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FINAL TECHNICAL REPORT

AFOSR-84-0165

The Reliability of Load Sharing System

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Principal Investigator

1. **Summary.** In a load sharing system, the failure of one or more components increases the load on other nonfailed components, thereby increasing their chances of failure, and the failure of the system. Examples of load sharing systems include many mechanical, thermal and electrical structures. System reliability is measured by the probability that the system is capable of performing its prescribed task or surviving for a prescribed duration. Traditional reliability models often postulate that component failures are statistically independent and, thus, do not describe load sharing systems.

The optimal replacement theory for fault tolerant load sharing systems was developed under the support of the grant. As components fail, the system deteriorates. Where failed components cannot be replaced individually, an algorithm was found that determines the optimal policy for replacing the entire system.

Load sharing systems arise as models for the failure process of certain composite materials such as Kevlar fibers embedded in an epoxy matrix, or for certain semi-crystalline polymer fibers themselves. Because of the astronomical numbers of components in these models, their analysis is quite difficult. A simple model that describes the failure process, but is far more easily understood and manipulated, has been developed and its accuracy compared with that of a more complex and less well understood model. The agreement is excellent. Because of its simplicity, the new model is more easily extended to cover a wider variety of load sharing systems.



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2. Research Objectives. In a load sharing system, the failure of one or more components increases the load on other nonfailed components, thereby increasing their chances of failure, and the failure of the whole system. Examples of load sharing systems include:

(a) Heat shields on a space re-entry vehicle. The failure of loss of some shields increases the burden on others;

(b) Fibrous composite materials. Many mechanical structures share loads among components. As one such example, consider a fibrous composite material consisting of parallel stiff fibers embedded in a plastic matrix. Failure of some fibers under an initial tensile load increases the stress on adjacent or nearby fibers.

(c) Networks of electric power generators, where the failure of one source causes the load on other sources to increase in order to meet total system demand.

Appropriate measures of system reliability in these examples are: (a) The probability that the system is capable of performing its prescribed task, such as supporting a specified mechanical load, or (b) The probability distribution of time to system failure. Traditional reliability analysis has concentrated on systems in which component failures are statistically independent. This assumption is not valid where load sharing among components appears, and thus traditional reliability techniques cannot be used to determine system performance.

Our research is directed towards developing and analyzing models and algorithms that predict system reliability from knowledge of component reliabilities and the load sharing and other features of the system. Such models are needed to guide experimentation and the interpretation of statistical data, to evaluate alternative system designs, and as a basis for the development of optimum policies for maintenance and repair of systems and components.

3. Status of Research. We have made substantial progress on three of our specific objectives:

(a) In an **optimal replacement model**, a system deteriorates over time, and based on its current state, one wishes to decide in an optimal manner whether or not to replace the system with a statistically identical new one. Optimal replacement problems are the special class of sequential decision models that are distinguished by having only two actions available at any state: (i) To replace the system with a new and statistically identical one, or (ii) To continue operation with the current system. Often there is a notion of **total failure**, at which point replacement is mandated.

Replacement models for systems having indistinguishable components that fail over time, while allowing the system to continue to perform at a reduced level, have been studied. Our contribution was to develop the model in which the individual system components are distinguishable so that it is important to know exactly which components are operating at any given time, and not merely their total number. Moreover, we allowed the individual component failure rates to depend on the state of the entire system. Such a feature is important in studying **load sharing systems** in which the load that a component had been carrying prior to its failure is, upon that component's failure, shifted to other components, thereby possibly increasing their failure rates. We were motivated to study this problem in part by the importance in modern practice of so-called **fault tolerant systems**, systems that can continue to operate even though some of their components have failed. In addition to simple redundancy, many such systems feature **function migration** in which the function served by a component is shifted to another component upon the first component's failure.

Our main contribution has been to develop an algorithm that determines an optimal replacement policy for a fault tolerant system having distinguishable components whose failure rates are system dependent. The rate at which the system produces income

is assumed to decrease as the system deteriorates, and the system replacement cost is assumed to rise. Our algorithm is of the so-called **greedy** type. It begins with the policy that continues to operate the system, (that is, does not replace it), only if all components are operational. The algorithm then enlarges this continuation set in a monotonic manner, state by state, to arrive at the optimal such continuation set. If there are N system components, then there are 2^N system states. The optimal policy will be specified by a class of these system states, the class of states in which the system is allowed to continue to operate. There are, then, 2^2 such possible continuation sets of states.

(b) If a modern high strength-low weight polymer material, such as a Kevlar fiber or Kevlar-epoxy composite, is suspended under a constant weight, it will eventually fail, an example of what is called **stress rupture** or **static fatigue**. The failure process begins at the atomic level where random fluctuations cause sufficient thermal energy to exist in a molecule to overcome certain local energy barriers, and a molecule may slip relative to other molecules or rupture at one of its atomic bonds. As molecules slip or rupture, neighboring molecules become overloaded, thus increasing their failure rates. Such molecule failures accumulate locally, forming minute cracks that are the irreversible changes in the microstructure that ultimately lead to macroscopic failure of the material. Because a Kevlar-epoxy strand can have a strength to weight ratio that is 50 times that of steel, these materials are of growing importance in aircraft and elsewhere. Nationwide, much research effort is being devoted to their study, but virtually all of this research is experimental or, more recently, simulation. In contrast, we are developing purely mathematical models that characterize the failure process. The key notion is the load sharing and local stress concentrations in the near vicinity of failed elements. We have made great progress over the past

decade, and our models are now providing results and insights that cannot be achieved by experimentation or simulation. Our long term goal is to work more closely with material scientists, both to communicate our results to them, and to improve our models. Unfortunately, the mathematical analysis of these models is extremely difficult, requiring a high level of proficiency in probability theory. During this past year, we have developed a simpler model that captures the essence of the failure process, but is far easier to understand. We have compared the accuracy of this simple model with that of a more complex model using a particular load sharing rule, the only one under which numerical results can be obtained in the more complex model. The agreement between the numerical predictions of the two models was excellent. In the simpler model it is possible to calculate reliabilities under a wider variety of more realistic load sharing assumptions. We hope that these mathematical models will become the accepted framework for describing the failure process in certain composite materials, both for design and experimentation.

(c) The size effect refers to the observation that larger material specimens are weaker, per volume, than smaller ones. The simplest example of this effect is a series system of independent components: "A chain is only as strong as its weakest link." If the strengths of n links are the independent and identically distributed random variables X_1, \dots, X_n , and

$$S_n = \min(X_1, X_2, \dots, X_n)$$

is the strength of the chain, then the size effect takes the form

$$\Pr\{S_n > x\} = [\Pr\{X_1 > x\}]^n.$$

While a series system exemplifies the size effect, it is wrong to believe that the size effect is limited to systems having independent elements in a series arrangement. Let G_V be the cumulative distribution function for the strength of a system of "size" V . In any particular application, size might refer to length, surface area or volume. Let W be a distribution function of a nonnegative random variable. We will say that the system satisfies the size effect with characteristic distribution W if, at least for large sizes, we have

$$1 - G_V(x) = [1 - W(x)]^V \text{ for } x \geq 0. \quad (1)$$

Of course, W is the cumulative distribution function for a system of unit size $V = 1$, but we do not mean to imply that W is the cumulative distribution function for the strength of an element. If the elements are in a purely series arrangement, then, of course, W corresponds to the element distribution function. In the more general case, (1) asserts that the system behaves as if it were a series arrangement of some fictitious elements whose strengths follow the cumulative distribution function W .

The theoretical and analytical models that I and my former colleagues at Cornell have developed over the past decade to

describe the failure process in certain composite materials find immediate relevance in the study of the size effect in general. Carrying out this generalization remains a most important objective. A monograph is currently being written by the principal investigator.

4. Publications.

"Optimal replacement for fault tolerant systems", by H. M. Taylor and B. Rodriguez. To appear: Naval Research Logistic Quarterly, Vol. 33, 1986.

"A model for the failure process of semi-crystalline polymer materials under static fatigue", by H. M. Taylor. To appear: Probability in the Engineering and Informational Sciences.

An Introduction to the Theoretical Study of the Size Effect on Material Strength, by H. M. Taylor. (In preparation.)