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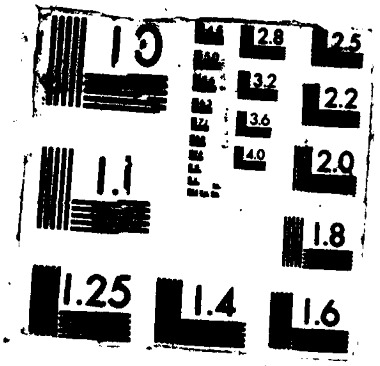
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THE C^2 CONTINUITY OF PIECEWISE CUBIC HERMITE POLYNOMIALS WITH UNEQUAL INTERVALS

AD-A183 351

C. N. SHEN

JULY 1987

US ARMY ARMAMENT RESEARCH, DEVELOPMENT
AND ENGINEERING CENTER
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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: ARCCB-TR-87019	2. GOVT ACCESSION NO. AD-A183351
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): THE C^2 CONTINUITY OF PIECEWISE CUBIC HERMITE POLYNOMIALS WITH UNEQUAL INTERVALS	5. TYPE OF REPORT & PERIOD COVERED: Final
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): C. N. Shen	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: US Army ARDEC, Benet Weapons Laboratory, SMCAR-CCB-TL, Watervliet, NY 12189-4050	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: AMCMS No. 6111.01.91A0.0, PRON No. 1A6AZ601NMLC
11. CONTROLLING OFFICE NAME AND ADDRESS: US Army ARDEC, Close Combat Armaments Center, Picatinny Arsenal, NJ 07806-5000	12. REPORT DATE: July 1987
	13. NUMBER OF PAGES: 12
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Presented at the Fourth Army Conference on Applied Mathematics and Computing, Cornell University, Ithaca, New York, 27-30 May 1986.
Published in Proceedings of the Conference.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Hermite Polynomials
Spline Functions
Data Smoothing
Laser Vision System

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Cubic hermite polynomials are usually C^2_u continuous. With the introduction of smoothing within the intervals, the second derivatives can be made continuous. This may be applied to the autonomous vehicle problem with unequal laser scanning.

In using a laser range finder to measure the range, the direction of these laser rays can be subjected to angular errors. These errors, in the direction

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20. ABSTRACT (CONT'D)

of the elevation angle, affect the determination of in-path slopes for navigation of autonomous vehicles. A nonuniform grid may be employed to compute by the spline function method with cubic hermite polynomials. For the purpose of smoothing, it is essential to obtain continuous second derivatives at the grid point from both sides. Keywords: Spline functions; Laser Vision Systems.

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INTRODUCTION

The smoothing of gradients can be obtained by using an optimization method for approximation involving spline functions. A nonuniform grid may be employed to compute by the spline function method with cubic hermite polynomials. Continuous second derivatives at the grid point from both sides are essential for the purpose of smoothing. This method can be applied to solve the following problems: whether the platform can climb on the estimated in-path slope or whether it will tip over the estimated cross-path slope.

RECURSIVE FILTERING AND SMOOTHING PROCEDURE

A spline function $s(\xi)$ is a solution to the optimization problem

$$J^* = \underset{h \in C^2}{\mathrm{Min.}} \left\{ \sum_{i=1}^{N} [h(\beta_i)-m_i]^T R_i^{-1} [h(\beta_i)-m_i] + \rho \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{h}]^2 d\xi \right\} \tag{1}$$

where for clarity and simplicity in discussion, we only consider the cubic spline case. A higher order polynomial spline can also be treated in a similar manner with more complicated computations.

A cubic spline, s, is a piecewise polynomial of class C^2 which has many good properties, such as the minimum norm property and local base property (refs 1,2). From the approximation theory, we know that for each set $A = \{a_1, \ldots, a_N, a'_1, a'_N\}$, there exists a unique cubic spline $s(\xi;A)$ such that

$$s(\beta_1;A) = a_i \quad , \quad i = 1,2,\ldots,N \tag{2}$$

$$\dot{s}(\beta_i;A) = a'_i, \; i = 1,N \tag{3}$$

where $\dot{s}$ is the first derivative of the function s. The above equations can be

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[1]Ahlberg, J. H., Nilson, E. N., and Walsh, J. L., The Theory of Splines and Their Applications, Academic Press, Inc., 1967.
[2]Schumaker, L. L., Spline Functions: Basic Theory, John Wiley & Sons, 1981.

thought of as boundary conditions for the piecewise cubic spline interpolation given a set of data (β_i, a_i), for $i = 1,2,...,N$. Thus, solving the problem in Eq. (1) is equivalent to determining a set of constraints A for the optimization problem:

$$J^* = \underset{A}{\text{Min}} \left\{ \sum_{i=1}^{N} [s(\xi_i;A)-m_i]^T R_i^{-1} [s(\xi_i;A)-m_i] + \rho \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{s}(\xi;A)]^2 d\xi \right\} \quad (4)$$

Instead of taking a direct approach to find an optimal set of constraints for the problem above, it is proposed to further transform this problem into a form which is convenient to be solved. From the theory of numerical analysis (ref 3), it is well known that a piecewise cubic Hermite polynomial $p(\xi)$ is in the family of C^1. For each set $B = A \cup A^c$, where A^c is a complement of A, i.e., $A^c = \{a'_i,\ i = 2,3,...,N-1\}$, then $B = \{a_i, a'_i,\ i=1,2,...,N\}$, there exists a unique piecewise cubic Hermite polynomial $p(\xi;A)$ such that

$$p(\beta_i;B) = a_i \quad , \quad i = 1,2,...,N \quad (5)$$

$$\dot{p}(\beta_i;B) = a'_i \quad , \quad i = 2,...,N \quad (6)$$

where $\dot{p}$ is the first derivative of p.

It should also be noted that for each set A, there are an infinite number of piecewise Hermite polynomials $p(\xi;A)$ such that

$$p(\beta_i;A) = a_i \quad , \quad i = 1,2,...,N \quad (7)$$

$$\dot{p}(\beta_i;A) = a'_i \quad , \quad i = 1,N \quad (8)$$

Let a set of $p(\xi;A)$ which satisfies the constraints in the equations above be P, i.e.,

$$P = \{p(\xi;A):(5),(6)\ \text{satisfied}\} \quad (9)$$

[3]Burden, R. L. et al., Numerical Analysis, Prindle, Weber, & Schmidt, 1978.

Referring to the paper by de Boor (ref 4), it is noted that there exists a unique cubic spline $s(\xi;A)$ in the set P. Also from the minimum norm property of a cubic spline, we have the following relation:

$$\sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{s}(\xi;A)]^2 \leq \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{p}(\xi;A)]^2 \tag{9}$$

That is

$$\sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{s}(\xi;A)]^2 = \inf_{p \in P} J_p(p) \tag{10}$$

where

$$J_p = \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{p}(\xi;A)]^2 \tag{11}$$

Since a cubic spline $s(\xi;A)$ is unique, a piecewise cubic Hermite polynomial $p(\xi;A)$ which minimizes the smoothing integral J_p in the above equation with respect to A^c becomes a cubic spline $s(\xi;A)$. To be more precise, we have the following theorem.

THEOREM: Let P represent a set of piecewise cubic Hermite polynomials p which satisfies the constraints below:

$$p(\beta_i;A^c) = a_i \quad , \quad i = 1,2,\ldots N \tag{12}$$

$$\dot{p}(\beta_i;A^c) = a'_i \quad , \quad i = 1,N \tag{13}$$

where $p \in C^1$, A, and A^c are the same as mentioned before. Then there exists a unique cubic spline $s(\xi)$ such that

$$\sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{s}(\xi)]^2 d\xi = \min_{A^c} \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{p}(\xi,A^c)]^2 d\xi \tag{14}$$

where $\ddot{s}$ and $\ddot{p}$ are the second derivatives of functions s and p and $s \in C^2$. A simple example with N = 3 is given next.

[4]de Boor, C., "Bicubid Spline Interpolation," J. Math Phys., Vol. 41, 1962, pp. 212-218.

EXAMPLE FOR C^2 CONTINUITY

For convenience and simplicity, we only consider a special case with N = 3. The node points are given as β_1, β_2, and β_3. The intervals are not equal, i.e.,

$$(\beta_2-\beta_1) \neq (\beta_3-\beta_2) \tag{15}$$

Let a set of piecewise cubic Hermite polynomials p be

$$P = [p(t;A^c) \quad , \quad p \in C^1\,[t_1,t_3],\ \dot{p}(t_2) = a,\ a \in A^c] \tag{16}$$

which satisfies the constraints in the equations below:

$$p(t_i;A^c) = a_i \quad , \quad \text{for } i = 1,2,3$$

$$\dot{p}(t_i;A^c) = a'_i \quad , \quad \text{for } i = 1,3 \tag{17}$$

In this special case, a set $A^c = a'_2 = a$.

We want to show here that the cubic Hermite polynomial $p(t;A^c)$, which is obtained by minimizing the smoothing integral, will become a cubic spline function $s(t) \in C^2[t_1,t_3]$

$$J^* = \underset{A^c}{\text{Min}}\ \{\int_{t_1}^{t_2^-} [\ddot{p}(t;A^2)]^2dt + \int_{t_2^+}^{t_3} [\ddot{p}(t;A^c)]^2dt\}$$

$$= \underset{a}{\text{Min}}\ \{\int_{t_1}^{t_2^-} [\ddot{p}(t;a)]^2dt + \int_{t_2^+}^{t_3} [\ddot{p}(t;a)]^2dt\} \tag{18}$$

From Eq. (A14) of the Appendix, the smoothing integral above can be written as

$$J(a) = (x_2-A_1x_1)^T B_1^{-1} (x_2-A_1x_1) + (x_3-A_2x_2)^T B_2^{-1} (x_3-A_2x_2) \tag{19}$$

where A_i, B_i^{-1}, and x_i are defined in the Appendix, and

$$x_i = (a_i, a'_i)^T \quad , \quad \text{with } a'_2 = a \quad , \quad i = 1,2,3 \tag{20}$$

$$\Delta_{i-1} = d_{i-1} = t_i-t_{i-1} \tag{21}$$

Using Eqs. (A11) and (A12), the functional J(a) is written as

$$\left[\begin{bmatrix} a_2 \\ a \end{bmatrix} - \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ a'_1 \end{bmatrix}\right]^T \begin{bmatrix} 12d_1^{-3} & -6d_1^{-2} \\ -6d_1^{-2} & 4d_1^{-1} \end{bmatrix} \left[\begin{bmatrix} a_2 \\ a \end{bmatrix} - \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ a \end{bmatrix}\right]$$

$$+ \left[\begin{bmatrix} a_3 \\ a'_3 \end{bmatrix} - \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a_2 \\ a \end{bmatrix}\right]^T \begin{bmatrix} 12d_2^{-3} & -6d_2^{-2} \\ -6d_2^{-2} & 4d_2^{-1} \end{bmatrix} \left[\begin{bmatrix} a_3 \\ a'_3 \end{bmatrix} - \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a_2 \\ a \end{bmatrix}\right]$$

$$\begin{aligned} J(a) = {} & 12d_1^{-3}\,(a_2-a_1-d_1a'_1)^2 - 12d_1^{-2}\,(a_2-a_1-d_1a'_1)(a-a'_1) \\ & + 4d_1^{-1}\,(a-a'_1)^2 + 12d_2^{-3}\,(a_3-a_2-d_2a)^2 \\ & -12d_2^{-2}\,(a_3-a_2-d_2a)(a'_3-a) + 4d_2^{-1}\,(a'_3-a)^2 \end{aligned} \tag{22}$$

Taking the partial derivative with respect to a yields

$$\begin{aligned} \frac{\partial J}{\partial a} = {} & -12d_1^{-2}\,(a_2-a_1-d_1a'_1) + 8d_1^{-1}\,(a-a'_1) \\ & + 24d_2^{-3}\,(a_3-a_2-d_2a)(-d_2) - 12d_2^{-3}\,(-d_2)(a'_3-a) \\ & -12d_2^{-2}\,(-1)(a_3-a_2-d_2a) - 8d_2^{-1}\,(a'_3-a) = 0 \end{aligned} \tag{23}$$

Solving the equation above for a, one obtains

$$a^* = [3d_1^{-2}(a_2-a_1)-d_1^{-1}a'_1+3d_2^{-2}a_3-d_2^{-2}\ 3a_2-d_2^{-1}a'_3]/[2(d_1^{-1}+d_2^{-1})] \tag{24}$$

To show that $p(t;a^*) \epsilon\ C^2[t_1,t_3]$, we only need to show that

$$\lim_{t\to t_2^-} \ddot{p}(t;a^*) = \lim_{t\to t_2^+} \ddot{p}(t;a^*) \tag{25}$$

That is, for a piecewise cubic Hermite polynomial p,

$$\ddot{p}_{1,2}(t_2;a^*) = \ddot{p}_{2,3}(t_2;a^*) \tag{26}$$

where $p_{1,2}$ is the cubic Hermite polynomial within the interval β_1 and β_2, and $p_{2,3}$ is the cubic Hermite polynomial within the interval β_2 and β_3.

Now from the definition of piecewise cubic Hermite polynomial in the Appendix, we have

$$\ddot{p}_{1,2}(t_2;a^*) = 6d_1^{-2}(a_1-a_2) + 2d_1^{-1}a'_1 + 4_1^{-1}\ a^* \tag{27}$$

By using Eq. (24), the above equation can be expressed as

$$\ddot{p}_{1,2}(t_2;a^*) = [-6a_2(d_1^{-1}+d_2^{-1}) + 6(a_1d_1^{-1}+a_3d_2^{-1}) + 2(a'_1-a'_3)]/(d_1+d_2) \tag{28}$$

In a like manner, omitting the detailed derivation, we obtain easily

$$\ddot{p}_{2,3}(t_2;a^*) = [-6a_2(d_1^{-1}+d_2^{-1}) + 6(a_1d_1^{-1}+a_3d_2^{-1}) + 2(a'_1-a'_3)]/(d_1+d_2) \tag{29}$$

Thus, Eq. (26) is always true, that is, the conclusion in the theorem is valid. It is proved that the C^2 continuity exists in the optimization procedure for piecewise cubic Hermite polynomials with unequal intervals.

CONCLUSION

For scanning in the direction of elevation angle from the top of a mast where a laser is located, the intervals needed in angles are small for far away targets, while the same are large for close-by objects. The smoothing algorithm discussed in this report indicates that piecewise cubic Hermite polynomials can be used for unequal intervals or nonuniform grids.

REFERENCES

1. Ahlberg, J. H., Nilson, E. N., and Walsh, J. L., The Theory of Splines and Their Applications, Academic Press, Inc., 1967.
2. Schumaker, L. L., Spline Functions: Basic Theory, John Wiley & Sons, 1981.
3. Burden, R. L. et al, Numerical Analysis, Prindle, Weber, & Schmidt, 1978.
4. de Boor, C., "Bicubid Spline Interpolation," J. Math Phys., Vol. 41, 1962, pp. 212-218.

APPENDIX

EVALUATION OF THE SMOOTHING INTEGRAL

A piecewise cubic Hermite polynomial in the interval $[\beta_{i-1},\beta_i]$ is represented in terms of the basis functions and the state vectors x_i, x_{i-1}, where the state vectors are defined as in Eq. (20). By changing the independent variable below,

$$t = \xi-\beta_{i-1} \tag{A1}$$

Then the smoothing integral in the interval $[\beta_{i-1},\beta_i]$ becomes

$$I_{i-1,i} = \int_0^{\Delta_{i-1}} [\ddot{p}_{i-1,i}(t)]^2 dt \tag{A2}$$

where $\Delta_{i-1} = t_i-t_{i-1} = \beta_i-\beta_{i-1}$, $\Delta_{i-1}\neq\Delta_i$.

With the change of the variable above, the second derivative of the Hermite polynomial can be written as

$$\ddot{p}_{i-1,i}(t) = \begin{bmatrix} \ddot{\phi}_{i,1}(t) \\ \ddot{\psi}_{i,1}(t) \\ \ddot{\phi}_{i,0}(t) \\ \ddot{\psi}_{i,0}(t) \end{bmatrix}^T \begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix} \tag{A3}$$

(A4)

where the second derivatives of the basis functions can be derived as follows.

Using the change of variables, we rewrite the basis functions as

$$\phi_{i,1}(t) = t^2(3\Delta_{i-1}-2t)/\Delta_{i-1}^3$$

$$\psi_{i,1}(t) = t^2(t-\Delta_{i-1})/\Delta_{i-1}^2$$

$$\phi_{i,0}(t) = (\Delta_{i-1}-t)^2(\Delta_{i-1}+2t)/\Delta_{i-1}^3$$

$$\psi_{i,0}(t) = t(\Delta_{i-1}-t)^2\Delta_{i-1}^2 \tag{A5}$$

Then, taking the second derivative with respect to t yields

$$\ddot{\phi}_{i,1}(t) = 6(\Delta_{i-1}-2t)/\Delta_{i-1}^3$$

$$\ddot{\psi}_{i,1} = (6t-2\Delta_{i-1})/\Delta_{i-1}^2$$

$$\ddot{\phi}_{i,0} = 6(2t-\Delta_{i-1})/\Delta_{i-1}^3$$

$$\ddot{\psi}_{i,0} = (6t-4\Delta_{i-1})/\Delta_{i-1}^2 \tag{A6}$$

Therefore, the integrand of the smoothing integral is expressed as

$$[\ddot{p}_{i-1,i}(t)]^2 = \begin{bmatrix} x_i \\ \\ x_{i-1} \end{bmatrix}^T \begin{bmatrix} \\ K_{i-1,i}(t) \\ \\ \end{bmatrix} \begin{bmatrix} x_i \\ \\ x_{i-1} \end{bmatrix} \tag{A7}$$

where $K_{i-1,i}$ is defined as

$$K_{i-1,i}(\mu) \stackrel{\Delta}{=}$$

$$\begin{bmatrix} \ddot{\phi}_{i,1}(\mu)\ddot{\phi}_{i,1}(\mu) , & \ddot{\phi}_{i,1}(\mu)\ddot{\psi}_{i,1}(\mu) , & \ddot{\phi}_{i,1}(\mu)\ddot{\phi}_{i,0}(\mu) , & \ddot{\phi}_{i,1}(\mu)\ddot{\psi}_{i,0}(\mu) \\ \ddot{\psi}_{i,1}(\mu)\ddot{\phi}_{i,1}(\mu) , & \ddot{\psi}_{i,1}(\mu)\ddot{\psi}_{i,1}(\mu) , & \ddot{\psi}_{i,1}(\mu)\ddot{\phi}_{i,0}(\mu) , & \ddot{\psi}_{i,1}(\mu)\ddot{\psi}_{i,0}(\mu) \\ \ddot{\phi}_{i,0}(\mu)\ddot{\phi}_{i,1}(\mu) , & \ddot{\psi}_{1,0}(\mu)\ddot{\psi}_{i,1}(\mu) , & \ddot{\psi}_{i,0}(\mu)\ddot{\phi}_{i,0}(\mu) , & \ddot{\psi}_{i,0}(\mu)\ddot{\psi}_{i,0}(\mu) \\ \ddot{\psi}_{i,0}(\mu)\ddot{\phi}_{i,1}(\mu) , & \ddot{\psi}_{1,0}(\mu)\ddot{\psi}_{i,1}(\mu) , & \ddot{\psi}_{i,0}(\mu)\ddot{\phi}_{i,0}(\mu) , & \ddot{\psi}_{i,0}(\mu)\ddot{\psi}_{i,0}(\mu) \end{bmatrix} \tag{A8}$$

By utilizing the above equation, the smoothing integral becomes

$$I_{i-1,i} = \begin{bmatrix} x_i \\ \\ x_{i-1} \end{bmatrix} \begin{bmatrix} \int_0^{\Delta_{i-1}} K_{i-1,i}(t)dt \end{bmatrix} \begin{bmatrix} x_i \\ \\ x_{i-1} \end{bmatrix} \tag{A9}$$

Evaluating the above integral, we obtain

$$\int_0^{\Delta_{i-1}} K_{i-1,i}(t)dt =$$

$$\begin{bmatrix} 12/\Delta_{i-1}^3 & -6/\Delta_{i-1}^2 & -12/\Delta_{i-1}^3 & -6/\Delta_{i-1}^2 \\ -6/\Delta_{i-1}^2 & 4/\Delta_{i-1} & 6/\Delta_{i-1}^2 & 2/\Delta_{i-1} \\ -12/\Delta_{i-1}^3 & 6/\Delta_{i-1}^2 & 12/\Delta_{i-1}^3 & 6/\Delta_{i-1}^2 \\ -6/\Delta_{i-1}^2 & 2/\Delta_{i-1} & 6/\Delta_{i-1}^2 & 4/\Delta_{i-1} \end{bmatrix} \qquad (A10)$$

Matrices B_{i-1}^{-1} and A_{i-1} are defined as follows:

$$A_{i-1} = \begin{bmatrix} 1 & \Delta_{i-1} \\ 0 & 1 \end{bmatrix} \qquad (A11)$$

$$B_{i-1}^{-1} = \begin{bmatrix} 12\Delta_{i-1}^{-3} & -6\Delta_{i-1}^{-2} \\ -6\Delta_{i-1}^{-2} & 4\Delta_{i-1}^{-1} \end{bmatrix} \qquad (A12)$$

where B_{i-1}^{-1} is a symmetric matrix. Equation (A10) can then be expressed as

$$\begin{bmatrix} B_{i-1}^{-1} & -B_{i-1}^{-1}A_{i-1} \\ (-B_{i-1}^{-1}A_{i-1})^T & A_{i-1}^T B_{i-1}^{-1} A_{i-1} \end{bmatrix} \qquad (A13)$$

where B_{i-1}^{-1} and A_{i-1} are functions of the variable Δ_{i-1}. By using the above notation, Eq. (A9) is rewritten as

$$
\begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix}^T
\begin{bmatrix} B_{i-1}^{-1} & -B_{i-1}^{-1}A_{i-1} \\ -A_{i-1}^T B_{i-1}^{-1} & A_{i-1}^T B_{i-1}^{-1} A_{i-1} \end{bmatrix}
\begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix}
$$

$$
= \begin{bmatrix} x_{i-1} \\ x_i \end{bmatrix}^T
\begin{bmatrix} A_{i-1}^T B_{i-1}^{-1} A_{i-1} & -A_{i-1}^T B_{i-1}^{-1} \\ -B_{i-1}^{-1} A_{i-1} & B_{i-1}^{-1} \end{bmatrix}
\begin{bmatrix} x_{i-1} \\ x_i \end{bmatrix}
$$

$$
= \begin{bmatrix} x_{i-1} \\ x_i \end{bmatrix}^T
\begin{bmatrix} -A_{i-1}^T \\ I \end{bmatrix} B_{i-1}
\begin{bmatrix} -A_{i-1} & I \end{bmatrix}
\begin{bmatrix} x_{i-1} \\ x_i \end{bmatrix}
$$

$$
= (x_i - A_{i-1}x_{i-1})^T B_{i-1}^{-1} (x_i - A_{i-1}x_{i-1}) \tag{A14}
$$

or

$$
I_{i-1,i} = \begin{bmatrix} x_{i-1} \\ x_i \end{bmatrix}
\begin{bmatrix} C_{i-1} & D_{i-1} \\ D_{i-1}^T & E_{i-1} \end{bmatrix}
\begin{bmatrix} x_{i-1} \\ x_i \end{bmatrix} \tag{A15}
$$

where

$$
C_{i-1} = \rho A_{i-1}^T B_{i-1}^{-1} A_{i-1} \tag{A16}
$$

$$
D_{i-1} = -\rho A_{i-1}^T B_{i-1}^{-1} \tag{A17}
$$

$$
E_{i-1} = \rho B_{i-1}^{-1} \tag{A18}
$$

Thus, the smoothing integral is transformed into the above quadratic form.

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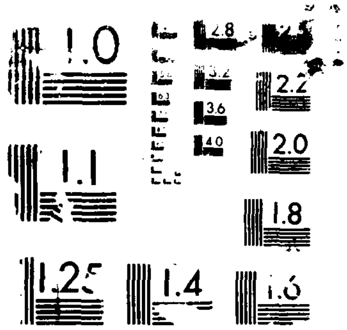
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THE C^2 CONTINUITY OF PIECEWISE CUBIC HERMITE POLYNOMIALS WITH UNEQUAL INTERVALS

by

C. N. SHEN

Please remove pages 1 through 4 from above publication and insert new pages enclosed. Corrections have been made to Equations 2, 9, 10, and 11 on pages 1 and 3.

US ARMY ARMAMENT RESEARCH, DEVELOPMENT,
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INTRODUCTION

The smoothing of gradients can be obtained by using an optimization method for approximation involving spline functions. A nonuniform grid may be employed to compute by the spline function method with cubic hermite polynomials. Continuous second derivatives at the grid point from both sides are essential for the purpose of smoothing. This method can be applied to solve the following problems: whether the platform can climb on the estimated in-path slope or whether it will tip over the estimated cross-path slope.

RECURSIVE FILTERING AND SMOOTHING PROCEDURE

A spline function $s(\xi)$ is a solution to the optimization problem

$$J^* = \min_{h \in C^2} \left\{ \sum_{i=1}^{N} [h(\beta_i)-m_i]^T R_i^{-1} [h(\beta_i)-m_i] + \rho \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [h]^2 d\xi \right\} \tag{1}$$

where for clarity and simplicity in discussion, we c ly consider the cubic spline case. A higher order polynomial spline can also be treated in a similar manner with more complicated computations.

A cubic spline, s, is a piecewise polynomial of class C^2 which has many good properties, such as the minimum norm property and local base property (refs 1,2). From the approximation theory, we know that for each set $A = \{a_1,\ldots,a_N, a'_1, a'_N\}$, there exists a unique cubic spline $s(\xi;A)$ such that

$$s(\beta_i;A) = a_i \quad , \quad i = 1,2,\ldots,N \tag{2}$$

$$\dot{s}(\beta_i;A) = a'_i, \ i = 1,N \tag{3}$$

where $\dot{s}$ is the first derivative of the function s. The above equations can be

[1]Ahlberg, J. H., Nilson, E. N., and Walsh, J. L., <u>The Theory of Splines and Their Applications</u>, Academic Press, Inc., 1967.
[2]Schumaker, L. L., <u>Spline Functions: Basic Theory</u>, John Wiley & Sons, 1981.

thought of as boundary conditions for the piecewise cubic spline interpolation given a set of data (β_i, a_i), for $i = 1,2,\ldots,N$. Thus, solving the problem in Eq. (1) is equivalent to determining a set of constraints A for the optimization problem:

$$J^* = \min_A \left\{ \sum_{i=1}^{N} [s(\xi_i;A)-m_i]^T R_i^{-1} [s(\xi_i;A)-m_i] + \rho \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [s(\xi;A)]^2 d\xi \right\} \quad (4)$$

Instead of taking a direct approach to find an optimal set of constraints for the problem above, it is proposed to further transform this problem into a form which is convenient to be solved. From the theory of numerical analysis (ref 3), it is well known that a piecewise cubic Hermite polynomial $p(\xi)$ is in the family of C^1. For each set $B = A \cup A^C$, where A^C is a complement of A, i.e., $A^C = \{a'_i,\ i = 2,3,\ldots,N-1\}$, then $B = \{a_i, a'_i,\ i=1,2,\ldots,N\}$, there exists a unique piecewise cubic Hermite polynomial $p(\xi;A)$ such that

$$p(\beta_i;B) = a_i \quad , \quad i = 1,2,\ldots,N \quad (5)$$

$$\dot{p}(\beta_i;B) = a'_i \quad , \quad i = 2,\ldots,N \quad (6)$$

where $\dot{p}$ is the first derivative of p.

It should also be noted that for each set A, there are an infinite number of piecewise Hermite polynomials $p(\xi;A)$ such that

$$p(\beta_i;A) = a_i \quad , \quad i = 1,2,\ldots,N \quad (7)$$

$$\dot{p}(\beta_i;A) = a'_i \quad , \quad i = 1,N \quad (8)$$

Let a set of $p(\xi;A)$ which satisfies the constraints in the equations above be P, i.e.,

$$P = \{p(\xi;A) : (5),(6) \text{ satisfied}\} \quad (9)$$

[3]Burden, R. L. et al., Numerical Analysis, Prindle, Weber, & Schmidt, 1978.

Referring to the paper by de Boor (ref 4), it is noted that there exists a unique cubic spline $s(\xi;A)$ in the set P. Also from the minimum norm property of a cubic spline, we have the following relation:

$$\sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{s}(\xi;A)]^2 d\xi \leq \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{p}(\xi;A)]^2 d\xi \tag{9}$$

That is

$$\sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{s}(\xi;A)]^2 d\xi = \inf_{p \in P} J_p(p) \tag{10}$$

where

$$J_p = \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{p}(\xi;A)]^2 d\xi \tag{11}$$

Since a cubic spline $s(\xi;A)$ is unique, a piecewise cubic Hermite polynomial $p(\xi;A)$ which minimizes the smoothing integral J_p in the above equation with respect to A^c becomes a cubic spline $s(\xi;A)$. To be more precise, we have the following theorem.

THEOREM: Let P represent a set of piecewise cubic Hermite polynomials p which satisfies the constraints below:

$$p(\beta_i;A^c) = a_i \quad , \quad i = 1,2,\ldots N \tag{12}$$

$$\dot{p}(\beta_i;A^c) = a'_i \quad , \quad i = 1,N \tag{13}$$

where $p \in C^1$, A, and A^c are the same as mentioned before. Then there exists a unique cubic spline $s(\xi)$ such that

$$\sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{s}(\xi)]^2 d\xi = \min_{A^c} \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{p}(\xi,A^c)]^2 d\xi \tag{14}$$

where $\ddot{s}$ and $\ddot{p}$ are the second derivatives of functions s and p and $s \in C^2$. A simple example with N = 3 is given next.

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[4]de Boor, C., "Bicubid Spline Interpolation," J. Math Phys., Vol. 41, 1962, pp. 212-218.

EXAMPLE FOR C^2 CONTINUITY

For convenience and simplicity, we only consider a special case with N = 3. The node points are given as β_1, β_2, and β_3. The intervals are not equal, i.e.,

$$(\beta_2-\beta_1) \neq (\beta_3-\beta_2) \tag{15}$$

Let a set of piecewise cubic Hermite polynomials p be

$$P = [p(t;A^c) \quad , \quad p \in C^1\ [t_1,t_3],\ \dot{p}(t_2) = a,\ a \in A^c] \tag{16}$$

which satisfies the constraints in the equations below:

$$p(t_i;A^c) = a_i \quad , \quad \text{for } i = 1,2,3$$

$$\dot{p}(t_i;A^c) = a'_i \quad , \quad \text{for } i = 1,3 \tag{17}$$

In this special case, a set $A^c = a'_2 = a$.

We want to show here that the cubic Hermite polynomial $p(t;A^c)$, which is obtained by minimizing the smoothing integral, will become a cubic spline function $s(t) \in C^2[t_1,t_3]$

$$J^* = \min_{A^c} \{\int_{t_1}^{t_2^-} [\ddot{p}(t;A^2)]^2dt + \int_{t_2^+}^{t_3} [\ddot{p}(t;A^c)]^2dt\}$$

$$= \min_{a} \{\int_{t_1}^{t_2^-} [\ddot{p}(t;a)]^2dt + \int_{t_2^+}^{t_3} [\ddot{p}(t;a)]^2dt\} \tag{18}$$

From Eq. (A14) of the Appendix, the smoothing integral above can be written as

$$J(a) = (x_2-A_1x_1)^TB_1^{-1}(x_2-A_1x_1) + (x_3-A_2x_2)^TB_2^{-1}(x_3-A_2x_2) \tag{19}$$

where A_i, B_i^{-1}, and x_i are defined in the Appendix, and

$$x_i = (a_i,a'_i)^T \quad , \quad \text{with } a'_2 = a \quad , \quad i = 1,2,3 \tag{20}$$

$$\Delta_{i-1} = d_{i-1} = t_i-t_{i-1} \tag{21}$$

Using Eqs. (A11) and (A12), the functional J(a) is written as

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