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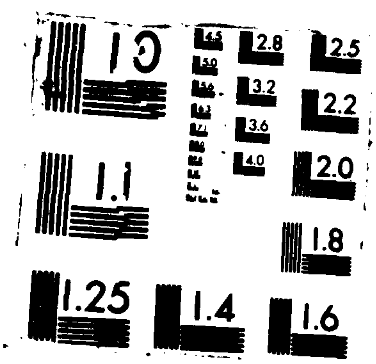
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This report describes the mathematical algorithms of a prototype version of the Aircraft Sustainability Model (ASM), a model designed to relate U.S. Air Force funding for reparable spares to aircraft sustainability measures.

The ASM is a multi-echelon, multi-indenture inventory model that is unique in its portrayal of the transient effects resulting from a dynamic demand environment. Furthermore, the ASM is unique in its ability to simultaneously optimize spare requirements for multiple days of a conflict scenario. Keywords: Aircraft maintenance; Spare parts; Inventory modeling.

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LMI

Executive Summary

PROTOTYPE AIRCRAFT SUSTAINABILITY MODEL

In FY87, the Air Force budget for Aircraft Replenishment Spares [Budget Program 1500, (BP-15)] was $3.8 billion. Of this total, $2.6 billion was for Peacetime Operating Stocks (POS), which support peacetime operations and provide readiness for war. The remaining $1.2 billion was for War Reserve Materiel (WRM) spares, which enable the Air Force to sustain the higher activity levels of a conflict.

The Aircraft Availability Model (AAM), part of the Air Force Logistics Capability Measurement System, allows Air Force decision-makers to analyze and evaluate the effect on aircraft readiness of differing funding levels and allocations to weapon systems. Such a capability is sorely needed for the WRM segment of the budget.

We present a new model of wartime sustainability that relates resources to fighting ability over a period of time. Specifically, it relates funding by weapon system to the probability – day by day – of being able to attain the flying levels specified in the Air Force War and Mobilization Plan (WMP). Moreover, the relationship between funding and sustainability is computed in a way that permits timely analysis of differing levels of WRM funding and probability. This model enables military planners to develop and evaluate budgets for WRM in a way that is rational and defensible.

The Aircraft Sustainability Model (ASM) optimizes logistic spares support simultaneously for multiple days of the WMP scenario. It does this by combining two systems. The first, the Marginal Analysis System (MAS), is a multi-echelon, multi-indenture model that optimizes logistics spares support for a single day of the scenario. This system is similar in structure to existing spares models, both of wartime sustainability and of peacetime readiness. It is driven by activity levels, item failure rates, and resupply times. Its primary output is a curve of the probability of achieving a prescribed level of activity on a specific day of the war as a function of the WRM spares budget.

Multiple runs of the MAS are used to analyze multiple days of the scenario. The ASM then uses a second system, the Cross-Linker, which accesses the output files from the MAS runs and combines them, to optimize spares support simultaneously over all chosen days of the scenario. The MAS analyses of each chosen day need only be performed once; the impacts of changes in funding level or sustainability profile can be quickly analyzed by a rerun of the Cross-Linker.

The ASM is capable of large-scale budget computations that cannot be made with present models of wartime sustainability. With the resulting ability to relate resources to sustainability, military planners will be able to make more informed budget decisions and to defend those decisions effectively.

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OVERVIEW

INTRODUCTION

Though models of peacetime weapon system readiness as a function of spares support are common and widely used, models of wartime sustainability are rare and of limited application. Determinations of Air Force-wide requirements, such as those used to size and defend peacetime budgets, are simply beyond the scope of existing wartime models.

A key problem with modeling wartime is the difficulty in quantifying sustainability. In the steady-state environment of peacetime, a single number, such as availability rate,[1] is enough to describe readiness. But, in the dynamic environment of war, no single number can adequately show how the system will perform over time. One mix of spares may yield better performance early in the war, while another mix may yield better performance later. Simply averaging system performance over a given period is not enough; in particular, the early days of a wartime scenario may be more crucial and therefore deserve more weight.

Another aspect of modeling sustainability is the difficulty of computing, in a *dynamic* environment, the number of units in the various logistics resupply pipelines. When the resupply processes take time, as they must, the transient effects caused by a surge in activity take time to filter through the system. Wartime sustainability models that compute pipeline quantities on the basis of straightforward extensions of steady-state techniques do not capture the transient effects correctly. It is essential to compute explicitly, *as a function of time*, both the mean and the variance of the number of units in the resupply pipelines .

The model presented here is unique in its treatment of both sustainability measures and logistics pipeline quantities in a dynamic environment. Resupply pipelines are computed through rigorous application of VARI-METRIC [1] theory to

[1]We define availability rate to be the probability that a randomly selected weapon system is not waiting for a spare part.

the dynamic environment, while the choice of a measure of sustainability is designed to fit the way wartime surges of aircraft activity are planned.

The U.S. Air Force's War and Mobilization Plan (WMP) includes projected levels of wartime flying activity. An appropriate measure of sustainability is the length of time that the Air Force can sustain the activity levels prescribed in the WMP until spares limitations force these levels to fall. Because random effects make this number impossible to predict with certainty, such a measure must be defined in terms of a level of confidence – e.g., "How long can we, with a 95-percent probability, fly the WMP program?" Nevertheless, once "flying the WMP" is defined in these terms, the length of time the Air Force can fly the WMP is an excellent measure of sustainability.

With the Aircraft Sustainability Model (ASM), the user can compute the minimum cost and the associated optimal spares mix to achieve a prescribed set of probabilities of flying the WMP for the duration of the scenario. Budget estimates for war reserve spares that are both rational and defensible can thus be prepared. Rather than be restricted to a prespecified confidence level in its definition of "flying the WMP," the model can accept varying levels, day by day. This feature enables the user to choose any profile of target probabilities of flying the WMP – not merely a constant percentage – as the objective function.

The ASM calculates the actual probabilities by integrating two main systems. The first, the Marginal Analysis System (MAS) computes, for a specific day, a curve of the cost versus the probability of flying the WMP (PWMP). The user builds this curve by computing the marginal worth of each component – its contribution, per dollar of investment – to the PWMP objective function. With multiple MAS runs, the user can assess the PWMP over a period of time. The second system, the Cross-Linker, merges the outputs of multiple marginal analysis runs to produce a curve of cost versus a vector of PWMPs for the days being merged.

APPROACH

The ASM is derived from the Aircraft Availability Model (AAM) of the Logistics Management Institute (LMI). This is a peacetime readiness model that has been used for years by Headquarters, U.S. Air Force in its analysis of funding requirements for peacetime spares. The AAM relies on a large body of well known steady-state mathematics appropriate for the relatively stable flying-hour programs of peacetime.

The AAM relates the Peacetime Operating Stock (POS) portion of the replenishment spares requirement contained in Budget Program 1500 (BP-15) to aircraft availability rates. (An available aircraft is defined as one that is not grounded for lack of a reparable spare.) The AAM produces a curve of the optimal funding/availability relationship; i.e., each point on the curve represents either the maximum achievable availability for a given level of funding or the minimal funding required to support a given level of availability. The AAM is both multi-echelon (makes optimal decisions regarding distribution of spares between bases and the depot) and multi-indenture (trades off the value of line replaceable units (LRUs) directly installed on the aircraft versus the constituent shop replaceable units (SRUs)). For more details on the AAM, see [2].

The ASM is the result of recent research aimed at extending the AAM's capability so that it can relate wartime sustainability to the War Reserve Materiel (WRM) portion of BP-15. The ASM can potentially become an integrated BP-15 requirements model, capable of measuring the effect of POS underfunding upon wartime capability. The requisite mathematics for handling the pipeline and backorder calculations in a dynamic environment is documented in reference [3], which outlines the mathematical underpinnings of the technique and documents the results from a prototype *assessment* model. On the basis of given asset positions for reparable components, that model projects the expected availability rates throughout a dynamic scenario provided by the user.

Though the ASM has assessment capabilities, it is a *requirements* model; it computes the WRM needed to fly the WMP, with a specified level of confidence, over a given period of time. As noted earlier, the ASM consists of two systems: the MAS and the Cross-Linker. The sustainability measure used by the MAS is the PWMP on a specific day. In calculating the PWMP, the user computes the probability that – assuming full cannibalization – the number of aircraft classed Not Mission Capable due to Supply (NMCS) is within the limit set by the user. Stated another way: It is the probability that enough aircraft are still available to fly the WMP. The MAS analyzes a single day of the surge, producing an optimal curve of cost versus probability for that day. As with the AAM, every point on this curve is backed up by an optimal spares mix (shopping list) that would, if included in the inventory, yield the highest possible level of sustainability for that budget.

By making multiple MAS runs for various days of a scenario, the user can investigate weapon system performance over a period of time. Then, using the Cross-Linker, the user can compute a budget or shopping list that is optimized, not for any individual day, but for a collection of days.

A common technique for computing a spares budget or shopping list for a wartime scenario would involve accumulation of day-by-day shopping lists: Run the MAS for the first day, "buy" the shopping list for the target probability, start the MAS model run for the second day (with the first day's shopping list included as assets in stock), and "buy" whatever additional assets are needed to reach the target probability for that second day. Repeat the process until all the days are "covered." Unfortunately, this final asset position is not optimized for the objective function (the set of PWMPs for the period).

A better procedure is to run the MAS for the days of interest and then use the Cross-Linker to combine the results. Each MAS run uses the same component data base. No shopping lists are generated by the MAS, and the curves of cost versus probability produced for the individual days are ignored. The Cross-Linker then merges the MAS outputs so as to produce a combined curve of cost versus probabilities.

The Cross-Linker works by defining a new, single objective function that is a linear combination of the PWMPs for the individual days. By merging the MAS outputs and optimizing on the linear combination of the PWMPs, the Cross-Linker produces a single curve of cost versus this new objective function. The Cross-Linker retains the individual pieces of this objective function, so that, instead of displaying the linear combination (which is of no particular interest), it displays all of the constituent PWMPs. That is, each "point" on the combined curve is displayed as a single cost value and a set of PWMPs.

The optimality of each point on the combined curve means that the spares mix for that point cannot be dominated by another spares mix. That is, any other spares mix that yields greater PWMPs for *every* day analyzed must cost more.[2] However, another spares mix may exist that yields greater PWMPs for *some* days, yet costs less. If the linear combination of PWMPs used by the Cross-Linker is not well balanced, the combined curve generated may not contain the best solution for a given set of PWMP *targets*. Though the first point (i.e., least cost) on the curve at which all the attained probabilities exceed the PWMP targets is undominated, some of the daily PWMPs may greatly exceed the corresponding targets, providing an opportunity for further economy. If day 5, for example, greatly exceeds its target, optimizing on a different linear combination (one that gives less weight to day 5) might produce a more balanced, economical solution.

In general, each set of coefficients for the linear combination of the daily PWMPs determines a feasible, undominated solution – the least-cost point on the associated curve at which all the daily probabilities meet or exceed the target. A search through the set of linear combinations will produce the global optimal solution.

Theoretically, we could analyze every day of the scenario, but practical considerations encourage us to implement this technique by analyzing a subset of those days. Exactly which days should be included in the subset is a matter for future research. However, we will presumably analyze a higher percentage of the earlier days than the later ones.

The prototype Cross-Linker system merges only two MAS runs and does not directly accept the target PWMPs as input. Rather, this version uses a "constant of proportionality," input by the user, to determine the linear combination. With the prototype Cross-Linker, the user runs the MAS for 2 days of the scenario and then runs the Cross-Linker on the output from those 2 days. The constant of proportionality simply represents the relative weight given the 2 days' PWMPs. If the balance between those two probabilities is not satisfactory, the constant can be changed and the Cross-Linker rerun. The process is repeated until the desired balance between the two PWMPs is achieved. We expect that further research will generate a suitable algorithm to automate this process.

[2]Actually, the optimality of this approach can be proven in a single level-of-indenture system only. The degree to which this method is suboptimal, given more than one indenture level, is expected to be minimal but will nonetheless be the subject of future research.

CHAPTER 2

THE SURGE PROTOTYPE

MODEL STRUCTURE

The ASM prototype was developed for IBM personal computers and compatibles. The source is written in FORTRAN 77 and was derived from the Demonstration Aircraft Availability Model (DAAM) [4].

The MAS is identical in structure to the DAAM, except for the dynamic pipeline computations and use of the PWMP as the objective function instead of aircraft availability. These differences have their principal effect on the input and output formats for the MAS. The dynamic pipeline computation and the formula for the probability of flying the WMP will be discussed in the section on MAS algorithms.

The MAS, like the DAAM and its ancestor, the AAM, uses a combination of marginal analysis programs and sorts to produce a curve of cost versus effectiveness (in this case, the probability of flying the WMP). The prototype version, which handles two levels of indenture, consists of an SRU marginal analysis program (SURGES), a sort, an LRU marginal analysis program (SURGEL), another sort, and a program to print the curve of cost versus the probability of flying the WMP (CURVES). We do not discuss the CURVES program here; it is virtually identical to the corresponding program in the DAAM. We use the same commercial sort as in the DAAM (SUPERSORT). For a more complete discussion of this basic model structure, see [4].

The prototype Cross-Linker consists of three programs: the Merger, a sort, and the CURVES program. The Merger reads the output from 2 days' MAS runs and generates a single file (UNSORTED) that is similar to the UNSORTED file generated by the MAS (and by the DAAM), except the records are extended to contain the data for both days. SUPERSORT is again used for the sort, and the CURVES program is the same as the CURVES programs in the MAS except that it prints two PWMPs. The technique for merging two MAS outputs will be discussed in the Merger Algorithms section.

MODEL ASSUMPTIONS

The ASM prototype is a two-indenture, two-echelon requirements model for a single weapon system. An aircraft is down for supply upon failure of a first-indenture LRU for which no spare is available. All failures occur at first-echelon sites (bases). The bases are presumed to be uniform with respect to demands, resupply times, and repair capabilities. If an item cannot be repaired at the base, it is shipped to the second echelon (depot) for possible repair. Replenishment from the depot is immediately requested. At the depot, the item may be repaired or condemned. In the latter case, a replenishment request is made from an outside source of supply. Sites at both echelons – base and depot – are presumed to operate under an $(s-1,s)$ inventory policy.

At either the base or the depot, repair of the LRU often consists of isolating the fault and replacing a defective SRU subassembly. Shortages of SRUs delay LRU repair; shortages of LRUs cause aircraft to be unavailable for use. The ASM makes the tradeoffs implicit in this indenture distinction and also distributes spare allocations optimally among sites.

The surge scenario consists of describing the operating tempo and the characteristics of the bases and depot. These characteristics include the number of bases, the resupply times, and the repair capability as prescribed by the probability of repair at each site. The user inputs the operating tempo to the ASM by specifying the day-by-day flying-hour program for the weapon system. Derivation of these flying hours from the more detailed conflict scenarios available to war planners (sortie rates, sortie durations, turn times, etc.) is presumed to be a preprocessing activity.

INPUT REQUIREMENTS

The dynamic nature of the typical wartime scenario requires substantial modifications to the input data structures from those input requirements of the DAAM or the AAM. The user first provides the name of the weapon system, the number of units deployed, and the number of deploying bases. All bases are assumed to be homogeneous with respect to demand rates, resupply times, and repair capabilities. The user also provides the variance-to-mean ratio option (VMOPTION) and the variance-to-mean ratio (VMR) for the underlying demand process (VMR = 1 for the default Poisson option), and the percentage of pipeline requirements that are to be automatically "purchased" without regard to considerations of marginal analysis (default = 100 percent).

The ASM has a more sophisticated VMR logic than did earlier versions of the AAM. Though a VMR of 1 (or any other constant) may be used – the model logic will be the same in those cases as in previous versions – the ASM contains a full implementation of VARI-METRIC [1]. If this option is chosen, the model will compute explicitly the VMR of pipelines and backorders. The variance of backorders at the depot and the variance of SRU backorders will be used to compute the VMR of the number of LRUs due in to base supply.

The prototype ASM model requires the user to input the surge scenario in terms of day-by-day flying hours. Specifically, an input file is set up to read:

- NDAYS – The dimension of the dynamic arrays of component data
- NDAYSFH – The dimension of the flying-hour program array
- FHP(T) – Flying-hour program on day T for T = 0, 1, . . . , NDAYSFH.

The scenario is portrayed with peacetime flying hours for day T = 0 and wartime factors starting with day T = 1. NDAYS and NDAYSFH are not necessarily the last day of the surge activity. The model assumes that all days preceding day 0 have the same parameters as day 0 and that after NDAYS or NDAYSFH the parameters are the same as day NDAYS or NDAYSFH, respectively. For example, if NDAYSFH = 6 and FHP(T) = 1, 6, 6, 6, 6, 6, and 4 for T = 0, 1, 2, 3, 4, 5, and 6, respectively, we get the flying-hour program of Figure 2-1.

Note that day T is depicted on the horizontal axis of Figure 2-1 as the interval:

$$\{t: T - 1 < t \leq T\}$$

All model computations and outputs are defined as of the *end* of each day's flying activity.

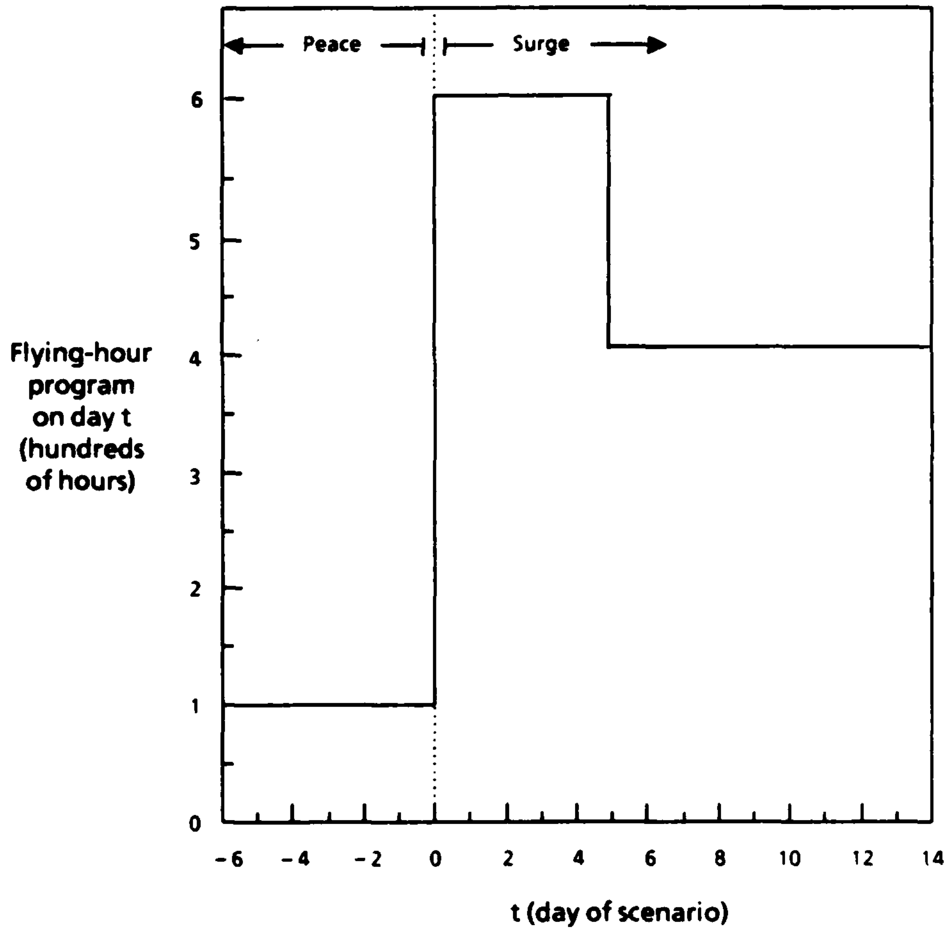

FIG. 2-1. SAMPLE FLYING PROFILE

In addition to the daily flying hours, the MAS model requires three scenario inputs:

- The number of days of warning before the start of the surge (NDAYWARN). At the start of the surge, many component parameters may change. Base and depot repairs may be expedited, shortening the repair time. Given a warning period before the start of the surge, those parameters change that much sooner. MAS models this explicitly; i.e., all component characteristics (see the next paragraph for a description of these component inputs) change NDAYWARN days earlier than they would if there were no warning. The component failure rate is driven by actual activity levels, not the anticipation of them, and is therefore an exception – the one exception – to this rule.

- The day of the war for which we are interested in computing performance (ITODAY).
- WMPNMCS(T), which specifies the maximum number of NMCS aircraft that can be grounded (with cannibalization), with the remaining aircraft inventory still able to meet the flying-hour objective, FHP(T), for day T. The user presumably derives the WMPNMCS values from an actual WMP document, together with knowledge of average sortie lengths and turn times for the weapon system under investigation.

The dynamics of the scenario are further defined for our component-specific input data, as shown in Table 2-1.

TABLE 2-1

DYNAMIC COMPONENT DATA

Name	Definition
BRT(T)	Base repair time for all units emerging from base repair on day T
OST(T)	Order-and-ship time from depot to base for all units arriving on day T
DRT(T)	Depot repair time for units emerging from depot repair on day T
FF(T)	Failures per flying hour on day T
BNRTS(T)	Base not-reparable-this-station rate for items failing on day T
CONPCT(T)	Condemnation percentage of all items failing on day T
BRTNHA(T) (SRU only)	Base repair time of the parent next-higher-assembly for units emerging from base repair on day T

Note: All arrays are defined over the range T = 0, 1, 2, . . . , NDAYS.

Note that resupply times are keyed to the emergence time – as opposed to the induction time – of the item. The reason for this untraditional approach is complex and is treated in the section on MAS Algorithms.

Finally, the user must provide the component-specific static information shown in Table 2-2.

TABLE 2-2

STATIC COMPONENT DATA

Name	Description
NSN	National Stock Number of the component
COST	Unit procurement cost
QPA	Quantity per application on the next-higher-assembly
FAP	Future application percentage
PLT	Procurement leadtime (months)
ASSETS	The starting spares level of the component
NHANSN	The NSN of the next-higher-assembly

The terminology and variable names are similar to those used in the Recoverable Consumption Item Requirements (D041) system, the primary input to the standard AAM.

MAS ALGORITHMS

This section describes the basics of our pipeline methodology, including the nature of first-in/first-out (FIFO) processes, the modeling effects of dynamic resupply times, and the proper modeling of the transient effects, explicitly considering the lag between the actual component failure and the time these failures are felt in the supply system.

We document the MAS algorithms through a series of examples and refer frequently to the results of reference [3].

Pipeline Calculations with Constant Resupply Times

This calculation is identical to the pipeline calculation in [3], except that here the continuous time notation has been adapted to the discrete (day-by-day) techniques we used in the actual implementation. Assume that a particular LRU fails according to a nonhomogeneous Poisson process with "intensity" function $\lambda(t)$

on day t. If $BSUPP_T$ is the total number in resupply to all bases on day T, then we have this fundamental decomposition of the random variable $BSUPP_T$:

$$BSUPP_T = BREP_T + DDEM_{[T-OST,T]} + DBO_{T-OST} \qquad \text{[Eq. 2-1]}$$

where OST is the constant (for now) order-and-ship time from the depot to the base, and:

$BREP_T$ = The number of LRUs in base repair at time T (including, perhaps, LRUs waiting for SRU repair parts)

$DDEM_{[T-OST,T]}$ = The number of depot demands in the interval [T−OST,T]

and:

DBO_{T-OST} = The number of depot backorders existing at time T−OST.

Note that the three random variables on the right side of Equation 2-1 are mutually independent with known distributions. An exact computation for the distribution of $BSUPP_T$ is possible[1] but difficult. Instead, we approximate it with a negative binomial (NB) distribution. The NB is specified by two parameters: the mean μ and variance σ^2. We find these by summing the means (pipelines) and variances for the three random variables $BREP_T$, $DDEM_{[T-OST,T]}$, and DBO_{T-OST}. We illustrate these computations with a series of examples of increasing complexity.

We assume the following wartime scenario: NDAYSFH = 6 (refer to the Input Requirements section for variable name definitions) and the flying-hour profile shown in Table 2-3. Flying hours are stated in hundreds. Table 2-3 describes a scenario with a 5-day surge at 6 times the peacetime rate (100 flying hours), followed by a sustained period at 4 times the peacetime rate. The first 15 days of this scenario were plotted in Figure 2-1.

[1] Actually, the problem is to evaluate the number in resupply to a specific base rather than to all bases. For the time being, we ignore this distinction. Equivalently, we may assume there is only one base. Extensions to multiple bases are addressed later in this report.

TABLE 2-3

SAMPLE SCENARIO

Day	0	1	2	3	4	5	6
FHP	1	6	6	6	6	6	4

Example 1: LRU with no subassemblies and constant resupply times. **Consider now a particular LRU with no subassemblies (and therefore no SRU delay) with the item characteristics as shown in Table 2-4.**

TABLE 2-4

BASE REPAIR PIPELINE FOR EXAMPLE 1

Constant resupply times	Non-dynamic failure characteristics
Base repair time = BRT = 5 days Order-and-ship time = OST = 3 days Depot repair time = DRT = 10 days	Failure factor = FF = 1 per 100 flying hours Not-reparable-this-station rate = NRTS = 0.5

Thus, half of all failures are fixed at base level; the others are depot repairable (no condemnations).

We begin with the base repair pipeline, BREPPIPE(T), for day T of the scenario. Recall the formulation in [3]:

$$\text{BREPPIPE}(T) = E(\text{BREP}_T) = \int_{T-\text{BRT}}^{T} \lambda(t)\,(1 - \text{NRTS})\,dt$$

In our discrete (day-by-day) application, the intensity on day t is:

$$\lambda(t) = \text{FF}(t) \times \text{FHP}(t)$$

and the integral above can be written as the summation:

$$\text{BREPPIPE(T)} = E(\text{BREP}_T) = \sum_{t=T-BRT+1}^{T} \lambda(t) \times (1 - \text{NRTS}) \qquad \text{[Eq. 2-2]}$$

which becomes, for our example, when T = 6:

$$\text{BREPPIPE(6)} = \sum_{t=2}^{6} [1 \times \text{FHP}(t)](1 - 0.5)$$

$$= (6)(0.5) + (6)(0.5) + (6)(0.5) + (6)(0.5) + (4)(0.5)$$

$$= 14.0$$

Simply put: To obtain the base repair pipeline on day T, accumulate the base repair inductions (failures × base repair rate) over the BRT days that end on day T. Note that a period of BRT days *ending* on day T encompasses days [T − BRT + 1] through T.

Recall our convention that all days preceding day 0 have the same (peacetime) parameters as day 0 and that all days after day 6 have the same parameters as day 6. The steady-state peacetime value for the base repair pipeline is, then:

$$\text{BREPPIPE(0)} = \sum_{t=-4}^{0} \text{FHP}(t) \times (.5)$$

$$= 5 \times \{(1)(0.5)\}$$

$$= 2.5.$$

It follows that BREPPIPE(T) as a function of T is as shown in Figure 2-2.

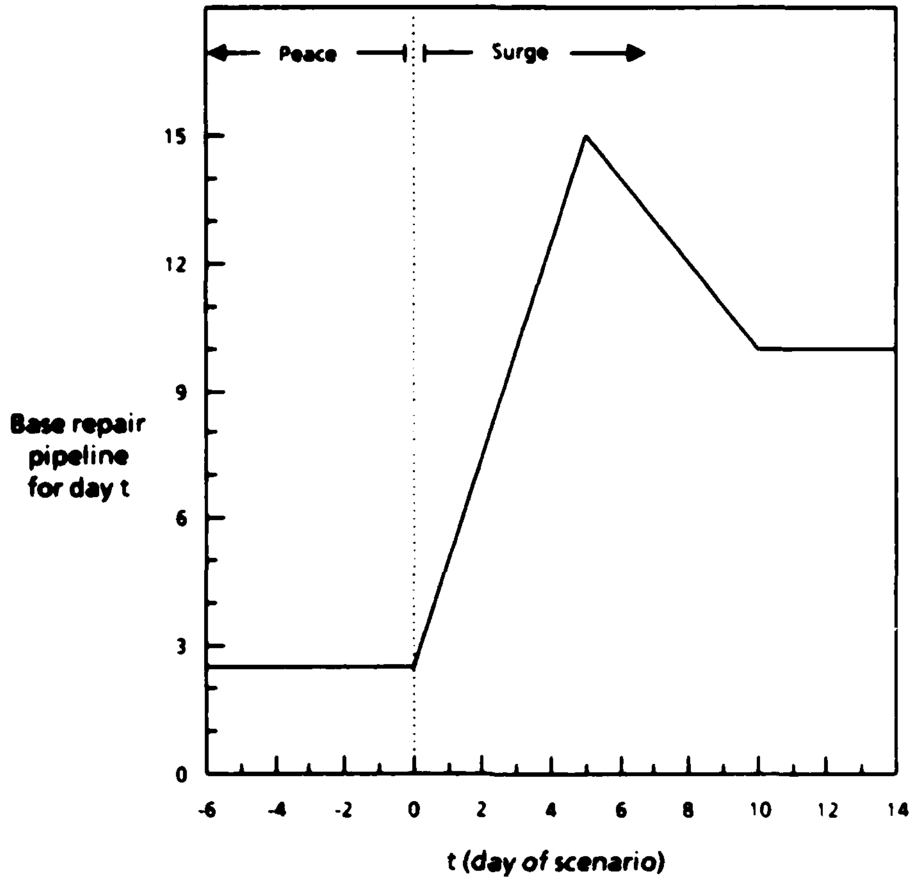

FIG. 2-2. BASE REPAIR PIPELINE FOR EXAMPLE 1

The calculation of the mean for $DDEM_{[T-OST,T]}$ is analogous to the BREPPIPE(T) calculation: Accumulate the depot repair inductions over the interval [T – OST,T]. In integral form:

$$DDEMPIPE(T) = DDEMPIPE(T) = E(DDEM_{[T-OST,T]}) = \int_{T-OST}^{T} \lambda(t)\,(NRTS)\,dt$$

while, in the context of our discrete setting,

$$DDEMPIPE(T) = E(DDEM_{[T-OST,T]}) = \sum_{t=T-OST+1}^{T} \lambda(t) \times (NRTS) \qquad [Eq.\ 2\text{-}3]$$

In our example, with T = 6 and OST = 3:

$$\text{DDEMPIPE}(6) = \sum_{T=6-3+1}^{6} \text{FF}(t) \times \text{FHP}(t) \times \text{NRTS}$$

$$= (6)(0.5) + (6)(0.5) + (4)(0.5)$$

$$= 8.0$$

To complete the calculation of the expected number in base resupply from Equation 2-1, we need to evaluate, for day T, the expected number (mean) of depot backorders at time T − OST. The mean and variance for depot backorders are computed with the standard formulas for inventory systems with an (s − 1,s) inventory policy [1].

If DREP_{T-OST} denotes the number in depot repair at time T − OST, the depot repair pipeline is calculated from:

$$\text{DREPPIPE}(T-OST) = E\left(\text{DREP}_{T-OST}\right) \qquad \text{[Eq. 2-4]}$$

$$= \sum_{t=T-OST-DRT+1}^{T-OST} \lambda(t) \times \text{NRTS}$$

For example, when T = 6, our sample component has a depot repair pipeline for day T − OST = 3 given by:

$$\text{DREPPIPE}(T-OST) = \text{DREPPIPE}(3) = \sum_{T=6-3-10+1}^{6-3} [\text{FF}(t) \times \text{FHP}(t)] \times \text{NRTS}$$

$$= 12.5$$

We take the expected values and variances on both sides of Equation 2-1 and, combining Equations 2-2 and 2-3, we obtain the mean and variance for BSUPP_T:

$$\text{BSUPPIPE}(T) = E(\text{BSUPP}_T) = \text{BREPPIPE}(T) \qquad \text{[Eq. 2-5]}$$

$$+ \text{DDEMPIPE}(T) + E(\text{DBO}_{T-OST})$$

and:

$$VSUPP(T) = V(BSUPP_T) = BREPPIPE(T) + DDEMPIPE(T) + V(DBO_{T-OST}) \quad \text{[Eq. 2-6]}$$

The derivation depends on these observations from [3]:

- The random variables $BREP_T$, $DDEM_{[T-OST,T]}$, and DBO_{T-OST} are mutually independent.
- Both $BREP_T$ and $DDEM_{[T-OST,T]}$ are nonhomogeneous Poisson variates with intensity functions $\lambda(t)\,(1-NRTS)$ and $\lambda(t)\,(NRTS)$, respectively.

Table 2-5 lists the values of the various pipelines across our hypothetical scenario. All are plotted in Figure 2-3 to show the time plot of the resupply pipeline, BSUPPIPE(T), as a function of time. This particular plot assumes that there are no depot spares; therefore, $E(DBO_{T-OST}) = DREPPIPE(T-OST)$ for all values of T.

Pipeline Calculations with Dynamic Resupply Times

Now, let us suppose we have an LRU with a BRT of 7 days in peacetime and 5 days in wartime. We first ask, "What does this mean?" What happens to the items already in base repair at the start of the war? We begin our treatment of this subject with a discussion of first-in/first-out (FIFO) processes.

FIFO Processes

We assume that all resupply processes are deterministic and FIFO. We define a deterministic resupply process as one in which all simultaneous failures of a particular component with identical sources of resupply have the same resupply time. Thus, all base-repairable failures of a particular component occurring on the same day will have the same resupply time. Because these times can vary from one day to another, "deterministic" does not mean "constant."

TABLE 2-5

SAMPLE PIPELINE CALCULATIONS FOR EXAMPLE 1

Day T	BREPPIPE(T)	DDEMPIPE(T)	DREPPIPE(T – OST)	BSUPPIPE(T)
– 3	2.5	1.5	5.0	9.0
– 2	2.5	1.5	5.0	9.0
– 1	2.5	1.5	5.0	9.0
0	2.5	1.5	5.0	9.0
1	5.0	4.0	5.0	14.0
2	7.5	6.5	5.0	19.0
3	10.0	9.0	5.0	24.0
4	12.5	9.0	7.5	29.0
5	15.0	9.0	10.0	34.0
6	14.0	8.0	12.5	34.5
7	13.0	7.0	15.0	35.0
8	12.0	6.0	17.5	35.5
9	11.0	6.0	19.0	36.0
10	10.0	6.0	20.5	36.5
11	10.0	6.0	22.0	38.0
12	10.0	6.0	23.5	39.5
13	10.0	6.0	25.0	41.0
14	10.0	6.0	24.0	40.0
15	10.0	6.0	23.0	39.0
16	10.0	6.0	22.0	38.0
17	10.0	6.0	21.0	37.0
18	10.0	6.0	21.0	36.0

Note: BREPPIPE(T) = base repair pipeline for day T
DDEMPIPE(T) = depot demands in interval [T – OST,T]
DREPPIPE(T) = depot repair pipeline for day T – OST
BSUPPIPE(T) = total base resupply pipeline for day T (assuming no spares)

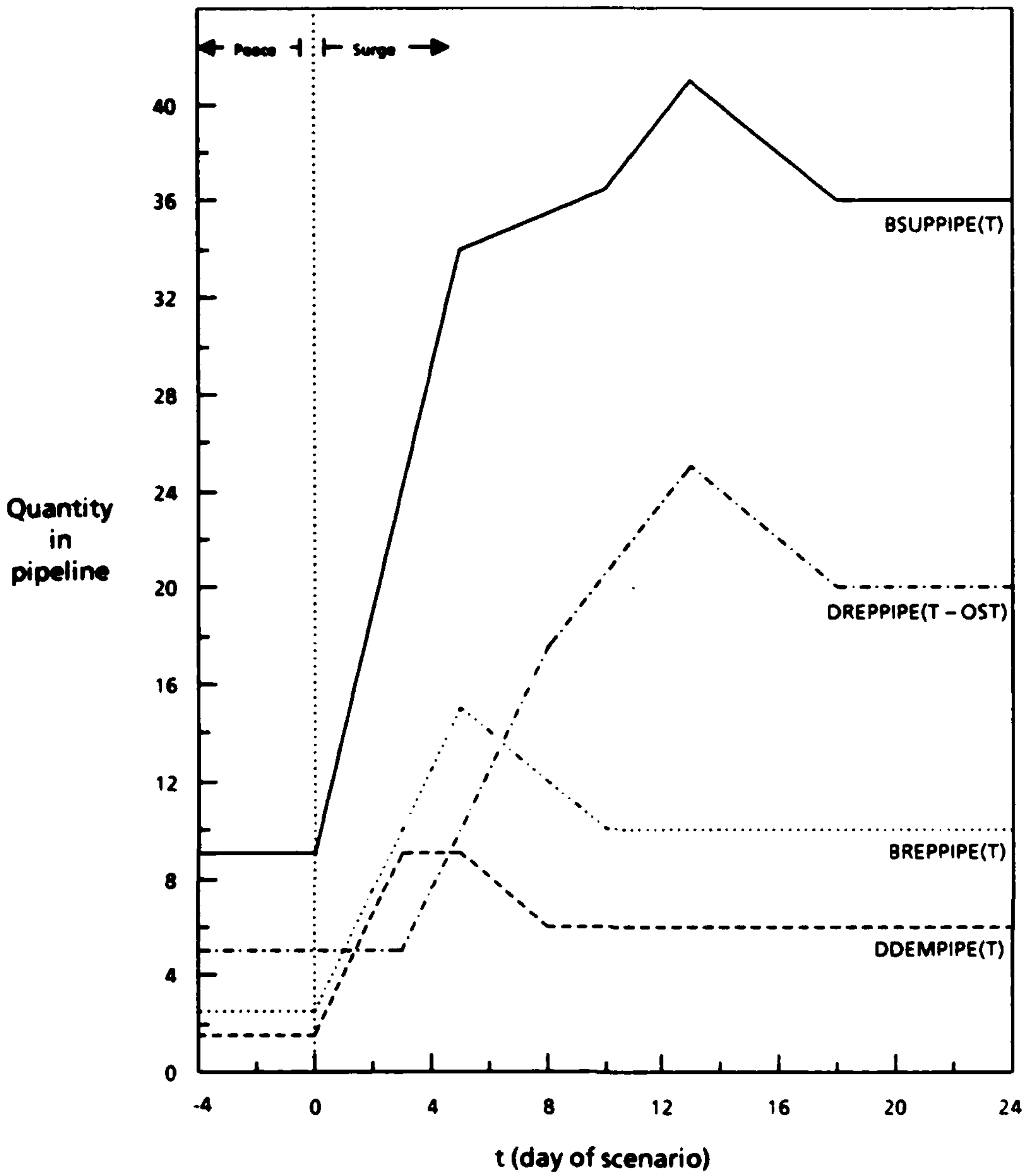

FIG. 2-3. RESUPPLY PIPELINES FOR EXAMPLE 1

We define the following functions:

E(t) = Emergence time (time at which the resupply action is completed) for an item that fails at time t

I(t) = Induction time (time at which the resupply action is initiated) for an item that emerges from resupply at time t.

Then a deterministic resupply process is one for which E(t) and I(t) are uniquely defined for all values of t. Such a process is FIFO if and only if:

$$E(t_1) < E(t_2) \text{ for all times } t_1, t_2 \text{ with } t_1 < t_2$$

It follows that:

$$I(t_1) < I(t_2) \text{ for all times } t_1, t_2 \text{ with } t_1 < t_2$$

and:

$$E(I(t)) = I(E(t)) = t \text{ for all values of } t$$

The resupply time is the difference between t and either E(t) or I(t). An item that is inducted at time t has resupply time $E(t) - t$. An item that emerges at time t has resupply time:

$$R(t) = t - I(t)$$

We use this definition of R(t), which defines the resupply time as a function of the *emergence* time, because we need to "look back" from time t to the point in time where the decomposition given by Equation 2-1 occurs.

The key to Equation 2-1 is that all depot demands made in the interval [T – OST,T] are still due in. If OST(T) is the order-and-ship time for an item *arriving* on day T at the base, we can generalize Equation 2-1. All the demands in [T – OST(T),T] are still due in (while demands made before T – OST(T) should have arrived or emerged, barring a backorder delay at T – R(T)).

Algebraically, $I(t_1) < I(t_2)$ for all $t_1 < t_2$ is the same as:

$$\frac{R(t_2) - R(t_1)}{t_2 - t_1} \leq 1$$

for all $t_1 < t_2$. That is, the resupply time (as a function of time) cannot rise faster than 1 day per day. This is best understood in terms of the limiting case where resupply has been suspended (frequent in surge scenarios).

Suppose, for example, on day 15, the value of OST is 10 days, implying that no depot demands made since day 5 have arrived, while all depot demands made by day 5 "should" have arrived (i.e., given no depot backorders). If depot resupply is suspended, the OST will rise by 1 per day so that on day 16 no depot demands made since day 5 will have arrived – meaning the OST is now 11 days. But it would be impossible for the OST to have grown to 12 days by day 16, because this would imply that the depot demands made on day 5 have not arrived by day 16 (when they had, in fact, already arrived by day 15).

Once depot resupply is resumed (say, on day 19), the OST (which has reached 14 days by then) falls back to 10 days. This transition back to 10 days could occur abruptly or gradually, depending on the cause of the suspension and the available shipping capacity. The computation is difficult and must be done with care.

Computing Pipelines for FIFO Processes

To evaluate the pipeline corresponding to a deterministic FIFO process, consider an example:

> *Example 2:* LRU with no subassemblies and non-dynamic characteristics as before:

$$FF = 1.0, NRTS = 0.5$$

Suppose the base repair time BRT(T) (see Table 2-6) and the wartime scenario are the same (see Table 2-3). To simplify notation, let:

$$T\_BR = T - BRT(T)$$

TABLE 2-6

PEACE-TO-WAR TRANSITIONAL BASE REPAIR TIMES FOR EXAMPLE 2

Day t	– 3	– 2	– 1	0	1	2	3	4	5	6
BRT(t)	7	7	7	7	6	5	4	3	3	3
I(t)	– 10	– 9	– 8	– 7	– 5	– 3	– 1	1	2	3

The expression for the base repair pipeline on day T, given by Equation 2-2, can then be generalized to:

$$BREPPIPE(T) = \int_{T\_BR}^{T} \lambda(t)(1 - NRTS)\,dt$$

or, in discrete form, as:

$$BREPPIPE(T) = \sum_{t=T\_BR+1}^{T} \lambda(t)(1 - NRTS) \qquad \text{[Eq. 2-7]}$$

This expression is valid for this reason: Though the repair time is not constant, all items inducted before T − BRT(T) have completed repair and all items inducted after T − BRT(T) are still undergoing repair. (This is why keying the repair time to the emergence time is convenient.)

For our example with T = 2:

$$BREPPIPE(2) = \sum_{t=2-BRT(2)+1}^{2} \lambda(t)(1 - NRTS)$$

$$= \sum_{t=-2}^{2} \lambda(t)(1 - NRTS)$$

$$= (1)(0.5) + (1)(0.5) + (1)(0.5) + (6)(0.5) + 6\,(0.5)$$

$$= 7.5$$

The complete graph of BREPPIPE as a function of time is shown in Figure 2-4.

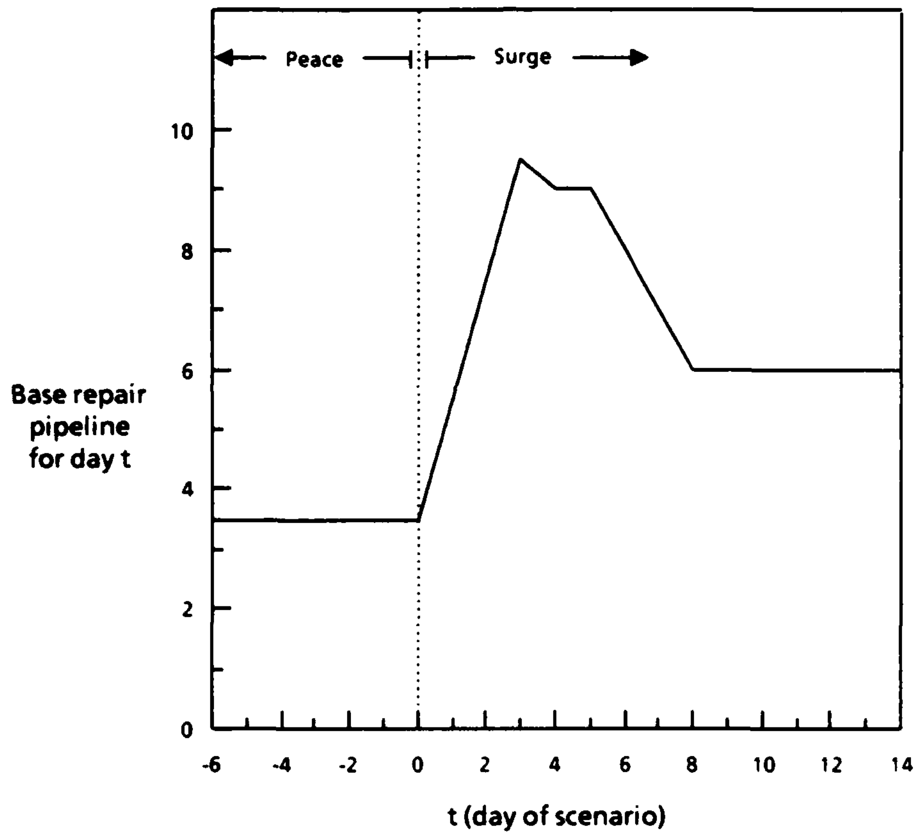

FIG. 2-4. BASE REPAIR PIPELINE FOR EXAMPLE 2

These approaches can be extended to time-dependent OSTs and DRTs, as well. Specifically, suppose:

OST(T) = Order-and-ship time for items arriving at the base at time T

DRT(T) = Depot repair time for items emerging from depot repair at time T

represent deterministic, FIFO resupply processes. Then, Equation 2-1 is changed only slightly:

$$BSUPP_T = BREP_T + DDEM_{[T-OST(T),T]} + DBO_{T-OST(T)}$$

or, using the notation T_OS = T − OST(T):

$$BSUPP_T = BREP_T + DDEM_{[T\_OS,T]} + DBO_{T\_OS} \qquad \text{[Eq. 2-8]}$$

The summands on the right-hand side of this equation are independent random variables, as in Equation 2-1.

The pipeline equations corresponding to the expected values of the three random variables in Equation 2-8 are:

$$\text{BREPPIPE(T)} = \sum_{t=T\_BR+1}^{T} \lambda(t)[(1 - \text{NRTS})] \qquad \text{[Eq. 2-9]}$$

$$\text{DDEMPIPE(T)} = E(\text{DDEM}_{[T\_OS,T]}) = \sum_{t=T\_OS+1}^{T} \lambda(t)\,\text{NRTS} \qquad \text{[Eq. 2-10]}$$

with the depot repair pipeline at time T_OS given by:

$$\text{DREPPIPE(T\_OS)} = E(\text{DREP}_{T\_OS}) = \sum_{t=T\_OS\_DR+1}^{T\_OS} \lambda(t)\text{NRTS} \qquad \text{[Eq. 2-11]}$$

where we have extended our notation recursively so that:

$$\text{T\_OS\_DR} = \text{T\_OS} - \text{DRT(T\_OS)}$$

Table 2-7 provides sample data for an LRU *(still with no subassemblies).*

TABLE 2-7

SAMPLE DATA FOR AN LRU WITH DYNAMIC RESUPPLY TIMES

Day t	0	1	2	3	4	5	6 = NDAYSFH . . .
FHP(T)	1	6	6	6	6	6	4 . . . .
FF(T)	1.0	1.0	1.0	1.0	1.0	1.0	1.0 . . . .
BRT(T)	7	6	5	4	3	3	3 . . . .
OST(T)	3	2	2	2	2	2	3 . . . .
DRT(T)	10	9	8	7	7	7	3 . . . .

We will the pipelines on day 6:

$$\text{BREPPIPE}(6) = \sum_{t=6-3+1}^{6} \lambda(t)(1-\text{NRTS})$$

$$= (6)(0.5) + (6)(0.5) + (4)(0.5)$$

$$= 8.0.$$

$$\text{DDEMPIPE}(6) = \sum_{t=6-3+1}^{6} \lambda(t)\,\text{NRTS}$$

$$= (6)(0.5) + (6)(0.5) + (4)(0.5)$$

$$= 8.0$$

$$\text{DREPPIPE}(6-3) = \sum_{t=3-7+1}^{3} \lambda(t)\,\text{NRTS}$$

$$= (4)(1)(0.5) + (6)(0.5) + (6)(0.5) + (6)(0.5)$$

$$= 11.0$$

We now extend these ideas to include levels of indenture.

Levels of Indenture

To this point, we have considered LRUs only. We next address the pipeline computations for SRUs. Consider, for the moment, the SRU/LRU relationship with constant base repair times. Let BRT and BRTNHA represent the base repair times for a specific SRU and its LRU parent (next-higher-assembly). BRTNHA can be written as:

$$\text{BRTNHA} = \text{BRTNHA}_{FIT} + \text{BRTNHA}_{RAT}$$

where:

BRTNHA_{FIT} = Fault isolation time for the LRU

BRTNHA_{RAT} = Reassembly time for the LRU.

The sequence of events is depicted in Figure 2-5.

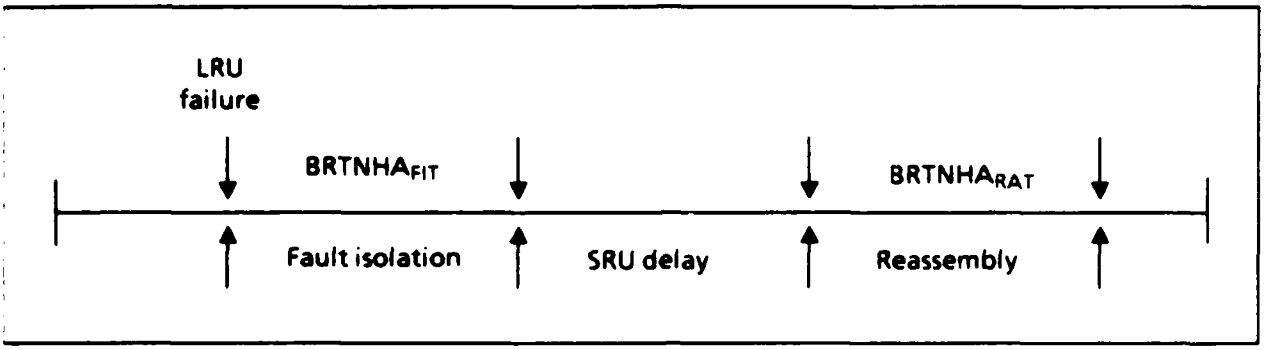

FIG. 2-5. LRU/SRU TIMELINE

SRUDELAY represents the time the LRU spends waiting for resupply of the SRU. This can range from zero to the full resupply time for the SRU. We will assume that the reassembly time is negligible compared to the fault isolation time.[2] Then, $BRTNHA = BRTNHA_{FIT}$, approximately. The main observation to be derived from Figure 2-6 is that SRU demands lag the corresponding LRU failure by the parent base repair time. Thus, to measure the status of the aircraft at time T, we compute SRU pipelines and resulting backorders at time $T - BRTNHA$.

Combining the theory of dynamic (but still FIFO) resupply processes with the lag effect just described is straightforward but notationally cumbersome. We illustrate all of the computations to this point with a comprehensive example:

> *Example 3*: Table 2-8 represents sample data for an SRU with dynamic resupply times (including resupply times for the next-higher-assembly).

We compute the SRU pipelines according to their *end item impact* on day T. The SRU base repair pipeline is the sum of the repair inductions over the interval:

$$\{t: T - BRT(T) < t \leq T\}$$

[2]In fact, the reassembly time need only be nondynamic for the following results to apply

TABLE 2-8

SRU DATA FOR EXAMPLE 3

(NRTS = 0.5, no condemnations)

Day T	0	1	2	3	4	5	6 = NDAYSFH
FHP(T)	1	6	6	6	6	6	4 ....
FF(T)	1.0	1.0	1.0	1.0	1.0	1.0	1.0 ....
BRT(T)	7	6	5	4	3	3	3 ....
BRTNHA(T)	9	8	7	7	7	7	7 ....
OST(T)	3	2	2	2	2	2	3 ....
DRT(T)	10	9	8	7	7	7	7 ....

But these inductions correspond to SRU failures over the interval:

$$\{t{:}T - BRT(T) - BRTNHA(T - BRT(T)) < t \le T - BRTNHA(T)\}$$

Therefore, the general form of the SRU base repair pipeline is given by:

$$BREPPIPE(T) = \sum_{t=T\_BR\_BN+1}^{T\_BN} \lambda(t)(1 - NRTS) \qquad \text{[Eq. 2-12]}$$

where T_BN = T − BRTNHA(T).

For T = 10, BRT(T) = 3, BRTNHA(T) = 7, BRTNHA(T − BRT(T)) = 7, and the cor-responding SRU base repair pipeline is:

$$BREPPIPE(10) = \sum_{t=10-3-7+1}^{10-7} \lambda(t)(1 - NRTS) = \sum_{t=1}^{3} \lambda(t)(1 - NRTS)$$

$$= 9.0$$

Similarly:

$$DDEMPIPE(T) = \sum_{t=T\_OS\_BN+1}^{T\_BN} \lambda(t)\,NRTS \qquad \text{[Eq. 2-13]}$$

$$DREPPIPE(T\_OST) = \sum_{t=T\_OS\_DR\_BN+1}^{T\_OS\_BN} \lambda(t)\,NRTS \qquad \text{[Eq. 2-14]}$$

For T = 10, we obtain,

$$DDEMPIPE(10) = \sum_{t=10-3-7+1}^{10-7} \lambda(t)NRTS = \sum_{t=1}^{3} \lambda(t)NRTS$$

$$= 9.0$$

and:

$$DREPPIPE(10-OST(10)) = DREPPIPE(7)$$

$$= \sum_{t=10-3-7-9+1}^{10-3-7} \lambda(t)NRTS$$

$$= \sum_{t=-8}^{0} \lambda(t)NRTS$$

$$= 4.5$$

All resupply pipelines for Example 3 are represented in Figure 2-6. As usual, the backorders for the SRU are evaluated at time T − OST(T) by means of the standard backorder computation with a DREPPIPE as given by Equation 2-14.

Other Extensions

Thus far, we have, for the sake of clarity in exposition, simplified some of the hypotheses concerning our wartime scenario. Now we describe briefly some extensions of this theory. They represent straightforward modifications of the standard AAM treatment and the dynamic theory discussed to this point in the report and discussed earlier [3].

Condemnations

Suppose the depot has a probability, CONPCT(T), of condemning a failed item on day T of the scenario. This is treated as an additional resupply pipeline segment at the depot. For an LRU, we compute the *condemnation pipeline* at time T − OST(T) by:

$$\text{CONPIPE}(T - \text{OST}(T)) = \sum_{t=t-\text{OST}(T)-\text{PLT}+1}^{T-\text{OST}(T)} \lambda(t)\,\text{NRTS}\;\text{CONPCT}(t) \quad \text{[Eq. 2-15]}$$

where PLT represents the procurement leadtime for the item (in days). CONPIPE is then added to the depot repair pipeline, and the depot backorder calculation is computed on the basis of this total. Treatment of condemnations for SRUs is analogous.

Days of Warning

Let NDAYWARN equal the number of days of warning before the start of the surge conflict. We model this by shifting the time dependence of the component characteristics by NDAYWARN days. For example, if the wartime base repair rate is 5 days and NDAYWARN is 3 days, MAS interprets the 5-day base repair rate as beginning on day −2; i.e., the last 3 days of peacetime base repairs are performed at the wartime rate. The other resupply times, as well as the NRTS and condemnation rates, are treated similarly. However, the flying hours and failure rates are not shifted.

Multiple Bases

The ASM assumes that all bases are uniform with respect to demand rates and resupply times. Suppose there are N uniform bases. We must calculate the backorders *at each base*, B1EBO. This calculation requires knowledge of the single-base

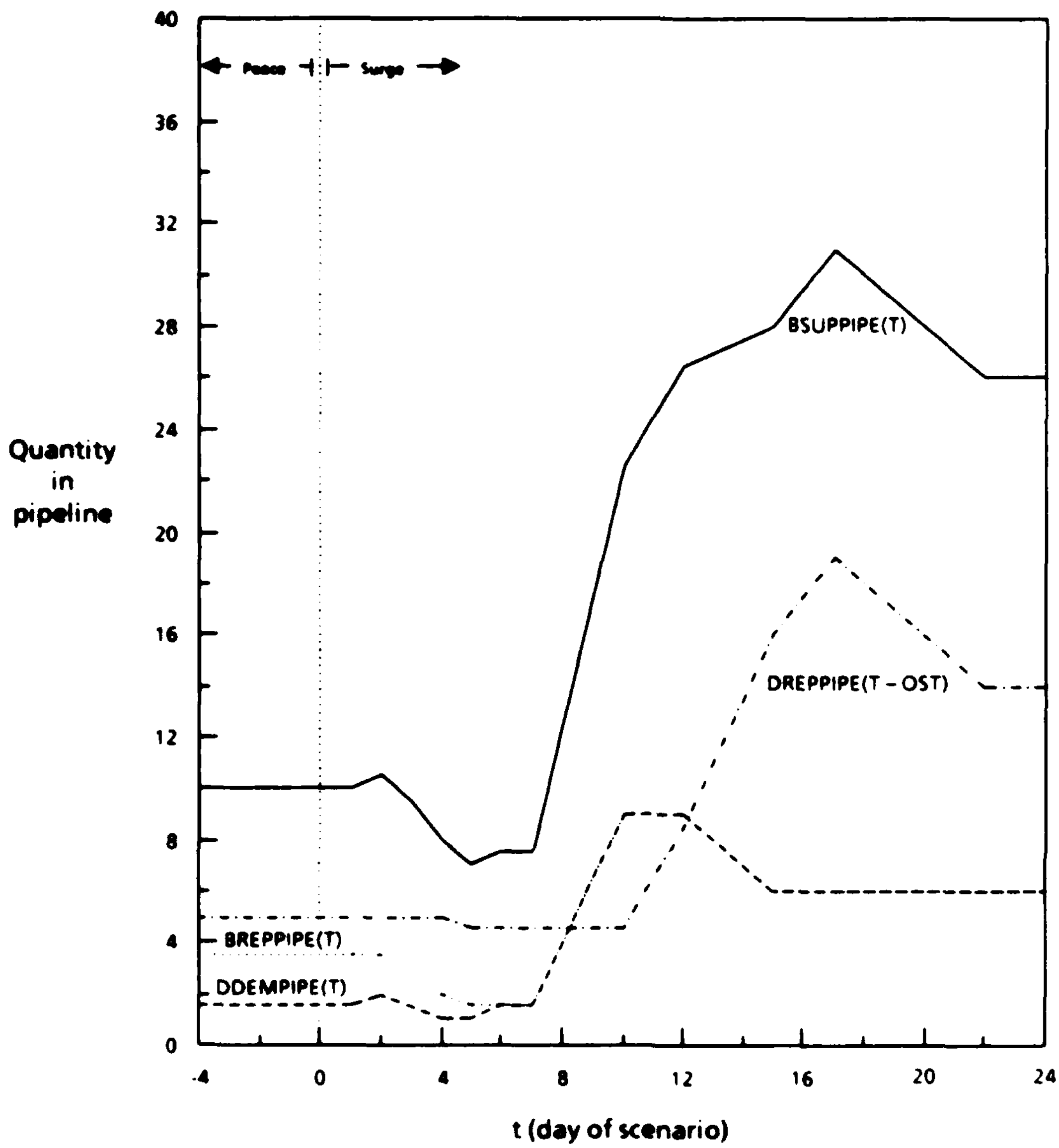

FIG. 2-6. RESUPPLY PIPELINES FOR EXAMPLE 3

resupply quantity, $B1SUPP_T$, for day T. Consider the single-indenture, constant resupply time, Equations 2-5 and 2-6, representing the mean and variance, respectively, for the total number in resupply (at all bases) for day T. Not surprisingly, the single base pipeline is given by:

$$B1SUPPIPE(T) = E(B1SUPP_T) = 1/N \times BSUPPIPE(T) \qquad \text{[Eq. 2-16]}$$

The variance calculation is more complex. Given d depot backorders and N uniform bases, the conditional distribution of backorders at a specific base is binomial (with "success probability" $p = 1/N$). This leads to the following equation for the variance of $B1SUPP_T$ (given constant OST):

$$V1SUPP(T) = V(B1SUPP_T) \qquad \text{[Eq. 2-17]}$$

$$= (BREPPIPE(T) + DDEMPIPE_{[T-OST,T]})/N$$

$$+ 1/N(1 - 1/N)\,E(DBO_{T-OST}) + 1/N^2\,V(DBO_{T-OST})$$

The extension to dynamic values for OST is as before. More details about this derivation are provided in [3].

THE OBJECTIVE FUNCTION – PROBABILITY OF FLYING THE WMP

The MAS computes curves relating the total spares cost to the PWMP in a way very similar to the way the AAM computes cost/availability curves. In large part, the reason for the efficiency of the technique is that the objective function is "separable"; i.e., the benefit or marginal worth of each spare unit of a specific component can be computed independently of the spares levels of all other components. This feature enables us to compute an entire "curve" of BP-15 costs against the PWMP for a given day. As a result, the user can output the cost associated with a set of success probabilities as a postprocessing activity. It is not necessary to rerun the marginal analysis for "what-if" analyses of the success probabilities.

The user determines safety levels for spares procurements by specifying the desired "success probability" p of performing the program. For example, if $p = 0.8$, MAS determines the spares levels necessary for:

$$Pr\{NMCS(T) \le WMPNMCS(T)\} = 0.8 \qquad \text{[Eq. 2-18]}$$

where NMCS(T) represents the number of aircraft that are not available (i.e., they are waiting for one or more reparable components), under the maximum-cannibalization assumption, on day T.

We evaluate Equation 2-18 by noting that, under the maximum-cannibalization assumption for LRUs:

$$Pr\{NMCS(T) \leq WMPNMCS(T)\} = \prod_i Pr\{BO_i(T) \leq QPA_i \times WMPNMCS(T)\} \quad \text{[Eq. 2-19]}$$

where:

$BO_i(T)$ = Backorders for LRU on day T

QPA_i = Quantity per application of LRU i.

This equation is discussed more fully in the [3].

MAS OUTPUT

The MAS output is the same as the output from the DAAM, except that availability is replaced by PWMP. The MAS model calculates for the given day, ITODAY, a curve of the probability that the weapon system can fly the program specified for that day, as a function of dollars spent. If the MAS were operated as a stand-alone system, this curve would be the principal output. But, because the Cross-Linker will be used most of the time, the file that serves as the input to the Cross-Linker (the RESULTS file) becomes important.

The RESULTS file contains the complete results of each component's marginal analysis, stored in component order. For each component, the file includes:

- A header record containing general component information plus the starting log of PWMP
- A subheader record containing the names of any subassemblies (LRUs without subassemblies do not have this record.
- A number of marginal records, each including the marginal cost of this buy, the marginal improvement in the log of PWMP per dollar (the sort value), the cumulative number of units of this component "bought" up through this marginal buy, and the sort value cutoff flag for the number of SRU subassemblies (if any) bought at this point.

This file is designed to speed generation of a shopping list and will be used for that purpose when the shopping-list part of the ASM is completed. But the file structure is ideal for input to the Cross-Linker, and that is its main role in the prototype ASM.

The curve of cost versus PWMP is contained in the SORTED file that is identical to the UNSORTED file, except that the records have been sorted. These files contain the marginal buy records from the marginal analysis. Each record consists of five fields:

- NSN – The National Stock Number of the component involved in this marginal buy.
- SV – The sort value (i.e., the marginal improvement in the log of PWMP per dollar for this marginal buy)
- GLCOST – The cost of this marginal buy
- NHANSN – The NSN of the next-higher-assembly (for LRUs, the next-higher-assembly is the weapon system)
- IBUDCODE – The budget code.

In addition to the marginal records described above, these files contain one starting PWMP record for each component. That record differs from those described above in that the SV is replaced by a large constant that causes the starting records to be sorted to the top of the file, and the GLCOST is replaced by the starting log of the PWMP.

MERGER ALGORITHMS

The prototype Merger reads the LRU RESULTS files from two MAS runs and generates a single UNSORTED file that is similar to the UNSORTED files produced by the MAS runs, except that each record contains the information for both days plus their weighted average, weighted by the constant of proportionality (C). Each record in the merged UNSORTED file consists of seven fields:

- NSN – The National Stock Number of the component involved in this marginal buy
- SV1 – The sort value from the first day being merged
- SV2 – The sort value form the second day being merged

- SVAVG – The weighted averaged of the two sort values:

$$C \times SV1 + (1-C) \times SV2$$

- GLCOST – The cost of this marginal buy
- NHANSN – The NSN of the next-higher-assembly (for LRUs, the next-higher-assembly is the weapon system)
- IBUDCODE – The budget code.

As with the files produced by the MAS, this file has one starting PWMP record for each component. These records are similar to the MAS starting records. The SVAVG field is replaced by a large constant that causes the starting records to be sorted to the top of the file, the SV1 field is replaced by the starting log of the first PWMP, the SV2 field is replaced by the starting log of the second PWMP, and the GLCOST is replaced by the weighted average of the starting logs of the PWMPs ($C \times SV1 + (1-C) \times SV2$).

The MAS RESULTS file contains complete results of the marginal analysis for each component in a format that is ideal for matching specific components in different RESULTS files. For each component, the file contains a header record, an SRU subheader (if applicable), and marginal analysis records that contain the benefit and cost data associated with each potential buy.

The Merger combines the LRU RESULTS files from the two MAS runs, producing the merged LRU UNSORTED file for input to the sort. The program reads a header record from each RESULTS file, matching the component names on the two records. When a match is found, the starting record is written to the UNSORTED file. Then the marginal records from the RESULTS files are read, and the merged UNSORTED records written, until all the records for that component are processed. If one RESULTS file has fewer marginal records for that component than the other, the Merger assumes zero SVs for the missing records. Then the Merger reads the next header record from each RESULTS file, and the process is repeated. This continues until the end of the RESULTS files.

CHAPTER 3

APPLICATIONS

USING THE MARGINAL ANALYSIS SYSTEM

Though one could run the MAS for each day of the scenario, that is not necessary. If a specific day is not run, that day's probability of flying the WMP will not be computed and can only be estimated by interpolation. But a shopping list generated by the Cross-Linker may, if the right days are run and cross-linked, still be adequate for the days that were skipped.

Normally, there will be a few critical days in the scenario where spares shortages are most likely to be a problem. These are the days that will be analyzed with the model.

Recalling the flying-hour program, depicted in Figure 2-1, two critical days would be day 6 and the last day of interest. If one can fly the WMP on day 60, it is likely that one can fly it on day 59; the extra day of flying at 4 times the peacetime program will presumably be continuing to wear down the resupply system. Similarly, if one can fly the WMP on day 6, flying the WMP on preceding days should be easier because the logistics pipelines are filling up fast during this initial phase of the surge.

Though critical days can be guessed by an examination of the scenario inputs, they can be determined precisely by MAS runs. Suppose it is desired that there be a 95-percent probability of flying the for days 1 through 6 and a 90-percent probability of flying the program for days 11 through 60. It may be that day 60 is the only critical one. That is, if the MAS is run for day 60, and a shopping list is generated corresponding to a 90-percent probability of flying the program on day 60, that shopping list may yield more than adequate probabilities of flying the program on all the earlier days.

To test this hypothesis, we generate the shopping list for a 90-percent PWMP on day 60. Then, using the PWMP Evaluator (a special MAS program written for just this purpose), evaluate the PWMP produced on day 6 by this (day 60) shopping list. If this PWMP is greater than the "target" for day 6 (95 percent), day 6 is dominated by day 60 and is not critical. If this is not the case, then day 6 is critical and, in fact, may dominate day 60. We could check this by running the day 6 shopping list first, buying the shopping list for a 95-percent PWMP on day 6, and evaluating that on day 60. The MAS is specifically designed to make it easy to run these kinds of tests.

RUNNING THE CROSS-LINKER

The prototype version of the Cross-Linker merges the output from two MAS runs, enabling the user to optimize simultaneously the probabilities of flying the WMP on the 2 days analyzed, subject to a single budget constraint. We envision a production Cross-Linker that will join the output from any number of MAS runs and will search automatically for the constants of proportionality to achieve a given set of probabilities of flying the WMP at a minimum total cost. However, the prototype version requires the user to input the (single) constant of proportionality that represents the relative weights given to the 2 days being joined. Studying the effect of changing that constant and understanding its role then becomes easier.

The only user-supplied inputs to the Cross-Linker are this constant and the days to be cross-linked. The Cross-Linker reads the RESULTS files from the two MAS runs, generates the merged UNSORTED file, sorts the file, and runs the CURVES program that prints the combined cost-versus-probabilities curve. A point from those curves (representing a single cost and two probabilities) may be selected and a shopping list generated or, if no point provided the desired pair of probabilities, a new constant may be selected and the system rerun.

The constant of proportionality (C) is the percentage of weight given to the first day; the remainder is allotted to the second. That is, the objective function that is optimized by the Cross-Linker is $(C \times PWMP_{Day\,1} + (1 - C) \times PWMP_{Day\,2})$. A value of zero would give all the weight to the second day; a value of 1 would give all the weight to the first day. A value of one-half would give equal weight to the 2 days (and is recommended as a starting point).

If the curve pair resulting from a specific constant yields too high a PWMP on the first day, relative to the second day, the constant is too high and should be lowered. Conversely, if the PWMP on the second day is too high relative to the first, the constant should be raised.

For example, suppose that we are cross-linking the MAS runs for days 10 and 60 and we want to find the minimum budget for which we will have a PWMP of 90 percent on both day 10 and day 60. If, with a constant of 0.5, the PWMP on day 10 is 93 percent at the budget level while the PWMP on day 60 is 90 percent, the constant is too high and should be lowered. (Note that day 10 is the "first" day in this example). A constant of zero might be tried next and, if that tilted the balance in the other direction (e.g., day 60 is at 92 percent when day 10 is at 90 percent), a constant of 0.25 might be tried.

This binary search process can be continued until some satisfactory convergence criterion has been met. How close we can come eventually to exactly 90 percent on both days is a topic for further research. In addition, we will try to find out whether some faster search technique is feasible.

CHAPTER 4

CONCLUSIONS

Development of the ASM thus far shows that a sustainability model that is a practical tool for budgeting and assessment is now a feasible goal. The ASM itself is still in the prototype stage with much yet to be done. Conceptual problems remain to be solved, interfaces with Air Force data bases must be constructed, and output products must be refined through the experience of Air Force planners.

The continuing development of the model includes these efforts:

- *Finding critical days.* As we pointed out in the section on MAS algorithms, we need to learn how to identify critical days. During this investigation, we will also develop a feel for how many critical days the model can handle practically.

- *Automating the Cross-Linker convergence.* The production Cross-Linker should be able to combine MAS output from more than 2 days and automatically find the least-cost budget spares mix to achieve a given set of target PWMPs. Research into appropriate multidimensional search techniques is underway.

In addition, we are working to solve several data problems:

- *Development of realistic surge scenarios.* LMI, with the assistance of the Air Staff and other organizations within the operations community of the Air Force, will develop realistic surge scenarios for a specific weapon system. This includes collecting the operating requirements (number of sorties, sortie lengths, turn times, etc.) and the base parameters (resupply times, repair capabilities, etc.). Reasonable unclassified scenarios should be developed for model-testing purposes, but the ASM is ultimately envisioned to be used with classified data. We will, therefore, be collecting surge requirements from actual (classified) WMP documents. In the long term, we must consider the impact of such factors as battle damage, which has historically been ignored in setting spares requirements.

- *Development of a weapon system data base.* We will be developing a data base for a single weapon system suitable for input to the ASM. These data will come largely from the D041 (which now includes both peacetime and wartime factors). The D041 data must be reformatted to portray the peacetime-to-wartime transition of resupply times and be so modified that the assets and demands of common components are prorated to the weapon system of interest.

- *Development of interfaces with other asset data*. The ASM is designed to be a *total requirements* model. By this we mean that it can determine the optimal *gross* requirement for spares needed in a user-prescribed conflict scenario. We must offset this requirement with existing POS and WRM assets in order to determine the *net* requirement. In fact, one key use of the ASM will be quantifying the tradeoffs involved in POS and WRM requirements.

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