for willingness to pay, it may not be necessary to know the levels (that is, the means) of any of the outcomes but only the change in the means due to the project. This may be an advantage in assessment.

Willingness to pay is illustrated in Fig. 2.5 for the case when the prices that CG faces in choosing his consumption are fixed and only his income varies in order to pay for the project. Before the project, CG is at point A in Fig. 2.5, with expected income $\overline{m^o}$ and expected income for the future individual at the level $\overline{z^o}$. After the project is adopted, he will be at point C. However, if starting from C he were paid the amount d in current dollars, he would end up at B. For linear indifference curves, B is indifferent to A. Thus, his willingness to pay is approximately $(-d)$.

The assumption made above, that the prices that current individuals face are fixed and only income varies in order to pay for the project, may be reasonable in some actual applications. This might be approximately the case in the helium storage example discussed earlier. Assume that current tax dollars are spent to store helium and the benefit in terms of lower helium prices occurs after all current individuals die. Then, the outcomes could be modeled simply as a change in each current individual's income and a change in an attribute measuring the benefit to future consumers.

2.5 Additional Interpretations of Willingness to Pay

In order to gain a more intuitive understanding of willingness to pay, we will look at two interpretations that arise under special

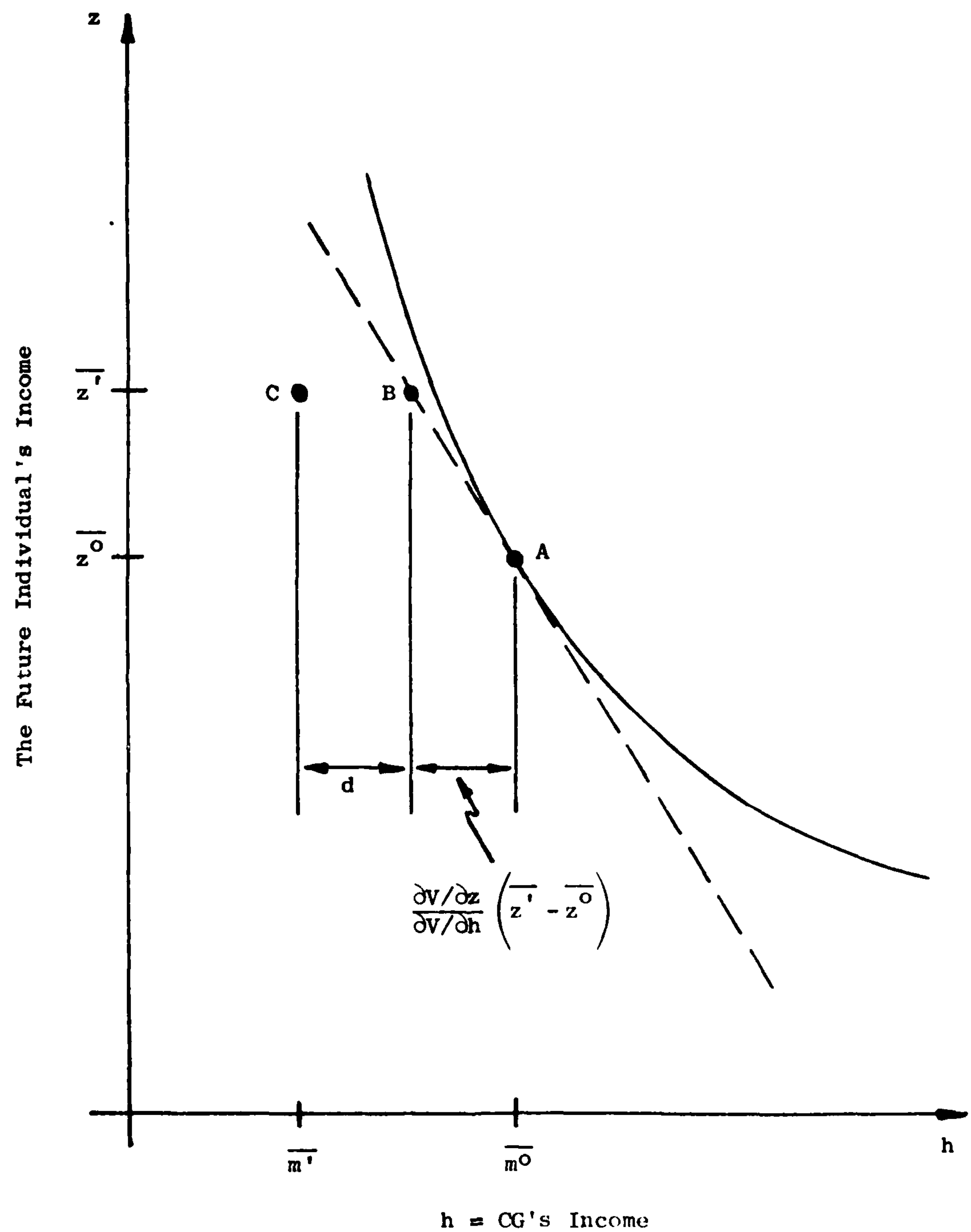

Fig. 2.5. THE FIRST-ORDER EXPRESSION FOR CG's WILLINGNESS TO PAY.

circumstances. The first of these is a set of assumptions such that willingness to pay is exactly equal to the quantity that economists call consumer surplus. Consider a case when all outcomes are deterministic and consist only of changes in the prices that CG pays for his own consumption. That is, there are no future outcomes beyond his lifetime. Then, Eq. (2.2) defining willingness to pay becomes:

$$h(p^o, m \mid p^b) = h(p', m \mid p^b) - w \tag{2.5}$$

The value function V is unnecessary because there are no tradeoffs with the future and we do not have to consider expectations because there is no uncertainty. Also, his income is fixed at m.

We saw earlier that the numeraire h is related to a compensating variation:

$$h(p, m \mid p^b) = m + CV_{p \rightarrow p^b} \mid_m$$

If we substitute this expression into both sides of Eq. (2.5) and choose as base prices the vector of prices with the project, p', then willingness to pay is equal to the following:

$$w = -CV_{p^o \rightarrow p'} \mid_m$$

The amount that CG would willingly pay for the project in addition to its outcomes is exactly what economists call consumer surplus. The quantity $-CV_{p^o \rightarrow p'} \mid_m$ is the amount he would pay to effect a change in prices from p^o to p', starting from an income m. Of course, this result was derived assuming no uncertainty and no outcomes beyond his lifetime. Thus, the definition of willingness to pay in this research is the extension of consumer surplus to include uncertainty and future outcomes.

A second interpretation of willingness to pay can be made in terms of expected values if we choose a particular representation of CG's value function as shown in Fig. 2.6. Consider the value V of certain outcomes $\mathcal{H}$ and $\mathfrak{Z}$ of the numeraire h and future income z. Let $V(\mathcal{H},\mathfrak{Z})$ be the number h^* that corresponds to $\overline{z^o}$ on the same indifference curve as the point $(\mathcal{H},\mathfrak{Z})$. That is, h^* satisfies the equation:

$$V\left(h^*,\overline{z^o}\right) = V(\mathcal{H},\mathfrak{Z})$$

Without the project, CG faces a lottery (p^o,m^o,z^o) which induces a lottery on value V. Using the value function which was defined above, his expected value is the expectation of the quantity h^* satisfying:

$$V\left(h^*,\overline{z^o}\right) = V\left(h(p^o,m^o\,|p^b),z^o\right) \tag{2.6}$$

We expand both sides of Eq. (2.6) in a Taylor's series about $(\overline{p^o},\overline{m^o},\overline{z^o})$:

$$V\left(h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right) + \frac{\partial V}{\partial h}\left[h^* - h\left(\overline{p^o},\overline{m^o}\,|p^b\right)\right] \doteq V\left(h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right)$$

$$+ \frac{\partial V}{\partial h}\frac{\partial h}{\partial m}\left(m^o - \overline{m^o}\right) + \sum_i \frac{\partial V}{\partial h}\frac{\partial h}{\partial p_i}\left(p_i^o - \overline{p_i^o}\right) + \frac{\partial V}{\partial z}\left(z^o - \overline{z^o}\right)$$

Solving for h^* and taking expectations, we obtain CG's expected value without the project:

$$\overline{h^*}\Big|_{p^o,m^o,z^o} \doteq h\left(\overline{p^o},\overline{m^o}\,|p^b\right)$$

With the project, CG faces a lottery (p',m',z') and his expected value is the expectation of the number h^* that satisfies:

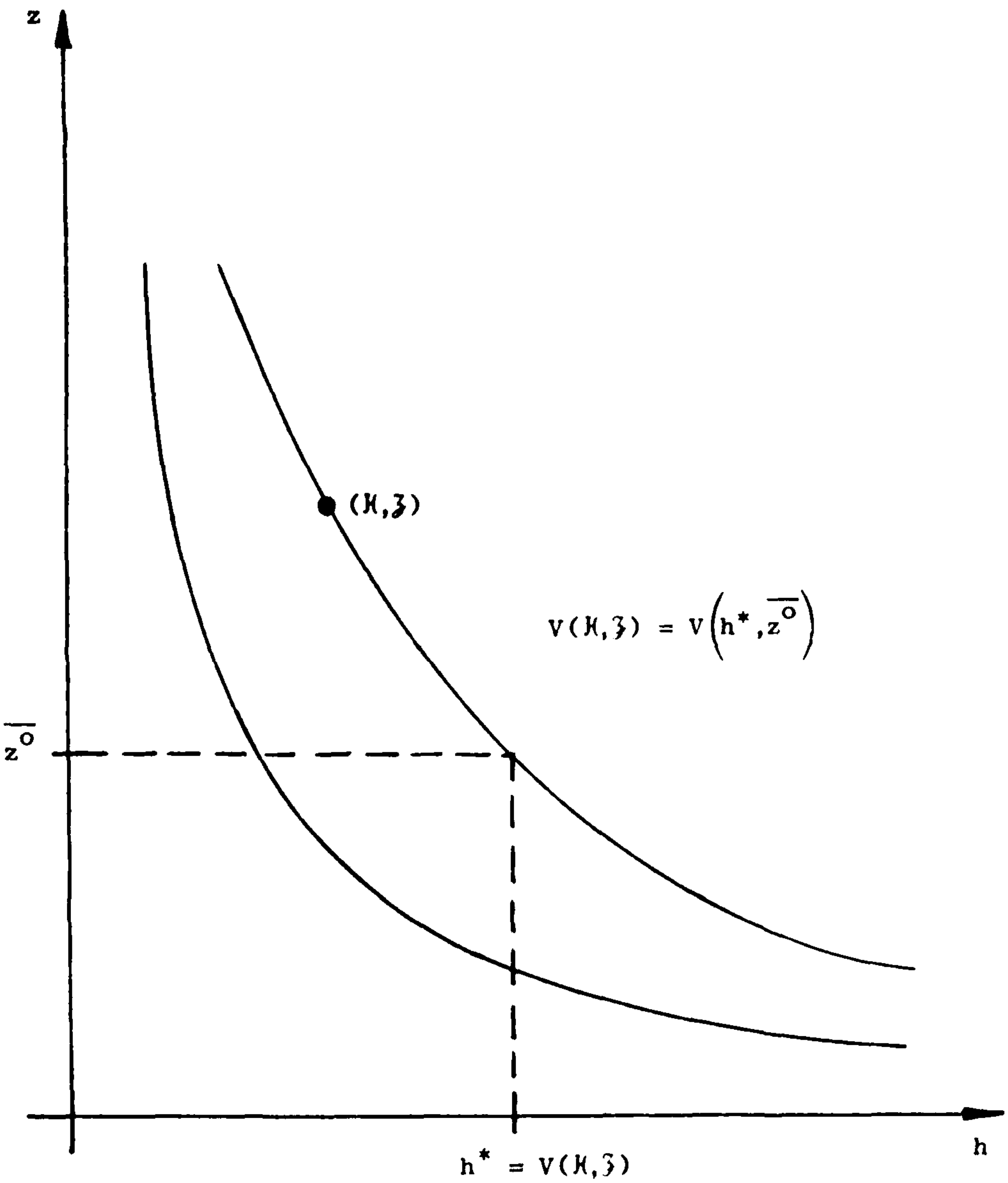

Fig. 2.6. DEFINITION OF THE VALUE FUNCTION V AS THE NUMERAIRE h^* CORRESPONDING TO $\overline{z^o}$.

$$V\left(h^*,\overline{z^o}\right) = V\left(h(p',m'|p^b),z'\right)$$

Following the same procedure as before, we find his expected value with the project:

$$\overline{h^*}\Big|_{p',m',z'} \doteq h\left(\overline{p^o},\overline{m^o}|p^b\right) + \left(\overline{m'} - \overline{m^o}\right) + \sum_i d_i\left(\overline{p_i^o} - \overline{p_i'}\right) + \frac{\partial V/\partial z}{\partial V/\partial h}\left(\overline{z'} - \overline{z^o}\right)$$

Thus, to a second-order approximation, the change in CG's expected value as a result of the project is equal to his willingness to pay for the project:

$$\overline{h^*}\Big|_{p',m',z'} - \overline{h^*}\Big|_{p^o,m^o,z^o} \doteq \left(\overline{m'} - \overline{m^o}\right) + \sum_i d_i\left(\overline{p_i^o} - \overline{p_i'}\right) + \frac{\partial V/\partial z}{\partial V/\partial h}\left(\overline{z'} - \overline{z^o}\right) \doteq w$$

When the value function is defined as above, willingness to pay for an expected-value decision maker can be interpreted as the change in his expected value.

2.6 The Aggregation of Preferences: The Social Brokerage Firm

Suppose there is a group of current citizens in a situation similar to CG's. That is, they all live for one fixed period, they are risk neutral, and they value their own consumption and the well-being of the future generation. How should this group of citizens decide whether to undertake jointly a project with future outcomes?

The aggregation of preferences to make a group decision will be addressed in the context of a "social brokerage firm." This concept is both a framework for thinking about aggregation and also a suggestion for actually implementing the methodology proposed in this research. The social brokerage firm is a private company operating in the following manner. It guarantees to make current citizens better off if they contract with the company. It undertakes projects with future outcomes when it can make a profit by doing so. The company charges or rebates to each current customer an amount such that he is better off, given both his compensation and the outcomes of the project. Thus, the company acts as a broker between current individuals by undertaking projects with future outcomes and redistributing expectations about the outcomes by charging or rebating to each customer.

The concept of a social brokerage firm is similar to that of a charity or benevolent association. For example, nature conservancies use voluntary contributions to purchase land that has special ecological value. Donors benefit directly from their own use of the land and indirectly from the use of the land by future generations. Landowners who sell their land to the conservancy benefit directly from the sale and indirectly from their contribution to others, including those in the future. Since participation is voluntary, both those who donate and those who sell their land are made better off.

It is important to note that the social brokerage firm is not selling claims to future outcomes. Each current individual can enjoy the fact that future individuals are being made better off, but he is not entitled to any specific share of the future outcomes, either directly or indirectly through his heirs. He is simply paying or being paid to change a lottery on his own current income and the well-being of other individuals. Thus, the social brokerage firm is not trading on what is commonly called the "futures" market.

The company can ask customers questions in order to determine their preferences and can ask experts questions about the uncertain state variables (assessment will be discussed in Chapter 5). Given this information, how should it decide whether to pursue a particular project? That is, when can the company make a profit by undertaking a project?

If outcomes were deterministic and they accrued only to the current generation, the condition for project acceptance would be the well-known Kaldor criterion of economics [16]. This criterion says that projects should be undertaken when it is possible to redistribute income after adopting the project so that everyone is better off. It is possible to do this redistribution if and only if total willingness to pay for the project is positive. In this case, the company could make a profit by retaining part of the amount by which individuals are made better off, that is, part of their willingness to pay.

Since we are including uncertainty and outcomes to future generations, we must modify the Kaldor criterion. Projects should still be undertaken by the social brokerage company when income can be redistributed to make everyone better off. Now, however, "everyone" includes only current individuals who contract with the company. Also, "better off" means that individuals have greater expected values. With the assumptions and notation used in this research, the Kaldor criterion under uncertainty can be written as:

> The new situation with the project is preferred if there exists a set of redistributions $\{\epsilon_k\}$ such that
>
> $$\sum_k \epsilon_k = 0 \tag{2.7}$$

and

$$\left\langle V_k\left(h_k\left(p',m_k'\,|\,p^b\right) - \epsilon_k, z'\right)\right\rangle > \left\langle V_k\left(h_k\left(p^o,m_k^o\,|\,p^b\right), z^o\right)\right\rangle \quad \text{for all } k \tag{2.8}$$

The subscript k refers to the k^{th} current individual.

Similar to the case under certainty, the Kaldor criterion under uncertainty can be expressed in terms of total willingness to pay. We can show that Eqs. (2.7) and (2.8), taken together, are equivalent to a positive value of total willingness to pay:

<u>Theorem 2.1</u>. The Kaldor criterion under uncertainty is satisfied if and only if

$$\sum_k w_k > 0$$

Thus, in order for the social brokerage firm to make a profit, it should only consider projects for which the company's customers would pay a positive net amount. In this case, it could retain as profits part of the total willingness to pay. A proof of this theorem is given in Appendix C.

To calculate total willingness to pay, expressions such as Eq. (2.4) for CG's willingness to pay are summed over individuals. The following expression results for aggregate willingness to pay:

$$\sum_k w_k \doteq \left(\overline{M'} - \overline{M^o}\right) + \sum_i D_i\left(\overline{p_i^o} - \overline{p_i'}\right) + \left(\overline{z'} - \overline{z^o}\right)\sum_k \left.\frac{\partial V_k/\partial z}{\partial V_k/\partial h_k}\right|_{\overline{p^o},\overline{m_k^o},\overline{z^o}}$$

where

$D_i = \sum_k d_{k_i}$ is the market demand for good i summed over individuals k

$M = \sum_k m_k$ is the total income of all individuals

Let z be the average income of an individual in the future generation. Then, the social brokerage company should undertake a project if and only if the sum of the following results of the project is positive:

- the change in the expected total income of its customers
- the change in the expected cost of the goods they consumed before the project
- the change in the expected average income of a future individual evaluated at the sum of marginal rates of substitution of the current customers

This expression is similar to the usual cost-benefit criterion except for the term involving future outcomes. Instead of discounting future outcomes at the market rate of interest, they are valued at the sum of marginal rates of substitution.

Chapter 3

A MORE GENERAL THEORY OF WILLINGNESS TO PAY

3.1 Summary of the Chapter

In Chapter 2, we calculate the amount that an expected-value individual would be willing to pay for a project with outcomes beyond his lifetime under very simple assumptions. For example, the individual's life consists of one period of fixed length, and he cares about only his own consumption and one attribute describing the future generation's well-being. In this case, his willingness to pay is the sum of changes in the expected value of his income, the expected cost of goods consumed, and the expected well-being of future individuals, evaluated in current dollars:

$$w \doteq \left(\overline{m'} - \overline{m^o}\right) + \sum_i d_i \bigg|_{\overline{p^o},\overline{m^o},\overline{z^o}} \left(\overline{p_i^o} - \overline{p_i'}\right) + \frac{\partial V/\partial z}{\partial V/\partial h}\bigg|_{\overline{p^o},\overline{m^o},\overline{z^o}} \left(\overline{z'} - \overline{z^o}\right) \tag{3.1}$$

In this chapter, we extend the assumptions to include more realistic cases. To begin with, we recognize that the individual's lifetime consists of more than one period, with goods defined by both the kind of good and the time of consumption. This leads to an interpretation of the numeraire h in terms of the present value equivalent defined by Pollard [28]. Then, we include additional attributes describing outcomes to "others." These may be added measures of the well-being of future individuals or they may describe outcomes to other individuals in the current generation.

When we allow current individuals to be non-expected-value decision makers, terms are added to the expression for willingness to pay that depend on the individual's risk aversion and the covariances of the uncertain state variables. Then, we assume that the length of each individual's lifetime is uncertain, adding the probabilities of being alive in future years as factors in the computation of willingness to pay. In the final section of this chapter, we compute willingness to pay when the changes in an individual's own consumption are estimated from changes in the quantities of goods he consumes rather than changes in the prices and income he faces. The result of these extensions are general expressions for willingness to pay that can be applied to a wide range of real problems.

3.2 Multiple Periods in the Current Individual's Lifetime

In Eq. (3.1), we model the current citizen's lifetime as a single time period. It is more realistic to assume that his life consists of many different periods, with the definitions of goods and prices including the time of consumption. This means that each good x_i, and each demand d_i, is the consumption of a particular commodity at a particular time in the individual's life. For simplicity in what follows, we will assume that there is only one commodity, equal to "total consumption," in each period. If we let the subscript i denote the time of consumption, then x_i is total consumption in period i. We can also take prices p_i to be current prices for one unit of x_i.

With these definitions, an interpretation of the numeraire h for an individual's lifetime consumption can be made in terms of the present value equivalent defined by Pollard. We first recall Pollard's definition using Fig. 3.1. This figure illustrates the present value equivalent when there are two periods, with consumption in the first period, x_1, having a price of one. If an individual is indifferent between owning an income vector with present value s (computed at prices p) or a consumption vector C, then s is the present value equivalent of C:

$$s = PVE(C,p)$$

This means that, with income s and prices p, the individual could purchase a consumption vector B which would be "equivalent" (indifferent to) C.

Next, we look back at Fig. 2.1 illustrating the numeraire h. We see in this figure that h is the present value equivalent of any consumption point on the indifference curve $\underline{v}(x_1,x_2) = c$, evaluated at prices p^b. If we choose the particular consumption point A, the individual would be indifferent to consuming A or having the income h to purchase consumption at prices p^b. Thus, h is precisely the quantity defined above as the present value equivalent, and we can write:

$$h(p,m|p^b) = PVE(A,p^b)$$

where $A = d(p,m)$ is the consumption demanded at prices p and income m. The value of h for a price p and income m is equal to the present value equivalent of the consumption A that would be demanded at that price and income. Thus, when the individual's lifetime is assumed to consist of many periods, h is just the present value equivalent of lifetime consumption expressed in terms of prices and income rather than the amounts of goods consumed. The numeraire we have chosen for an individual's

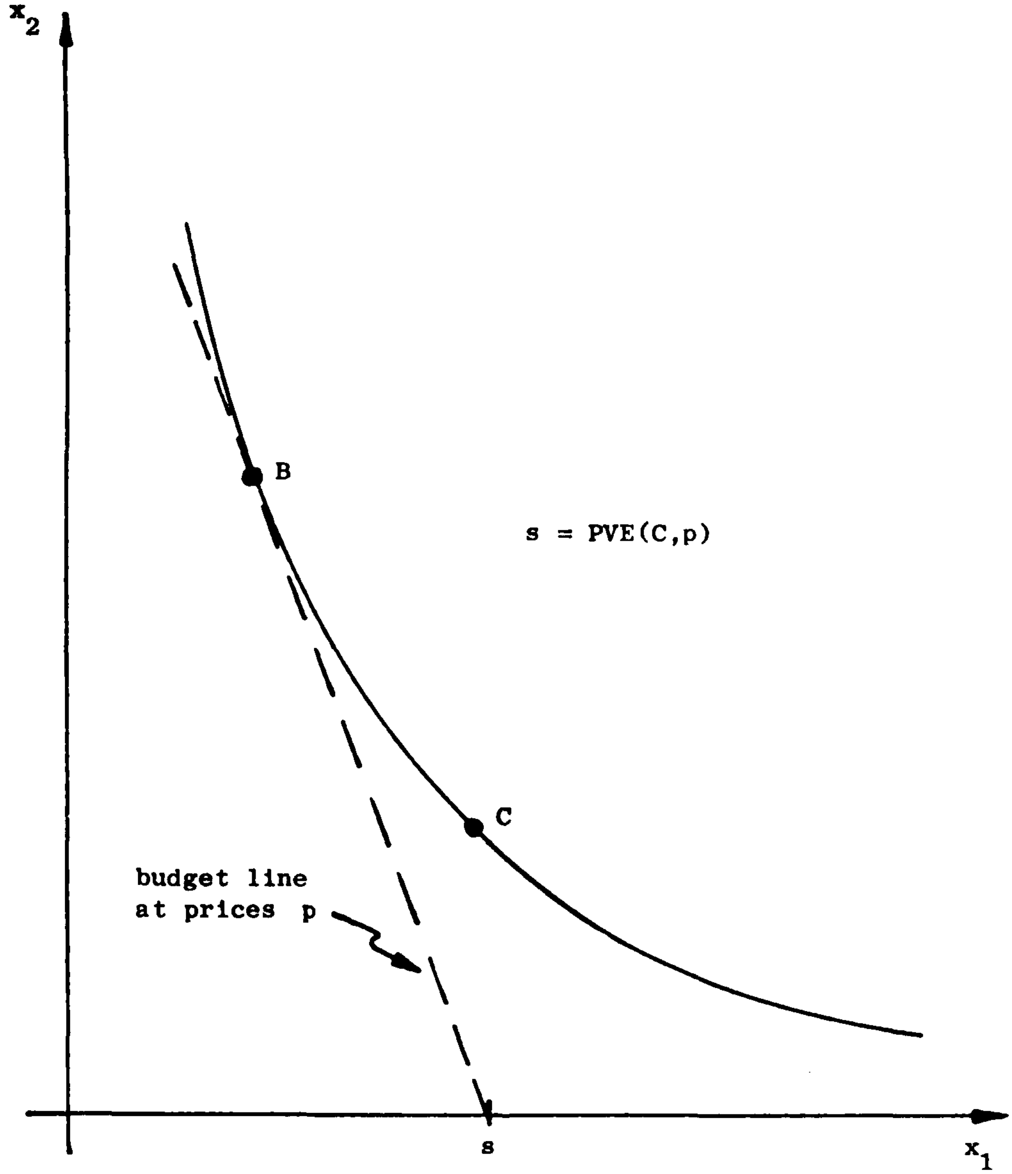

Fig. 3.1. ILLUSTRATION OF THE PRESENT VALUE EQUIVALENT OF A CONSUMPTION VECTOR C.

lifetime consumption is the present value equivalent, with a change of arguments.

As pointed out by Barrager [4], the present value equivalent is not necessarily equal to the present value. For example, in Fig. 3.2, h is the present value equivalent of both B and C at base prices p^b. The quantity h is also the present value of B computed at prices p^b. However, the present value of C at prices p^b is γ, which is not equal to h.

How does this new interpretation of the numeraire h affect the computation of willingness to pay? First of all, the definitions of goods and prices must include the time of consumption, as mentioned previously. We assume that prices represent current prices for future consumption or what are sometimes called discounted prices rather than prices that will actually be paid in the future. Also, the quantity m is the <u>present value</u> of the individual's lifetime income. This value is computed using the prices of a numeraire good, usually "dollars," in each time period. Thus, the terms

$$\sum_i d_i\left(\overline{p_i'} - \overline{p_i^o}\right) \quad \text{and} \quad \left(\overline{m'} - \overline{m^o}\right)$$

in the expression for willingness to pay are both measured in current dollars, as we would expect since these are the units for willingness to pay.

3.3 <u>Multiple Attributes Describing Outcomes to "Others"</u>

In Eq. (3.1), we assume that each current individual values only his own consumption and a single attribute that measures the well-being of a

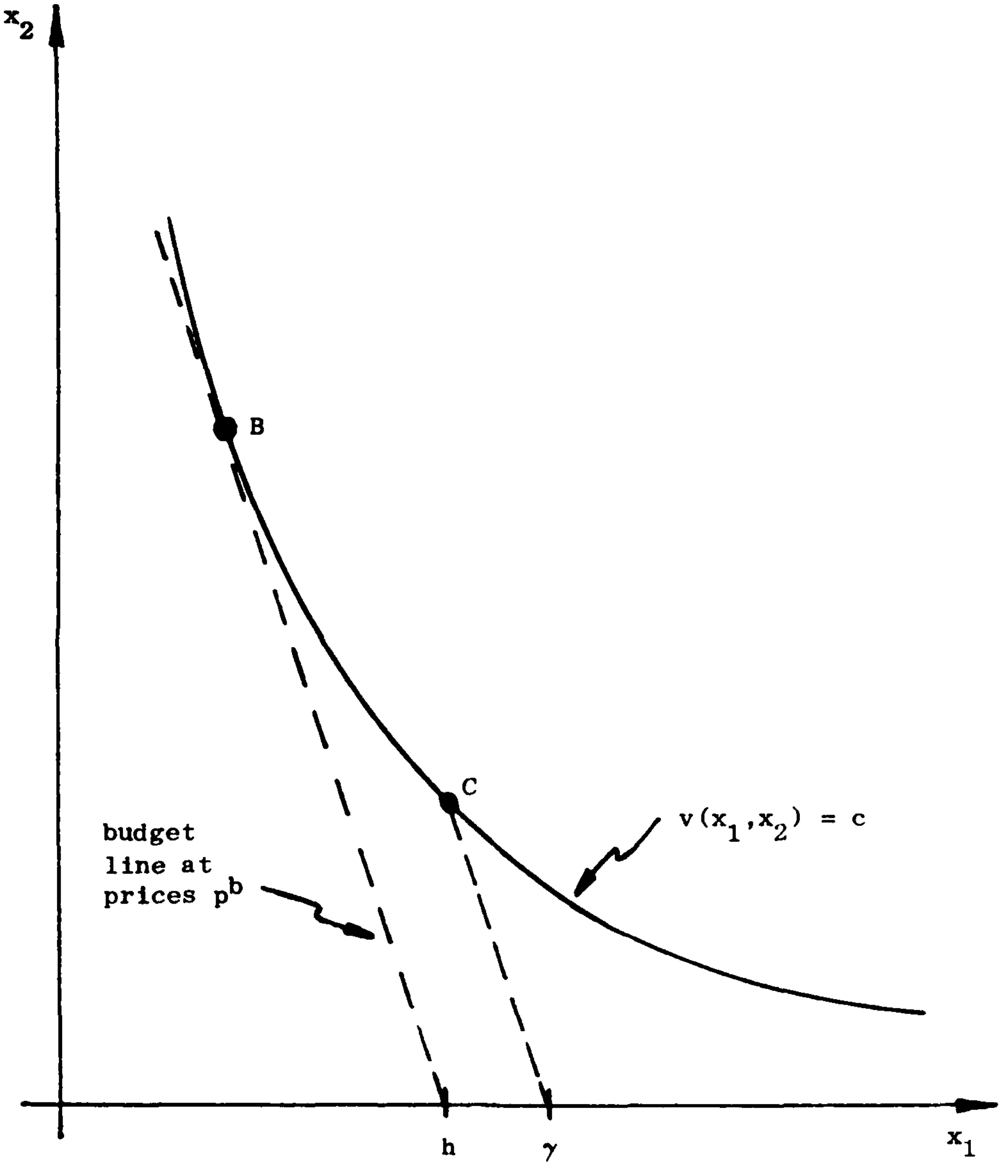

Fig. 3.2. EXAMPLE SHOWING THAT THE PRESENT VALUE EQUIVALENT IS NOT THE SAME AS THE PRESENT VALUE.

future generation by its dollar income. There are two reasons why this is an unrealistic model of current individuals' preferences. First of all, each individual may also be concerned about the well-being of other individuals in the same generation as himself. This would include not only relatives and friends but also others he does not know. An individual may be willing to pay something to make his family, friends, or even his "generation" better off.

A more fundamental shortcoming of Eq. (3.1) is that it assumes that what current individuals value about the well-being of others is their dollar income. In other words, the amount an individual is willing to pay to make others better off depends on how much better off the "others" think they are, as measured by their income. Preliminary interviews that have been conducted to assess current individuals' preferences (described in Chapter 5) indicate that this is not a realistic assumption. Current individuals appear to value attributes describing outcomes to "others" for which income is not an appropriate surrogate. For example, a current individual may value the amount of leisure time that future individuals spend on various activities. The hours of leisure time depend not only on income in the future, but also on future preferences.

In general, a current individual will value several attributes describing the well-being of "others" in the same generation and "others" in the future. The two cases are conceptually the same. Regardless of whether the "others" are in the same generation or a different one, the individual is valuing the well-being of someone different than himself. In order to extend Eq. (3.1) to include this case, it will be assumed that the tradeoffs that an individual makes among his own consumption goods are preferentially independent of all attributes accruing to "others." This means, for example, that the housing changes an individual would trade for the opportunity to eat away from home twice a week do not depend on how well off anyone else may be. His preferences between the different things that he consumes are independent of the well-being of his neighbors or the future generation.

This independence condition means that a numeraire exists for an individual's own consumption and that his preferences depend on his consumption only through this numeraire. Using the numeraire that was discussed in the previous section, we can then consider the individual's tradeoffs between current dollars and a set of attributes measuring the well-being of others. Thus, the only change in our previous results when we include the current individual's concern about the rest of his generation or additional characteristics of the future generation is the addition of attributes in the individual's value function. The variable z describing the well-being of others in the expression for willingness to pay becomes a vector of attributes rather than a scalar. If we denote the elements of z by z_j, then the term

$$\sum_j \left.\frac{\partial V/\partial z_j}{\partial V/\partial h}\right|_{\overline{p^o},\overline{m^o},\overline{z^o}} \left(\overline{z'_j} - \overline{z^o_j}\right)$$

appears in the expression for the individual's willingness to pay.

When z is a vector of attributes, there are constraints on the tradeoffs the individual would be willing to make between the elements z_j, if his preferences are consistent. These constraints can be expressed in terms of the marginal rates of substitution $(\partial V/\partial z_j)/(\partial V/\partial h)$. For example, let z_1 describe outcomes to other people who are alive after the individual's lifetime and let z_2 describe outcomes to other people who are alive at the same time. Then, we can write:

$$\frac{\partial V/\partial z_1}{\partial V/\partial h} = \frac{\partial V/\partial z_2}{\partial V/\partial h} \cdot \frac{\partial V/\partial z_1}{\partial V/\partial z_2}$$

The rate at which the individual would trade his own current dollars for outcomes to others in the future is the product of the rate at which he would trade his own current dollars for outcomes to others who are alive at the same time, multiplied by the rate at which he would trade outcomes to others who are alive at the same time for outcomes to others in the future. This and other similar consistency conditions are derived in Chapter 5. We may be able to take advantage of these conditions in modeling and assessing individuals' preferences for outcomes to "other" individuals.

Before considering further extensions, let us summarize the first sections of this chapter by writing the resulting expression for an individual's willingness to pay. When we include more than one period in the individual's lifetime and more than one attribute describing outcomes to others, we obtain the following first-order expression for an expected-value individual:

$$w \doteq \left(\overline{m'} - \overline{m^o}\right) + \sum_i d_i\Big|_{\overline{p^o},\overline{m^o},\overline{z^o}} \left(\overline{p_i^o} - \overline{p_i'}\right) + \sum_j \frac{\partial V/\partial z_j}{\partial V/\partial h}\Bigg|_{\overline{p^o},\overline{m^o},\overline{z^o}} \left(\overline{z_j'} - \overline{z_j^o}\right) \tag{3.2}$$

This expression looks identical to that in Eq. (3.1) except for additional attributes z_j describing outcomes to others. However, the interpretation of the terms has changed. The quantity $\bar{m}$ is the expected present value of the individual's lifetime income. The product $d_i \bar{p}_i$ is the expected present cost of the i^{th} good which he consumes. The marginal rate of substitution between the individual's own current dollars and the i^{th} attribute describing outcomes to others, $(\partial V/\partial z_j)/(\partial V/\partial h)$, is the answer to a question such as "How much would you pay to raise the per capita GNP at a specific future date after your lifetime by 5¢?". Thus, the amount that an individual would be willing to pay is approximately equal to the sum of

- the change in the expected present value of his lifetime income
- the change in the expected present cost of the goods that he consumed before the project
- the changes in the expected levels of others' well-being, evaluated in current dollars.

We note that the sum of the first two terms, that is, the change in the present value of income plus the change in the present cost of goods, is equal to the change in the individual's present value equivalent h as a result of the project.

3.4 Second-Order Terms in the Approximation for Willingness to Pay

In Eq. (3.1), only first-order terms were included in the expression for willingness to pay. These terms involved the means of the uncertain state variables, since the mean is the first-order approximation to the entire probability distribution. In the cost-benefit approach, the analysis of uncertainty would end with the replacement of variables by their expectations. But, in the formulation proposed in this research, uncertainty can be modeled more accurately by adding second-order terms in the Taylor's series expansion of willingness to pay.

In order that these second-order terms have a simple form, we must make additional assumptions about preferences. We will assume that the individual's overall value function V can be put in the form of $V(h,z) = hf(z) + g(z)$ for some functions f and g. This means that, for each fixed level of well-being for everyone else, an individual's value function is linear in his own current dollars. This assumption is consistent with the analysis of decisions that affect only the individual's own consumption, where we often use "dollars" as a value function. Although the condition may not hold exactly, it is an acceptable approximation over the range of marginal changes that is being considered in this research.

Given the assumption on the value function V, we can solve for the second-order terms in the individual's willingness to pay for a project. These terms all have a similar form. Consider the set of variables $\{p_1,\ldots,p_n,m,z_1,\ldots,z_m\}$. For each pair of variables from this set, a second-order term is added that consists of the change in the covariance of the variables due to the project plus the product of the changes in the means, all multiplied by the corresponding derivative of V. For example, for the pair of prices p_i and p_j, the following term is added to the first-order result given in Chapter 2:

$$\frac{1}{2}\,\frac{\partial^2 V/\partial p_i \partial p_j}{\partial V/\partial h}\left[\mathrm{cov}\left(p_i',p_j'\right) - \mathrm{cov}\left(p_i^o,p_j^o\right) + \left(\overline{p_i'} - \overline{p_i^o}\right)\left(\overline{p_j'} - \overline{p_j^o}\right)\right]$$

Using the list of derivatives of V in Appendix D, we can express the coefficient in terms of the demand function:

$$\frac{1}{2}\left(d_i \frac{\partial d_j}{\partial m} - \frac{\partial d_i}{\partial p_j}\right)\left[\mathrm{cov}\left(p_i', p_j'\right) - \mathrm{cov}\left(p_i^o, p_j^o\right) + \left(p_i' - p_i^o\right)\left(p_j' - p_j^o\right)\right]$$

For each pair of state variables, a second-order term of the form illustrated above is added to the first-order results in Chapter 2. The resulting total expression for an expected-value individual's willingness to pay is the sum of terms in the left column of Table 3.1. The derivatives and all terms involving demand functions in this table are evaluated at the initial point $(\overline{p^o}, \overline{m^o}, \overline{z^o})$. The notes in the left margin indicate the source of each term, that is, to which state variables it corresponds.

In an actual application, it would be necessary to determine from market data and questions about the individuals' preferences which of the second-order terms were most significant. Procedures for this assessment are discussed in Chapter 5.

The coefficients of the terms listed in Table 3.1 indicate the sensitivity of willingness to pay to changes in the means and variances of the corresponding state variables. For example, when the variance of a single price p_i' increases by $\Delta V(p_i')$, willingness to pay increases by an amount Δw given by the following:

$$\Delta w = \frac{1}{2}\left(d_i \frac{\partial d_i}{\partial m} - \frac{\partial d_i}{\partial p_i}\right) \Delta V(p_i')$$

When the i^{th} good is a normal good (not inferior), this expression is positive because the demand function d_i increases with income and decreases with price. This means that an expected-value individual would pay a positive amount for increased variance in the prices p_i' that he

Table 3.1

INDIVIDUAL WILLINGNESS TO PAY TO CHANGE FROM (p^o, m^o, z^o) TO (p', m', z')

	Expected Value	**Risk Averse** **(Add this risk premium to expected value results)**
Income Changes $(m' \neq m^o)$	$w = \overline{m'} - \overline{m^o}$	$-\frac{1}{2} r_v \frac{\partial V}{\partial m}\left[{}^{v}(m') - {}^{v}(m^o)\right]$
Prices Change $(p' \neq p^o)$	$+\sum_i d_i\left(\overline{p_i^o} - \overline{p_i'}\right)$ $+\frac{1}{2}\sum_{i,j}\left(d_i \frac{\partial d_j}{\partial m} - \frac{\partial d_i}{\partial p_j}\right)\left[\mathrm{cov}\left(p_i'p_j'\right) - \mathrm{cov}\left(p_i^o p_j^o\right) + \left(\overline{p_i'} - \overline{p_i^o}\right)\left(\overline{p_j'} - \overline{p_j^o}\right)\right]$	$-\frac{1}{2} r_v \frac{\partial V}{\partial m}\sum_{i,j} d_i d_j\left[\mathrm{cov}\left(p_i'p_j'\right) - \mathrm{cov}\left(p_i^o p_j^o\right)\right]$
Prices and Income Change	$-\sum_i \frac{\partial d_i}{\partial m}\left[\mathrm{cov}\left(p_i'm'\right) - \mathrm{cov}\left(p_i^o m^o\right) + \left(\overline{p_i'} - \overline{p_i^o}\right)\left(\overline{m'} - \overline{m^o}\right)\right]$	$+ r_v\left\{\frac{\partial V}{\partial m}\sum_i d_i\left[\mathrm{cov}\left(p_i'm'\right) - \mathrm{cov}\left(p_i^o m^o\right)\right]\right\}$
Others' Well Being Changes $(z' \neq z^o)$	$+\sum_i \frac{\partial V/\partial z_i}{\partial V/\partial m}\left(\overline{z_i'} - \overline{z_i^o}\right)$ $+\frac{1}{2}\sum_{i,j}\frac{\partial^2 V/\partial z_i \partial z_j}{\partial V/\partial m}\left[\mathrm{cov}\left(z_i'z_j'\right) - \mathrm{cov}\left(z_i^o z_j^o\right) + \left(\overline{z_i'} - \overline{z_i^o}\right)\left(\overline{z_j'} - \overline{z_j^o}\right)\right]$	$-\frac{1}{2} r_v \sum_{i,j}\left(\frac{1}{\partial V/\partial m}\right)\frac{\partial V}{\partial z_i}\frac{\partial V}{\partial z_j}\left[\mathrm{cov}\left(z_i'z_j'\right) - \mathrm{cov}\left(z_i^o z_j^o\right)\right]$
Prices, Income, and Others' Well Being Change	$+\sum_i \frac{\partial^2 V/\partial m \partial z_i}{\partial V/\partial m}\left[\mathrm{cov}\left(z_i'm'\right) - \mathrm{cov}\left(z_i^o m^o\right) + \left(\overline{z_i'} - \overline{z_i^o}\right)\left(\overline{m'} - \overline{m^o}\right)\right]$ $-\sum_i d_i \frac{\partial^2 V/\partial m \partial z_j}{\partial V/\partial m}\left[\mathrm{cov}\left(p_i'z_j'\right) - \mathrm{cov}\left(p_i^o z_j^o\right) + \left(\overline{p_i'} - \overline{p_i^o}\right)\left(\overline{z_j'} - \overline{z_j^o}\right)\right]$	$- r_v \sum_i \frac{\partial V}{\partial z_i}\left[\mathrm{cov}\left(z_i'm'\right) - \mathrm{cov}\left(z_i^o m^o\right)\right]$ $+ r_v \sum_{i,j} d_i \frac{\partial V}{\partial z_j}\left[\mathrm{cov}\left(p_i'z_j'\right) - \mathrm{cov}\left(p_i^o z_j^o\right)\right]$

*For V(m) linear in m

pays when the project is adopted. This result occurs because the value function V is convex in p_i' for normal goods (Fig. 3.3). Additional variance in the price, with the mean held constant, increases the expected value of V.

A similar interpretation can be made of the coefficients in Table 3.1 corresponding to other state variables. Consider, for example, a single attribute z_i' describing others' well-being after the project is undertaken. If the variance of z_i' changes by $\Delta^v(z_i')$, willingness to pay changes by an amount Δw given by the following:

$$\Delta w = \frac{1}{2}\,\frac{\partial^2 V/\partial z_i^2}{\partial V/\partial h}\,\Delta^v(z_i')$$

If more of the attribute z_i is preferred to less, but the marginal value is decreasing, then the coefficient will be negative since $\partial^2 V/\partial z_i^2 < 0$. This means that an individual would have to <u>be paid</u> to accept increased variance in the level of well-being for others that results when the project is adopted. This result occurs because, under our assumptions, the value function V is concave in z_i' (Fig. 3.4). Additional variance in z_i', with the mean held constant, decreases the expected value of V.

A final example is the change in willingness to pay resulting from changes in the mean and variance of the income with the project, m'. If the mean of the individual's income changes by $\Delta\overline{m'}$, then $\Delta w = \Delta\overline{m'}$. Willingness to pay increases with the mean income resulting from the project. However, it is not affected by uncertainty in income. This result occurs because we have assumed that the value function V is linear in the numeraire h, or equivalently that value is linear in income.

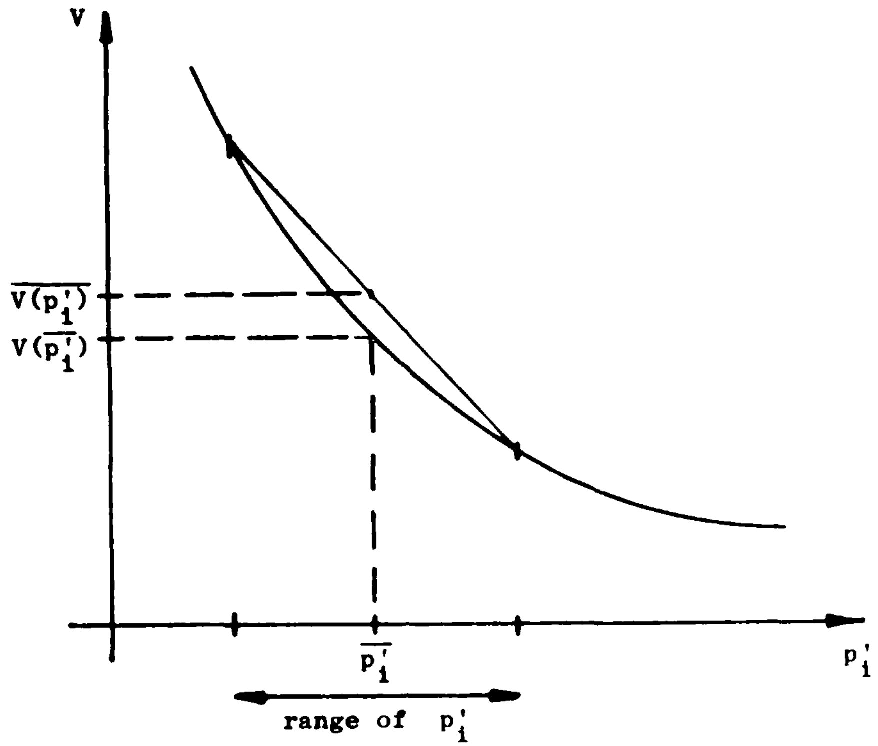

Fig. 3.3. THE INDIVIDUAL'S VALUE FUNCTION V IS CONVEX IN THE PRICE p_i'.

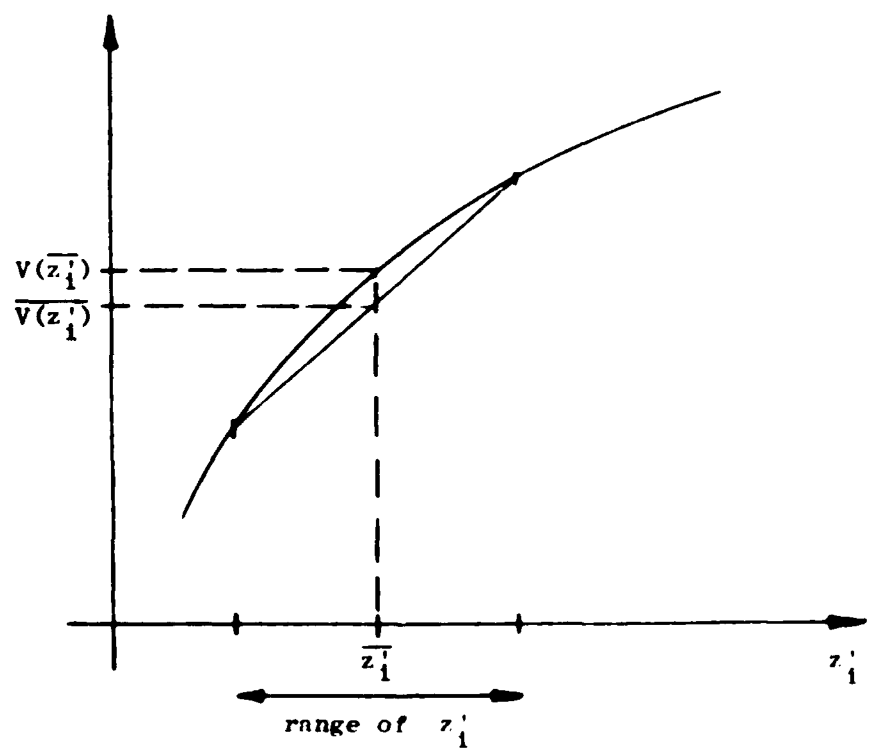

Fig. 3.4. THE INDIVIDUAL'S VALUE FUNCTION V IS CONCAVE IN THE OTHERS' WELL BEING z_i'.

Similar interpretations can be made of the changes in willingness to pay when the means or variances of any other state variables change.

3.5 Non-Expected-Value Individuals

In Eq. (3.1), it was assumed that the individual was risk neutral. In that case, his willingness to pay for a project was the amount that could be subtracted from the present value equivalent of his own consumption resulting from the project and leave him with the same expected value as he would have without the project. If the individual is not risk neutral, he will be indifferent if his certain equivalent remains the same. Thus, we replace expected value by certain equivalent in the equation for w to calculate willingness to pay for a non-expected-value individual.

$$\tilde{\left\langle V\left[h(p^o, m^o | p^b), z^o\right]\right\rangle} = \tilde{\left\langle V\left[h(p', m' | p^b) - w, z'\right]\right\rangle} \qquad (3.3)$$

Both sides of Eq. (3.3) are evaluated using the following approximation for the certain equivalent:

$$\tilde{\langle} V(h,z) \rangle \doteq \langle V(h,z) \rangle - \frac{1}{2} r_v \Big|_{\langle V(h,z) \rangle} {}^{v}\langle V(h,z) \rangle \qquad (3.4)$$

The term r_v, which is equal to $-u''(V)/u'(V)$, is the risk-aversion coefficient for the individual's cardinal, von Neumann-Morgenstern utility function $u(V)$. The mean and variance of V in Eq. (3.4) are computed using a Taylor's series expansion, as was done previously. Using these approximations, we can solve Eq. (3.3) for the individual's willingness to pay.

As expected, we obtain the same first and second-order terms in the expression for willingness to pay that were described in Chapter 2 and the previous sections of this chapter. We also get additional terms involving the variances and covariances of the random variables due to risk aversion or risk preference. These added terms are "risk premiums" or additional amounts the individual is willing to pay or be paid because of his attitude toward uncertainty.

Consider the set of variables $(p_1, \ldots, p_n, m, z_1, \ldots, z_m)$. For each pair of variables from this set, a risk premium is added that consists of the change in the covariance of the variables due to the project multiplied by the risk-aversion coefficient and the corresponding derivatives of the value function. For example, for the pair of prices p_i and p_j, the risk premium is given by the following expression:

$$- \frac{1}{2} r_v \bigg|_{\langle V(h,z) \rangle} \frac{\partial V}{\partial h} \frac{\partial V}{\partial p_i} \frac{\partial V}{\partial p_j} \left[\operatorname{cov}\left(p_i', p_j'\right) - \operatorname{cov}\left(p_i^o, p_j^o\right) \right]$$

Using the fact that $\partial V/\partial p_i = -d_i$ and assuming $i = j$, this reduces to:

$$- \frac{1}{2} r_v \bigg|_{\langle V(h,z) \rangle} \frac{\partial V}{\partial h} d_i^2 \left[{}^{v}\langle p_i' \rangle - {}^{v}\langle p_i^o \rangle \right]$$

The term multiplying the change in variance will be negative for a risk-averse individual. This means that he must <u>be</u> paid a risk premium to accept additional variance in the price p_i' resulting from the project, because of his risk aversion.

Similar interpretations can be made of the risk premiums corresponding to the other state variables. The resulting total risk premium for an individual with non-expected-value preferences is the sum of terms in the right column of Table 3.1. This premium must be added to the willingness to pay of an expected-value decision maker, given in the left column of the table, to calculate total willingness to pay for non-expected-value preferences. Total willingness to pay is derived in Appendix E.

3.6 The Relation between Willingness to Pay and the Certain Equivalent

In Chapter 2, we saw that, for a particular representation of the value function $V(h,z)$, willingness to pay for an expected-value individual is equal to the change in his expected value. It is not surprising therefore that, for a general risk attitude, willingness to pay is equal to the change in certain equivalent.

To show this, we represent the individual's value function in the following way. The value $V(\mathfrak{H},\mathfrak{Z})$ corresponding to certain values $\mathfrak{H}$ and $\mathfrak{Z}$ of a numeraire h for the individual's own consumption and an attribute z for outcomes to others is the number h^* such that

$$V\left(h^*,\overline{z^o}\right) = V(\mathfrak{H},\mathfrak{Z}) \tag{3.5}$$

Then, we use the approximation for the certain equivalent,

$$\tilde{\langle h^* \rangle} \doteq \langle h^* \rangle - \frac{1}{2}\, r_v\Big|_{\langle h^* \rangle} \, {}^{v}\langle h^* \rangle$$

and an approximation for the risk-aversion coefficient:

$$r_v\Big|_{\langle h^* \rangle} \doteq r_v\Big|_{h(\overline{p^o},\overline{m^o},\overline{z^o})}$$

With these assumptions, the change in certain equivalent from a lottery (p^o, m^o, z^o) to a lottery (p', m', z') is the change in expected value minus one half the risk-aversion coefficient times the change in variance:

$$\tilde{\langle h^* \rangle}\Big|_{p', m', z'} - \tilde{\langle h^* \rangle}\Big|_{p^o, m^o, z^o} = \langle h^* \rangle\Big|_{p', m', z'} - \langle h^* \rangle\Big|_{p^o, m^o, z^o}$$

$$- \frac{1}{2} r_v\Big|_{h\left(\overline{p^o}, \overline{m^o}, \overline{z^o}\right)}$$

$$\cdot \left\{ {}^v\langle h^* \rangle\Big|_{p', m', z'} - {}^v\langle h^* \rangle\Big|_{p^o, m^o, z^o} \right\}$$

We evaluate the mean and variance of h^* using Taylor's series expansions of Eq. (3.5). When we do this, the second-order approximation for the change in certain equivalent is the same as the expression for willingness to pay in Table 3.1. That is, to a second-order approximation, the amount an individual would pay for a project is equal to the change in his certain equivalent when the project is adopted. The details of this derivation are given in Appendix F.

3.7 Uncertainty in the Length of the Individual's Life

Implicit in this research so far is the assumption that the individual lives for a fixed period. Specifically, the length of his life was assumed to be known in the computation of the present value equivalent that was used as a numeraire for lifetime consumption. Thus, the willingness to pay that was calculated in Chapter 2 and the earlier sections of Chapter 3 should really be written as $w|n_o$, where n_o is the number of years that the individual lives.

If we want to calculate willingness to pay when lifetime is uncertain, we can integrate over the probability distribution on the length of life n:

$$w = \sum_{n_o} [w|n_o] \cdot [\{n\}(n_o)]$$

To do this, we must first incorporate the length of life in the expressions we derived for willingness to pay in Table 3.1. A method for doing this will be explained using a simple example that may often be an appropriate model for actual decision problems.

Assume that the outcomes of a project are estimated in terms of dollars per person alive in each year. This might be reasonable, for example, in the helium storage decision discussed earlier. The immediate costs of the project are paid by taxes, resulting in an outcome that can be measured as dollars per taxpayer. Also, the benefits of lower prices in the future change the consumption of each future individual by some current dollar equivalent.

Denote by $\overline{d_i}$ for $i = 1,\ldots,t$ the expected net dollars per person in year i that result from the project, where the time horizon is t years. If a current individual is alive in year i, then he will be affected directly by any dollars paid or received. Thus, one contribution to his willingness to pay for the project, given that he is alive in year i, is the change in his income. The dollars paid or received are discounted at the market rate of interest r to put them in present value terms, since income is evaluated in dollars of present value:

$$\frac{\overline{d_i}}{(1 + r)^{i-1}}$$

An individual who is alive in year i also benefits indirectly from the dollars that are received by other individuals about whom he cares. Let y_i be the average dollar income in year i for other people, given that the individual is alive in year i. Then, dollars received by others contribute to the individual's willingness to pay at his marginal rate of substitution between his own dollars and dollars per person in year i, given that he is alive:

$$\frac{\partial V/\partial y_i}{\partial V/\partial h} \overline{d_i}$$

If the individual is not alive in year i, then the outcome will not affect him directly but will affect the well-being of future individuals living after him. Let z_i be the average dollar income in year i for other people, given that the individual is not alive in year i. In this case, dollars received by others contribute to the individual's willingness to pay at his marginal rate of substitution between his own dollars and dollars per person in year i, given that he is not alive:

$$\frac{\partial V/\partial z_i}{\partial V/\partial h} \overline{d_i}$$

Thus, for a stream of outcomes $d_1, \ldots, d_t$, an individual who lives exactly n_o years would pay a total of:

$$w|n_o \doteq \sum_{i \leq n_o} \left[\frac{\overline{d_i}}{(1 + r)^{i-1}} + \frac{\partial V/\partial y_i}{\partial V/\partial h} \overline{d_i} \right] + \sum_{i > n_o} \frac{\partial V/\partial z_i}{\partial V/\partial h} \overline{d_i}$$

If we sum over the probability distribution of lifetime n, we obtain:

$$w \doteq \sum_{n_o=1}^{t} \left(\sum_{i \leq n_o} \left[\frac{\overline{d_i}}{(1 + r)^{i-1}} + \frac{\partial V/\partial y_i}{\partial V/\partial h} \overline{d_i} \right] + \sum_{i > n_o} \frac{\partial V/\partial z_i}{\partial V/\partial h} \overline{d_i} \right) \{n\}(n_o)$$

Writing out a few terms and rearranging them in the following way,

$$w \doteq \left[\frac{\overline{d_1}}{(1 + r)^o} + \frac{\partial V/\partial y_1}{\partial V/\partial h} \overline{d_1} \right] \left(\{n\}(1) + \ldots + \{n\}(t) \right)$$

$$+ \left[\frac{\overline{d_2}}{(1 + r)^1} + \frac{\partial V/\partial y_2}{\partial V/\partial h} \overline{d_2} \right] \left(\{n\}(2) + \ldots + \{n\}(t) \right) + \ldots$$

$$+ \left[\frac{\overline{d_t}}{(1 + r)^{t-1}} + \frac{\partial V/\partial y_t}{\partial V/\partial h} \overline{d_t} \right] \left(\{n\}(t) \right)$$

$$+ \frac{\partial V/\partial z_2}{\partial V/\partial h} \overline{d_2} \left(\{n\}(1) \right) + \frac{\partial V/\partial z_3}{\partial V/\partial h} \overline{d_3} \left(\{n\}(1) + \{n\}(2) \right) + \ldots$$

$$+ \frac{\partial V/\partial z_t}{\partial V/\partial h} \overline{d_t} \left(\{n\}(1) + \ldots + \{n\}(t - 1) \right)$$

we see that willingness to pay can be written as:

$$w \doteq \sum_{i=1}^{t} \left[\frac{\overline{d_i}}{(1 + r)^{i-1}} + \frac{\partial V/\partial y_i}{\partial V/\partial h} \overline{d_i} \right] \left(\sum_{j \geq i} \{n\}(j) \right) + \sum_{i=2}^{t} \frac{\partial V/\partial z_i}{\partial V/\partial h} \overline{d_i} \left(\sum_{j < i} \{n\}(j) \right)$$

Now we define q_i as the probability of being alive in year i. It is related to the probability of living exactly n_o years, $\{n\}(n_o)$, by the following:

$$q_i = 1 - \sum_{j<i} \{n\}(j) = \sum_{j \geq i} \{n\}(j)$$

The second equality comes about because $\sum_{j=1}^{t} \{n\}(j) = 1$. Noting that $q_1 = 1$, that is, the current individual is alive in the first period by definition, then we can write:

$$w \doteq \sum_{i=1}^{t} \left(\left[\frac{\overline{d_i}}{(1 + r)^{i-1}} + \frac{\partial V/\partial y_i}{\partial V/\partial h} \overline{d_i} \right] q_i + \frac{\partial V/\partial z_i}{\partial V/\partial h} \overline{d_i}(1 - q_i) \right) \qquad (3.6)$$

Each outcome of $\overline{d_i}$ dollars per person in year i contributes to a current individual's willingness to pay an amount equal to the sum of

- the change in the expected present value equivalent of his own consumption, multiplied by the probability that he will be alive in year i
- the change in the expected average well-being of other individuals who are alive at the same time, evaluated in current dollars, multiplied by the probability that he will be alive in year i
- the change in the expected average well-being of future individuals, evaluated in current dollars, multiplied by the probability that he will be dead in year i

These contributions are added up over all years to calculate an individual's total willingness to pay for a project when his lifetime is uncertain, given in Eq. (3.6). Willingness to pay depends on the probabilities that he will be alive in each future year and, thus, on age and other socioeconomic characteristics.

Equation (3.6) has interesting interpretations in the cases of very short or very long time periods. In the short term, a current individual is quite likely to be alive $(q_i \doteq 1)$. If he also does not care much about other people who are alive at the same time, then Eq. (3.6) reduces to discounting at the market rate of interest:

$$w \doteq \sum_{i=1}^{t} \frac{\overline{d_i}}{(1 + r)^{i-1}}$$

Thus, present-value computations give a reasonable measure of willingness to pay in the very short term, if individuals do not care much about other people who are alive at the same time.

In the very long term, current individuals are unlikely to be alive ($q_i \doteq 0$). In this case, outcomes are valued at the marginal rate of substitution between current dollars and the attribute describing the well-being of future individuals:

$$w \doteq \sum_{i=1}^{t} \frac{\partial V/\partial z_i}{\partial V/\partial h} \overline{d_i}$$

The computation in this equation does not necessarily correspond to discounting at any fixed rate. Discounting outcomes in the very distant future is not appropriate. In order to value future outcomes, we must know <u>what</u> attributes z_i current individuals value about the future, as well as how much they value the future.

3.8 Expression of Willingness to Pay as a Function of Quantity Changes in an Individual's Own Consumption

In Chapter 2, we express the numeraire for CG's own consumption in terms of the prices and income he faces in order that the corresponding terms of willingness to pay can be estimated from price and income changes. Sometimes, however, it may be easier to estimate changes in the quantities of goods consumed. In order to express the part of willingness to pay due to alterations in an individual's own consumption in terms of quantity changes, we must change arguments in the numeraire for lifetime consumption.

To do this, we start with the individual's direct value function on his own consumption $\underline{v}(x_1,\ldots,x_n)$. Then, we choose as the numeraire a particular transformation of the value function denoted by $\underline{h}(x|p^b)$. The quantity $\underline{h}$ is the income an individual needs at fixed base prices p^b to purchase a consumption vector that leaves him indifferent to x. A graphical representation is given in Fig. 3.5 for the case of two goods. With prices p^b and income $\underline{h}$, the individual demands a vector d which leaves him on the same indifference curve as x. Thus, the income $\underline{h}$ is the solution to the equation:

$$\underline{v}\left[d\left(p^b,\underline{h}(x|p^b)\right)\right] = \underline{v}(x) \tag{3.7}$$

where d is the vector of ordinary Marshallian demand functions.

The numeraire $\underline{h}$ is simply the numeraire h that we used in Chapter 2, with a change of arguments. The two definitions are related through the demand function:

$$\underline{h}(x|p^b) = h(p,m|p^b)$$

where $x = d(p,m)$ is the consumption demanded at prices p and income m. Thus, all of the previous interpretations of the numeraire can be extended. Most importantly, when the individual's life is assumed to consist of more than one period, $\underline{h}$ is exactly equal to Pollard's definition of the present value equivalent:

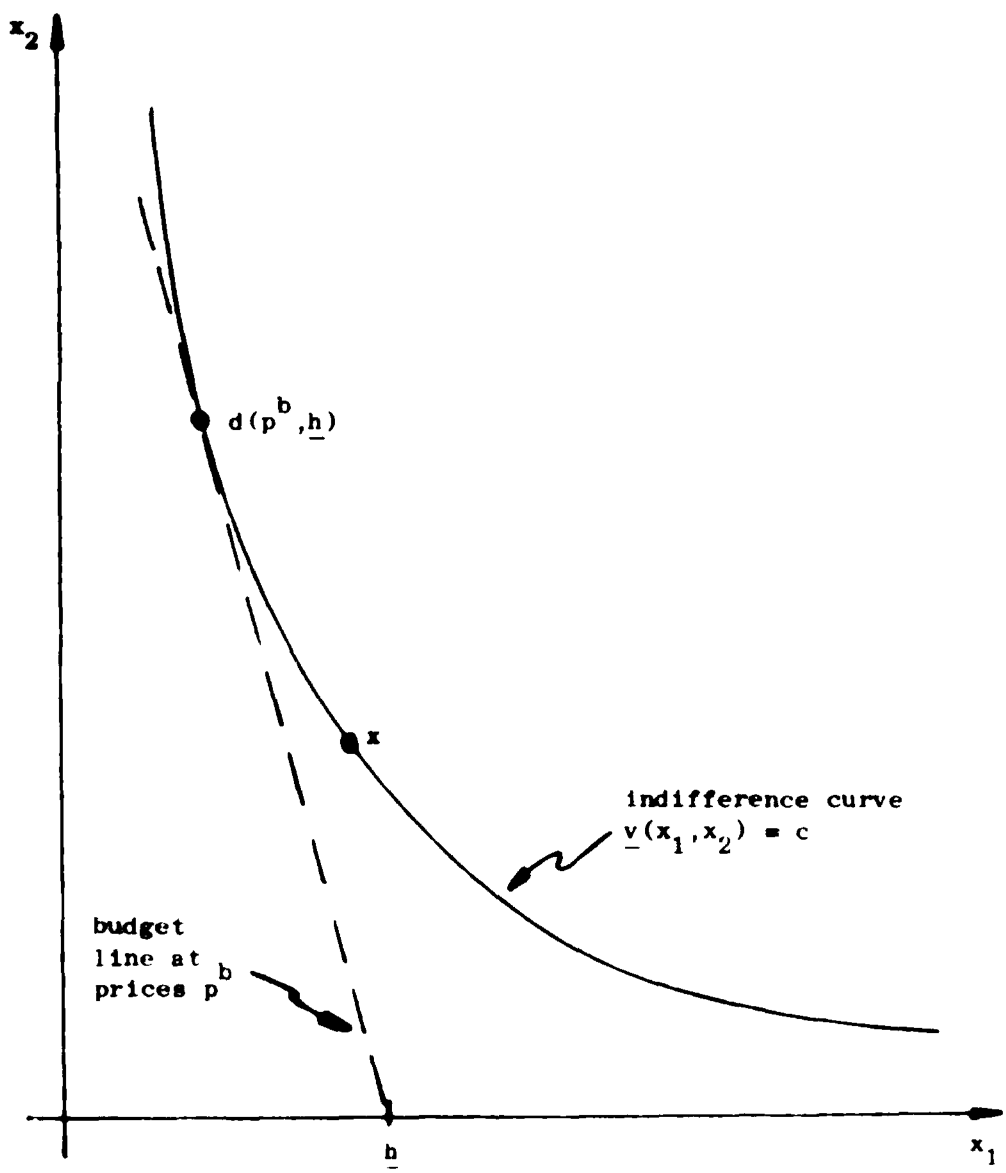

Fig. 3.5. INTERPRETATION OF THE NUMERAIRE $\underline{h}$ IN THE TWO GOOD CASE (x_1 HAS A PRICE OF ONE).

$$\underline{h}(x|p^b) = PVE(x,p^b)$$

The present value equivalent, $\underline{h}$, provides an ordering of consumption vectors that is independent of the base price vector p^b. The general shape of $\underline{h}$ is shown in Fig. 3.6, which corresponds to the case of two goods and hyperbolic consumption indifference curves of the form $\underline{v}(x_1,x_2) = x_1x_2$. For these preferences, $\underline{h}$ is given by the following expression,

$$\underline{h}(x|p^b) = 2\sqrt{p_1^b p_2^b x_1 x_2}$$

which is derived by solving Eq. (3.7) for $\underline{h}$.

The function $\underline{h}$ increases, and the individual is better off with increasing consumption of either good. Expressions for the present value equivalent for the general hyperbolic, exponential, and posynomial direct value functions are given by Barrager [4].

Just as was done previously, the numeraire $\underline{h}$ is combined with the vector of attributes describing outcomes to "others" in the individual's overall value function $V(\underline{h},z)$. This value function can then be used to calculate an individual's willingness to pay for changes in both his own consumption and the well-being of others, including people in future generations:

$$(x^o,z^o) \xrightarrow[\text{individual pay?}]{\text{What would an}} (x',z')$$

The numeraire $\underline{h}$ is measured in current dollars, so w is the solution to the equation

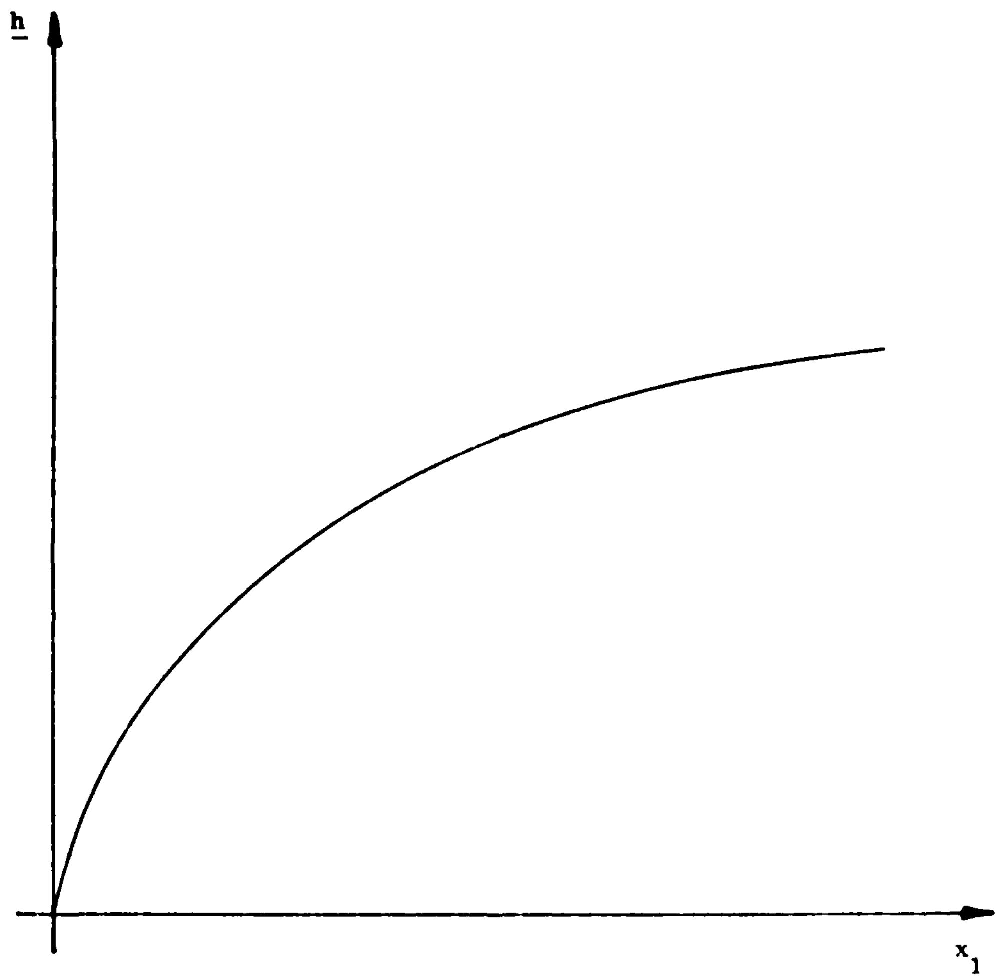

Fig. 3.6. THE NUMERAIRE $\underline{h}$ AS A FUNCTION OF CONSUMPTION FOR HYPERBOLIC INDIFFERENCE CURVES $\underline{v}(x_1, x_2) = x_1 x_2$.

$$\left\langle V\left[\underline{h}(x^o|p^b), z^o\right]\right\rangle = \left\langle V\left[\underline{h}(x'|p^b) - w, z'\right]\right\rangle$$

for expected-value individuals or

$$\tilde{\left\langle V\left[\underline{h}(x^o|p^b), z^o\right]\right\rangle} = \tilde{\left\langle V\left[\underline{h}(x'|p^b) - w, z'\right]\right\rangle}$$

for non-expected-value individuals. These expressions implicitly define willingness to pay as a function of the uncertain state variables with and without the project:

$$w = w(x^o, x', z^o, z')$$

Again, we expand in Taylor's series about the vector of mean values without the project, $(\overline{x^o}, \overline{z^o})$. We also choose as base prices in the definition of $\underline{h}$ the prices at which $\overline{x^o}$ would be demanded if income were $\underline{h}$. That is, $\overline{x^o}$ and p^b are related by

$$\overline{x^o} = d(p^b, \underline{h})$$

We take advantage of the fact that the derivatives of $\underline{h}$ are particularly simple when they are evaluated at the initial point. For example,

$$\left.\frac{\partial \underline{h}(x|p^b)}{\partial x_i}\right|_{\overline{x^o}, \overline{z^o}} = p_i\bigg|_{\overline{x^o}, \overline{z^o}} = p_i^b$$

Since the procedure is the same as that already described, we will simply present the results. The first-order expression for willingness to pay is the following:

$$w \doteq \sum_i p_i^o\left(\overline{x_i'} - \overline{x_i^o}\right) + \sum_i \frac{\partial V/\partial z_i}{\partial V/\partial h}\Bigg|_{\overline{x^o},\overline{z^o}} \left(\overline{z_i'} - \overline{z_i^o}\right)$$

The term $p_i\bar{x}_i$ is the expected cost of the good i, and $(\partial V/\partial z_i)/(\partial V/\partial h)$ is the marginal rate of substitution between an individual's own current dollars and the i^{th} attribute describing outcomes to others. The amount an individual would be willing to pay for a project is approximately equal to the sum of

- the change in the expected present cost of his own consumption, evaluated at the prices without the project
- the change in the expected levels of others' well-being, evaluated in current dollars

This equation is very similar to the earlier expression for willingness to pay in Eq. (3.2). The difference is that, in the current case, the part of willingness to pay due to changes in an individual's own consumption is measured in terms of changes in the quantities consumed rather than changes in prices and income. When we include second-order terms and the risk premium for non-expected-value preferences, we get the results listed in Table 3.2. They are similar in interpretation to the expressions given earlier in Table 3.1 and slightly simpler in form because there are fewer variables describing an individual's own consumption. All derivatives and terms involving the demand functions in this table are evaluated at the initial point $(\bar{x}^0, z^0)$. This table is the analog of Table 3.1 for the case when changes in an individual's own consumption are estimated in terms of changes in the quantities consumed.

Table 3.2

INDIVIDUAL WILLINGNESS TO PAY* TO CHANGE FROM (x^o, z^o) TO (x', z')

	Expected Value	Risk Averse (Add this risk premium to expected value results)
Quantity Change [illegible]	$\sum_i p_i(x_i' - x_i^o)$ $+\frac{1}{2}\sum_{i,j}\frac{\partial p_i}{\partial x_j}\left[\mathrm{cov}\left(x_i'x_j'\right) - \mathrm{cov}\left(x_i^o x_j^o\right) + \left(\bar{x}_i' - \bar{x}_i^o\right)\left(\bar{x}_j' - \bar{x}_j^o\right)\right]$	$-\frac{1}{2} r_v \sum_{i,j} \frac{\partial V}{\partial m} p_i p_j \left[\mathrm{cov}\left(x_i'x_j'\right) - \mathrm{cov}\left(x_i^o x_j^o\right)\right]$
[illegible] Being [illegible]	$\sum_i \frac{\partial V}{\partial w_i}\Big/\frac{\partial V}{\partial m}\left(z_i' - z_i^o\right)$ $+\frac{1}{2}\sum_{i,j}\frac{\partial^2 V/\partial w_i \partial w_j}{\partial V/\partial m}\left[\mathrm{cov}\left(z_i'z_j'\right) - \mathrm{cov}\left(z_i^o z_j^o\right) + \left(\bar{z}_i' - \bar{z}_i^o\right)\left(\bar{z}_j' - \bar{z}_j^o\right)\right]$	$-\frac{1}{2} r_v \sum_{i,j} \left(\frac{1}{\partial V/\partial m}\right) \frac{\partial V}{\partial w_i} \frac{\partial V}{\partial w_j} \left[\mathrm{cov}\left(z_i'z_j'\right) - \mathrm{cov}\left(z_i^o z_j^o\right)\right]$
Quantities and Other Being Large [illegible]	$+\sum_{i,j} \frac{\partial p_i}{\partial w_j}\left[\mathrm{cov}\left(x_i'z_j'\right) - \mathrm{cov}\left(x_i^o z_j^o\right) + \left(\bar{x}_i' - \bar{x}_i^o\right)\left(\bar{z}_j' - \bar{z}_j^o\right)\right]$	$-r_v \sum_{i,j} p_i \frac{\partial V}{\partial w_j} \left[\mathrm{cov}\left(x_i'z_j'\right) - \mathrm{cov}\left(x_i^o z_j^o\right)\right]$

*For [illegible]

3.9 A Summary of the Proposed Approach

Although we have not yet discussed the assessment of data, we can summarize the methodology that is being proposed in the following steps:

(1) Divide the individuals who are affected by the project into socioeconomic groups such that individuals in each group have similar preferences and will be affected similarly by the project.

(2) Describe the outcomes of the project on each socioeconomic group as changes in the probability distributions of

- prices and income (or consumption) received by current individuals during their lifetime

- outcomes to other individuals including those alive within an individual's lifetime and those alive at a later date

(3) Assess data from experts and current individuals who would be affected by the project.

(4) Calculate willingness to pay for individuals in each socioeconomic group, including uncertainty in their lifetime.

(5) Add over socioeconomic groups to compute total willingness to pay for the project.

Chapter 4

THE RELATIONSHIP BETWEEN THE PROPOSED APPROACH AND SOCIAL DISCOUNTING: THE HELIUM STORAGE EXAMPLE

4.1 Introduction

The approach proposed in this research is a method for making decisions when the decisions influence outcomes beyond the lifetimes of current citizens. Thus, we could ask how the approach compares with the technique of discounting that is used in cost-benefit analysis. If we made a decision by the proposed method, what discount rates would be implied? To illustrate the methodology and compare it with discounting, in this chapter we analyze the preference issues involved in a current decision problem: the decision to store helium underground for the future.

At the current time, helium is being vented to the air as a by-product of natural gas production because the potential supply exceeds the demand (Fig. 4.1). It is anticipated that, after the year 2000, the demand for helium in advanced electric power applications will rise dramatically. However, the helium supply from natural gas will diminish as the gas is depleted, necessitating the production of helium from air at much higher prices to meet the increased demand. Helium could be purchased now and stored until the future, thus making it available at lower cost to a future generation.

4.2 The Assumptions and Data Used in the Analysis of the Helium Decision

We assume that the project being evaluated is one where the government undertakes helium storage on behalf of current citizens, using

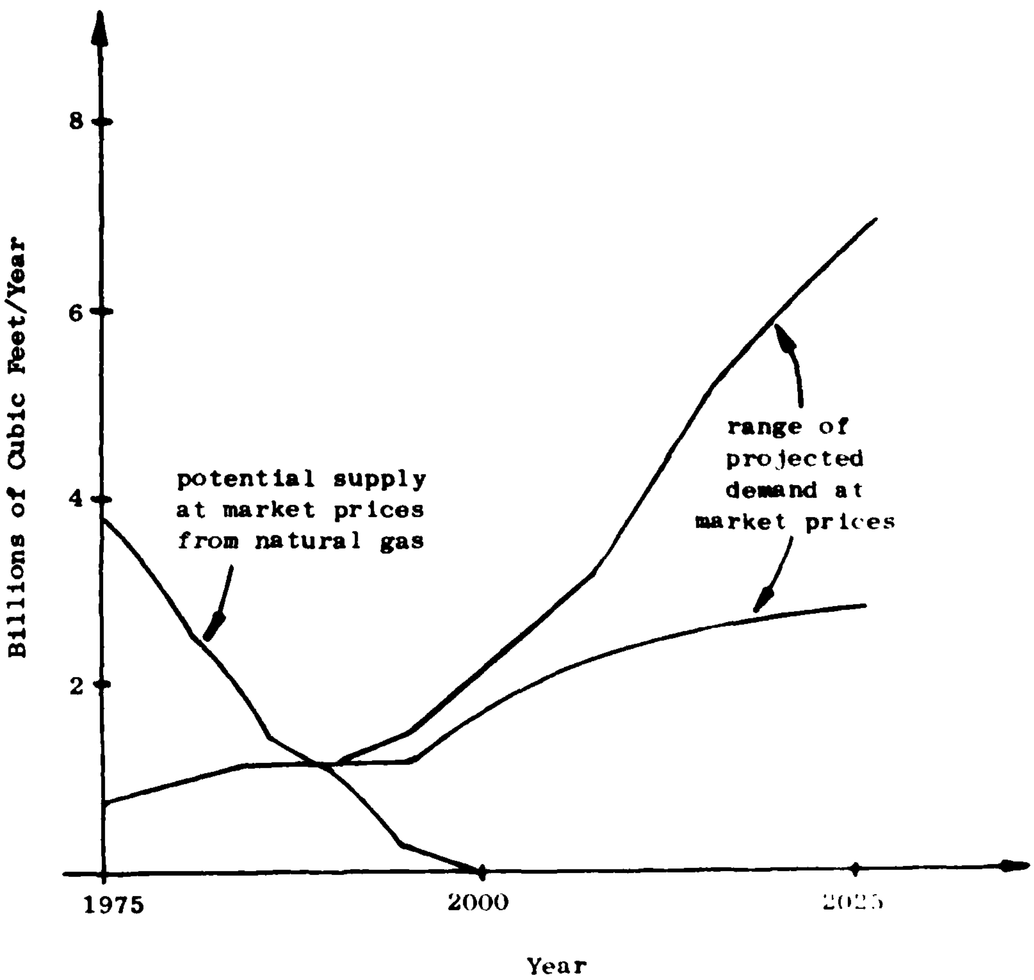

Fig. 4.1. HELIUM SUPPLY AND DEMAND.

to finance the project.* The current generation pays all of the costs of helium storage. The outcomes in this case can be viewed as a simple transfer from one generation to the next. The current population pays the cost of storing additional helium, and then helium is available at a lower price than it would be otherwise at some time in the future. Of course, we could consider more complicated alternatives where we transfer some of the costs of helium storage to the future generation. However, in order to focus on the tradeoffs between ourselves and the future, we will consider a project with very simple outcomes.

Since we are not doing a complete analysis of the helium decision, but only studying the preference issues that are involved, we will use existing estimates of the benefits of helium storage. Figure 4.2 shows the expected net cash flow to the U.S. Treasury from additional helium storage by the Federal government.** In this case, the government is first purchasing helium in the private market on long term contracts, then storing it, and later selling the stored helium to federal and private users. These figures are from a simulation model of the helium market over time by Westinghouse Research Laboratory [14]. We are not presenting the Westinghouse model as the correct analysis of the benefits of helium storage. We are simply using the [illegible] as an input to our analysis.

The Westinghouse figures for the net [illegible] helium storage are in 1973 dollars. The [illegible]

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*We distinguish this situation from the [illegible] its own resources to undertake helium [illegible] pany would act as an individual decision [illegible] on the basis of the present [illegible] the future sales of helium.

**See also Appendix [illegible]

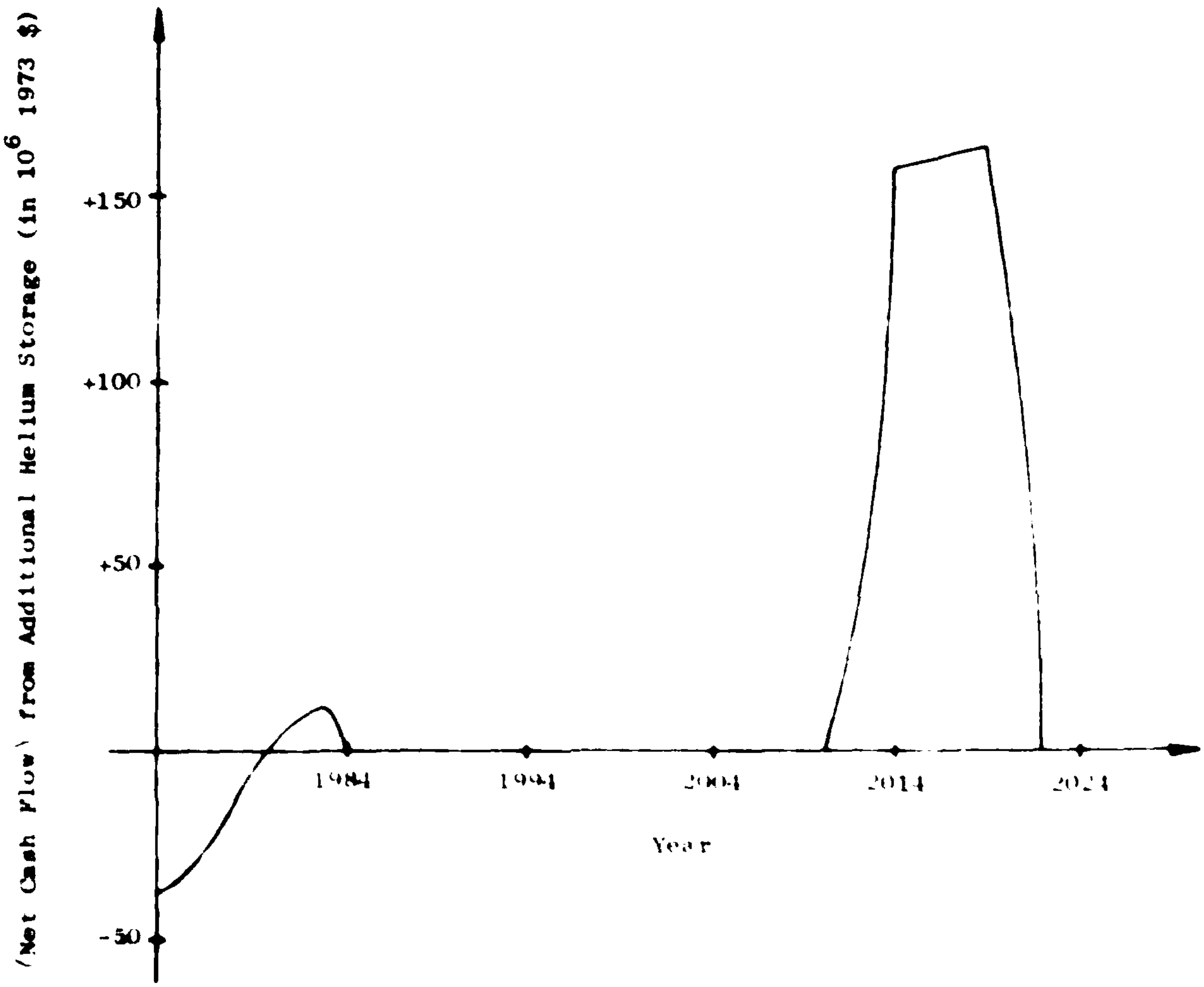

Fig. 4.2 BENEFITS OF ADDITIONAL HELIUM STORAGE UNDER NOMINAL [illegible]

of helium as determined by the equilibrium between supply and demand in each year. They also correspond to a particular policy for pricing the stored helium, which we will call the nominal policy.* Figure 4.2 shows that, under this policy, stored helium would be sold and consumers would benefit from lower helium prices for a ten-year period, starting about forty years from now.

The small positive cash flow just before 1984 in Fig. 4.2 indicates that, under the nominal pricing policy, a small portion of the additional stored helium is sold between 1980 and 1984. According to the author of the Westinghouse simulation study, this result occurs because, in the simulation, the purchase and storage of additional helium by the government reduces the amount available in the private market. Between 1980 and 1984, the government's selling price for stored helium undercuts the price in the private market, so private users purchase some of the additional helium stored by the government.

In order to analyze the preference issues in the helium storage decision, we assume that all of the outcomes of the project would be evenly distributed among the population. That is, each person alive in a given year would receive an equal share of the net cash flow to the treasury via his taxes. Thus, changes in an individual's own consumption consist of changes in the present value of his income. Present value is computed using 3% as the individual's real rate of time preference for income received within his lifetime. All income is evaluated in 1973 dollars.

Each individual is also concerned about the effects of the helium project on other people. Outcomes to "others" are evaluated in 1973

*The "nominal policy" is called policy 2 in the Westinghouse report.

dollars per person. We assume a current individual feels the same about others who are alive during his lifetime and others who are alive in the future, after his lifetime. All "others" are the same, regardless of when they live.

Thus, we have assumed that current individuals value only their own consumption and the average well-being of "other" individuals, including those living in the future. We also assume that all current individuals make the same tradeoff between their own income and current dollars per other person. However, a range of marginal rates of substitution is examined.

The population is divided into age categories, and mortality data for each category is taken from the Vital Statistics of the United States [33].* Population projections over the life of the project are taken from the population reports of the Census Bureau [6].** It is also assumed that [illegible] gross national product grows at a real rate of three percent per year [illegible] life of the project, in accord with historical data. Although [illegible] of population and GNP are treated deterministically in this [illegible] uncertainty in these variables could easily be included.

[illegible] of Willingness to Pay for Additional Helium Storage

[illegible] an expression was developed for an individual's willing- [illegible] of outcomes to himself and others, assuming that [illegible] Consider a single outcome of d_i dollars per [illegible] note by w_i the individual's willingness to [illegible] w_i is given by the i^{th} term of Eq. (3.6):

$$w_i \doteq \left[\frac{\overline{d_i}}{(1+r)^{i-1}} + \frac{\partial V/\partial y_i}{\partial V/\partial h}\overline{d_i}\right] q_i + \frac{\partial V/\partial z_i}{\partial V/\partial h}\overline{d_i}(1-q_i) \qquad (4.1)$$

This is one current individual's willingness to pay for an uncertain outcome of d_i dollars per person in year i.

The variable y_i in Eq. (4.1) refers to others who are alive at the same time and z_i refers to others who are alive in the future. We have assumed that the current individual makes no distinction between these two group of "others." Let z denote dollars per person received by others in any year. Then, the marginal rates of substitution for all others must be equal:

$$\frac{\partial V/\partial y_i}{\partial V/\partial h} = \frac{\partial V/\partial z_i}{\partial V/\partial h} = \frac{\partial V/\partial z}{\partial V/\partial h} \quad \text{for all years } i$$

Using this expression, Eq. (4.1) reduces to:

$$w_i \doteq \left(\frac{\overline{d_i}}{(1+r)^{i-1}}\right) q_i + \frac{\partial V/\partial z}{\partial V/\partial h}\overline{d_i} \doteq \left(\frac{q_i}{(1+r)^{i-1}} + \frac{\partial V/\partial z}{\partial V/\partial h}\right)\overline{d_i} \qquad (4.2)$$

A current individual's willingness to pay for the uncertain outcome d_i is proportional to the expected outcome $\overline{d_i}$. The proportionality constant depends on the probability q_i that he will be alive in year i. Thus, when each individual considers outcomes in future years, he effectively multiplies them by factors that depend on his mortality distribution. The factors that are applied to dollars per person in each year by individuals of various ages in 1974 are plotted in Fig. 4.3. All of the curves in this figure assume a marginal rate of substitution of 0.1 between an individual's own income and current dollars per other person.

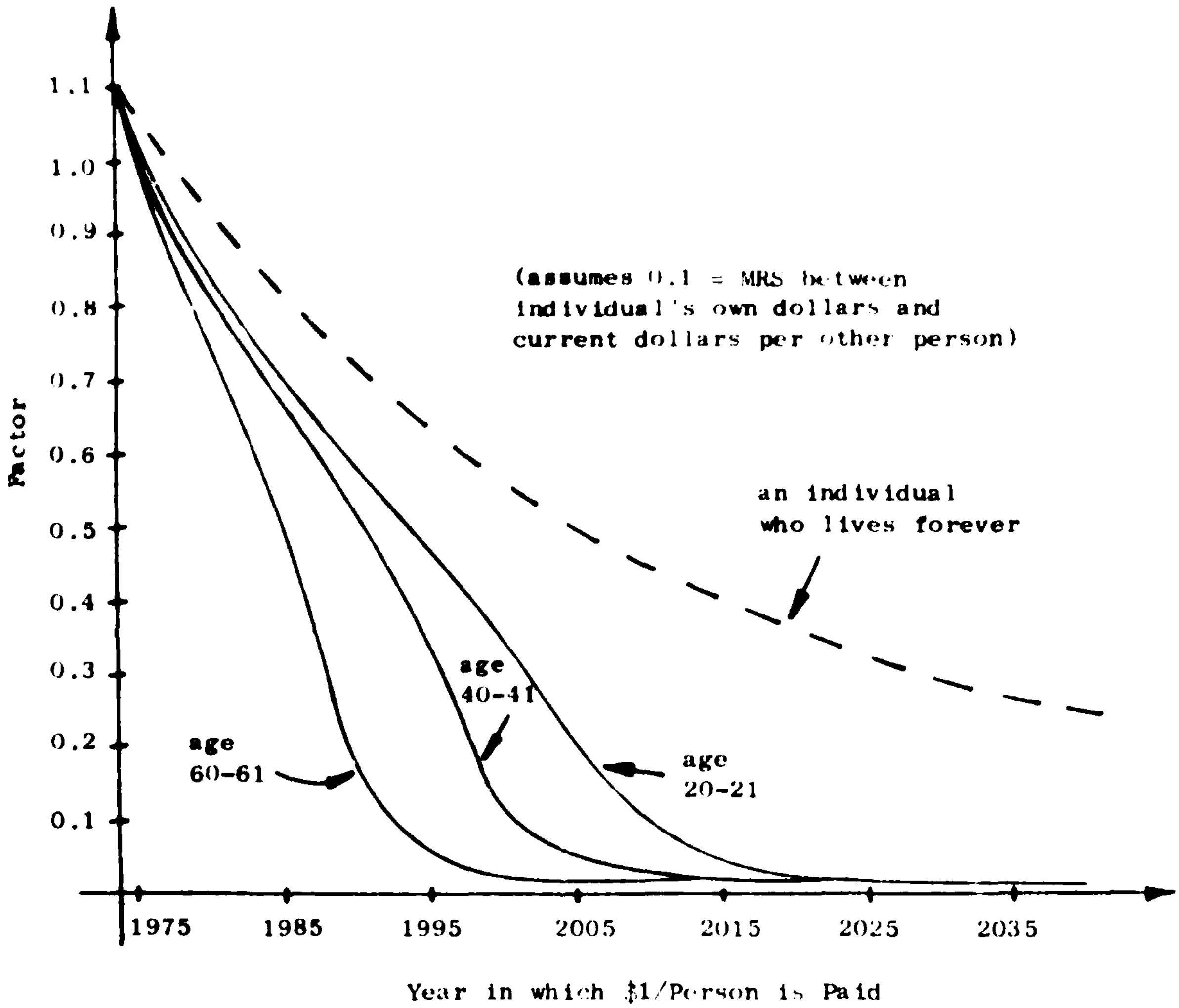

Fig. 4.3. FACTORS BY WHICH DOLLARS/PERSON ARE MULTIPLIED BY INDIVIDUALS OF VARIOUS AGES IN 1974.

As shown in this figure, older persons "discount" more. That is, the factor by which they multiply dollars per person is smaller because they are less likely to be alive in a future year to receive the outcomes themselves. The dashed line would be the factor for an individual who lived forever, so that all future outcomes were within his lifetime.

The factors for all ages are equal to 1.1 in the initial year. Since all individuals are alive in this year, they simply add the dollar they receive to the dollars per person received by others, multiplied by the marginal rate of substitution of 0.1. The sum gives an effective "discount" factor of 1.1.

Each individual's willingness to pay for outcomes in year i is approximated using Eq. (4.2). Then, the total willingness of society to pay for helium storage is computed by integrating over the age distribution and adding over all years of the project. Of course, total willingness to pay depends on how much individuals care about others, that is, the marginal rate of substitution between an individual's own current dollars and dollars per other person. In Fig. 4.4, total willingness to pay for helium storage is plotted as a function of the marginal rate of substitution, assuming the nominal policy is followed for selling stored helium.

As individuals care less about others, including those in the future, willingness to pay approaches a constant value of -$90,000,000. This number reflects the contribution of the project to the present value of current individuals' own lifetime consumption. As the rate of substitution for current dollars per other person increases, the amount current individuals would pay for the project increases. Additional helium storage under this policy is desirable only if the marginal rate of substitution

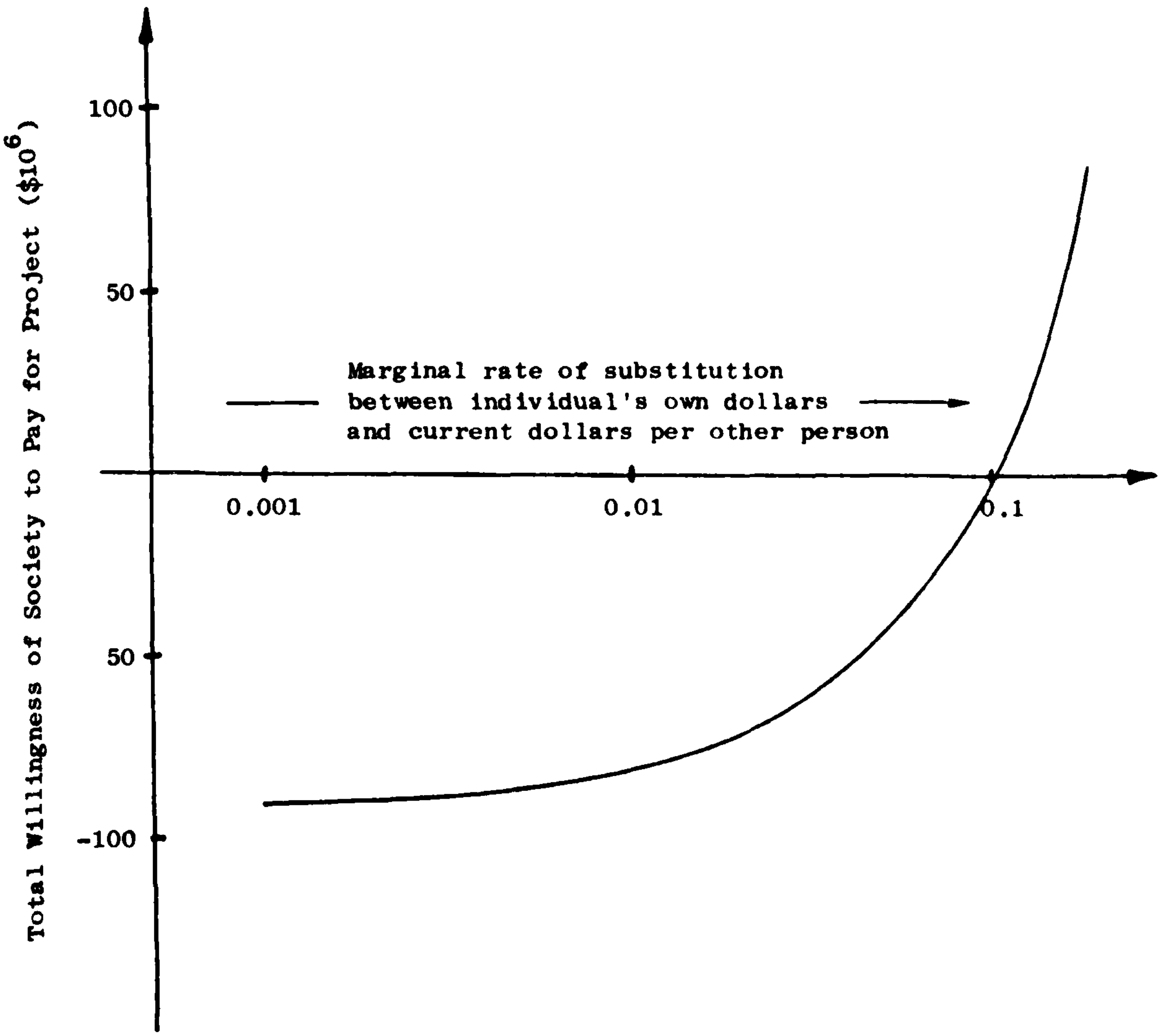

Fig. 4.4. MRS SENSITIVITY PROFILE FOR NOMINAL POLICY.

is at least 0.1. This means that a current citizen would have to be willing to sacrifice 10¢ now for each additional dollar per person paid to <u>others</u>, including those in the future.

The plot of willingness to pay as a function of the marginal rate of substitution (Fig. 4.4) is analogous to a risk sensitivity profile. It shows how the worth of a project varies with individuals' preferences. The "MRS sensitivity profile" can be used to rank alternatives or, in some cases, to eliminate them. For example, in Fig. 4.5, the expected net cash flow from additional helium storage is plotted assuming that an alternate policy for managing the stored helium is followed.* Under the alternate policy, the helium is sold from storage in approximately sixty years rather than in forty years, as with the nominal policy.

The MRS sensitivity profile for additional helium storage is plotted in Fig. 4.6 for the two pricing policies. It is clear from this figure that the nominal policy dominates the alternate policy in the sense that additional helium storage under the nominal policy is worth more for all marginal rates of substitution. This result occurs because the magnitude of the outcomes is approximately the same under both policies, but the individual is less likely to receive the benefits himself when they occur at a later date. Also, even if he were alive to receive the benefits, they would be worth less to him when received later. Helium storage under the alternate policy is worth less to him even though he feels the same about others alive forty years from now as others alive sixty years from now.

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*The "alternate policy" is called policy 3 in the Westinghouse report.

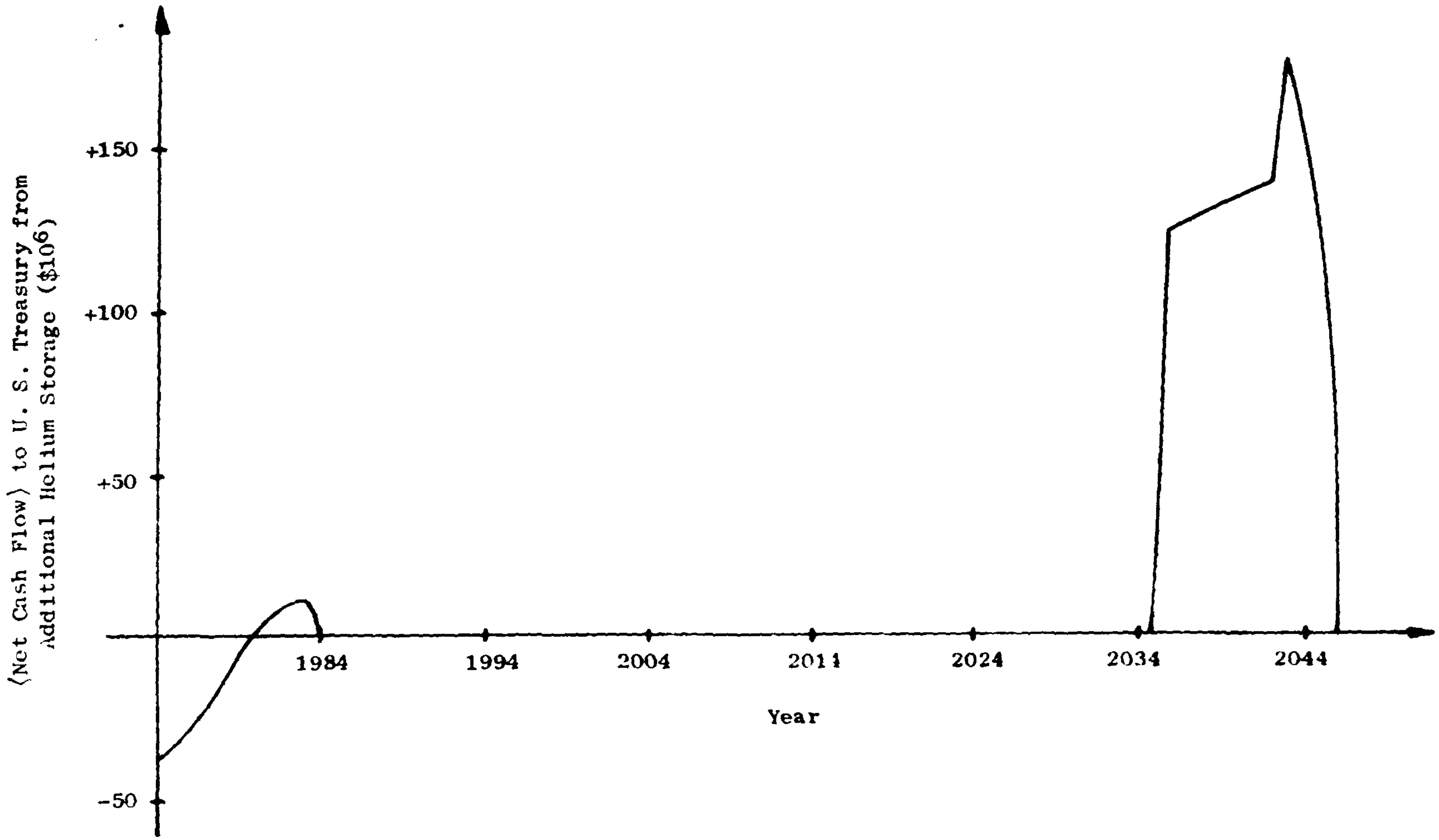

Fig. 4.5. BENEFITS OF ADDITIONAL HELIUM STORAGE UNDER ALTERNATE POLICY.

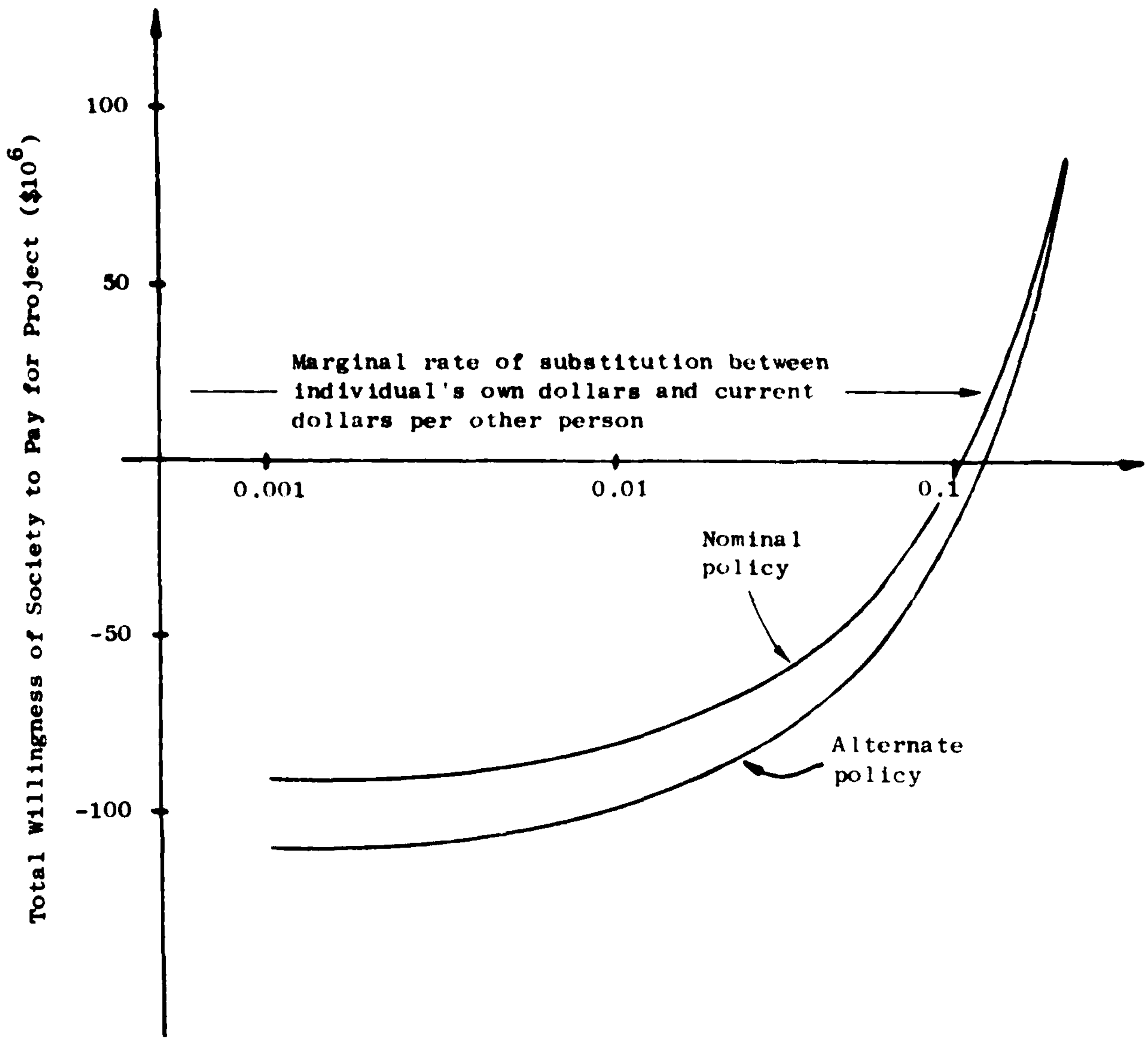

Fig. 4.6. MRS SENSITIVITY PROFILE FOR THE TWO PRICING POLICIES.

4.4 The Social Discount Factors Implied by the Proposed Methodology

The methodology we have proposed for evaluating projects with future outcomes is an alternative to discounting techniques in cost-benefit analysis. The two methods can be compared by computing the social discount factors and discount rates that would be implied by the proposed methodology. However, before this is done, we have to define carefully what we mean by a social discount factor or discount rate.

We will first define the social discount factor for year i in such a way that the discounted value of the dollar outcome in year i (the cost-benefit measure) is equal to the sum of current individuals' willingness to pay for the future outcome (the measure we are proposing). Let $\overline{D_i}$ be the expected total dollars received by all individuals in year i. That is,

$$\overline{D_i} = \overline{d_i} \cdot (\text{number of people alive in year } i)$$

where each individual receives d_i dollars. Also, let W_i be the sum of all current individuals' willingness to pay for outcomes in year i:

$$W_i = \sum_j w_{ij}$$

where

w_{ij} = willingness of j^{th} current individual to pay for outcomes that occur in year i

Then, the social discount factor f_i for year i is the number by which we multiply the total expected dollars, $\overline{D_i}$, to equal total willingness to pay, W_i:

$$\overline{D_i} f_i = W_i$$

In Fig. 4.7, the social discount factor f_i is plotted as a function of year i. The three curves correspond to different marginal rates of substitution between current individuals' own dollars and current dollars per other person. We see from this figure that, as current individuals care less about others, including those in the future, that is, as their marginal rate of substitution decreases, outcomes in all future years are discounted more. Also, for all marginal rates of substitution, years further in the future are worth less than earlier years because current individuals are less likely to be alive to receive the benefits themselves. As the probability of being alive decreases, they see the outcomes accruing to other individuals and effectively discount them more.

In the long term, when all current individuals are unlikely to be alive, we might expect the discount factor to be constant and equal to the marginal rate of substitution. Instead, the discount factor decreases slowly and is less than the corresponding marginal rate of substitution. This result occurs because the population is increasing, so that current citizens see the future outcomes being spread over a larger number of people. A fixed number of dollars implies fewer dollars per person as the population increases. If the population were constant, the social discount factors would be equal to the corresponding marginal rates of substitution in the long term.

In the initial year, when all current individuals are alive to receive the outcomes of the project, each discount factor is equal to one plus the corresponding marginal rate of substitution. For each dollar per person that is paid, a current individual is willing to pay the one dollar he receives directly, plus his marginal rate of substitution multiplied by the dollar per person that others receive.

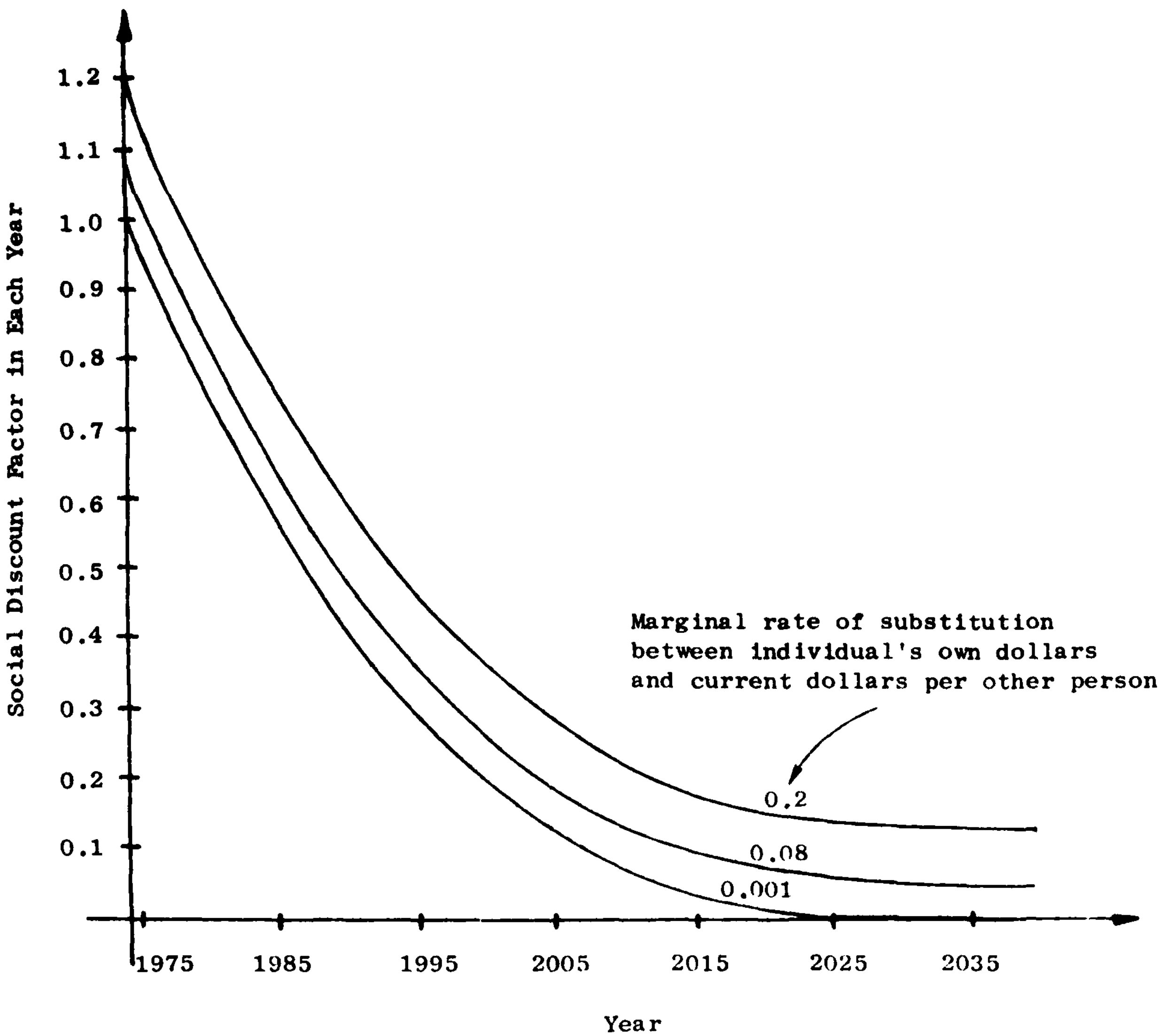

Fig. 4.7. THE SOCIAL DISCOUNT FACTOR AS A FUNCTION OF TIME.

4.5 The Social Discount Rates Implied by the Proposed Methodology

It is interesting to express our results in terms of implied social discount rates. Like the discount factors, the discount rates are defined in such a way that the discounted value of the dollar outcome in year i is equal to the sum of current individuals' willingness to pay for the outcome.

Let r_t be the "single period" discount rate used to discount outcomes from period t to the previous period t - 1. Then, we start by calculating r_1 that makes the discounted value of the expected dollar outcome in year 1, $\overline{D_1}$, equal to the total that everyone together is willing to pay now for the future outcome, W_1:

$$\frac{\overline{D_1}}{1 + r_1} = W_1$$

Then, given r_1, we find r_2 that makes the discounted value of the outcomes in year 2 equal to willingness to pay for them:

$$\frac{\overline{D_2}}{(1 + r_1)(1 + r_2)} = W_2$$

In this expression, we discount the outcome from year 2 to year 1 at r_2, then from year 1 to the current time at r_1. In general, we calculate r_t knowing the set of discount rates $r_1, \ldots, r_{t-1}$, expected outcomes in year t, and total willingness to pay now for outcomes in year t:

$$\frac{\overline{D_t}}{(1 + r_1) \ldots (1 + r_{t-1})(1 + r_t)} = W_t$$

Figure 4.8 shows the social discount rate that is computed as we have described. First, we note that the rate is a function of time and not a constant. This means that, in order for cost-benefit analysis to value the project in the same way as the approach we have suggested, the discount rate would have to vary in each year. The social discount rate also depends on individuals' marginal rates of substitution, as shown in the figure. This occurs because the value placed on outcomes to other individuals, including those in the future, affects willingness to pay for the project. As individuals care less about others, the discount rate implied by their willingness to pay increases.

In the first years of the project, the implied discount rate may be negative if individuals care enough about others. Outcomes are actually "worth" more than the total number of dollars paid or received because each individual is willing to pay for changes in the well-being of others as well as himself. Negative discount rates correspond to the discount factors greater than 1 in Fig. 4.7.

Initially, the discount rate increases with time because individuals are less likely to be alive in each succeeding year. As the probability of being alive decreases, they see the outcomes accruing to other individuals and effectively discount them at a higher rate. Eventually, when outcomes occur after the lifetimes of most current individuals, the discount rate falls. It approaches a constant value equal to the long term population growth rate. Under the assumptions we have made, individuals care about the <u>average</u> well-being of future individuals, evaluated in dollars per person. Thus, they effectively discount a fixed number of dollars because it provides fewer dollars per person as the population increases.

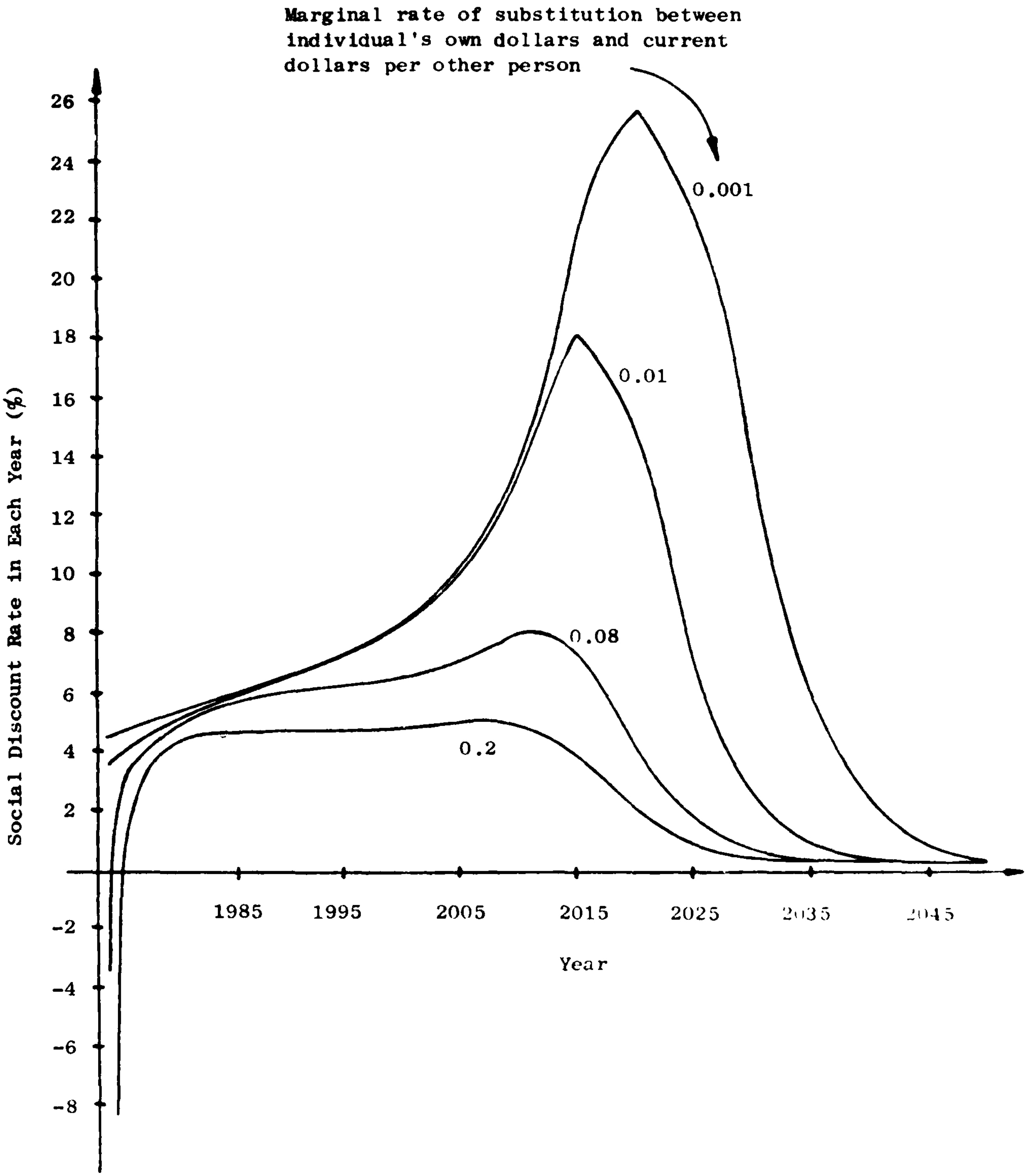

Fig. 4.8. THE SOCIAL DISCOUNT RATE AS A [illegible]
ARE EVALUATED IN DOLLARS PERSON.

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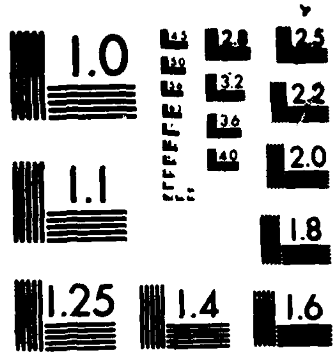

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When an individual cares little about others (MRS = 0.001), we might expect the discount rate in the first year to be equal to the rate of 3% used to compute the present value of an individual's income when he is alive. However, two factors make the initial discount rate for small marginal rates of substitution different from 3%. The most important factor is that each individual has some chance of dying within even the first year, so he effectively discounts outcomes by more than 3%. A smaller factor is the increase in population size, so that current citizens see outcomes being spread over a larger number of people in only a short time.

If population size is held constant, the implied discount rates in Fig. 4.9 result. It is obvious from a comparison of this figure with Fig. 4.8 that changing population size is a much smaller influence on the discount rate than mortality. The general shape of the discount rate as a function of time results mainly from the assumptions that individuals care about the well-being of others and that individuals have finite uncertain lifetimes.

The numerical value of the discount rate varies with the units in which future outcomes are measured. In the example thus far, future outcomes are measured in dollars per person alive in the future. If, instead, they are measured as a percent change in the gross national product, different discount rates result. In Fig. 4.10, the discount rate is plotted for several marginal rates of substitution between current dollars and percent change in the future GNP. The discount rate is different from that given in Fig. 4.8 because GNP per person is not constant over time.

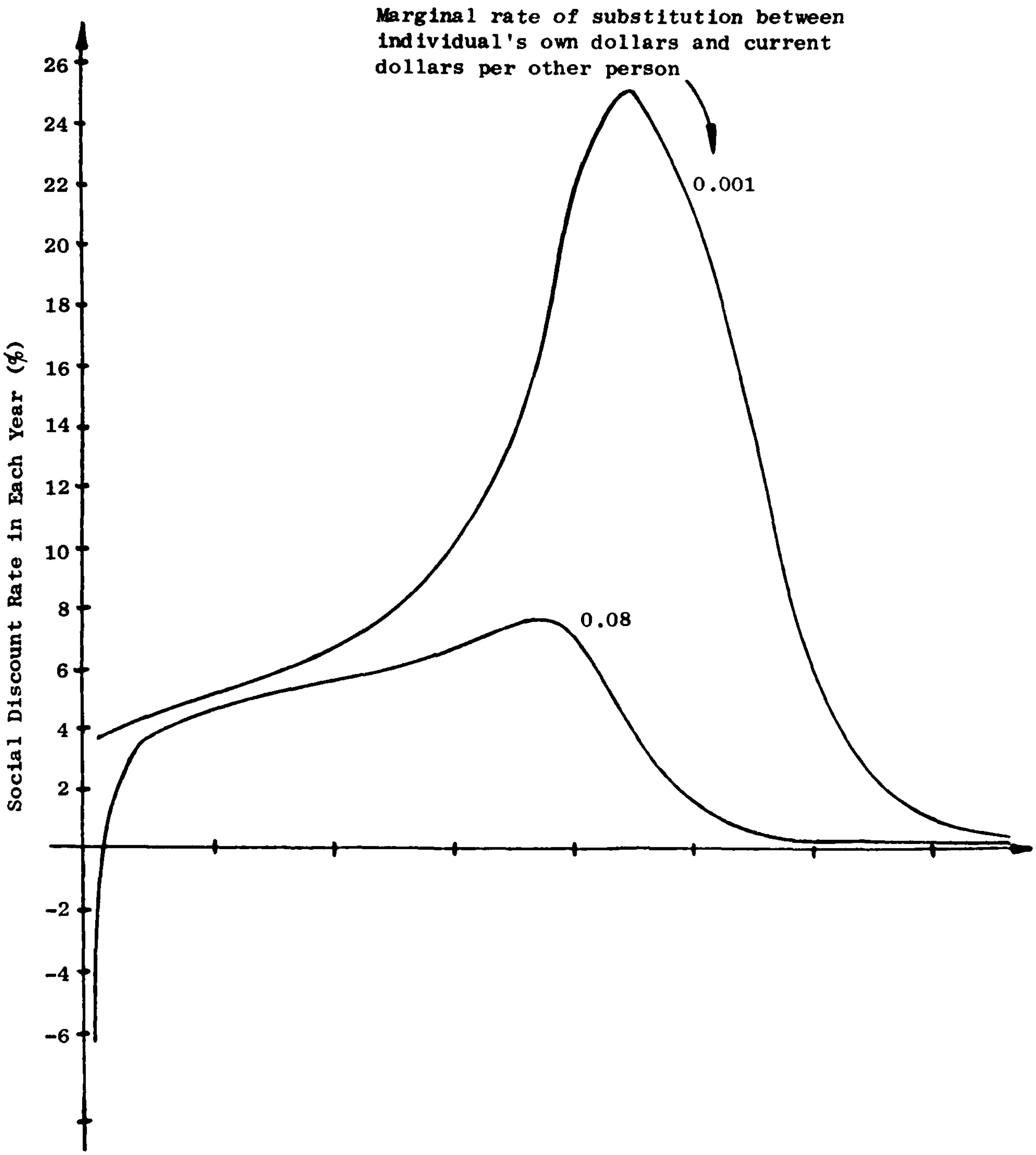


Fig. 4.9. THE SOCIAL DISCOUNT RATE AS A FUNCTION OF TIME WHEN THE TOTAL POPULATION REMAINS CONSTANT.

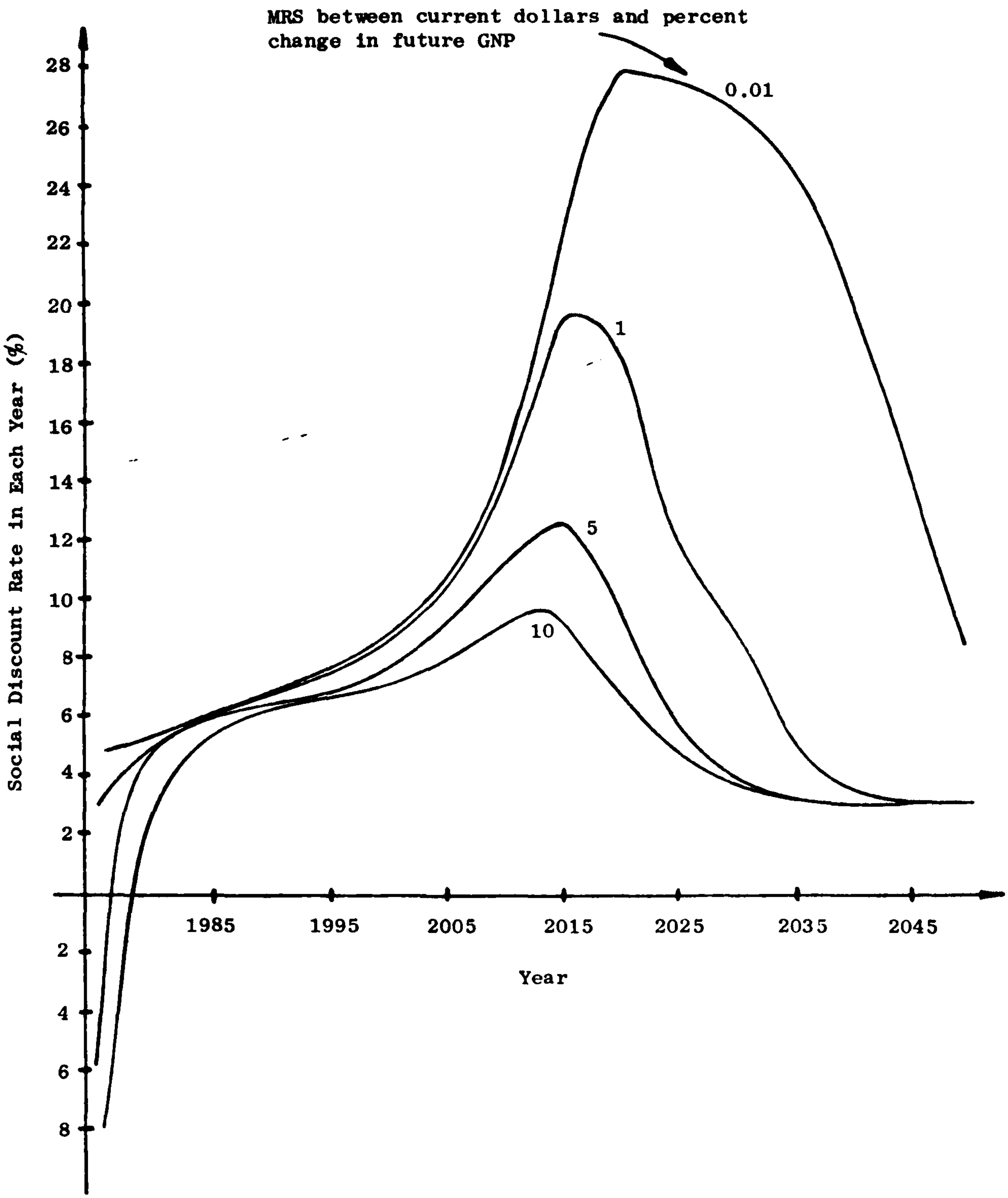


Fig. 4.10. THE SOCIAL DISCOUNT RATE AS A FUNCTION OF TIME WHEN FUTURE OUTCOMES ARE EVALUATED AS A PERCENT CHANGE IN GNP.

If we change the definition of the future attributes, we must also measure individuals' marginal rates of substitution in the new units. For example, to calculate Fig. 4.10, we might ask the question "How much would you pay per percent increase in the GNP forty years from now if you knew you would not be alive then?" This emphasizes that the social discount rate depends not only on how <u>much</u> people value the future but <u>what</u> it is about the future that they value. When the way in which we describe the future changes, the implied social rate of discount varies.

The social discount rate implied in a given year also depends on the distribution of the year's outcomes among individuals alive at the time. There are two ways in which distributional effects can influence the discount rate. First, a different distribution of the outcomes that are received directly by current individuals as changes in their own consumption would change total willingness to pay. This result occurs because individuals with different preferences would pay different amounts for equal changes in the probability distributions of the state variables. Second, if current individuals value the income distributions that exist during or after their lifetime, then changes in those distributions affect willingness to pay.

Given our assumptions, the implied discount rates do not depend on the cash flow of the project. This will be true whenever each individual's willingness to pay for outcomes in a particular year is proportional to the expected cash flow in the year. The proportionality constant can vary with the year but not with the size of the cash flow. In the example just presented, an individual's willingness to pay for outcomes in year i, given in Eq. (4.2), is proportional to the expected cash flow $\overline{d_i}$.

4.6 Conclusion of this Chapter

The point of the detailed discussion of the social discount rate that is implied by our methodology is that discounting is not the appropriate way to make a decision when the decision affects outcomes beyond the lifetimes of current citizens. If we want the decision to be consistent with the amounts individuals are willing to pay, we would have to use discount rates that vary with time, individuals' preferences, the units in which future outcomes are evaluated, and the distribution of outcomes among individuals. It is true that, if we had all of this information and it did not vary from one project to the next, we could compute discount "functions" that could be used in cost-benefit analysis of any project. However, this would force the cost-benefit approach to give the desired answer by using a complicated discount rate that varies with many parameters. It is more straightforward to base the decision directly on individuals' preferences and the amounts they are willing to pay. A discount rate is implied by the correct decision-making procedure but is really irrelevant to it.

Chapter 5

A PROCEDURE FOR ASSESSING THE REQUIRED INFORMATION

Three types of information are needed to apply the proposed approach. Demographic data, information about the uncertain state variables, and information about current individuals' preferences are required. The first two items would generally be provided entirely by experts designated by the decision maker, while at least part of the preference information would be obtained by interviewing current individuals who would be affected by the project.

5.1 Demographic Data

In order to compute willingness to pay for the project, current individuals are grouped into socioeconomic groups, such as age-income categories. The categories are chosen so that individuals in each group would experience similar changes in prices and income within their lifetimes as a result of the project and would be willing to make similar tradeoffs between themselves and "other" individuals. The number of current individuals in each group is part of the required demographic data. We also need to know the probabilities that individuals within each group would be alive at various dates in the future, which can be computed from mortality tables.

It may also be necessary to assess additional demographic data in order to estimate the vector of attributes z describing outcomes to "other" individuals. For example, if current individuals value the <u>average</u> well-being of people in the future, then we may need to estimate the future population. In general, demographic data describing the future will be uncertain.

5.2 Information About the State Variables

In Table 5.1, the information that is required about the uncertain state variables to calculate willingness to pay is listed. In this table, it is assumed that outcomes within an individual's life are measured in terms of the prices and income he faces. The subscripts i and j refer to different goods and k to different individuals.

The only information we need about the uncertain state variables to approximate willingness to pay to the first order are the changes in the expected values of the state variables as a result of undertaking the project. For example, if p_i is the current price for the i^{th} good, we only need to know the difference in $\overline{p_i}$ with and without the project:

$$\overline{p_i'} - \overline{p_i^o}$$

We do not need to assess the entire distribution of either p_i' or p_i^o, nor do we need to know the expected value with the project, $\overline{p_i'}$. Since all we need is the <u>change</u> that results from the project, the assessment task is considerably simplified.

The assessment of changes in the expected prices of goods and changes in the expected present value incomes of individuals is straightforward. A more challenging task involves the changes in the expected state variables describing outcomes to "other" individuals. We must first know what individuals value about the well-being of others, both others in the same generation and others in the future. Do current individuals care about only the total dollar benefit to others (as assumed in cost-benefit analysis), or do they value the average well-being or the specific income distribution? We must also know how individuals

Table 5.1

DATA REQUIRED TO APPLY THE METHODOLOGY

	First Order Approximation	Second Order Approximation	Risk Aversion
State Variables	$\left(\overline{p_i'} - \overline{p_i^o}\right)$ for all i $\left(\overline{m_k'} - \overline{m_k^o}\right)$ for all k $\left(\overline{z_j'} - \overline{z_j^o}\right)$ for all j	$\mathrm{cov}\left(p_i', p_j'\right) - \mathrm{cov}\left(p_i^o, p_j^o\right)$ for all i,j ${}^{v}\langle m_k' \rangle - {}^{v}\langle m_k^o \rangle$ for all k $\mathrm{cov}\left(z_i', z_j'\right) - \mathrm{cov}\left(z_i^o, z_j^o\right)$ for all i,j $\mathrm{cov}\left(p_i', m_k'\right) - \mathrm{cov}\left(p_i^o, m_k^o\right)$ for all i,k $\mathrm{cov}\left(p_i', z_j'\right) - \mathrm{cov}\left(p_i^o, z_j^o\right)$ for all i,j $\mathrm{cov}\left(m_k', z_j'\right) - \mathrm{cov}\left(m_k^o, z_j^o\right)$ for all k,j	
Preferences	$\frac{\partial V_k/\partial z_i}{\partial V_k/\partial h}$ for all i,k D_j for all j	$\frac{\partial^2 V_k}{\partial h \partial z_i}, \frac{\partial^2 V_k}{\partial z_i \partial z_j}$ for all i,j,k $\frac{\partial D_i}{\partial M}, \frac{\partial D_i}{\partial p_j}$ for all i,j	$\left(r_v\right)_k$ for all k

conceptualize the "future" beyond their lifetime. What distinguishes different periods in the future and what state variables describe each period? The answers to these questions may vary for individuals in different socioeconomic groups.

The state variables that are chosen to describe outcomes to other individuals must satisfy some simple conditions. First, each variable should be defined so that more of the attribute is preferred to less, and each variable must pass the clairvoyant's test [10]. The vector of variables z should completely describe outcomes of the project to "others" in the sense that the individual would be willing to decide if he favored the project knowing the effect of the project on the value of z and on his own consumption.

If we want to include second-order terms in the approximation of willingness to pay, the assessment task is more involved. We need to assess the change in covariances of each pair of state variables that results from the project, as listed in the second column of Table 5.1. To do this, we can use an approximation for the covariance of two variables s_i and s_j given by Owen [27]:

$$\operatorname{cov}(s_i, s_j) = \left.\frac{\partial\langle s_j | s_i\rangle}{\partial s_i}\right|_{\overline{s_i}} {}^{v}\langle s_i\rangle + \frac{1}{2}\left.\frac{\partial^2\langle s_j | s_i\rangle}{\partial s_i^2}\right|_{\overline{s_i}} \left\langle (s_i - \overline{s_i})^3 \right\rangle$$

Often, the second term can be ignored because the third moment is small, for instance, when the distribution is approximately symmetric. Using this approximation, we can calculate, for example, the change in covariances of the prices p_i and p_j:

$$\mathrm{cov}\left(p_i',p_j'\right) - \mathrm{cov}\left(p_i^o,p_j^o\right) = \left.\frac{\partial\langle p_j'|p_i'\rangle}{\partial p_i'}\right|_{\overline{p_i'}} {}^{v}\langle p_i'\rangle - \left.\frac{\partial\langle p_j^o|p_i^o\rangle}{\partial p_i^o}\right|_{\overline{p_i^o}} {}^{v}\langle p_i^o\rangle \tag{5.1}$$

To compute the change in covariance, we need to know the variance of p_i with and without the project. We can ask the expert the variances directly, or we can measure the variance after assessing the probability distributions on p_i^o and p_i'. We also need the slope of the conditional means of p_j^o and p_j'. We can ask the expert to sketch the conditional means and then measure the slopes, or we may be able to ask him the rate of change of the conditional means directly. We note that Eq. (5.1) can be written with i and j reversed due to the symmetry of the covariance:

$$\mathrm{cov}\ (p_i,p_j) = \mathrm{cov}\ (p_j,p_i)$$

Depending on the way in which we have defined the goods i and j, it may be easier to assess the required data with the change in covariance expressed in this alternative form.

In assessing all of the information about the state variables, we follow the guidelines given by Spetzler for eliminating biases and errors in assessment [32].

5.3 First-Order Preference Information

Part of the information on preferences could be provided by experts. In particular, since the first derivatives of individuals' value functions are demand functions, expert opinion could be the source of data concerning the demands for goods and services. Current individuals would then be interviewed to determine the additional derivatives of their value functions involved in computing willingness to pay. Of course, it would usually be too expensive to interview everyone affected by a project, so a scheme for sampling individuals from socioeconomic categories that classify people according to preferences would probably be used. In the rest of this section, we will simply talk about "the individual," with the understanding that we would have to repeat the assessment of preference information for each socioeconomic group.

In order to measure the present value equivalent of outcomes within an individual's lifetime, we need to know the interest rate for computing present value. This is, the individual's time preference for dollars paid to him at a future date, given that he will be alive then. If outcomes are expressed in current dollars, without inflation, then this could be related to the real rate of interest on long term annuities. In the helium storage example, we used 3%.

To calculate willingness to pay to a first-order approximation, we need to measure the individual's marginal rates of substitution between his own current dollars and each attribute describing outcomes to others (Table 5.1). It is convenient to assess this information by asking questions about the quantity λ_{h,z_i} defined as follows:

$$\lambda_{h,z_i} = \frac{\partial V/\partial h}{\partial V/\partial z_i}$$

In this and following expressions, we have dropped the subscript k denoting the individual. The quantity λ_{h,z_i} is the answer to the question "By how many units would the attribute z_i have to be increased for you to sacrifice one current dollar?" Alternatively, we could ask "By how many units could the attribute z_i decrease if we paid you one current dollar?" The answers should be the same since they both define the rate at which the individual would trade the well-being of other individuals for his own well-being.

To calculate willingness to pay, the quantity λ_{h,z_i} is evaluated at the vector of mean values $(\overline{p^o},\overline{m^o},\overline{z^o})$. That is, the questions above are conditioned by the attributes being set at the expected values without the project. It is important to make this point clear to the individual to help him distinguish questions of value from uncertainty about the outcomes. By fixing the point of evaluation, we are eliminating uncertainty and asking him questions about his deterministic tradeoffs.

If z_i refers to outcomes received by others after the current individual's lifetime, then the questions above are also conditioned on the fact that the individual would not be alive. For example, if z_i is evaluated in units of dollars per person alive in forty years, then we ask "By how many dollars per person would we have to increase income in forty years, given that you won't be alive to receive the increase, for you to pay one dollar now?"

As mentioned before, we also need to know the market demands for those goods and services that current individuals receive within their lifetime, whose prices or quantities are changed by the project. These demands are evaluated at the vector of mean outcomes without the project $(\overline{p^o},\overline{m^o},\overline{z^o})$. This information would generally come from economic experts.

5.4 Second-Order Preference Information

If we want to include second-order terms in the approximation of willingness to pay, we must assess the second derivatives of the individual's value function listed in Table 5.1. Our procedure for doing this is based on the preference parameters defined by Keelin [18]. Rather than explaining the theory in detail, we will simply describe the sequence of assessments in this section and delay the mathematics to Appendix G.

(1) We first assess the cross derivative $\partial^2 V/\partial h \partial z_i$ for each of the attributes z_i describing outcomes to other individuals. Intuitively, the second derivative of the value function is the rate at which the individual's tradeoff between himself and others (that is, the first derivative) changes with his own well-being. Thus, we ask the individual his substitution rate λ_{h,z_i} at two different values of his wealth, as illustrated in Fig. 5.1.

First, we assess λ at the expected values without the project:

$$\lambda^o = \lambda_{h,z_i}\Big|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} = \frac{\partial V/\partial h}{\partial V/\partial z_i}\Big|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}}$$

Next, we ask the individual his value of λ if we make him $\$\Delta$ better off, that is, if we increase his present value income by $\$\Delta$:

$$\lambda^\Delta = \lambda_{h,z_i}\Big|_{h\left(\overline{p^o},\overline{m^o}\right)+\Delta,\overline{z^o}}$$

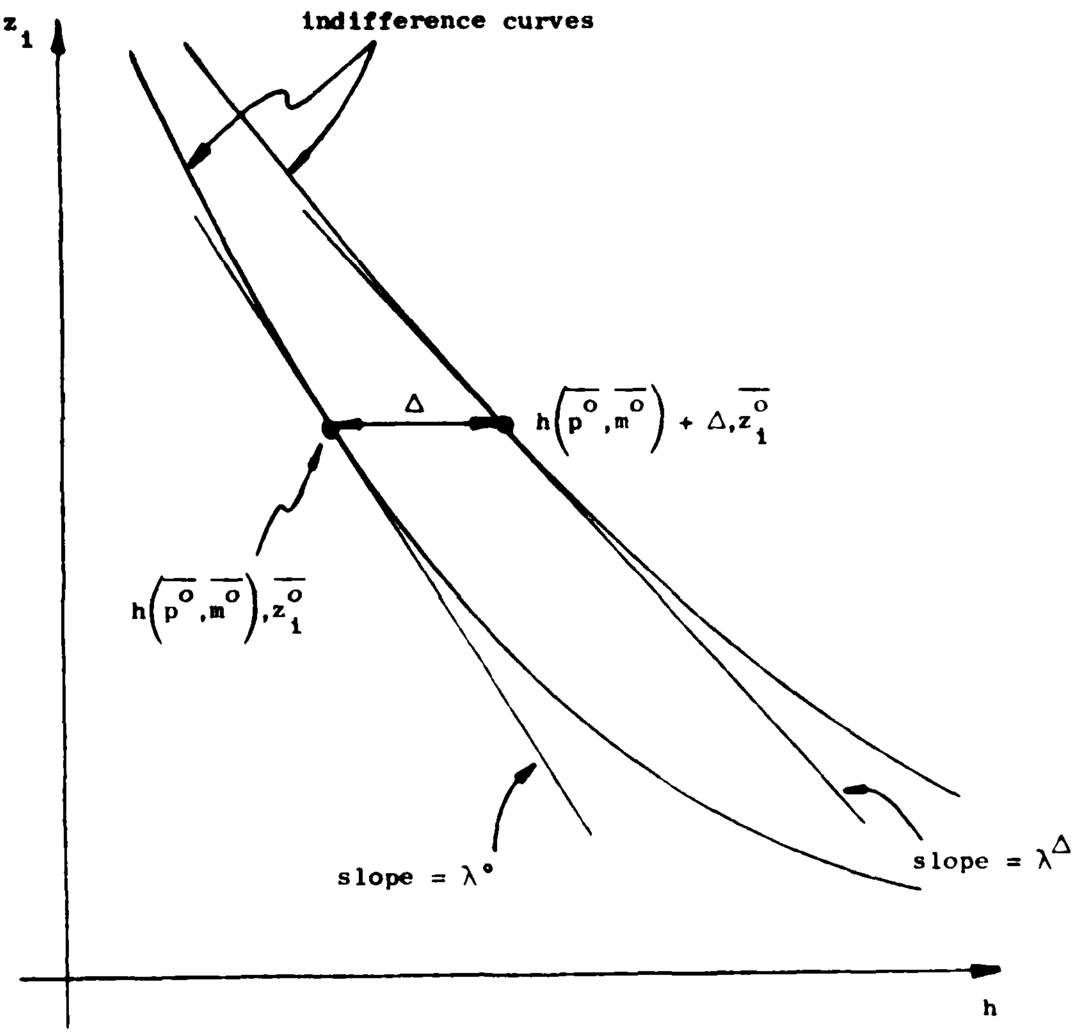


Fig. 5.1. THE ASSESSMENT OF DATA NEEDED TO CALCULATE THE DERIVATIVES $\partial^2 V/\partial h \partial z_i$ FOR ALL z_i.

Then, as shown in Appendix G, we can solve for the desired cross derivative in terms of the individual's answers and the increment Δ:

$$\left.\frac{\partial^2 V}{\partial h \partial z_i}\right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} = \frac{\frac{1}{\lambda^{\Delta}} - \frac{1}{\lambda^{o}}}{\Delta}$$

The quantity λ^{Δ} will usually be smaller than λ^{o} because, as an individual becomes wealthier, he would be willing to accept less of an increase in the well-being of others for his own sacrifice of one current dollar. If this is the case, the resulting derivative will be positive.

We note that the derivative that we calculate is related to the quantity Z_{h,z_i} which Keelin defines as the "marginal value reduction coefficient." For the assumptions we have made about the individual's value function, we get:

$$Z_{h,z_i} = \frac{\frac{\partial^2 V}{\partial h \partial z_i}}{\partial V/\partial z_i} = \frac{\frac{\lambda^{o}}{\lambda^{\Delta}} - 1}{\Delta}$$

In Section 5.5, we indicate how this coefficient can be used in checking the consistency of the individual's preferences.

(2) Next, we assess the second derivative $\partial^2 V/\partial z_i^2$ for each of the attributes z_i. Starting at the expected values without the project, $(\overline{p^o},\overline{m^o},\overline{z^o})$, we let the individual's present value equivalent wealth decrease by Δ and let λ^{o} be the substitution rate which we have already assessed. We ask the individual whether he would be willing to be compensated at his marginal rate of substitution for changes in the

attribute z_i (Fig. 5.2). That is, would he accept the following trade:

$$h\left(\overline{p^o},\overline{m^o}\right),\overline{z_i^o} \longrightarrow h\left(\overline{p^o},\overline{m^o}\right) - \Delta,\overline{z_i^o} + \lambda^o\Delta$$

If his indifference curves are convex, as in the figure, he refuses. Then, we ask him what part $\emptyset$ of the decrease in his wealth we would have to give back so that he would be indifferent.

His answer allows us to calculate a quantity S_{h,z_i} which Keelin calls the "substitution aversion coefficient":

$$S_{h,z_i}\Big|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} \doteq \frac{2\emptyset}{\Delta^2}$$

Given our assumptions on the value function V, this coefficient can also be written in terms of the derivatives of the value function:

$$S_{h,z_i} = -\frac{\frac{\partial V}{\partial h}\frac{\partial^2 V}{\partial z_i^2}}{\left(\frac{\partial V}{\partial z_i}\right)^2} + 2\,\frac{\frac{\partial^2 V}{\partial h\,\partial z_i}}{\frac{\partial V}{\partial z_i}}$$

Thus, we can solve for the desired second derivative in terms of the individual's answer $\emptyset$ and the previous assessments λ^o and λ^Δ:

$$\frac{\partial^2 V}{\partial z_i^2}\Bigg|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} = 2\,\frac{\left(\frac{1}{\lambda^\Delta} - \frac{1}{\lambda^o}\right)}{\Delta\lambda^o} - 2\,\frac{\emptyset}{\Delta^2\left(\lambda^o\right)^2}$$

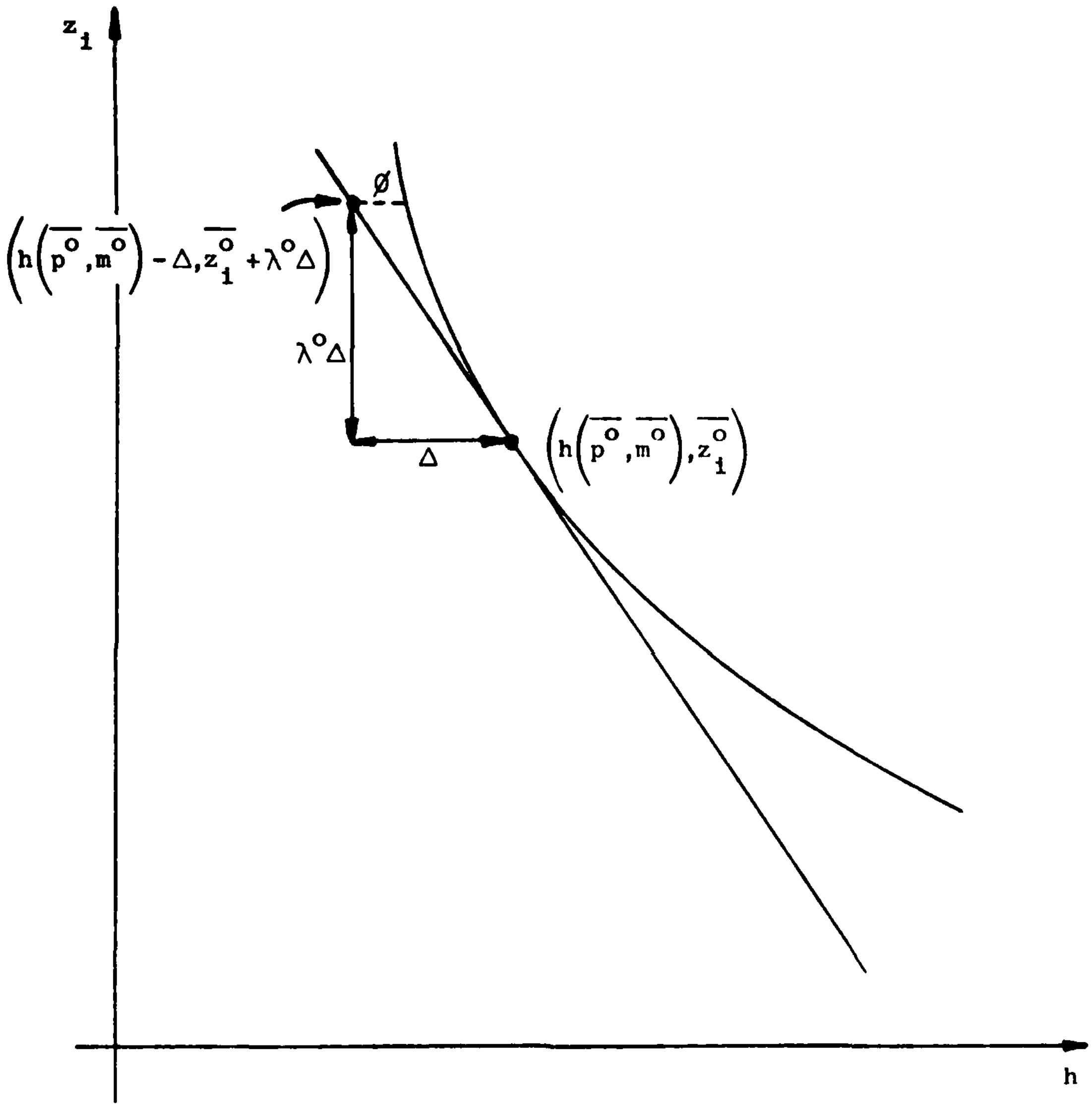


Fig. 5.2. THE ASSESSMENT OF DATA NEEDED TO CALCULATE THE DERIVATIVES $\partial^2 V/\partial z_i^2$ FOR ALL z_i.

(3) Finally, we assess the cross derivative between two attributes describing outcomes to "other" individuals, $\partial^2 V/\partial z_i \partial z_j$, for each pair of attributes z_i and z_j. To do this, we must first ask the individual the rate λ_{z_i,z_j} at which he would trade between the two attributes:

$$\lambda^1 = \lambda_{z_i,z_j}\Big|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} = \frac{\partial V/\partial z_i}{\partial V/\partial z_j}\Big|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}}$$

In this expression, all outcomes are set to their expected values without the project $(\overline{p^o},\overline{m^o},\overline{z^o})$. The quantity λ^1 is the answer to the question "By how many units would the attribute z_j have to be increased, if z_i were decreased by one unit, for you to be indifferent?"

Then, we repeat step 2 above using λ^1 as the substitution rate. That is, we ask the individual if he would be willing to be compensated at the marginal rate λ^1 for changes in the attribute z_j. Would he accept the trade

$$\overline{z_i^o},\overline{z_j^o} \longrightarrow \overline{z_i^o} - \Delta,\overline{z_j^o} + \lambda^1\Delta$$

given that all other variables are fixed. When he refuses, we ask what fraction $\emptyset$ of the decrease in z_i we would have to return to make him indifferent. This allows us to calculate the substitution aversion coefficient for the attributes z_i and z_j, denoted s_{z_i,z_j}:

$$s_{z_i,z_j}\Big|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} \doteq \frac{2\emptyset}{\Delta^2} \qquad (5.2)$$

The coefficient can also be written in the form:

$$S_{z_i,z_j} = -\frac{\dfrac{\partial^2 V}{\partial z_i^2}}{\dfrac{\partial V}{\partial z_i}} - \frac{\dfrac{\partial V}{\partial z_i}\dfrac{\partial^2 V}{\partial z_j^2}}{\left(\dfrac{\partial V}{\partial z_j}\right)^2} + 2\,\frac{\dfrac{\partial^2 V}{\partial z_i \partial z_j}}{\dfrac{\partial V}{\partial z_j}} \tag{5.3}$$

Since we already know the values of $\partial V/\partial z_i$, $\partial^2 V/\partial z_i^2$, $\partial V/\partial z_j$, and $\partial^2 V/\partial z_j^2$ from the first part of our assessment, we can solve Eqs. (5.2) and (5.3) for the desired derivative $\partial^2 V/\partial z_i \partial z_j$.

As part of the second-order approximation to willingness to pay, we also need to know the derivatives of the market demands for those goods and services that current individuals receive within their lifetimes, whose prices or quantities are changed by the project. In particular, we need the rate at which market demands vary with both prices and the total income of individuals who are affected by the project (Table 5.1). These derivatives are evaluated at the vector of mean outcomes without the project $(\overline{p^o},\overline{m^o},\overline{z^o})$. This information would generally come from experts or econometric studies.

5.5 Consistency Conditions

There are two ways in which we might want the preference information we assess from individuals to be consistent. The first requirement is that the answers be internally consistent in the sense that they do not contradict each other. We can use the ordinal preference identities given by Keelin to derive various tests for this type of consistency.

Keelin derives the following relationships between the preference measures which we used in the two preceding sections:

$$\lambda_{a,b} = \lambda_{a,c} \cdot \lambda_{c,b}$$

$$S_{a,b} = Z_{a,b} + \lambda_{a,b} \cdot Z_{b,a}$$

$$S_{a,b} = \lambda_{a,b} \cdot S_{b,a}$$

In these expressions, a, b, and c denote the variables with respect to which tradeoffs are considered and derivatives are taken. By considering various combinations of variables, we can check the internal consistency of the data. For example, let z_1 denote outcomes to other people alive after the individual's lifetime and let z_2 denote outcomes to other people who are alive at the same time as the individual. Then, we require that:

$$\lambda_{h,z_1} = \lambda_{h,z_2} \cdot \lambda_{z_2,z_1}$$

where all terms are evaluated at the same point. That is, the rate at which the individual would trade outcomes to others in the future for his own current dollars is the product of the rate at which he would trade outcomes to others who are alive at the same time for his own current dollars, multiplied by the rate at which he would trade outcomes to others in the future for outcomes to others who are alive at the same time.

The second consistency requirement that we would like to place on the preferences that the individual expresses is that they be consistent with other decisions he makes involving those preferences. For example, the individual's marginal rate of substitution between outcomes to others who are alive at the same time and his own current dollars should reflect the amount he actually contributes to charities. For this reason, we would ask the individual questions about other decisions and tradeoffs he has made before beginning the assessment procedure. We may also be able to develop a set of guidelines that specify what other decisions (such as charity contributions) an individual should make as a function of what it is he values about outcomes to others and how much he values them. As the assessment progresses, answers falling outside these guidelines would warrant further investigation.

5.6 Risk Aversion

In order to compute willingness to pay for non-expected-value individuals, we must assess each individual's risk aversion coefficient r_v on value V:

$$r_v \Big|_{\langle V[h(p^o,m^o),z^o]\rangle}$$

The risk aversion is evaluated at the mean value the individual would expect without the project, $\bar{V}$. We can simplify the assessment by making some approximations. First, we note that, using a formula given by Keelin [18], the risk aversion on present value equivalent dollars h is related to risk aversion on value V(h,z) by the following:

$$r_h = r_v \frac{\partial V}{\partial h} - \frac{\partial^2 V/\partial h^2}{\partial V/\partial h}$$

By our previous assumption that V is linear in h, we can write r_v simply as:

$$r_v \Big|_{\langle V[h(p^o,m^o),z^o]\rangle} = \frac{r_h}{\partial V/\partial h}\Big|_{\langle V[h(p^o,m^o),z^o]\rangle}$$

Next, we approximate the mean of V by the value at the expected outcomes without the project:

$$\left\langle V\left[h(p^o,m^o),z^o\right]\right\rangle \doteq V\left[h(\overline{p^o},\overline{m^o}),\overline{z^o}\right]$$

The individual's value function is unique only to a monotonically increasing transformation. As discussed in Appendix G, we choose V so that:

$$\left.\frac{\partial V}{\partial h}\right|_{h(\overline{p^o},\overline{m^o}),\overline{z^o}} = 1$$

The result of these simplifications is that we can assess the required risk-aversion coefficient approximately by assessing the risk aversion on current dollars at the mean values without the project:

$$\left.r_V\right|_{\langle V[h(p^o,m^o),z^o]\rangle} \doteq \left.r_h\right|_{h(\overline{p^o},\overline{m^o}),\overline{z^o}}$$

This makes the assessment of the risk-aversion coefficient quite routine.

5.7 A Summary of the Assessment Procedure

In Tables 5.2a, 5.2b, and 5.2c, the procedure we have just discussed is summarized in a flow chart. This flow chart provides the details of the assessment step that was listed in Section 3.9, "Summary of the Proposed Approach."

Table 5.2a

ASSESSMENT FLOW CHART

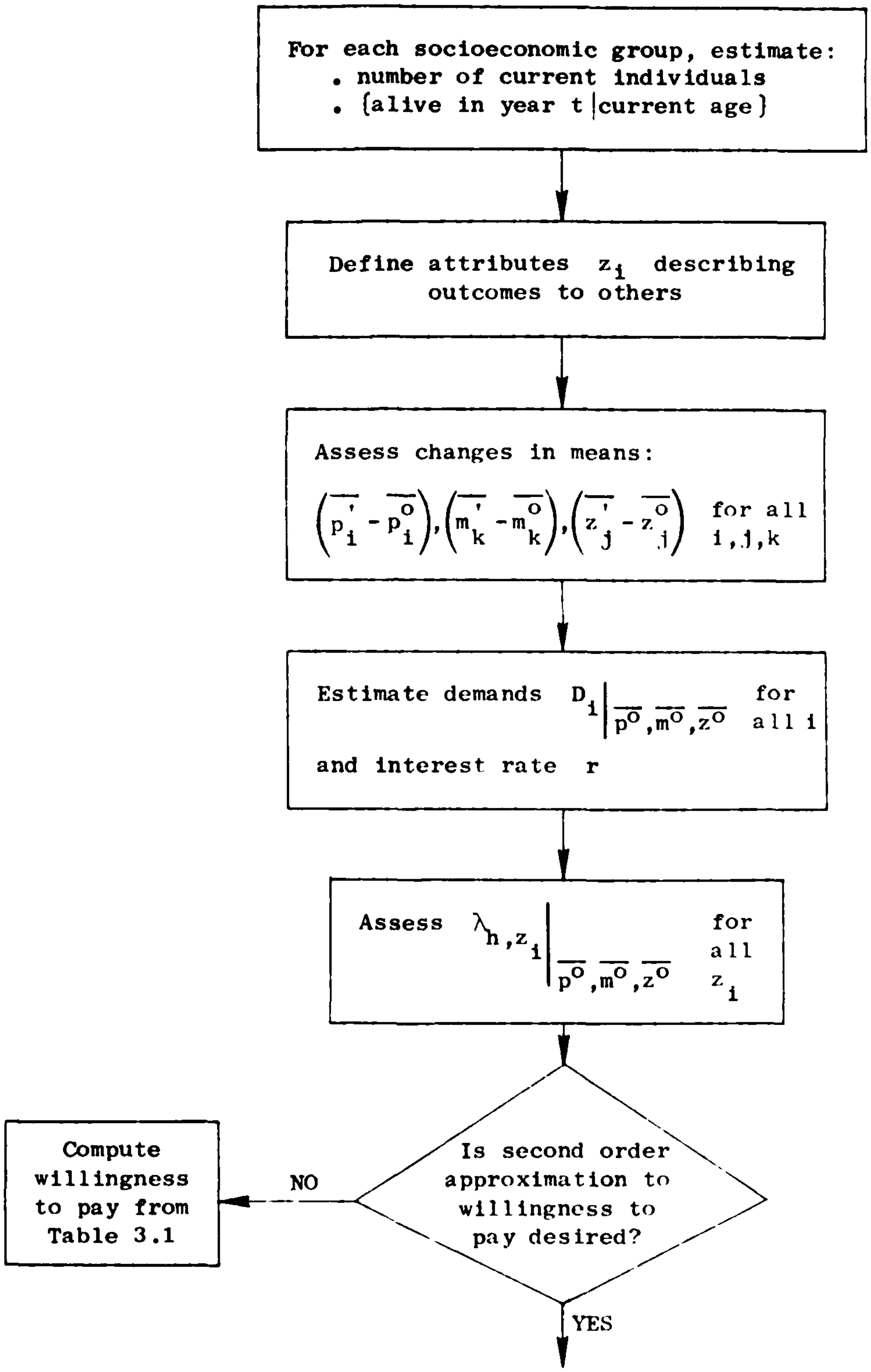


Table 5.2b

ASSESSMENT FLOW CHART

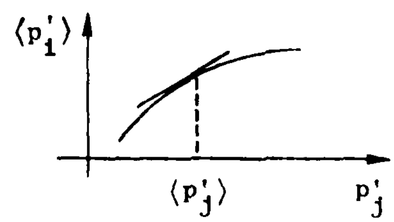
Second order approximation desired

↓

Assess ${}^{v}\langle p_i^o \rangle$, ${}^{v}\langle p_i' \rangle$, ${}^{v}\langle m_k^o \rangle$, ${}^{v}\langle m_k' \rangle$, ${}^{v}\langle z_j^o \rangle$, ${}^{v}\langle z_j' \rangle$ for all i,j,k

↓

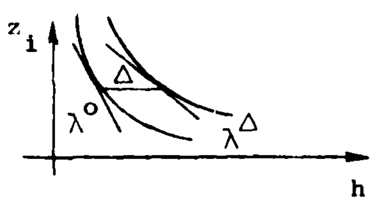
Assess slopes of conditional means for all pairs of state variables, e.g.:



↓

Compute covariances for all pairs of state variables using Eq. (5.1)

↓



Assess $\lambda^o, \lambda^\Delta, \Delta$ for all z_i

↓

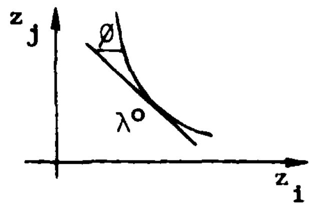
$$\left.\frac{\partial^2 v}{\partial h \partial z_i}\right|_{\overline{p^o},\overline{m^o},\overline{z^o}} = \frac{\frac{1}{\lambda^\Delta} - \frac{1}{\lambda^o}}{\Delta}$$ for all z_i

↓

Assess $\left.\lambda_{z_i,z_j}\right|_{\overline{p^o},\overline{m^o},\overline{z^o}}$ for all pairs z_i, z_j

Table 5.2c

ASSESSMENT FLOW CHART



Assess $\emptyset$ for each pair z_i, z_j

↓

Solve Eqs. (5.2) and (5.3) for

$\dfrac{\partial^2 V}{\partial z_i \partial z_j}$ for all pairs z_i, z_j

↓

Estimate $\dfrac{\partial D_i}{\partial M}, \dfrac{\partial D_i}{\partial p_j}$ for all i, j

↓

Consistency checks:

$\lambda_{a,b} \overset{?}{=} \lambda_{a,c}\lambda_{c,b}$

$S_{a,b} \overset{?}{=} Z_{a,b} + \lambda_{a,b} Z_{b,a}$

$S_{a,b} \overset{?}{=} \lambda_{a,b} S_{b,a}$

↓

$0 \approx$ ⟨ .5: ρ ; .5: $-\rho/2$ ⟩ $\qquad r_v \big|_{(v)} \doteq 1/\rho$

↓

Compute willingness to pay from Table 3.1

5.8 An Application of the Assessment Procedure

A small experimental program was undertaken to determine whether individuals can understand and answer questions of the type required by the assessment procedure. Ten individuals were interviewed to find out what they valued about the future and how much they valued the future. In this section, we discuss the results of these interviews and make additional suggestions for the practical application of the assessment procedure, based on the results.

Each interview began with a discussion of the purpose of the interview and the procedure to be used. Next, individuals were asked to describe briefly three alternative scenarios for the world's economic, political, and social state in fifty years, ranging from a very "good" scenario to a "bad" one. Then, the concept of a "clairvoyant" was introduced. The clairvoyant assured the interviewee that his second scenario actually would occur. The purpose of this step was to eliminate uncertain factors about the state of the world in fifty years, so that deterministic tradeoffs could be addressed.

The individual was then told that the current generation had undertaken a project that would affect individuals in fifty years. The interviewee was asked to describe what he would see happening to future individuals if the project were "beneficial" and if it were "detrimental." Quantifiable variables were found to represent each qualitative benefit and detriment that the individual mentioned. A majority of the interviewing effort was spent identifying the attributes which described outcomes to others. When the list of attributes was completed for an individual, an attempt was made to consolidate the list.

To the extent that time permitted, individuals were asked to quantify their marginal rates of substitution between their own current dollars and one or more of the attributes with which they described the future outcomes. The interviewees were asked how they had arrived at their answers. Finally, internal consistency of their answers and consistency with the individual's charitable contributions were reviewed, and the interviewees were asked to resolve any differences.

5.8.1 The Results of the Interviews

The answers that individuals gave pointed out one of the main advantages of the approach we are proposing when compared to cost-benefit techniques. Cost-benefit analysis evaluates outcomes to future generations in terms, such as the gross national product, that are surrogates for future willingness to pay. However, only one of ten interviewees was willing to use a measure of future willingness to pay as the sole attribute describing outcomes to future individuals. The remaining interviewees wanted to know not only how well off the future generation thinks they are, in dollars of income, but what they do with the additional income. That is, how do future individuals spend the additional income on health, food, environmental quality, cultural activities, etc.

Table 5.3 is a list of all of the attributes describing outcomes to future individuals which the ten interviewees discussed. The attributes are grouped into broad categories. Most individuals wanted to know the value of several of the attributes, for the entire world, even though the hypothetical project directly influenced only individuals in the United States.

Table 5.3

ATTRIBUTES DESCRIBING OUTCOMES TO FUTURE INDIVIDUALS

<table>
<tr><th>Economic Performance and Structure</th><th>Environment</th></tr>
<tr><td rowspan="5"><u>Health</u>
. number of people having various diseases/year
. mortality rates for each age group

<u>Food</u>
. average percent of income required to purchase minimum diet
. number of people who eat a minimum diet

<u>Education</u>
. percent literacy

<u>Housing</u>
. average percent of income required to purchase minimum housing

<u>General</u>
. percent of population at various incomes
. hours of leisure time/person
. population size
. gross national product
. gross national product/person
. average percent of income that is disposable
. percent unemployed</td><td>. average air and water pollution indices
. number of species of plants and animals
. number of genetic diseases/year
. average level of carcinogens in air and water
. number of acres of undeveloped land
. reserves of depletable resources
. percent of lakes and streams that are "drinkable"</td></tr>
<tr><th>Political Structure</th></tr>
<tr><td>. number of people killed in wars/conflicts
. percent of gross national product spent on defense, education, welfare
. average response to "What percent of the time is government responsive to needs?"</td></tr>
<tr><th>Social/Cultural/Personal</th></tr>
<tr><td>. number of child abuses/year
. number of violent crimes/year
. percent of people who answer yes to "Are material goals less important than humanistic goals?"
. number of library trips/person
. result of nationwide psychological test that predicts whether people will steal when they have the opportunity to do so
. percent of literary and artistic works produced that survive for 250 years</td></tr>
</table>

A few of the attributes were very difficult to put in quantifiable terms. They were finally expressed in terms of the results of surveys taken in the future. For example, one interviewee wanted to know the results of a nationwide psychological test designed to predict whether individuals would steal if they had the opportunity to do so. Another person wanted to know the percent of future individuals who answer yes to the question "Are materialistic goals less important than humanistic goals?"

Theoretically, all of the attributes in Table 5.3 could be influenced by a project that has future outcomes. However, the linkage between the first-order effects of the project and these attributes may be indirect. For example, consider the helium storage project described in Chapter 5. Figure 5.3 shows how the quantity of helium that is stored is linked to one future attribute from Table 5.3.

The direct effect of the project is to release helium to the market, thereby increasing the future supply of helium at any given price. According to the equilibrium between supply and demand, the price of helium decreases and the quantity consumed increases. Then, assuming that helium is used in advanced electric power applications, the price of electricity to future consumers declines.

A decline in electricity prices may affect future individuals consumption of many other goods. However, not all attributes of future consumption are valued by current individuals. Assume, for example, that current individuals care about the amount of leisure time in the future. If we know the elasticity of the future demand for leisure to the future price of electricity, we can calculate the change in the leisure time of future individuals as a result of the decreased price of electricity.

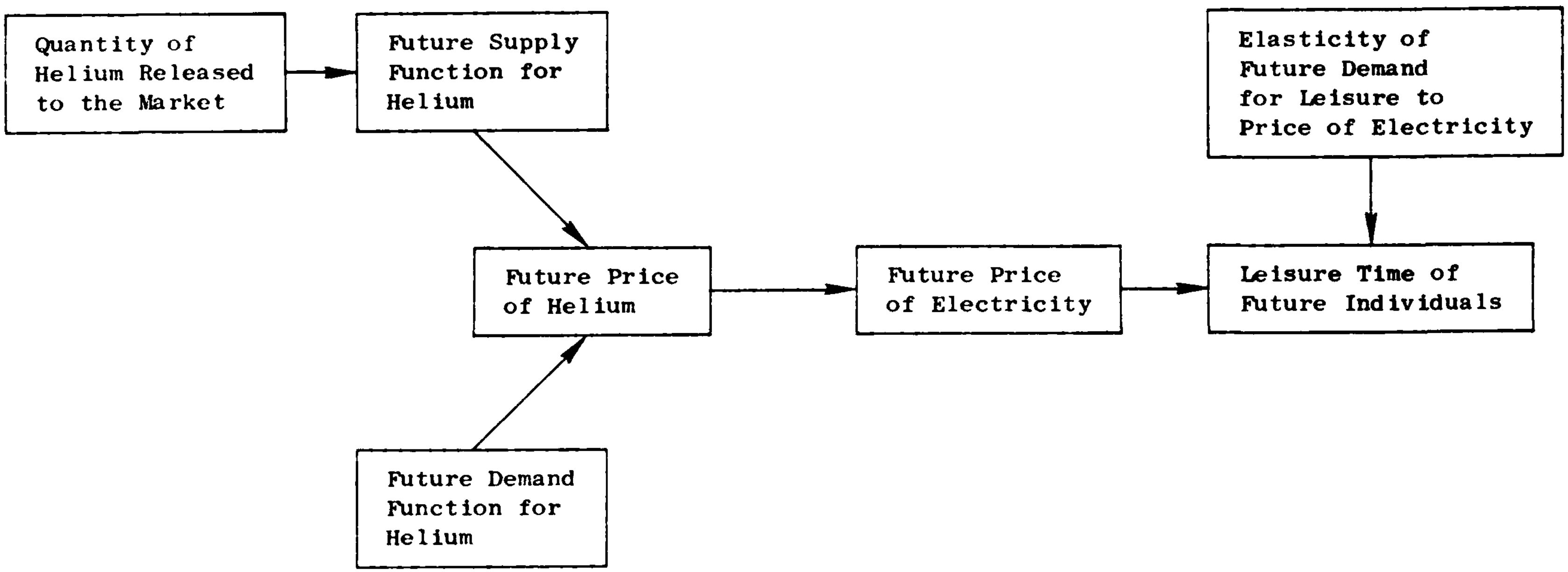


Fig. 5.3. THE LINKAGE BETWEEN STORED HELIUM AND THE LEISURE TIME OF FUTURE INDIVIDUALS.

We note that these calculations require some information about future preferences, such as the price elasticities of future demand functions. This information is required because, as the interviews indicate, current individuals value more than just future willingness to pay for the direct benefits of a project. The current generation wants to know how future individuals use the additional income opportunity, and this information depends on future preferences.

After determining what future attributes an interviewee valued, marginal rates of substitution were assessed between an individual's own current dollars and one or more attributes. This was done by asking each interviewee what change would constitute a "significant" improvement or detriment in an attribute and how much they would pay for this change. Table 5.4 lists some of the answers received. Column 1 gives the future attribute. The changes that are given as percents in column 2 are changes over a fifty-year period measured relative to the expected outcome without the project. For example, a 20% decrease in the number of violent crimes means that, for a fifty-year period starting fifty years from now, the expected number of violent crimes in each year will be 20% less than the number expected without the project. The numbers in column 3 are the present values that individuals would pay for the changes in column 2.

Interviewees were also asked what they valued about outcomes to other individuals who are alive now. Not surprisingly, the attributes that describe outcomes to "other" individuals are the same, whether those others are in the current generation or a future generation. A few individuals were asked how the amount they would pay for a change in an attribute that affects others would differ if the outcome were received now rather than in fifty years. Their answers indicated that outcomes to

Table 5.4

THE AMOUNTS CURRENT INDIVIDUALS WOULD PAY FOR CHANGES IN FUTURE ATTRIBUTES

Future Attribute	Change in the Attribute (from the Expected Value Without the Project) Over a Fifty-Year Period	Current Dollars Paid for the Change
1. Number of violent crimes/year	-20%	2000
2. Number of people who eat a minimum diet	+20%	300
3. Number of genetic diseases/year	-10%	20,000
4. Gross national product	+10%	10
5. Gross national product/person	+$100	25
6. Average air pollution index	-100 points*	80
7. Number of lakes and streams that are drinkable	+50%	2000

*Measured by the EPA Pollutant Standards Index, which has a scale from 0 to 500, 0 being the cleanest air.

other individuals who are alive now are worth about three times as much as outcomes to others in fifty years.

Because of the limited scope of the interviews, it was difficult to determine if the results were consistent with individuals' charitable contributions. Several interviewees said that they did not contribute significantly because they did not know of charities that benefited the attributes that they valued. When individuals did contribute time or money to charities, the charities generally benefited the attributes which the individuals valued.

5.8.2 The Biases that were Observed and Suggestions for Correcting Them

All of the ten people who were interviewed exhibited biases in their responses. In fact, the major obstacle in obtaining answers was not the individual's ability to understand the questions but the biases that entered into the answers. From the experience of these initial interviews, we can make some suggestions for designing an interview procedure that avoids or compensates for these biases.

Previous research on subjective probability assessment distinguishes between motivational and cognitive biases [32]. The same distinction is useful here. Motivational biases are conscious or subconscious adjustments in a response motivated by the perceived rewards for the response. In this case, the two motivational biases that were observed related to government involvement in the project and the importance of the "problem" which the project addresses.

Several of the interviewees indicated that their answers would be different, depending on whether the government or a charitable organization were undertaking the project. They felt that the government could not be trusted to keep the costs of a project within projected levels. Since the individuals expected their own share of the costs to be greater than the government predicted, they consciously reduced the amount they would pay for the benefits. The solution to this problem is to have the clairvoyant assure the interviewee that the outcomes are as predicted. However, some interviewees had difficulty understanding the concept of the clairvoyant.

A second motivational bias arose whenever any information was given about the nature of the project. It appeared that individuals increased the amounts they would pay for a fixed benefit, when the project was perceived to be related to an important "problem." For example, several individuals wanted to know if the project would help the "energy crisis" in the future. This bias can be avoided by omitting details about the project and focusing attention on the outcomes with a question such as "If the project were one you considered beneficial, what would you see happening as a result?"

Cognitive biases are conscious or subconscious adjustments in a response introduced by the way that a subject is processing his perceptions. One such bias arises from a confusion between value and uncertainty. Most of the interviewees initially adjusted the amount they would pay for a change in an attribute to account for uncertainty in the future state of the world. This bias is handled with the concept of the clairvoyant, as discussed earlier. The clairvoyant tells the interviewee which one of the scenarios he described will actually occur, thus eliminating uncertainty about the state of the world.

A second cognitive bias arises when an individual adjusts the amount he would pay to account for his short-term financial state. Several of the interviewees indicated that their answers would have been different if they had been asked the same questions just before or after payday. This bias is removed by telling the individual that he may take out a loan or pay in installments if he wishes to pay more than he can afford at the current time.

Another bias may arise if the units for measuring either the change in the future attribute or the amount the individual is willing to pay are picked by the interviewer. If the interviewee is not comfortable with thinking in those units, his answers may not represent his preferences. For example, most individuals preferred to think about percentage changes in an attribute rather than absolute changes. This bias is easily corrected by letting the interviewee pick the units.

A final type of cognitive bias is reflected in the fact that individuals' preferences appeared to be changing during the course of the interview. What was probably happening was that their preferences were forming during the interview. Several of the individuals stated that they had never thought much about the value of outcomes to other people. As the interview progressed, some individuals changed what they valued about the future as well as how much they valued the future. This bias might be eliminated by conducting the interview in two segments, giving the individual time to formulate his preferences between the two segments.

Chapter 6

SUGGESTIONS FOR FURTHER RESEARCH

Outcomes in the future may be viewed as public goods in the sense that one individual's enjoyment of the future does not detract from another individual's enjoyment, and future outcomes cannot be appropriated. The future is a type of externality. Using this idea, the methodology in this research can be interpreted as a solution to decision problems when there are externalities. Thus, the basic formulation could be applied to other cases of external effects. For example, the attribute z could represent the ambient level of air pollutants or the number of acres of wilderness area. One suggestion for further research is to explore decisions about other external effects using a formulation similar to the one in this research.

We assume in this research that the basis for social decisions is the amounts that individuals are willing to pay. Although other analysts start with the same assumption, they usually do not base their analysis on the fundamental preferences of the individual for the outcomes he receives. For example, cost-benefit analysis often defines the outcomes of a project in each period in units of expected present value dollars. If there are also "intangible" benefits and costs, expected present values may not be consistent with individuals' preferences for all of the outcomes. A consideration of fundamental preferences may admit entirely new solutions. As an example, Howard [11] has computed the "value of life" to be used in social decisions, starting with the assumption that the value of an individual's life should be the value to the individual himself. This same assumption could be applied to a wide range of "extraordinary" outcomes.

It became clear in the course of this research that there is a great deal of confusion in the cost-benefit literature over the treatment of uncertainty. This research implicitly assumes that the resolution of future lotteries is delayed until the future and uncertainty affects each individual through his risk aversion. In cost-benefit analysis, on the other hand, assumptions are often made that imply that lotteries are resolved immediately. The effects of uncertainty are usually included simply by using expected values. The more realistic formulation used in this research could offer new interpretations of issues such as risk bearing and irreversibility that arise in connection with public decision making. One specific example is the question of "option values" for cost-benefit analysis of irreversible decisions. At least one recent computation of option values [1] can be interpreted from a decision analysis perspective as a comparison of the values of lotteries with immediate and delayed resolution. The option value that is computed in this case is simply the quantity which Pollard [28] discusses as the value of immediate resolution of the outcomes of a lottery that pays in the future. A review of the cost-benefit treatment of uncertainty could produce solutions to other problems as well.

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Appendix A

THE NUMERAIRE h CORRESPONDING TO SEVERAL PREFERENCE FORMS

Preference	Direct Value Function $\underline{v}(x_1,\ldots,x_n)$	Demand for Good k $d_k(p,m)$	Numeraire $h(p,m\mid p^b)$
General Hyperbola (Cobb Douglas)	$\sum_i a_i \ln x_i$	$m\left(\frac{a_k}{p_k}\right)$	$m\prod_i\left(\frac{p_i^b}{p_i}\right)^{\frac{a_i}{\Sigma a_i}}$
Exponential	$-\sum_i a_i e^{-\gamma_i x_i}$	$\dfrac{m-\sum_{i\neq k}\left(\frac{p_i}{\gamma_i}\right)\ln\frac{a_i\gamma_i p_k}{p_i a_k\gamma_k}}{\gamma_k\sum_i\left(\frac{p_i}{\gamma_i}\right)}$	$m\left(\dfrac{\sum_i p_i^b/\gamma_i}{\sum_i p_i/\gamma_i}\right)+\left(\sum_i p_i^b/\gamma_i\right)\ln\left\{\dfrac{\sum_k a_k\sum_{i\neq k}\left(\frac{a_i\gamma_i p_k^b}{a_k\gamma_k p_i^b}\right)^{\frac{p_i^b/\gamma_i}{\sum_i p_i^b/\gamma_i}}}{\sum_k a_k\sum_{i\neq k}\left(\frac{a_i\gamma_i p_k}{a_k\gamma_k p_i}\right)^{\frac{p_i/\gamma_i}{\sum_i p_i/\gamma_i}}}\right\}$
[illegible]	$-\sum_i a_i x_i^{-\alpha}$ where $0 < \alpha < \infty$; $\sum_i a_i x_i^{-\alpha}$ where $0 > \alpha > -1$	$\dfrac{m}{p_k+\left(\frac{p_k}{a_k}\right)^{\frac{1}{1+\alpha}}\sum_i p_i\left(\frac{a_i}{p_i}\right)^{\frac{1}{1+\alpha}}}$	$m\left[\dfrac{\sum_k a_k\left\{p_k^b+\left(\frac{p_k^b}{a_k}\right)^{\frac{1}{1+\alpha}}\sum_{i\neq k}p_i^b\left(\frac{a_i}{p_i^b}\right)^{\frac{1}{1+\alpha}}\right\}^{\alpha}}{\sum_k a_k\left\{p_k+\left(\frac{p_k}{a_k}\right)^{\frac{1}{1+\alpha}}\sum_{i\neq k}p_i\left(\frac{a_i}{p_i}\right)^{\frac{1}{1+\alpha}}\right\}^{\alpha}}\right]^{\frac{1}{\alpha}}$

Appendix B

DERIVATIVES OF THE NUMERAIRE $h(p,m|p^b)$

(1) $h(p,m|p^b)\Big|_{p=p^b} = m$ from the definition of h

(2) $$\frac{\partial h(p,m|p^b)}{\partial m}\Bigg|_{p=p^b} = \frac{\partial h(p^b,m|p^b)}{\partial m} = \frac{\partial m}{\partial m} = 1$$

(3) (a) By definition $v[p^b,h(p,m|p^b)] = v(p,m)$ (*)

Differentiating both sides of * with respect to p_i gives

$$\frac{\partial v(p^b,h)}{\partial h}\,\frac{\partial h(p,m|p^b)}{\partial p_i} = \frac{\partial v(p,m)}{\partial p_i}$$

$$\frac{\partial h(p,m|p^b)}{\partial p_i} = \frac{\partial v(p,m)/\partial p_i}{\partial v(p,m)/\partial m}\left(\frac{\partial v(p,m)/\partial m}{\partial v(p^b,h)/\partial h}\right)$$

(b) Differentiating both sides of * with respect to m gives

$$\frac{\partial v(p^b,h)}{\partial h}\,\frac{\partial h(p,m|p^b)}{\partial m} = \frac{\partial v(p,m)}{\partial m}$$

$$\frac{\partial h(p,m|p^b)}{\partial m} = \frac{\partial v(p,m)/\partial m}{\partial v(p^b,h)/\partial h}$$

(c) It is well known that

$$\frac{\partial v(p,m)/\partial p_i}{\partial v(p,m)/\partial m} = -d_i(p,m)$$

where d_i is the ordinary Marshallian demand for good i

(d) Combining parts a, b, and c gives

$$\frac{\partial h(p,m|p^b)}{\partial p_i} = -d_i(p,m)\ \frac{\partial h(p,m|p^b)}{\partial m}$$

$$\left.\frac{\partial h(p,m|p^b)}{\partial p_i}\right|_{p=p^b} = -d_i(p,m)\Big|_{p=p^b}$$

(4) $$\left.\frac{\partial^2 h(p,m|p^b)}{\partial m^2}\right|_{p=p^b} = \frac{\partial^2 m}{\partial m^2} = 0$$

(5) $$\frac{\partial^2 h(p,m|p^b)}{\partial p_i\,\partial m} = -\frac{\partial d_i(p,m)}{\partial m}\ \frac{\partial h(p,m|p^b)}{\partial m} - d_i(p,m)\ \frac{\partial^2 h(p,m|p^b)}{\partial m^2}$$

$$\left.\frac{\partial^2 h(p,m|p^b)}{\partial p_i\,\partial m}\right|_{p=p^b} = -\left.\frac{\partial d_i(p,m)}{\partial m}\right|_{p=p^b}$$

(6) $$\frac{\partial^2 h(p,m|p^b)}{\partial p_i\,\partial p_j} = -\frac{\partial d_i(p,m)}{\partial p_j}\ \frac{\partial h(p,m|p^b)}{\partial m} - d_i(p,m)\ \frac{\partial^2 h(p,m|p^b)}{\partial m\,\partial p_j}$$

$$\left.\frac{\partial^2 h(p,m|p^b)}{\partial p_i\,\partial p_j}\right|_{p=p^b} = -\frac{\partial d_i(p,m)}{\partial p_j} + d_i(p,m)\ \left.\frac{\partial d_j(p,m)}{\partial m}\right|_{p=p^b}$$

Appendix C

THE RELATIONSHIP BETWEEN THE KALDOR CRITERION AND AGGREGATE WILLINGNESS TO PAY

Theorem. The Kaldor Criterion is satisfied if and only if $\sum_k w_k > 0$.

Proof.

(1) Let $\sum_k w_k = \delta > 0$. Define $\epsilon_k = w_k - \delta/n < w_k$ where n is the number of individuals. That is, subtract a constant increment from each individual's willingness to pay. Then,

$$\sum_k \epsilon_k = \sum_k w_k - \sum_k \frac{\delta}{n} = \delta - \delta = 0$$

Since w_k satisfies

$$\left\langle V_k\left(h_k(p',m_k'|p^b) - w_k, z'\right)\right\rangle = \left\langle V_k\left(h_k(p^o,m_k^o|p^b), z^o\right)\right\rangle$$

and $V_k(h_k,z)$ is monotonically increasing in h_k, then adding a constant increment to h_k increases expected value:

$$\left\langle V_k\left(h_k(p',m_k'|p^b) - w_k + \delta/n, z'\right)\right\rangle > \left\langle V_k\left(h_k(p^o,m_k^o|p^b), z^o\right)\right\rangle$$

$$\left\langle V_k\left(h_k(p',m_k'|p^b) - \epsilon_k, z'\right)\right\rangle > \left\langle V_k\left(h_k(p^o,m_k^o|p^b), z^o\right)\right\rangle$$

(2) Let the Kaldor criterion be satisfied. Then, there exists $\{\epsilon_k\}$ such that

$$\left\langle V_k\left(h_k(p',m_k'|p^b) - \epsilon_k, z'\right)\right\rangle > \left\langle V_k\left(h_k(p^o,m_k^o|p^b), z^o\right)\right\rangle$$

$$\left\langle V_k\left(h_k(p',m_k'|p^b) - \epsilon_k, z'\right)\right\rangle > \left\langle V_k\left(h_k(p',m_k'|p^b) - w_k, z'\right)\right\rangle$$

Since $V_k(h_k,z)$ is monotonically increasing in h_k, then $\epsilon_k < w_k$ $\forall k$. Thus,

$$0 = \sum_k \epsilon_k < \sum_k w_k$$

Appendix D

DERIVATIVES OF THE VALUE FUNCTION V(h,z)

(1) $$\left.\frac{\partial V}{\partial m}\right|_{p=p^b} = \left.\frac{\partial V}{\partial h}\frac{\partial h}{\partial m}\right|_{p=p^b} = \left.\frac{\partial V}{\partial h}\right|_{p=p^b}$$

(2) $$\left.\frac{\partial V}{\partial p_i}\right|_{p=p^b} = -d_i \left.\frac{\partial V}{\partial h}\frac{\partial h}{\partial m}\right|_{p=p^b} = -d_i \left.\frac{\partial V}{\partial h}\right|_{p=p^b}$$

(3) $$\left.\frac{\partial^2 V}{\partial m^2}\right|_{p=p^b} = \left.\frac{\partial^2 V}{\partial h^2}\right|_{p=p^b} = 0$$

(4) $$\left.\frac{\partial^2 V}{\partial p_i \partial p_j}\right|_{p=p^b} = d_i \frac{\partial V}{\partial h}\frac{\partial d_j}{\partial m} + d_i d_j \frac{\partial^2 V}{\partial h^2}\left(\frac{\partial h}{\partial m}\right)^2 - \left.\frac{\partial d_i}{\partial p_j}\frac{\partial V}{\partial h}\frac{\partial h}{\partial m}\right|_{p=p^b}$$

$$= d_i \frac{\partial V}{\partial h}\frac{\partial d_j}{\partial m} - \left.\frac{\partial V}{\partial h}\frac{\partial d_i}{\partial p_j}\right|_{p=p^b}$$

(5) $$\left.\frac{\partial^2 V}{\partial p_i \partial m}\right|_{p=p^b} = -d_i \frac{\partial^2 V}{\partial h^2}\left(\frac{\partial h}{\partial m}\right)^2 - \frac{\partial d_i}{\partial m}\frac{\partial V}{\partial h}\frac{\partial h}{\partial m} - d_i \left.\frac{\partial V}{\partial h}\frac{\partial^2 h}{\partial m^2}\right|_{p=p^b} = -\left.\frac{\partial d_i}{\partial m}\frac{\partial V}{\partial h}\right|_{p=p^b}$$

(6) $$\left.\frac{\partial^2 V}{\partial p_i \partial z_j}\right|_{p=p^b} = -d_i \left.\frac{\partial^2 V}{\partial h \partial z_j}\frac{\partial h}{\partial m}\right|_{p=p^b} = -d_i \left.\frac{\partial^2 V}{\partial h \partial z_j}\right|_{p=p^b}$$

(7) $$\left.\frac{\partial^2 V}{\partial m \partial z_j}\right|_{p=p^b} = \left.\frac{\partial^2 V}{\partial h \partial z_j}\frac{\partial h}{\partial m}\right|_{p=p^b} = \frac{\partial^2 V}{\partial h \partial z_j}$$

Appendix E

DERIVATION OF WILLINGNESS TO PAY TO CHANGE FROM (p^o, m^o, z^o) TO (p', m', z')

(1) Willingness to pay is the amount w such that:

$$\tilde{\left\langle V\left[h(p^o, m^o \mid p^b), z^o\right]\right\rangle} = \tilde{\left\langle V\left[h(p', m' \mid p^b) - w, z'\right]\right\rangle} \qquad (*)$$

and the certain equivalent is approximately:

$$\tilde{\langle v \rangle} \doteq \langle v \rangle - \frac{1}{2}\, r_v\Big|_{\langle v \rangle}\ {}^{v}\langle v \rangle \qquad (**)$$

(2) First, expand $V[h(p,m \mid p^b) - w, z]$ around $(\overline{p^o}, \overline{m^o}, \overline{z^o})$:

$$V\left[h(p,m \mid p^b) - w, z\right] = V\left[h\left(\overline{p^o}, \overline{m^o} \mid p^b\right), \overline{z^o}\right] + \frac{\partial V}{\partial h}(-w) - \frac{\partial V}{\partial h}\sum_i d_i\left(p_i - \overline{p_i^o}\right)$$

$$+ \frac{\partial V}{\partial h}\left(m - \overline{m^o}\right) + \sum_i \frac{\partial V}{\partial z_i}\left(z_i - \overline{z_i^o}\right)$$

$$+ \frac{1}{2}\sum_{i,j}\left(d_i \frac{\partial V}{\partial h}\frac{\partial d_j}{\partial m} - \frac{\partial V}{\partial h}\frac{\partial d_i}{\partial p_j}\right)\left(p_i - \overline{p_i^o}\right)\left(p_j - \overline{p_j^o}\right)$$

$$+ \frac{1}{2}\sum_{i,j}\frac{\partial^2 V}{\partial z_i \partial z_j}\left(z_i - \overline{z_i^o}\right)\left(z_j - \overline{z_j^o}\right) - \sum_i \frac{\partial V}{\partial h}\frac{\partial d_i}{\partial m}\left(p_i - \overline{p_i^o}\right)\left(m - \overline{m^o}\right)$$

$$- \sum_{i,j} d_i \frac{\partial^2 V}{\partial h \partial z_j}\left(p_i - \overline{p_i^o}\right)\left(z_j - \overline{z_j^o}\right) + \sum_i \frac{\partial^2 V}{\partial h \partial z_i}\left(m - \overline{m^o}\right)\left(z_j - \overline{z_j^o}\right)$$

In this expression, all derivatives are evaluated at $(\overline{p^o}, \overline{m^o}, \overline{z^o})$. Next, we find the expected value of V:

$$\left\langle V\left[h(p,m|p^b) - w,z\right]\right\rangle = V\left[h\left(\overline{p^o},\overline{m^o}|p^b\right),\overline{z^o}\right] + \frac{\partial V}{\partial h}(-w) - \frac{\partial V}{\partial h}\sum_i d_i\left(\overline{p_i} - \overline{p_i^o}\right)$$

$$+ \frac{\partial V}{\partial h}\left(\overline{m} - \overline{m^o}\right) + \sum_i \frac{\partial V}{\partial z_i}\left(\overline{z_i} - \overline{z_i^o}\right)$$

$$+ \frac{1}{2}\sum_{i,j}\left(d_i \frac{\partial V}{\partial h}\frac{\partial d_j}{\partial m} - \frac{\partial V}{\partial h}\frac{\partial d_i}{\partial p_j}\right)\overline{\left(p_i - \overline{p_i^o}\right)\left(p_j - \overline{p_j^o}\right)}$$

$$+ \frac{1}{2}\sum_{i,j}\frac{\partial^2 V}{\partial z_i \partial z_j}\overline{\left(z_i - \overline{z_i^o}\right)\left(z_j - \overline{z_j^o}\right)} - \sum_i \frac{\partial V}{\partial h}\frac{\partial d_i}{\partial m}\overline{\left(p_i - \overline{p_i^o}\right)\left(m - \overline{m^o}\right)}$$

$$- \sum_{i,j} d_i \frac{\partial^2 V}{\partial h \partial z_j}\overline{\left(p_i - \overline{p_i^o}\right)\left(z_j - \overline{z_j^o}\right)} + \sum_i \frac{\partial^2 V}{\partial h \partial z_j}\overline{\left(m - \overline{m^o}\right)\left(z_j - \overline{z^o}\right)}$$

(3) To find the variance, first square the expected value of V:

$$\left\langle V\left[h(p,m|p^b) - w,z\right]\right\rangle^2 = V^2\left[h\left(\overline{p^o},\overline{m^o}|p^b\right),\overline{z^o}\right] + \left(\frac{\partial V}{\partial h}\right)^2 w^2$$

$$- 2w\frac{\partial V}{\partial h} V\left[h\left(\overline{p^o},\overline{m^o}|p^b\right),\overline{z^o}\right] + \left(\frac{\partial V}{\partial h}\right)^2 \sum_{i,j} d_i d_j\left(\overline{p_i} - \overline{p_i^o}\right)\left(\overline{p_j} - \overline{p_j^o}\right)$$

$$+ \left(\frac{\partial V}{\partial h}\right)^2\left(\overline{m} - \overline{m^o}\right)^2 + \sum_{i,j}\frac{\partial V}{\partial z_i}\frac{\partial V}{\partial z_j}\left(\overline{z_i} - \overline{z_i^o}\right)\left(\overline{z_j} - \overline{z_j^o}\right)$$

$$- 2\left(\frac{\partial V}{\partial h}\right)^2 \sum_i d_i\left(\overline{p_i} - \overline{p_i^o}\right)\left(\overline{m} - \overline{m^o}\right) + 2\frac{\partial V}{\partial h}\sum_{i,j} d_i \frac{\partial V}{\partial z_j}\left(\overline{p_i} - \overline{p_i^o}\right)\left(\overline{z_j} - \overline{z_j^o}\right)$$

$$+ 2\frac{\partial V}{\partial h}\sum_i \frac{\partial V}{\partial z_i}\left(\overline{m} - \overline{m^o}\right)\left(\overline{z_i} - \overline{z_i^o}\right) - 2V\left[h\left(\overline{p^o},\overline{m^o}|p^b\right),\overline{z^o}\right]\frac{\partial V}{\partial h}\sum d_i\left(\overline{p_i} - \overline{p_i^o}\right)$$

$$+ 2V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]\frac{\partial V}{\partial h}\left(\overline{m}-\overline{m^o}\right) + 2V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]\sum_i \frac{\partial V}{\partial z_i}\left(\overline{z_i}-\overline{z_i^o}\right)$$

$$+ 2\left(\frac{\partial V}{\partial h}\right)^2 w \sum_i d_i\left(\overline{p_i}-\overline{p_i^o}\right) - 2\left(\frac{\partial V}{\partial h}\right)^2 w\left(\overline{m}-\overline{m^o}\right) - 2w\,\frac{\partial V}{\partial h}\sum_i \frac{\partial V}{\partial z_i}\left(\overline{z_i}-\overline{z_i^o}\right)$$

$$+ V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]\Bigg\{\sum_{i,j}\left(d_i\frac{\partial V}{\partial h}\frac{\partial d_j}{\partial m} - \frac{\partial V}{\partial h}\frac{\partial d_i}{\partial p_j}\right)\overline{\left(p_i-\overline{p_i^o}\right)\left(p_j-\overline{p_j^o}\right)}$$

$$+ \sum_{i,j}\frac{\partial^2 V}{\partial z_i\partial z_j}\overline{\left(z_i-\overline{z_i^o}\right)\left(z_j-\overline{z_j^o}\right)} - 2\sum_i \frac{\partial V}{\partial h}\frac{\partial d_i}{\partial m}\overline{\left(p_i-\overline{p_i^o}\right)\left(m-\overline{m^o}\right)}$$

$$- 2\sum_{i,j} d_i\frac{\partial^2 V}{\partial h\partial z_j}\overline{\left(p_i-\overline{p_i^o}\right)\left(z_j-\overline{z_j^o}\right)} + 2\sum_i \frac{\partial^2 V}{\partial h\partial z_j}\overline{\left(m-\overline{m^o}\right)\left(z_j-\overline{z_j^o}\right)}\Bigg\}$$

(4) *Then, square the expression for* V *and take expectations:*

$$\left\langle V^2\left[h(p,m\,|p^b) - w,z\right]\right\rangle = V^2\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right] + \left(\frac{\partial V}{\partial h}\right)^2 w^2$$

$$- 2w\,\frac{\partial V}{\partial h}\,V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right] + \left(\frac{\partial V}{\partial h}\right)^2\sum_i d_i d_j\overline{\left(p_i-\overline{p_i^o}\right)\left(p_j-\overline{p_j^o}\right)}$$

$$+ \left(\frac{\partial V}{\partial h}\right)^2\overline{\left(m-\overline{m^o}\right)^2} + \sum_i \frac{\partial V}{\partial z_i}\frac{\partial V}{\partial z_j}\overline{\left(z_i-\overline{z_i^o}\right)\left(z_j-\overline{z_j^o}\right)}$$

$$- 2\left(\frac{\partial V}{\partial h}\right)^2\sum_i d_i\overline{\left(p_i-\overline{p_i^o}\right)\left(m-\overline{m^o}\right)} - 2\,\frac{\partial V}{\partial h}\sum_{i,j} d_i\frac{\partial V}{\partial z_j}\overline{\left(p_i-\overline{p_i^o}\right)\left(z_j-\overline{z_j^o}\right)}$$

$$+ 2\,\frac{\partial V}{\partial h}\sum_i \frac{\partial V}{\partial z_i}\overline{\left(m-\overline{m^o}\right)\left(z_i-\overline{z_i^o}\right)} - 2V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]\frac{\partial V}{\partial h}\sum_i d_i\left(\overline{p_i}-\overline{p_i^o}\right)$$

$$+ 2V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]\frac{\partial V}{\partial h}\left(\bar{m} - \overline{m^o}\right) + 2V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]\sum_i \frac{\partial V}{\partial z_i}\left(\overline{z_i} - \overline{z_i^o}\right)$$

$$+ 2\left(\frac{\partial V}{\partial h}\right)^2 w \sum_i d_i\left(\overline{p_i} - \overline{p_i^o}\right) - 2\left(\frac{\partial V}{\partial h}\right)^2 w\left(\bar{m} - \overline{m^o}\right) - 2\frac{\partial V}{\partial h} w \sum_i \frac{\partial V}{\partial z_i}\left(\overline{z_i} - \overline{z_i^o}\right)$$

$$+ V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]\left\{\sum_{i,j}\left(d_i \frac{\partial V}{\partial h}\frac{\partial d_j}{\partial m} - \frac{\partial V}{\partial h}\frac{\partial d_i}{\partial p_j}\right)\overline{\left(p_i - \overline{p_i^o}\right)\left(p_j - \overline{p_j^o}\right)}\right.$$

$$+ \sum_{i,j}\frac{\partial^2 V}{\partial z_i \partial z_j}\overline{\left(z_i - \overline{z_i^o}\right)\left(z_j - \overline{z_j^o}\right)} - 2\sum_i \frac{\partial V}{\partial h}\frac{\partial d_i}{\partial m}\overline{\left(p_i - \overline{p_i^o}\right)\left(m - \overline{m^o}\right)}$$

$$\left. - 2\sum_{i,j} d_i \frac{\partial^2 V}{\partial h \partial z_j}\overline{\left(p_i - \overline{p_i^o}\right)\left(z_j - \overline{z_j^o}\right)} + 2\sum_i \frac{\partial^2 V}{\partial h \partial z_j}\overline{\left(m - \overline{m^o}\right)\left(z_j - \overline{z_j^o}\right)}\right\}$$

(5) Compute the variance as $\langle V^2\rangle - \langle V\rangle^2$:

$$\overset{v}{\left\langle V\left[h(p,m\,|p^b) - w, z\right]\right\rangle} = \left(\frac{\partial V}{\partial h}\right)^2 \sum_{i,j} d_i d_j \operatorname{cov}(p_i,p_j) + \left(\frac{\partial V}{\partial h}\right)^2 \overset{v}{\langle m\rangle}$$

$$+ \sum_{i,j}\frac{\partial V}{\partial z_i}\frac{\partial V}{\partial z_j}\operatorname{cov}(z_i,z_j) - 2\left(\frac{\partial V}{\partial h}\right)^2 \sum_i d_i \operatorname{cov}(p_i,m)$$

$$- 2\frac{\partial V}{\partial h}\sum_{i,j} d_i \frac{\partial V}{\partial z_j}\operatorname{cov}(p_i,z_j) + 2\frac{\partial V}{\partial h}\sum_i \frac{\partial V}{\partial z_i}\operatorname{cov}(m,z_i)$$

(6) Using the approximation

$$r_V\Big|_{\langle V\rangle} \doteq r_V\Big|_{V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]}$$

and the expression (**) for the certain equivalent, we can write equation (*) defining willingness to pay as:

$$\left\langle V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]\right\rangle - \left\langle V\left[h\left(p',m'\,|p^b\right) - w, z'\right]\right\rangle = -\frac{1}{2}\, r_V\Big|_{V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]}$$

$$\cdot \left\{ {}^{v}\!\left\langle V\left[h\left(p',m'\,|p^b\right) - w, z'\right]\right\rangle - {}^{v}\!\left\langle V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]\right\rangle \right\}$$

$$w = \sum_i d_i\left(\overline{p_i^o} - \overline{p_i'}\right) + \left(\overline{m'} - \overline{m^o}\right) + \sum_i \frac{\partial V/\partial z_i}{\partial V/\partial h}\left(\overline{z_i'} - \overline{z_i^o}\right)$$

$$+ \frac{1}{2}\sum_{i,j}\left(d_i\frac{\partial d_j}{\partial m} - \frac{\partial d_i}{\partial p_j}\right)\left[\mathrm{cov}\left(p_i',p_j'\right) - \mathrm{cov}\left(p_i^o,p_j^o\right) + \left(\overline{p_i'} - \overline{p_i^o}\right)\left(\overline{p_j'} - \overline{p_j^o}\right)\right]$$

$$+ \frac{1}{2}\sum_{i,j}\frac{\partial^2 V/\partial z_i\partial z_j}{\partial V/\partial h}\left[\mathrm{cov}\left(z_i',z_j'\right) - \mathrm{cov}\left(z_i^o,z_j^o\right) + \left(\overline{z_i'} - \overline{z_i^o}\right)\left(\overline{z_j'} - \overline{z_j^o}\right)\right]$$

$$- \sum_i \frac{\partial d_i}{\partial m}\left[\mathrm{cov}\left(p_i',m'\right) - \mathrm{cov}\left(p_i^o,m^o\right) + \left(\overline{p_i'} - \overline{p_i^o}\right)\left(\overline{m'} - \overline{m^o}\right)\right]$$

$$- \sum_{i,j} d_i\,\frac{\partial^2 V/\partial h\partial z_j}{\partial V/\partial h}\left[\mathrm{cov}\left(p_i',z_j'\right) - \mathrm{cov}\left(p_i^o,z_j^o\right) + \left(\overline{p_i'} - \overline{p_i^o}\right)\left(\overline{z_j'} - \overline{z_j^o}\right)\right]$$

$$- \frac{1}{2}\, r_V\Big|_{V\left[h\left(\overline{p^o},\overline{m^o}\,|p^b\right),\overline{z^o}\right]}\left\{\left(\frac{\partial V}{\partial h}\right)\sum_{i,j} d_i d_j\left[\mathrm{cov}\left(p_i',p_j'\right) - \mathrm{cov}\left(p_i^o,p_j^o\right)\right]\right.$$

$$\left. + \left(\frac{\partial V}{\partial h}\right)\left[{}^{v}\langle m'\rangle - {}^{v}\langle m^o\rangle\right] + \sum_{i,j}\frac{(\partial V/\partial z_i)\,\partial V/\partial z_j}{\partial V/\partial h}\left[\mathrm{cov}\left(z_i',z_j'\right) - \mathrm{cov}\left(z_i^o,z_j^o\right)\right]\right.$$

$$- 2\left(\frac{\partial V}{\partial h}\right) \sum_{i} d_i \left[\operatorname{cov}\left(p_i', m'\right) - \operatorname{cov}\left(p_i^o, m^o\right)\right]$$

$$- 2 \sum_{i,j} d_i \frac{\partial V}{\partial z_j} \left[\operatorname{cov}\left(p_i', z_j'\right) - \operatorname{cov}\left(p_i^o, z_j^o\right)\right]$$

$$+ 2 \sum_{i} \frac{\partial V}{\partial z_i} \left[\operatorname{cov}\left(m', z_i'\right) - \operatorname{cov}\left(m^o, z_i^o\right)\right]\Bigg\}$$

Appendix F

THE RELATION BETWEEN WILLINGNESS TO PAY AND THE CERTAIN EQUIVALENT

(1) Define the individual's value function $V(\mathfrak{H},\mathfrak{Z})$ as the number h^* such that:

$$V\left(h^*,\overline{z^o}\right) = V(\mathfrak{H},\mathfrak{Z})$$

Compute the expected value of h^* as follows:

(a) Expand both sides of this equation around $(\overline{p^o},\overline{m^o},\overline{z^o})$ using the fact that $\partial^2 V/\partial h^2 = \partial^2 V/\partial m^2 = 0$:

$$V\left[h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}\right] + \frac{\partial V}{\partial h}\left[h^* - h\left(\overline{p^o},\overline{m^o}\right)\right] = V\left[h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}\right] + \sum_i \frac{\partial V}{\partial h}\frac{\partial h}{\partial p_i}\left(p_i - \overline{p_i^o}\right)$$

$$+ \frac{\partial V}{\partial h}\frac{\partial h}{\partial m}\left(m - \overline{m^o}\right) + \sum_i \frac{\partial V}{\partial z_i}\left(z_i - \overline{z_i^o}\right) + \frac{1}{2}\sum_{i,j}\frac{\partial^2 V}{\partial p_i \partial p_j}\left(p_i - \overline{p_i^o}\right)\left(p_j - \overline{p_j^o}\right)$$

$$+ \frac{1}{2}\sum \frac{\partial^2 V}{\partial z_i \partial z_j}\left(z_i - \overline{z_i^o}\right)\left(z_j - \overline{z_j^o}\right) + \sum_{i,j}\frac{\partial^2 V}{\partial p_i \partial z_j}\left(p_i - \overline{p_i^o}\right)\left(z_j - \overline{z_j^o}\right)$$

$$+ \sum_i \frac{\partial^2 V}{\partial m \partial z_i}\left(m - \overline{m^o}\right)\left(z_j - \overline{z_j^o}\right) + \sum_i \frac{\partial^2 V}{\partial p_i \partial m}\left(p_i - \overline{p_i^o}\right)\left(m - \overline{m^o}\right)$$

(b) Solve for h^*, take expectations, and use the derivatives of h listed in Appendix B:

$$\overline{h^*}\Big|_{p,m,z} = h\left(\overline{p^o},\overline{m^o}\right) + \sum_i d_i\left(\overline{p_i^o} - \overline{p_i}\right) + \left(\overline{m} - \overline{m^o}\right) + \sum_i \frac{\partial V/\partial z_i}{\partial V/\partial h}\left(\overline{z_i} - \overline{z_i^o}\right)$$

$$+ \frac{1}{2} \sum_{i,j} \left(d_i \frac{\partial d_j}{\partial m} - \frac{\partial d_i}{\partial p_j} \right) \left[\overline{\left(p_i - \overline{p_i^o}\right)\left(p_j - \overline{p_j^o}\right)} \right]$$

$$+ \frac{1}{2} \sum_{i,j} \frac{\partial^2 V / \partial z_i \partial z_j}{\partial V / \partial h} \left[\overline{\left(z_i - \overline{z_i^o}\right)\left(z_j - \overline{z_j^o}\right)} \right]$$

$$- \sum_{i,j} d_i \frac{\partial^2 V / \partial h \partial z_j}{\partial V / \partial h} \left[\overline{\left(p_i - \overline{p_i^o}\right)\left(z_j - \overline{z_j^o}\right)} \right]$$

$$+ \sum_{i} \frac{\partial^2 V / \partial h \partial z_i}{\partial V / \partial h} \left[\overline{\left(m - \overline{m^o}\right)\left(z_i - \overline{z_i^o}\right)} \right] - \sum_{i} \frac{\partial d_i}{\partial m} \left[\overline{\left(p_i - \overline{p_i^o}\right)\left(m - \overline{m^o}\right)} \right]$$

(2) Compute the variance of h^* as follows:

(a) Square the expression for $\overline{h^*}$ above, keeping only second-order terms:

$$\left(\overline{h^*}\right)^2 \Bigg|_{p,m,z} = h^2\left(\overline{p^o}, \overline{m^o}\right) + \sum_{i,j} d_i d_j \left(\overline{p_i^o} - \overline{p_i}\right)\left(\overline{p_j^o} - \overline{p_j}\right) + \left(\overline{m} - \overline{m^o}\right)^2$$

$$+ \sum_{i,j} \frac{\partial V / \partial z_i}{\partial V / \partial h} \frac{\partial V / \partial z_j}{\partial V / \partial h} \left(\overline{z_i} - \overline{z_i^o}\right)\left(\overline{z_j} - \overline{z_j^o}\right) + 2h\left(\overline{p^o}, \overline{m^o}\right) \sum_{i} d_i \left(\overline{p_i^o} - \overline{p_i}\right)$$

$$+ 2h\left(\overline{p^o}, \overline{m^o}\right)\left(\overline{m} - \overline{m^o}\right) + 2h\left(\overline{p^o}, \overline{m^o}\right) \sum_{i} \frac{\partial V / \partial z_i}{\partial V / \partial h} \left(\overline{z_i} - \overline{z_i^o}\right)$$

$$+ h\left(\overline{p^o}, \overline{m^o}\right) \sum_{i,j} \left(d_i \frac{\partial d_j}{\partial m} - \frac{\partial d_i}{\partial p_j} \right) \left[\overline{\left(p_i - \overline{p_i^o}\right)\left(p_j - \overline{p_j^o}\right)} \right]$$

$$+ h\left(\overline{p^o}, \overline{m^o}\right) \sum_{i,j} \frac{\partial^2 V / \partial z_i \partial z_j}{\partial V / \partial h} \left[\overline{\left(z_i - \overline{z_i^o}\right)\left(z_j - \overline{z_j^o}\right)} \right]$$

$$- 2h\left(\overline{p^o},\overline{m^o}\right) \sum_{i,j} d_i \frac{\partial^2 V/\partial h \partial z_j}{\partial V/\partial h} \left[\overline{\left(p_i - \overline{p_i^o}\right)\left(z_j - \overline{z_j^o}\right)}\right]$$

$$+ 2h\left(\overline{p^o},\overline{m^o}\right) \sum_{i} \frac{\partial^2 V/\partial h \partial z_i}{\partial V/\partial h} \left[\overline{\left(m - \overline{m^o}\right)\left(z_i - \overline{z_i^o}\right)}\right]$$

$$- 2h\left(\overline{p^o},\overline{m^o}\right) \sum_{i} \frac{\partial d_i}{\partial m} \left[\overline{\left(p_i - \overline{p_i^o}\right)\left(m - \overline{m^o}\right)}\right] + 2\left(\bar{m} - \overline{m^o}\right) \sum_i d_i\left(\overline{p_i^o} - \overline{p_i}\right)$$

$$+ 2 \sum_{i,j} d_i\left(\overline{p_i^o} - \overline{p_i}\right) \frac{\partial V/\partial z_j}{\partial V/\partial h} \left(\overline{z_j} - \overline{z_j^o}\right) + 2\left(\bar{m} - \overline{m^o}\right) \sum_i \frac{\partial V/\partial z_i}{\partial V/\partial h} \left(\overline{z_i} - \overline{z_i^o}\right)$$

(b) Square the expression for h^* and then take expectations:

$$\left(h^*\right)^2\Big|_{p,m,z} = h^2\left(\overline{p^o},\overline{m^o}\right) + \sum_{i,j} d_i d_j \overline{\left(\overline{p_i^o} - p_i\right)\left(\overline{p_j^o} - p_j\right)} + \overline{\left(m - \overline{m^o}\right)^2}$$

$$+ \sum_{i,j} \frac{\partial V/\partial z_i}{\partial V/\partial h} \frac{\partial V/\partial z_j}{\partial V/\partial h} \overline{\left(z_i - \overline{z_i^o}\right)\left(z_j - \overline{z_j^o}\right)} + 2h\left(\overline{p^o},\overline{m^o}\right) \sum_i d_i\left(\overline{p_i^o} - \overline{p_i}\right)$$

$$+ 2h\left(\overline{p^o},\overline{m^o}\right)\left(\bar{m} - \overline{m^o}\right) + 2h\left(\overline{p^o},\overline{m^o}\right) \sum_i \frac{\partial V/\partial z_i}{\partial V/\partial h} \left(\overline{z_i} - \overline{z_i^o}\right)$$

$$+ h\left(\overline{p^o},\overline{m^o}\right) \sum_{i,j} \left(d_i \frac{\partial d_j}{\partial m} - \frac{\partial d_i}{\partial p_j}\right)\left[\overline{\left(p_i - \overline{p_i^o}\right)\left(p_j - \overline{p_j^o}\right)}\right]$$

$$+ h\left(\overline{p^o},\overline{m^o}\right) \sum_{i,j} \frac{\partial^2 V/\partial z_i \partial z_j}{\partial V/\partial h} \left[\overline{\left(z_i - \overline{z_i^o}\right)\left(z_j - \overline{z_j^o}\right)}\right]$$

$$- 2h\left(\overline{p^o}, \overline{m^o}\right) \sum_{i,j} d_i \frac{\partial^2 V/\partial h \partial z_j}{\partial V/\partial h} \left[\overline{\left(p_i - \overline{p_i^o}\right)\left(z_j - \overline{z_j^o}\right)}\right]$$

$$+ 2h\left(\overline{p^o}, \overline{m^o}\right) \sum_{i} \frac{\partial^2 V/\partial h \partial z_i}{\partial V/\partial h} \left[\overline{\left(m - \overline{m^o}\right)\left(z_i - \overline{z_i^o}\right)}\right]$$

$$- 2h\left(\overline{p^o}, \overline{m^o}\right) \sum_{i} \frac{\partial d_i}{\partial m} \left[\overline{\left(p_i - \overline{p_i^o}\right)\left(m - \overline{m^o}\right)}\right] + 2 \sum_{i} d_i \left[\overline{\left(\overline{p_i^o} - p_i\right)\left(m - \overline{m^o}\right)}\right]$$

$$+ 2 \sum_{i,j} d_i \frac{\partial V/\partial z_j}{\partial V/\partial h} \left[\overline{\left(\overline{p_i^o} - p_i\right)\left(z_j - \overline{z_j^o}\right)}\right]$$

$$+ 2 \sum_{i} \frac{\partial V/\partial z_i}{\partial V/\partial h} \left[\overline{\left(m - \overline{m^o}\right)\left(z_i - \overline{z_i^o}\right)}\right]$$

(c) Taking the difference of b-a gives the variance of h^*:

$$ {}^v\langle h^* \rangle \Big|_{p,m,z} = \sum_{i,j} d_i d_j \operatorname{cov}(p_i, p_j) + {}^v/m + \sum_{i,j} \frac{\partial V/\partial z_i}{\partial V/\partial h} \frac{\partial V/\partial z_j}{\partial V/\partial h} \operatorname{cov}(z_i, z_j)$$

$$- 2 \sum_{i} d_i \operatorname{cov}(p_i, m) - 2 \sum_{i,j} d_i \frac{\partial V/\partial z_j}{\partial V/\partial h} \operatorname{cov}(p_i, z_j)$$

$$+ 2 \sum_{i} \frac{\partial V/\partial z_i}{\partial V/\partial h} \operatorname{cov}(m, z_i)$$

(3) Compute the change in certain equivalents using the approximation

$$\tilde{h}^* \Big|_{p,m,z} \doteq \overline{h^*} \Big|_{p,m,z} - \frac{1}{2} r_h\left(\overline{h^*}\right) {}^v h^* \Big|_{p,m,z}$$

We also note that, as Keelin explained [15], the individual's risk aversion on h^* is related to his risk aversion on value V by

$$r_h = r_V \frac{\partial V}{\partial h} - \frac{\partial^2 V/\partial h^2}{\partial V/\partial h} = r_V \frac{\partial V}{\partial h}$$

since we have assumed V is linear in h. We also approximate the expected numeraire by the numeraire at $(\overline{p^o},\overline{m^o},\overline{z^o})$:

$$r_V\left(V\left(\overline{h^*(p,m,z)},\overline{z^o}\right)\right) = r_V\left(V\left(h(\overline{p^o},\overline{m^o}),\overline{z^o}\right)\right)$$

Thus, the change in certain equivalent from the initial lottery (p^o,m^o,z^o) to the final lottery (p',m',z') is given by:

$$\begin{aligned}
\tilde{\ }\langle h^*\rangle\Big|_{p',m',z'} - \tilde{\ }\langle h^*\rangle\Big|_{p^o,m^o,z^o} &\doteq \overline{h^*}\Big|_{p',m',z'} - \overline{h^*}\Big|_{p^o,m^o,z^o} \\
&- \frac{1}{2} r_V\left(V\left(h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}\right)\right)\frac{\partial V}{\partial h}\left[{}^v\langle h^*\rangle\Big|_{p',m',z'} - {}^v\langle h^*\rangle\Big|_{p^o,m^o,z^o}\right] \\
&= \sum_i d_i\left(\overline{p_i^o} - \overline{p_i'}\right) + \left(\overline{m'} - \overline{m^o}\right) + \sum_i \frac{\partial V/\partial z_i}{\partial V/\partial h}\left(\overline{z_i'} - \overline{z_i^o}\right) \\
&+ \frac{1}{2}\sum_{i,j}\left(d_i \frac{\partial d_j}{\partial m} - \frac{\partial d_j}{\partial p_i}\right)\left[\operatorname{cov}\left(p_i',p_j'\right) - \operatorname{cov}\left(p_i^o,p_j^o\right) + \left(\overline{p_i'} - \overline{p_i^o}\right)\left(\overline{p_j'} - \overline{p_j^o}\right)\right] \\
&+ \frac{1}{2}\sum_{i,j}\frac{\partial^2 V/\partial z_i \partial z_j}{\partial V/\partial h}\left[\operatorname{cov}\left(z_i',z_j'\right) - \operatorname{cov}\left(z_i^o,z_j^o\right) + \left(\overline{z_i'} - \overline{z_i^o}\right)\left(\overline{z_j'} - \overline{z_j^o}\right)\right]
\end{aligned}$$

$$- \sum_{i,j} d_i \frac{\partial^2 V/\partial h \partial z_j}{\partial V/\partial h} \left[\text{cov}\left(p_i', z_j'\right) - \text{cov}\left(p_i^o, z_j^o\right) + \left(\overline{p_i'} - \overline{p_i^o}\right)\left(\overline{z_j'} - \overline{z_j^o}\right) \right]$$

$$+ \sum_{i} \frac{\partial^2 V/\partial h \partial z_i}{\partial V/\partial h} \left[\text{cov}\left(m', z_i'\right) - \text{cov}\left(m^o, z_i^o\right) + \left(\overline{m'} - \overline{m^o}\right)\left(\overline{z_i'} - \overline{z_i^o}\right) \right]$$

$$- \sum_{i} \frac{\partial d_i}{\partial m} \left[\text{cov}\left(p_i', m'\right) - \text{cov}\left(p_i^o, m^o\right) + \left(\overline{p_i'} - \overline{p_i^o}\right)\left(\overline{m'} - \overline{m^o}\right) \right]$$

$$- \frac{1}{2} r_v \left(V\left(h\left(\overline{p^o}, \overline{m^o} \right), \overline{z^o} \right) \right) \frac{\partial V}{\partial h} \Bigg\{ v\langle m' \rangle - v\langle m^o \rangle$$

$$+ \sum_{i,j} d_i d_j \left[\text{cov}\left(p_i', p_j'\right) - \text{cov}\left(p_i^o, p_j^o\right) \right]$$

$$+ \sum_{i,j} \frac{\partial V/\partial z_i}{\partial V/\partial h} \frac{\partial V/\partial z_j}{\partial V/\partial h} \left[\text{cov}\left(z_i', z_j'\right) - \text{cov}\left(z_i^o, z_j^o\right) \right]$$

$$- 2 \sum_{i} d_i \left[\text{cov}\left(p_i', m'\right) - \text{cov}\left(p_i^o, m^o\right) \right]$$

$$- 2 \sum_{i,j} d_i \frac{\partial V/\partial z_j}{\partial V/\partial h} \left[\text{cov}\left(p_i', z_j'\right) - \text{cov}\left(p_i^o, z_j^o\right) \right]$$

$$+ 2 \sum_{i} \frac{\partial V/\partial z_i}{\partial V/\partial h} \left[\text{cov}\left(m', z_i'\right) - \text{cov}\left(m^o, z_j^o\right) \right] \Bigg\}$$

This expression is equal to willingness to pay as given in Table 3.1.

Appendix G

EXPECTED NET CASH FLOW FROM ADDITIONAL HELIUM STORAGE
(in 10^6 1973 Dollars)

Year	Nominal Policy	Alternate Policy
1974	-37,492	-37,492
1975	-36,974	-33,974
1976	-30,314	-29,310
1977	-23,682	-23,682
1978	-13,274	-13,274
1979	-5,594	-5,594
1980	-54	-54
1981	4,586	4,586
1982	8,839	8,839
1983	13,308	13,308
1984-2010	0	0
2011	12,211	0
2012	42,729	0
2013	72,004	0
2014	157,013	0
2015	158,512	0
2016	160,011	0
2017	161,509	0
2018	163,008	0
2019	163,324	0
2020	133,150	0
2021	107,891	0
2022-2034	0	0
2035	0	1,350
2036	0	126,112
2037	0	128,140
2038	0	130,792
2039	0	132,742
2040	0	134,692
2041	0	136,720
2042	0	138,670
2043	0	178,216
2044	0	161,308
2045	0	130,111
2046-2050	0	0

Appendix H

AGE AND MORTALITY DATA

Age	Population in 1974 (in Thousands)	Mortality (Deaths/Year Per 1000 Living at Specified Age)
under 1	3485	16.75
1	3378	1.00
2	3290	0.79
3	3419	0.64
4	3582	0.53
5	3811	0.46
6	3952	0.41
7	4012	0.37
8	4052	0.34
9	4128	0.30
10	4282	0.27
11	4127	0.28
12	4183	0.33
13	4101	0.45
14	4095	0.60
15	4029	0.78
16	3890	0.95
17	3825	1.10
18	3766	1.20
19	3560	1.26
20	3495	1.32
21	3328	1.39
22	3441	1.43
23	3424	1.43
24	2688	1.41
25	2761	1.38
26	2813	1.36
27	2898	1.35
28	2577	1.37
29	2429	1.40
30	2439	1.45
31	2306	1.50
32	2264	1.56
33	2184	1.64
34	2236	1.74
35	2235	1.86
36	2143	1.99
37	2223	2.16
38	2209	2.31
39	2296	2.55
40	2112	2.79

Age	Population in 1974 (in Thousands)	Mortality (Deaths/Year Per 1000 Living at Specified Age)
41	2381	3.05
42	2412	3.35
43	2379	3.69
44	2427	4.07
45	2475	4.50
46	2417	4.95
47	2412	5.40
48	2456	5.85
49	2357	6.31
50	2447	6.79
51	2247	7.33
52	2183	7.96
53	2116	8.69
54	2111	9.51
55	2110	10.39
56	2035	11.32
57	2008	12.34
58	1928	13.46
59	1892	14.66
60	1928	15.98
61	1754	17.38
62	1730	18.77
63	1611	20.11
64	1593	21.46
65	1537	22.85
66	1458	22.85
67	1401	22.85
68	1262	22.85
69	1334	22.85
70	1271	34.41
71	1121	34.41
72	1052	34.41
73	1017	34.41
74	983	34.41
75	915	53.23
76	830	53.23
77	768	53.23
78	683	53.23
79	620	53.23
80	611	79.18
81	526	79.18
82	431	79.18
83	378	79.48
84	336	79.48
85 +	1511	190.8

Appendix I

POPULATION PROJECTION DATA

Year	Population (in Millions)
1974	212
1980	223
1985	234
1990	245
1995	254
2000	262
2005	270
2010	279
2015	287
2020	294
2025	300
2030	304
2035	308
2040	312
2045	315
2050	318

Note: Population for intermediate years was interpolated from above data.

Appendix J

THE DERIVATION OF THE FORMULAS FOR CALCULATING SECOND-ORDER PREFERENCE INFORMATION FROM ASSESSED DATA

(1) (a) To calculate the cross derivatives $\partial^2 V/\partial h \partial z_i$, we first assess λ^o, λ^Δ, and Δ as described in Section 5.4.

(b) For a value function of the form $V(h,z) = hf(z) + g(z)$, we have

$$\lambda^o = \left.\frac{\partial V/\partial h}{\partial V/\partial z_i}\right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} = \left.\frac{f\left(\overline{z^o}\right)}{h\dfrac{\partial f}{\partial z_i} + \dfrac{\partial g}{\partial z_i}}\right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}}$$

$$\lambda^\Delta = \left.\frac{\partial V/\partial h}{\partial V/\partial z_i}\right|_{h\left(\overline{p^o},\overline{m^o}\right)+\Delta,\overline{z^o}} = \left.\frac{f\left(\overline{z^o}\right)}{(h+\Delta)\dfrac{\partial f}{\partial z_i} + \dfrac{\partial g}{\partial z_i}}\right|_{h\left(\overline{p^o},\overline{m^o}\right)+\Delta,\overline{z^o}}$$

(c) Keelin shows that the "marginal value reduction coefficient" is given by

$$Z_{h,z_i} = -\frac{\partial^2 V/\partial h^2}{\partial V/\partial h} + \frac{\partial^2 V/\partial h \partial z_i}{\partial V/\partial z_i}$$

For a value function of the form $V(h,z) = hf(z) + g(z)$, we have

$$Z_{h,z_i} = \frac{\partial^2 V/\partial h \partial z_i}{\partial V/\partial z_i} = \frac{\partial f/\partial z_i}{h\dfrac{\partial f}{\partial z_i} + \dfrac{\partial g}{\partial z_i}}$$

(d) Thus, we can see that

$$\frac{\frac{\lambda^{o}}{\lambda^{\Delta}} - 1}{\Delta} = \left. \frac{(h+\Delta)\frac{\partial f}{\partial z_i} + \frac{\partial g}{\partial z_i}}{\Delta\left(h\frac{\partial f}{\partial z_i} + \frac{\partial g}{\partial z_i}\right)} - \frac{h\frac{\partial f}{\partial z_i} + \frac{\partial g}{\partial z_i}}{\Delta\left(h\frac{\partial f}{\partial z_i} + \frac{\partial g}{\partial z_i}\right)} \right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}}$$

$$= \left. \frac{\partial f/\partial z_i}{h\frac{\partial f}{\partial z_i} + \frac{\partial g}{\partial z_i}} \right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}}$$

$$= \left. \frac{\partial^2 V/\partial h \partial z_i}{\partial V/\partial h}\lambda^o \right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} \qquad \text{from part c.}$$

(e) Given a preference ordering represented by a value function $V(h,z)$, the same ordering is represented by any monotonic increasing function of V. We choose the particular transformation h^* discussed in Sections 2.5 and 3.6. That is,

$$V(h,z) = h^* \quad \text{such that}$$

$$V\left(h^*,\overline{z^o}\right) = V(h,z) \tag{A.1}$$

This transformation guarantees that the "willingness to pay" which we calculate is the dollar amount the individual would actually pay, that is, the change in certain equivalent. For a value function of the form $V(h,z) = hf(z) + g(z)$, we calculate h^* from Eq. (A.1):

$$h^* f\left(\overline{z^o}\right) + g\left(\overline{z^o}\right) = hf(z) + g(z)$$

$$h^* = \frac{hf(z)}{f\left(\overline{z^o}\right)} + \frac{g(z) - g\left(\overline{z^o}\right)}{f\left(\overline{z^o}\right)}$$

Thus,

$$\left.\frac{\partial V}{\partial h}\right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} = \left.\frac{\partial h^*}{\partial h}\right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} = 1$$

(f) Combining parts d and e gives

$$\left.\frac{\partial^2 V}{\partial h \partial z_i}\right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} = \frac{\frac{1}{\lambda^{\Delta}} - \frac{1}{\lambda^{o}}}{\Delta}$$

(2) (a) To calculate the derivatives $\partial^2 V/\partial z_i^2$, we first assess $\emptyset$ as discussed in Section 5.4.

(b) Keelin shows that the "substitution aversion coefficient" is approximated by:

$$\left. S_{h,z_i}\right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} \doteq \frac{2\emptyset}{\Delta^2}$$

(c) He also shows that S_{h,z_i} can be written as

$$S_{h,z_i} = -\frac{\frac{\partial^2 V}{\partial h^2}}{\frac{\partial V}{\partial h}} - \frac{\frac{\partial V}{\partial h}\frac{\partial^2 V}{\partial z_i^2}}{\left(\frac{\partial V}{\partial z_i}\right)^2} + 2\,\frac{\frac{\partial^2 V}{\partial h \partial z_i}}{\frac{\partial V}{\partial z_i}}$$

Using the form of the value function, $V(h,z) = hf(z) + g(z)$, and the results of part 1, we have

$$S_{h,z_i}\Bigg|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} = - \frac{\dfrac{\partial^2 V}{\partial z_i^2}}{\left(\dfrac{1}{\lambda^o}\right)^2} + 2 \left. \frac{\dfrac{\lambda^o}{\lambda^\Delta} - 1}{\Delta} \right|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}}$$

(d) Combining parts b and c gives

$$\frac{\partial^2 V}{\partial z_i^2}\Bigg|_{h\left(\overline{p^o},\overline{m^o}\right),\overline{z^o}} = \left(2 \frac{\dfrac{\lambda^o}{\lambda^\Delta} - 1}{\Delta} - \frac{2\emptyset}{\Delta^2}\right)\left(\frac{1}{\lambda^o}\right)^2$$

$$= 2 \frac{\dfrac{1}{\lambda^\Delta} - \dfrac{1}{\lambda^o}}{\Delta\lambda^o} - 2 \frac{\emptyset}{\left(\Delta\lambda^o\right)^2}$$

(3) The calculation of the derivatives $\partial^2 V/\partial z_i \partial z_j$ is described in Section 5.4.

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