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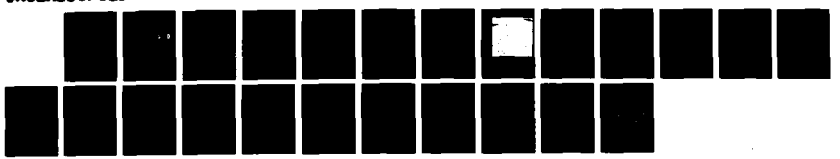

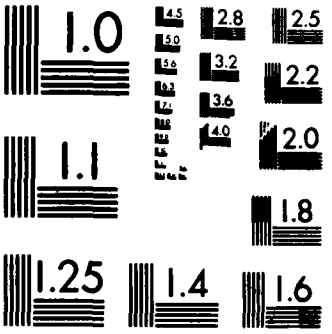


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Final Report for 1984-85 to the Air Force Office of Scientific Research:

# TEXTURE PERCEPTION AND SHAPE FROM TEXTURE

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## ABSTRACT

This proposal summarizes the progress made during the year 1984-85 under grant AFOSR 82-0317. We have examined the problem of extracting simple, perceptually significant representations of natural textures, and developed a system for lowest level perceptual grouping of dots in dot pattern representation. We have also developed procedures for deriving a "scale-space" representation of natural textures in terms of discs.

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## INTRODUCTION

The objectives of our ongoing research are two-fold [AHUJA84]. The first part of our research concerns perceptual grouping in dot textures [AHUJA82, WERTH38]. The goal here is to segment a given dot pattern into its perceptual components (i.e., to identify regions and curves defined by dots that coincide with the segmentation provided by humans.

In the second part of our research, we are developing a computational theory for extracting three-dimensional shape of a homogeneously textured surface from its images [STEVENS81, WITKIN81]. The goal of the second part of the research is to separate the spatial variation in the transformed texture into its two components: one, characteristic of the original texture before imaging, and the other introduced by the distance and orientation changes in the imaging process. Since we are not imposing any constraints on the complexity of the original texture, the problem in general involves separating the ambient, homogeneous, possibly anisotropic, part of the texture from a smooth, nonhomogeneous, geometric distortion due to distance gradients and geometry of the textured surface. The algorithm developed in this part of our research will interface with the first part. We have worked on both of these parts simultaneously during the year 1984-85. Following is a summary of the progress we have made during this year in each of the two areas.

## PERCEPTUAL SEGMENTATION OF DOT PATTERNS

The structure of our current grouping algorithm is shown in Figure 1. The first step of the algorithm (box A in figure 1) consists of three independent modules (boxes II, BI, and CI) running in parallel. Each of these modules responds to a certain aspect of the stimulus. The first one (II) identifies interior points, the second one (BI) identifies border points, and the third (CI) identifies curves. These modules were developed prior to 1984-85. The second step (B) corrects possible errors that might exist in the results of each of the three modules. The third step (C) combines the results of the border correction (BC) and interior correction modules (IC) performing a more global analysis. Steps B and C have been implemented this year (1984-85). We now summarize steps B

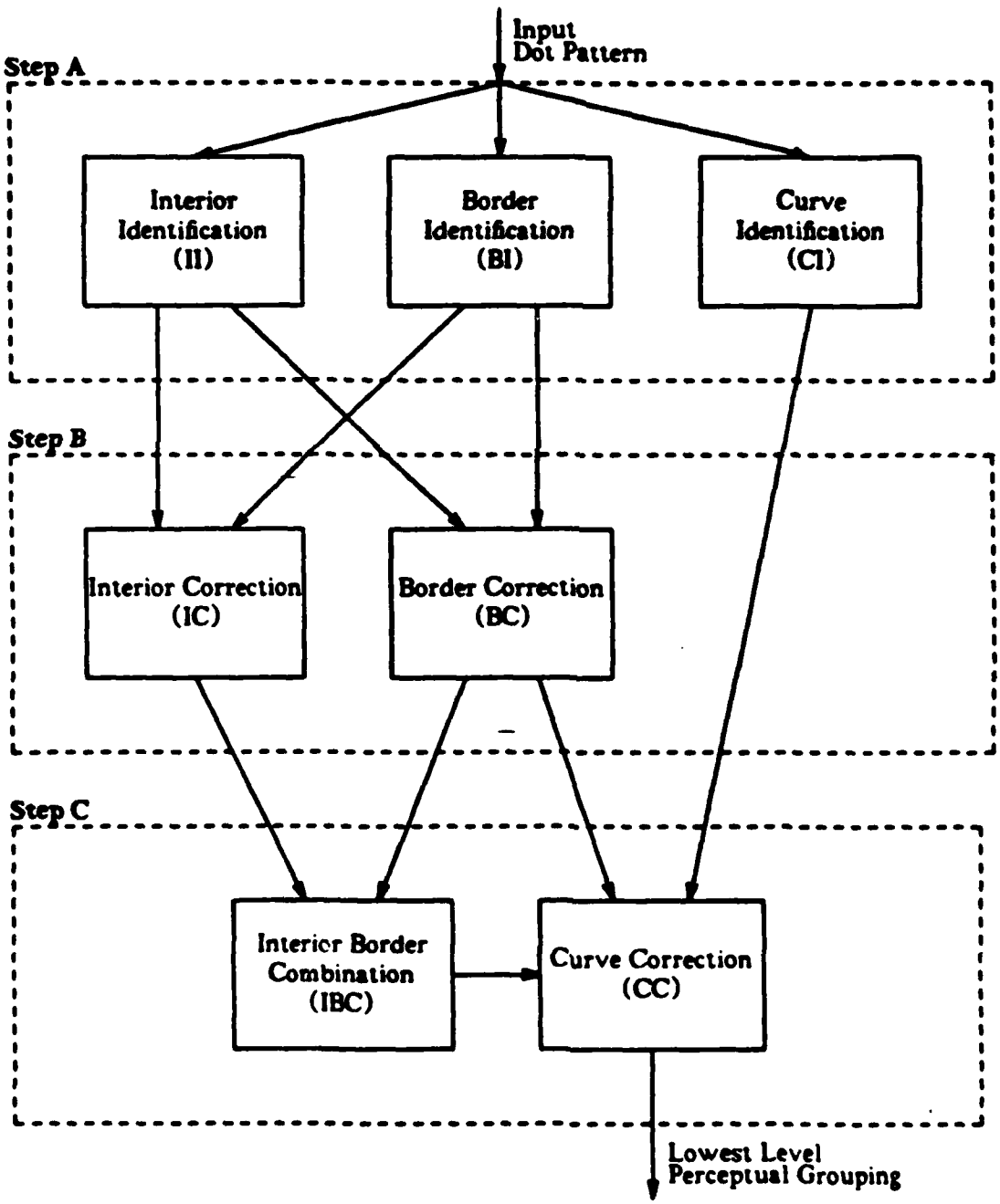


Figure 1. Structure of the perceptual grouping algorithm.

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and C; details are available in the enclosed copies of publications.

To perform the corrections, first the results of the modules (II, BI, and CI) are cross compared (figure 1). A module changes the labels of its input if doing so improves the measure of border smoothness and increases agreement with the results of other modules. The results reflect properties of more extended spatial context of dots and edges than computed by the individual modules. Once the correction of the interior and border identifications is completed, then the necessary changes are made and the correction process is iterated on the new set of identification until there are no changes. This iteration is necessary in order to propagate the effect of the label changes spatially.

The corrected results from step B are combined with the aid of assumptions about more global properties such as closure of borders. To do this, first the borders around the points labeled as interior by the module IC are identified. This results in border segments around interior regions. Then, the intersection of these identifications and the results of the module BC is taken. This results in those Delaunay edges being identified as border that have confirmation from two independent processes. The result is a set of border segments and a set of interior points next to them. A connected component analysis is carried out on the regions surrounded or separated by the border segments (step C). If a set of border segments defines a closed curve, no further processing is done on that region. If the border is not closed, then the process attempts to extend it with the eventual goal of closing it and, at the same time, ensuring that the border segment being extended is smooth. After the border completion is accomplished the borders are smoothed by performing single point changes in which the border grabs a single point either from the interior or the exterior if it makes the border smoother. A component interaction module then checks (step D) if any two components can be merged together, thus making the border smoother without altering the interior properties of the components.

## SURFACE SHAPE FROM TEXTURE

Texture variations provide strong cues for the three dimensional arrangement and structure of the surfaces visible in an image. Two types of distortions occur during the imaging process: increasingly large areas of surface are compressed onto a fixed area of the image as the textured surface recedes away from the viewer, and an anisotropic compression of the texture elements due to foreshortening occurs as the surface tilts away from the frontal plane. We have investigated how to exploit textural cues to infer the relative distance and orientation of the textured surfaces depicted in an image. We have looked for methods that would work on images of natural (as opposed to human-made) textures, that handle sub- and super-texture appropriately, and that do not rely on specific texture models. We do not address the texture segmentation/discrimination problem, assuming that it is known which parts of the image correspond to different homogeneously textured 3D surfaces.

We have found that a "scale-space" representation, which represents $\nabla^2 G$ image properties over a continuous range of scales, is useful for identifying textural features of all different sizes. We have derived measurements that can be performed in a $\nabla^2 G$ scale-space in order to characterize the size of texture elements without knowledge of the actual shape of the texture elements. The equations for the measurements are derived by mathematical analysis of the $\nabla^2 G$ and $\frac{\partial}{\partial \sigma} \nabla^2 G$ responses to images consisting of ideal bars and disks. The $\nabla^2 G$ response to more complex images cannot be analyzed in this way because the convolution integrals do not have closed form solutions. Nevertheless, it can be hoped that, due to the smoothing properties of the Gaussian, the $\nabla^2 G$ response to components of real textural primitives will be similar to the response expected for patterns of idealized bars and disks. Tests on real images bear out this hope. We have worked mainly with the image shown in figure 2.

Our method models texture elements with equivalent disks by identifying the size and location of circular disks which best fit the scale-space behavior of the texture sample in question. (We can compute the diameter of a disk. The location of disk centers is approximated by local maxima

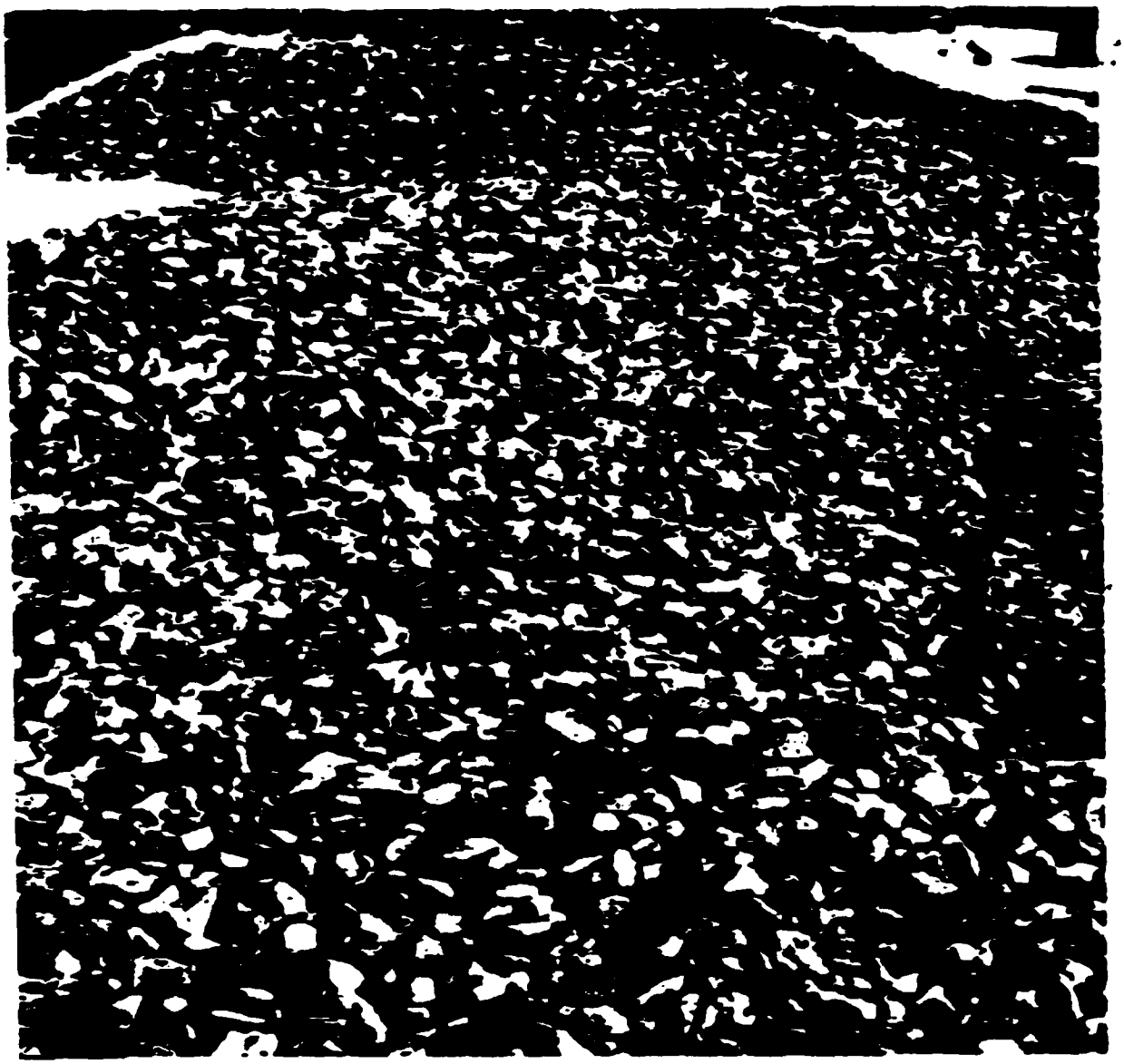

Figure 2. An image of a surface with rocks. This image is being used to develop our shape from texture algorithms.

in the $\nabla^2 G$ response.) Elongated texture elements appear as chains of disks; this representation is reminiscent of the medial axis transform.

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# APPENDIX

**Papers on past work**

# EXTRACTING PERCEPTUAL STRUCTURE IN DOT PATTERNS

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## ABSTRACT

This paper describes an approach to structural segmentation of dot patterns such that the results are in agreement with segmentation performed by humans. Voronoi neighborhoods are used to represent the geometric structure in the dot pattern. A set of experts then, in parallel, look for structural components such as borders, interiors, and curves. The experts have their expertise expressed in terms of the properties of and interaction among Voronoi neighborhoods. The interaction is accomplished through a relaxation, constraint-propagation process. The results of these experts have errors due to the lack of local evidence for the global role of a dot. Each result then is corrected such that 1) it agrees with the results of other experts and 2) it provides locally smooth borders. Except for occasional gaps in the borders the union of the corrected results represents a good approximation to the perceptual structure in the dot pattern. Connected component analysis is carried out to identify these gaps. The gaps are filled to close the component contours ensuring local border smoothness.

## 1. INTRODUCTION

Dot patterns have been studied in the past in different contexts including human perception [5, 14, 17] and as simpler cases of multidimensional clustering algorithms [2, 4, 6, 12, 18]. In the context of visual psychophysics, dot patterns provide a set of stimuli whose spatial properties can be controlled precisely. Dots can be regarded as tokens whose shapes, sizes, intensities, colors, etc. are kept constant and whose most important properties are their positions. Hence, they form an ideal set of stimuli to study the effects of only the spatial distribution of tokens in grouping processes in a tractable fashion. The simplicity of the stimulus may help probe selected parts of the early vision in humans and gain some understanding about the processes present here.

One of the most important aspects of early human vision is that it imposes a structure onto the stimulus in the form of perceptual grouping. It identifies elements in the visual field that in some sense "go together." This phenomenon was first pointed out by the Gestalt psychologists [29] and has important consequences. First, it improves the efficiency of the process of parsing the visual field into surfaces and objects and in the recognition of those objects by reducing the amount of data these processes must handle. Second, it might result in percepts that do not exist in the objective data but are introduced as a result of the internal biases of the system itself. Thus, grouping together the broken edge segments from an edge detector, would result in one long border of a region which no edge detector working on an intensity image could produce because of the intensity distribution. An understanding of how this grouping is accomplished and what kinds of rules govern it will probably increase our understanding of how the early human visual system is structured.

This work was supported by the Air Force Office of Scientific Research under Contract AFOSR 82-0317

This paper will present an algorithm to extract groupings in dot patterns and the resulting spatial structure. Section 2 will review briefly past research efforts in this field. Section 3 will present the algorithm for grouping. Finally, Section 4 will present some concluding remarks.

## 2. REVIEW

The research on dot patterns and grouping of dot patterns has been conducted in fields including psychology and computer science. Perceptual structure in both dynamic and static dot patterns has been studied.

In the perception of static dot patterns, different properties relevant to groupings of elements in the visual field such as spacing of elements were studied by Uttal et al [26], Pomerantz and Schwaitzberg [19], Julesz [10, 11]. The detectability of dotted lines in a noisy background was studied by Uttal et al [26] and the importance of dot spacing in this detectability was shown. The statistical properties of dot patterns, their local spatial properties such as number of terminations, etc. and the importance of these properties in the discriminability of textures [10, 11].

In computer science the work on dot patterns is divided into two major areas: 1) defining the neighbors of a dot, and 2) clustering algorithms. The research in each of these areas will be discussed next.

The crucial information that is used by perceptual processes that perform the grouping of the dots into "meaningful" clusters is the spatial relationship that a dot has with its "neighbors." Hence, the concept of "neighbor" and the concept of the "neighborhood" of a dot are of crucial importance to the theory of perceptual grouping processes. In the past, there has been a number of definitions of neighbor.

A fixed circular neighborhood of a dot with a globally defined radius R is used by Koontz [12] and Patrick and Shen [18]. The concept of k-nearest neighbors of a point has been used by Zucker and Hummel [31], Velasco [27], and Jarvis and Patrick [8]. Both of these have the disadvantage of being very inflexible.

O'Callaghan [16] gives a definition of neighbor based on both distances and angles. In his definition two points may be within the right range of distance to be considered neighbors, but if they are "hidden" from each other by a third point they are not considered to be neighbors.

Other definitions of neighbor include the minimum spanning tree (MST) [30], Gabriel graphs (GG) [21] and the relative neighborhood graphs (RNG) [22, 25]. Ahuja [1] suggests the use of Voronoi neighborhoods [28], which have more intuitively appealing properties than the previous definitions, in the processing of dot patterns.

Early clustering algorithms used various criteria based on pairwise similarity measures of dots in order to measure goodness of clustering [4, 9, 12]. These similarity measures were based on the

more traditional definitions of neighbors of a dot. Graph-theoretical approaches have been used in order to segment a dot pattern into clusters [6, 7, 25, 30]. Several methods have been developed which, for two-dimensional patterns, base their measure of similarity on neighbors of points taking into consideration local geometric structure of the point distribution [6-8, 14, 15, 17, 23-25]. Some researchers have used algorithms [23, 24] which are formulated in a relaxation labeling scheme [20], thus making it unnecessary to use thresholds. A consideration of the human visual system, its assumptions about the physical world, its biases, namely the gestalt principles of the way patterns are perceived by humans was lacking in most of these works with some exceptions [3, 13-15, 17]. In some of the work restrictions were made about the kinds of patterns that the algorithm was supposed to work on. Some of these restrictions were that the patterns would contain only uniform clusters [14, 15, 17, 23], or that they would contain only varying density clusters [24] or only curvilinear clusters [3, 13]. We now go on to describe the grouping algorithm in the next section.

## 3. GROUPING ALGORITHM

The algorithm to be described in this paper for performing grouping of dot patterns is based on the Voronoi tessellation of dot patterns and the geometrical properties of the Voronoi polygons, that are the outcome of this tessellation. These geometrical properties reflect information about the spatial distribution of dots in the neighborhood of a given dot.

The current system is designed to work in two major phases. The first phase takes the dot pattern as its input and produces the lowest level groupings. The second phase starts with the output groupings produced by the first phase and produces the hierarchical structure of these groups of dots (if any exists) in a recursive manner. The following section will describe the details of the first phase.

### 3.1. The First Phase of Segmentation

The first phase of the segmentation mentioned above works in three steps. The general structure of this phase, its modules and their interconnections are shown in figure 1. The first step (box A in figure 1) consists of three independent modules (boxes II, BI, and CI) running in parallel. Each of these modules responds to a certain aspect of the stimulus. The first one (II) identifies interior points, the second one (BI) identifies border points, and the third one (CI) identifies curves. The second step (B) corrects possible errors that might exist in the result of each of the three modules. The third step (C) combines the results of the border correction (BC) and interior correction modules (IC) performing a more global analysis. The outputs of all these modules obtained for the sample pattern in figure 5 are shown in figures 6-11.

Before going into any further details of the system that does the grouping, we will discuss the properties of the Voronoi polygons in the tessellation used by the system to perform the various groupings. There are a number of measures that are computed reflecting the geometric properties of the spatial distribution of dots. Each of these measures and its significance are described below. In this paper we will use the terms *polygon* and *cell*, the region surrounded by the polygon, interchangeably.

The first property is the area of the Voronoi neighborhood of a dot. This is a measure reflecting information about the dot density in the neighborhood of a dot. In the interiors of homogeneous clusters, the density does not change. Recalling the way the Voronoi tessellation is constructed, we observe that this uniformity of density will result in the Voronoi neighborhood being uniform in area in the interior regions.

The second property is the eccentricity of the Voronoi polygons. This measure is a scaled vector indicating how much a dot is off the center of gravity of its Voronoi polygon. The significance of the eccentricity measure is that it is related to the change of density of dots. The interiors of uniform clusters are

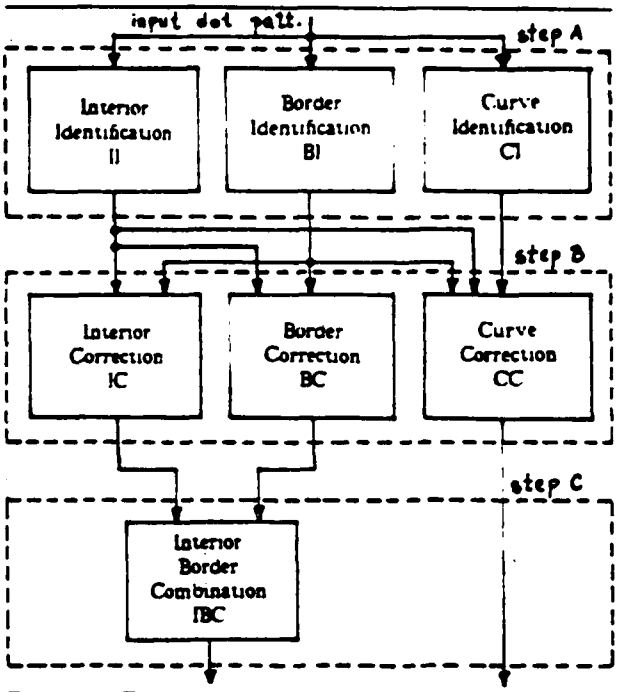


Figure 1 — The modules and their interconnections that make up this system.

expected to have cells with very low eccentricities because of the lack of variation in the density. The interiors of varying density clusters will have cells with high eccentricities due to the density variation; the directions of the eccentricities will be pointing towards the increasing density direction. Thus, the eccentricity vectors of the cells in the interiors of varying density clusters will be aligned most of the time. At the borders of clusters, the eccentricity directions in most cases, will be expected to point towards the interiors of the clusters because of the sudden increase in the dot density; i.e., from the very low density in the intercluster space to the comparatively high density in the interior of the cluster. This observation also holds on the borders of bars which are clusters without interior points.

The third property, Gabriel measure, measures the "neighborliness" of two Voronoi neighbors. If the line joining two Voronoi neighbors $i$ and $j$ intersects the edge shared by the corresponding Voronoi polygons then $i$ and $j$ are perfect neighbors. That is, there is no third point $k$, such that the point $i$ is hidden from the point $j$ by the point $k$, and vice versa. If, on the other hand, there is such a third point $k$, and the line $(i,j)$ crosses the Voronoi cell for point $k$, then $i$ is hidden from $j$ by $k$ and the Gabriel measure indicates the amount by which this is true. The deeper the line $(i,j)$ crosses cell $k$, the worse neighbors $(i,j)$ are and the lower the Gabriel measure is. This is important on the borders of clusters, where if the two points are not perfect neighbors and have a low Gabriel measure, then the border follows through the intervening point instead of the two points.

The fourth measure computed is the compactness measure. This is a measure indicating how wedge shaped the Voronoi neighborhood of a dot is. These types of cells are seen at points where the borders of two clusters gradually approach each other. The wedge-like shape of the cells is caused by the nonuniform distribution of the neighbors of a point around it, and the wedge-like shapes of the polygons indicate that the points are possibly on the border of a cluster.

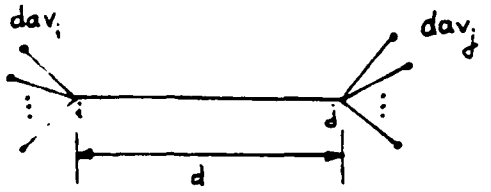


Figure 2 — The distance measure for the Delaunay edge $(i,j)$. $dist_{i,j}$, is defined in terms of the length of the Delaunay edge $(i,j)$, $d$, and the average Delaunay edge lengths on its two endpoints, $dav_i$ and $dav_j$. Let $D=min(dav_i,dav_j)$. Then $dist_{i,j}=1-D/d$ if $d>D$, and $dist_{i,j}=0$ otherwise.

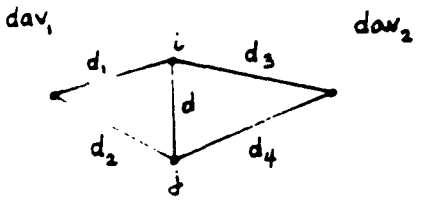


Figure 3 — Squeezedness measure, $sq_{i,j}$, is defined in terms of the length of the Delaunay edge $(i,j)$, $d$, and the average Delaunay edge lengths on its two sides laterally, $dav_1$ and $dav_2$. Let $D=min(dav_1,dav_2)$. Then $sq_{i,j}=1-d/D$ if $d<D$, and $sq_{i,j}=0$ otherwise.

The fifth measure is a distance measure defined for a Delaunay edge. It measures the change in the distance between two dots compared to their surroundings. The computation of this measure is shown in figure 2. This measure is useful mostly in the regions containing curvilinear clusters or single-point clusters, because around such clusters, measures such as areas, eccentricities, etc. are very unreliable. Distance, and Gabriel, and squeezedness (to be described below) measures are mostly the only ones used to describe these contexts. Of course, if the clusters are well separated, this measure is also helpful for identifying edges between clusters with interiors.

The sixth measure computed is the elongations of the Voronoi cells and the directions of their minor axes. When there is a cluster in a pattern where the density of dots is uniform but direction sensitive (i.e. the density in one direction is different from that in another direction) the Voronoi polygons tend to be elongated. The direction of the minor axis tends to be aligned with the higher density direction. As the disparity of the densities in two directions increases, the Voronoi cells become more elongated. This measure is computed using the second order moments of area of a cell.

Another measure, squeezedness, is computed which is very closely related to the elongation, but it is a property of the Delaunay edges rather than Voronoi polygons, and it is based upon the distances of the surrounding Delaunay edges. Its computation is shown in figure 3. This measure, similar to the distance measure, is mostly useful around curvilinear clusters. Namely, the Delaunay edges that lie on a curvilinear cluster are expected to be shorter compared to the Delaunay edges extending on the two sides of the curve laterally.

### 3.1.1. Interior Identification

The interior identification is formulated in a probabilistic relaxation scheme with the dots being labeled as either INTERIOR or NONINTERIOR. This formulation is based upon the local geometrical properties of the Voronoi polygons resulting from the Voronoi tessellation.

The main task is to formulate the local compatibilities between pairs of dots. This formulation is based on the assumption that certain properties of the polygons will be true for the interior points. Specifically, in the interiors of homogeneous clusters, the areas of Voronoi polygons will be approximately the same and the eccentricities of the cells will be low. In the interiors of nonhomogeneous clusters the eccentricities will be high but they will be pointing in the same direction, namely, in the increasing density direction. These facts, used conservatively, will result in the most obvious interior points being identified.

The compatibilities are defined for the four possible combinations of labeling two points as INTERIOR or NONINTERIOR. In order to define these compatibilities one has to consider all the possible cases of combinations and in what contexts they can occur. In this case all the possible contexts in which interior and noninterior combinations can occur are shown in figure 4. For each of these cases an expression is written which measures how compatible the label combination for two points is in the given context. For example, for the case shown in figure 4(a), one would have the expression $\min(1-ecc_i,1-ecc_j,1-\Delta A_{i,j})$. In this expression $ecc_i$ and $ecc_j$ are the eccentricity magnitudes for the Voronoi polygons of the cells of dots i and j, respectively. $\Delta A_{i,j}$ is the area difference of the two polygons normalized to the range [0,1] and is defined as $abs(A_i-A_j)/max(A_i,A_j)$. In this case $A_i$ and $A_j$ are the areas of the polygons for the dots i and j, respectively. The intuitive meaning of the expression is that in the interior of a homogeneous clusters the eccentricity magnitudes and the area differences are expected to be small. If that is the case with the two dots, the above expression will have a high value and will have a positive contribution to the INTERIOR-INTERIOR compatibility value. After the expressions for all these cases have been derived, for a particular combination of labels, the expressions for all its cases are combined by a fuzzy OR operation. That is, for two points i and j to be compatible with labels INTERIOR-INTERIOR, they must have the context shown in case (a), or case (b), of figure 4. Similarly, for other combinations of labels for the two points, the expressions for different cases are combined to get the compatibility expressions.

Once these compatibilities are defined, the relaxation labeling scheme is run which results in the assignment of probabilities to each point of having a certain label (step A, module II in figure 1). Most of these probabilities converge to either very high or very low values resulting in unambiguous labelings (even though they may be the wrong labels). The few points with more ambiguous probabilities are assigned the label with the stronger probability. If this turns out to be the wrong label, it is hoped that the later phases will correct this taking into consideration a larger context (step B in figure 1), information coming from other modules independently, and Gestalt assumptions such as border smoothness (steps B and C), and closure (step C). These corrections are performed when the results of the three modules are being processed for possible corrections and merges (steps B and C in figure 1).

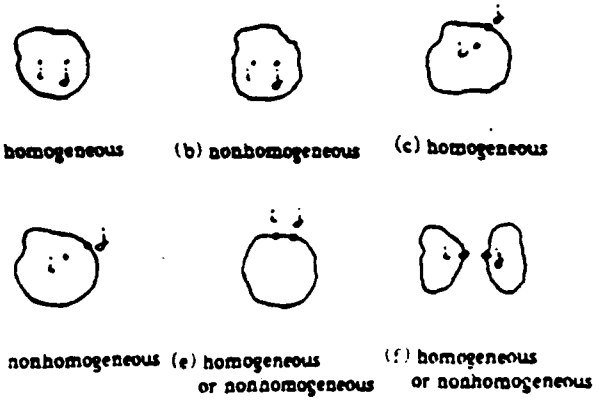


Figure 4 — The possible contexts in which INTERIOR and NON-INTERIOR compatibilities are computed.

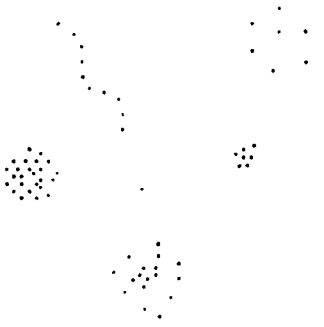

Figure 5 — A sample dot pattern to be segmented.

### 3.1.2. Border and Curve Identification

The method of identification of borders and curves is similar to the identification of the interior points. In this case the objects to be labeled are the Delaunay edges. They are labeled as either BORDER or NONBORDER in the border identification module and CURVE or NONCURVE in the curve identification module.

The compatibilities in these modules are also formulated by looking at all the possible contexts and identifying the different cases and formulating the constraints between properties of the cells and edges that must hold. The possible contexts for these cases are identified similar to the interior identification process. The expressions for each of these cases are identified based on these contexts and combined to get the final compatibility relations.

### 3.1.3. Label Corrections

As a result of the previous step (step A in figure 1), the dots and the Delaunay edges are labelled as INTERIOR-NONINTERIOR, BORDER-NONBORDER, or CURVE-NONCURVE. Some of these labels may not be correct due to lack of local evidence, ambiguities, etc. These incorrect labels need to be corrected by using information from a larger context. The results of the modules (II, BI, and CI) and the agreement among these results is used to obtain the information from the larger context. The criterion that borders be smooth is also used to decide whether a labeling of a dot or a Delaunay edge needs to be corrected. The context that is considered is larger because the border segment necessary for computing smoothness or the agreement measure possibly extends beyond the neighborhood of the object being considered for correction.

This step consists of three modules (IC, BC, and CC in figure 1). Each one corrects one set of identifications from the previous step concurrently and independently using the information from the previous step as shown in figure 1. A module changes the labels of its input if doing so improves the measure of border smoothness and increases agreement with the results of other modules. The correction process is formulated in a probabilistic relaxation scheme with the labels { CHANGE, NO-CHANGE } on the objects. The objects are the dots for the correction of dot labels, and the Delaunay edges for the correction of border identifications. However, not all of the dots or the Delaunay edges are considered for correction. The most confident ones (i.e. the dots or Delaunay edges whose identifications from the two independent modules II and BI are in agreement) are omitted. Only the objects whose identifications from the two independent modules conflict are considered for correction. This increases the efficiency of the correction process. An example of computing compatibilities $r_{ij}$ for objects $i$ and $j$ is as follows:

$$r_{ij} = \frac{((1-curv_i)+(1-curv_j)+0.5(agr_i+agr_j))}{3}$$

In this expression, $curv$ stands for the value of the curvature of a border segment. The border segment whose curvature is computed depends on the identification of the object being labelled. For example, if the object is a dot that is identified as interior, then the border segment around the point is considered for curvature computation. Note that if the interior point is surrounded by all interior points and does not have a border passing by it, then it has a confident identification and will not be considered for correction to begin with. If the dot is identified as border then the border segment going through the dot is considered for curvature computation. In each of these cases a small portion of the border segment is considered, for example, a segment of five dots. The expression $agr$ is a measure of the agreement of the results of the two independent modules. Therefore, this computation reflects the expectation that the curvatures of borders around the objects $i$ and $j$ be minimized (i.e. the borders be smooth) and the agreements of results between different modules be high.

Once the correction of the interior and border identifications is completed, then the necessary changes are made and the correction process described above is iterated on the new set of identifications. This iteration is necessary in order to propagate the effect of the newly changed labels. This iteration proceeds until there are no more label changes. These corrected results are then combined to get a final segmentation in the next step.

### 3.1.4. Combining the Results

The corrected results from step B are combined with the aid of assumptions about more global properties such as closure of borders. A connected component analysis is done in order to perform this task which is described below.

First, the borders around the points labeled as interior by the module IC are identified. This is done by identifying those Delaunay edges as borders that have both endpoints noninterior and the common neighbors of whose endpoints, located on the two sides of the edge, have different labels. This results in border segments that surround the interior regions. Then, the intersection of these identifications and the results of the module BC is taken. This results in those Delaunay edges being identified as border that have confirmation from two independent processes. The result is a set of border segments and a set of interior points next to them. Each of these border segments is given a label (e.g. they are numbered). The interior regions then are assigned the labels of all the border

the process attempts to extend it with the eventual goal of closing it and, at the same time, ensuring that the border segment being extended is smooth. The resulting forks and dangling border segments are cleaned. If there still remain border segments around a region that are not closed, they are extended as smoothly as possible so that some of these segments will either merge with each other to form longer border segments to be further processed, or they will become closed, thus ending the processing of that region. While combining the results of the correction step (step B in figure 1), one must be careful in handling the regions which are bar-like (i.e. two segments of parallel borders with no interior region between them).

Figure 6 — The result of interior identification (II) module run on the pattern. The borders that surround the identified interior regions are shown in the figure.

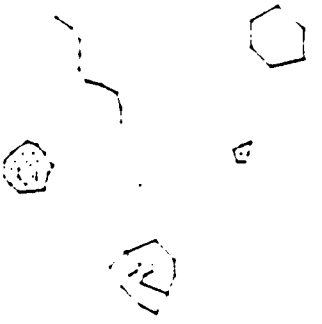

Figure 7 — The result of border identification (BI) module run on the pattern.

Figure 8 — The result of curv identification (CI) module run on the pattern.

segments that surround them. That is, a point $P$ is assigned the label of a border segment $B$, if there exists a path $p_1p_2\cdots p_k$ such that $p_1=P$, $p_k$ is on the border segment $B$, and all the points $p_1p_2\cdots p_{k-1}$ are labelled interior. The result is that all the interior points have one or more border segment labels assigned to them. The goal is to have all these border segments to form a closed contour. Note that the number of final border segment labels assigned to each interior region may be more than one since a component may have holes in it.

The combination process proceeds with the regions that have only one label assigned to them. If the border is closed no further processing is done on that region. If the border is not closed, then

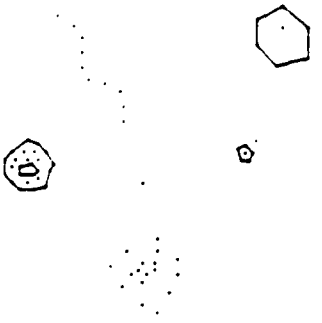

Figure 9 — The result of the interior correction (IC) module run on the result in figure 6 using information also from figure 7.

Figure 10 — The result of border correction (BC) module run on the result in figure 7 using information also from figure 6.

Figure 11 — The partial result of combining the two results in figures 9 and 10.

These are important because they might be part of a neck in a cluster and if they are not considered at this stage then problems arise in trying to close the borders being extended. The problems will be due to the fact that if these border segments are not merged with the border segments of regions with interior points then there will be gaps in the border of the entire cluster and the closure of it will be missed. To avoid this difficulty the border segments of these regions with no interiors are merged with the border segments of the regions with interiors if possible. The contexts in which there is a transition from a region with interiors to a region without an interior in a cluster can occur are limited. Therefore, this contextual knowledge along with the criterion of border smoothness are used when merging the border segments.

## 4. CONCLUSION

We have presented an algorithm for obtaining groupings in dot patterns. The algorithm works bottom up and is highly parallel. Some of the modules have not been completed and are currently being worked on. These are the two modules interior-border-combination (IBC) and curve correction (CC) in figure 1. The algorithm obtains the lowest level groupings present in the dot pattern. However, in patterns which contain hierarchical structures in them, the groups of dots obtained as a result of the segmentation at this level can be treated as tokens to be further grouped recursively. Future plans include extending the algorithm to be able to obtain the hierarchical structures in such patterns.

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# REPRESENTING GEOMETRIC STRUCTURE IN DOT PATTERNS

by

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## ABSTRACT

A representation of basic geometrical properties of dots is crucial in obtaining perceptual structure of dot patterns. This paper describes the use of Voronoi neighborhoods and their geometrical properties to infer geometrical structure in dot patterns. The distribution of dots characterizing different structural components such as curves, bars, single-point clusters, nonempty clusters with uniform and varying density interiors are studied. The effects of dot distribution in each of these components on the geometrical properties of the Voronoi neighborhood of a dot are described.

## 1. INTRODUCTION

This paper concerns representation of geometric structure in dot patterns. Since the perceived structure in a dot pattern is completely determined by the relative spatial locations of dots, a definition of "neighbors" of a dot and the "neighborhood" around a dot is crucial in processing the dot patterns. In [1] we discussed the importance of the notion of the neighborhood of a dot, and specifically, the possible use of the Voronoi neighborhood in a variety of applications. In this paper, we will describe in detail some of the geometric properties of the Voronoi neighborhoods that we have used in our research to infer structural information in a dot pattern.

Section 2 reviews previous work on dot pattern processing and definitions of neighbors of a dot. Section 3 reviews the definition of Voronoi tessellation and Voronoi neighborhood. Section 4 describes in detail the geometrical properties that can be used to infer structural information in dot patterns. Many of these properties have already been used in our programs for perceptual segmentation of dot patterns. Finally, Section 5 presents concluding remarks.

## 2. REVIEW OF PAST WORK

A major part of the past work on defining the neighborhood of a point has been concerned mainly with the following question: given an arbitrary point in a dot pattern, which other points should be treated as its neighbors? The definitions used in the past have included all the points covered by a circular neighborhood with a globally defined radius $R$ [4], k-nearest neighbors of a dot [12], minimal spanning trees [11], relative neighborhood graphs [7, 9], and Gabriel graphs [8]. In all of these, the geometrical information used pertains to pairs of dots and is one dimensional in nature except for the circular neighborhood which has the disadvantage of being insensitive to data. Minimal spanning trees are global in nature and a small change in one part may result in drastic changes in parts of the graph that are far away. O'Callaghan defines a more intuitive definition of a neighborhood. He uses two thresholds, $T_\rho$ and $T_\theta$, to decide whether two points $P_i$ and $P_j$ are neighbors. If $P_{near}$ is the nearest neighbor of $P_i$ then $P_j$ is a neighbor of $P_i$ if a) the distance ratio $d(P_i, P_j)/d(P_i, P_{near})$ is not greater than $T_\rho$ and b) for any neighbor $P_k$ of $P_i$ the angle $P_j P_i P_k$ is different than 180 degrees by more than $T_\theta$ (i.e. $P_j$ is not behind any neighbor $P_k$ of $P_i$). In [1], Ahuja examined the use of the neighborhood of a point which associates with a point not only other points as its neighbors, but also a part of the Euclidean plane around it, thus, giving the problem a two-dimensional character. He closely examined one specific definition of neighborhood called the Voronoi neighborhood, which is based upon the Voronoi tessellation defined by a set of points.

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## 3. VORONOI NEIGHBORHOODS

### 3.1. Voronoi Tessellation

Suppose that we are given a set $S$ of three or more points in the Euclidean plane. Assume that these points are not all colinear, and that no four points are cocircular. Consider an arbitrary pair of points $P$ and $Q$. The bisector of the line joining $P$ and $Q$ is the locus of points equidistant from both $P$ and $Q$ and divides the plane into two halves. The half plane $H_Q^P$ ($H_P^Q$) is the locus of points closer to $P$ ($Q$) than to $Q$ ($P$). For any given point $P$ a set of such half planes is obtained for various choices of $Q$. The intersection $\bigcap_{Q \in S, Q \neq P} H_Q^P$ defines a polygonal region consisting of points closer to $P$ than any other point. Such a region is called the Voronoi [10] polygon associated with the point. The set of complete polygons is called the *Voronoi diagram* of $S$ [5]. The Voronoi diagram together with the incomplete polygons in the convex hull define a *Voronoi tessellation* of the entire plane. The Voronoi tessellation for an example pattern is shown in figure 1. Two points are said to be *Voronoi neighbors* if the Voronoi polygons enclosing them share a common edge. The dual representation of the Voronoi tessellation is the *Delaunay graph* which is obtained by connecting all the pairs of points which are Voronoi neighbors as defined above.

### 3.2. Neighborhood of a Point

We will consider as the neighborhood of a point $P$ (the region enclosed by) the Voronoi polygon containing $P$. Considering the way a Voronoi polygon is constructed, this is an intuitively appealing approach. The local environment of a point in a given pattern is reflected in the geometrical characteristics of its Voronoi polygon. This presents a convenient way to compare the local environments of different points. Since the perceived structure in a dot pattern results from the relative spatial arrangement of points, the geometric properties of Voronoi polygons may be useful for describing and detecting structure in dot patterns. Such an approach lends a fully two-dimensional character to the problem in that the dot pattern is converted into a planar image or a mosaic.

The advantages of the Voronoi neighborhood compared to the definitions given in the previous section are that the Voronoi neighborhood is (i) intuitive, (ii) adaptive, and (iii) two-dimensional in character. It is adaptive in the sense that the assignment of neighbors does not depend on the scale of the dot pattern and neighbors are assigned to dots that reflect the local density variations. Also, the number of neighbors of a dot is not fixed and may vary depending on the structure in the vicinity of the dot.

## 4. GEOMETRIC PROPERTIES

It is useful to look at the possible structural patterns in a dot pattern and to see how the various possible distributions are reflected in the geometric properties of the Voronoi neighborhoods. One type of structural component in a dot pattern is a cluster of dots with nonempty interior. These types of clusters may have an interior having either a uniform density or a varying density. Another component is a cluster with no interior, or a bar. Besides these there are the curvilinear structures and single-point clusters. All of these and their spatial configurations are reflected in the shapes of the Voronoi neighborhoods or properties of the Delaunay edges.

The Voronoi neighborhoods of the points which reside within the interior of a homogeneous cluster will have similar shapes and sizes. For different clusters, these interior polygons may differ in their geometrical properties. The border cells of a cluster will be open if there is no other cluster to bound them. The cells of the border points of a cluster that have neighbors in a nearby cluster will differ from interior cells. For example, they may be elongated if the distance between cross cluster neighbors is larger than within cluster neighbors, or the nucleus of the cell may be located well off its center. Clearly, a globular cluster will have a larger number of interior cells than will a more elongated cluster. For dot patterns containing varying density clusters, the interior cells will be compressed in the direction of increasing density. This results in the dots being off the center of the cell in the direction of increasing density. Certain border cells will assume wedge-like shapes where borders of two clusters gradually approach each other. This is caused by the uneven distribution of the neighbors of a point. Such properties will be further described in detail in the next section.

Others have suggested in the past the use of certain properties of the Voronoi polygons for use in dot pattern processing. Sibson [6] has suggested the use of the areas and nucleus-vertex distances of the Voronoi polygons, and the distances between neighboring points as statistics of a point pattern. Chapman [2] uses the distribution of the areas of the Voronoi polygons to infer the structure of dot patterns describing geographic concentrations of economic activity. He relates the degree of the local clustering, or nonrandomness, to an entropy measure computed from the histogram of the cell areas. In the remainder of this section we will

describe each of the geometric properties in detail and their computation. The terms *polygon* and *neighborhood* which is the two-dimensional area surrounded by the Voronoi polygon will be used interchangeably.

The first property is the area of a Voronoi polygon of a dot. This is a measure reflecting information about the dot density in the local neighborhood of a dot. In the interiors of homogeneous clusters, the density does not change. Recalling the way the Voronoi tessellation is constructed, we observe that this uniformity of density will result in the Voronoi neighborhood areas being uniform in the interior regions.

The second property is the eccentricity of the Voronoi polygons. This measure is a scaled vector indicating how much a dot is off the center of gravity of its Voronoi polygon. The situation and the computation of the eccentricity is shown in figure 2. The significance of the eccentricity measure is that it is related to the change of density of dots. The interiors of uniform clusters are expected to have cells with very low eccentricities because of the lack of variation in the density. The interiors of varying density clusters will have cells with high eccentricities due to the density variation; the directions of the eccentricities will be pointing towards the increasing density direction. Thus, the eccentricity vectors of the cells in the interiors of varying density clusters will be aligned most of the time. At the borders of clusters, the eccentricity directions in most cases, will be expected to point towards the interiors of the clusters because of the sudden increase in the dot density; i.e., from the very low density in the intercluster space to the comparatively high density in the interior of the cluster. This observation also holds on the borders of bars which are clusters without interior points.

The third property is Gabriel measure, whose computation is shown in figure 3. It measures the "neighborliness" of two Voronoi neighbors. If the line joining two Voronoi neighbors $i$ and $j$ intersects the edge shared by the corresponding Voronoi polygons then $i$ and $j$ are perfect neighbors. That is, there is no third point $k$, such that the point $i$ is hidden from the point $j$ by the point $k$, and vice versa. If, on the other hand, there is such a third point $k$, and the line $(i,j)$ crosses the Voronoi cell for point $k$, then $i$ is hidden from $j$ by $k$ and the Gabriel measure indicates the amount by which this is true. The deeper the line $(i,j)$ crosses cell $k$, the worse neighbors $(i,j)$ are and the lower the Gabriel measure is. This is important on the borders of clusters, where if the two points are not perfect neighbors and have a low Gabriel measure, then the border follows through the intervening point instead of the two points. For example, in figure 3 if Gabriel measure is low enough, the border passes through points $(i,j,k)$ instead of going through $(i,j)$ directly.

The fourth property computed is the compactness of the Voronoi cells. In the interiors of clusters, even though the density of points may vary, the points are surrounded uniformly by other points all around. This results in the interior angles of the Voronoi polygons of such points being uniform. Such cells are called "compact." When the borders of two clusters gradually approach each other, the cluster exterior sides of such borders have a nonuniform distribution of dots. The cluster interior sides of such borders, however, have a uniform distribution of dots. This results in the dot distribution around a point lying on such a border segment to be uneven which, in turn, is reflected in the shape of the Voronoi polygon of the dot. An example of such a case is the point $A$ and its Voronoi neighborhood in figure 1. Such a polygon has a wedge-like shape, and is "noncompact." The computation of the compactness measure is shown in figure 4.

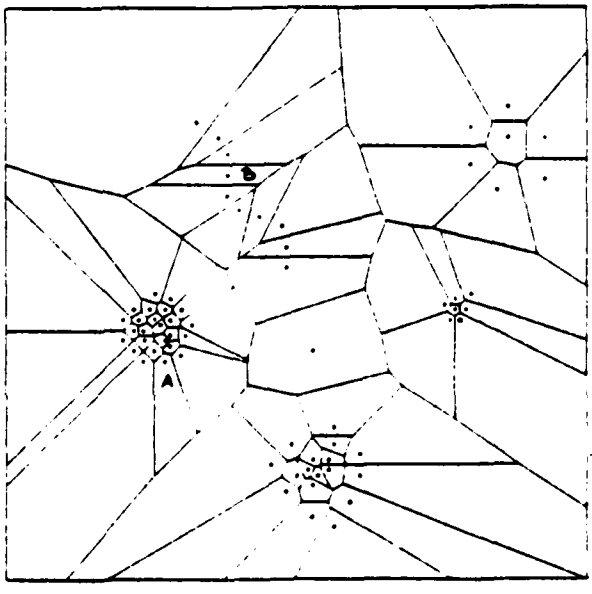


Figure 1 — The Voronoi tessellation of an ex-

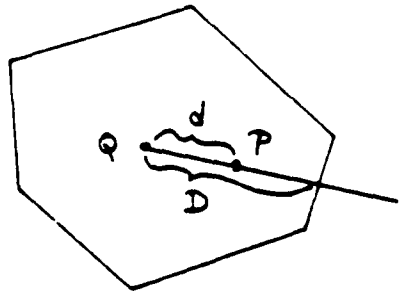


Figure 2 — Eccentricity of a cell belonging to point $P$ is defined as $d/D$. Eccentricity direction is in the direction of $QP$. Here $Q$ is the centroid of the cell.

Another property of the Voronoi cells is their elongation and the direction of their minor or major axis. This measure gains importance in clusters which have uniform distribution of dots but the density is direction sensitive. That is, the dot density along a given direction is different than that along another direction. In such cases, the Voronoi polygons tend to be more squeezed along the higher density direction than along lower density direction, thus, resulting in a more elongated cell. The major and minor axes directions of such cells indicate the directions along which the dot densities are smallest and greatest, respectively. The cells along a curve will also tend to be elongated (as illustrated by point $B$ in figure 1) due to the fact that on the two sides of the curve the dot density is very low compared to the dot density along the curve. Thus, such elongated cells are one of the indications of the existence of curvilinear structures.

There are many ways the elongation of a polygon can be computed. One possibility is the ratio of the area of the polygon to its perimeter squared. Another, which we have used, uses the information from the computation of moments of area [3] to obtain both the amount of elongation and the directions of the major and minor axes. The computation is as follows:

$$elong = \left| \frac{\left[(\mu_{20}-\mu_{02})^2+4\mu_{11}^2\right]^{\frac{1}{2}}}{\left[(\mu_{20}-\mu_{02})^2+4\mu_{11}^2\right]^{\frac{1}{2}}+\mu_{20}+\mu_{02}} \right|^{\frac{1}{2}}$$

$$\alpha = \frac{1}{2}\tan^{-1}\left|\frac{2\mu_{11}}{\mu_{20}-\mu_{02}}\right|$$

Here the $\mu_{i,j}$s are the second order moments of area that are scale and translation invariant; *elong* is the amount of elongation of the cell and $\alpha$ is the angle that the major axis of the cell makes with the x-axis. The elongation measure itself is rotation invariant.

So far all of the properties discussed were two-dimensional properties of the Voronoi polygons. Most of these properties make sense when used for clusters that have interiors. For curves or single-point clusters the two-dimensional properties are no longer well behaved. For example, the area differences are not meaningful when used in regions with curvilinear structures, or on borders of clusters. In such regions one dimensional measures such as distances, etc., are more useful. In the following paragraphs, we will describe several such properties.

The first such property is a measure based on the differences in the length of a Delaunay edge and the average distances of its endpoints to their Voronoi neighbors. The computation of this property is shown in figure 5. In the intercluster space the Delaunay edges have endpoints on the borders of clusters, on single-point clusters, or on curves. If one of the endpoints is on the border of a cluster then the distances on the interior of that cluster will have relatively small distances compared to the distance in the intercluster space. Thus at least one side of a Delaunay edge having an average distance which is small compared to the length of that Delaunay edge is an indication that the edge is most probably in the intercluster space. In this case the measure will be high indicating this high probability.

Another measure, "squeezedness," is computed which is very closely related to the elongation, but it is a property of the Delaunay edges rather than Voronoi polygons, and it is based upon the distances of the

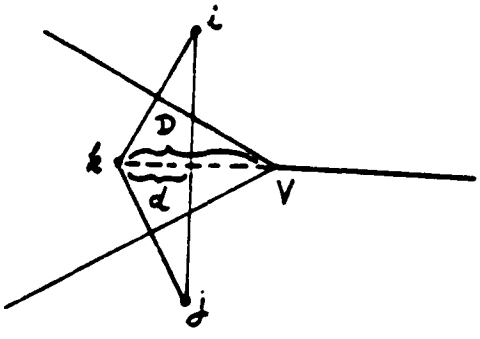


Figure 3 — The Gabriel measure for the Delaunay edge $(i,j)$, $gabr_{i,j} = d/D$ if the line $(i,j)$ intersects the line $(k,V)$, and $gabr_{i,j} = 1$

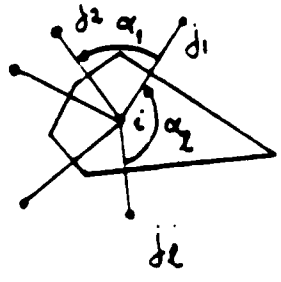


Figure 4 — Compactness measure is computed in terms of the angles $\alpha_i$. Let $\alpha_{av} = \frac{\sum_{i=1}^{l} \alpha_i - \alpha_{max}}{l-1}$. Then the compactness of the cell for point $i$, $w_i = \frac{\alpha_{max}-\alpha_{av}}{\ldots}$

surrounding Delaunay edges. Figure 6 shows its computation. This measure is also mostly useful for curvilinear clusters. Namely, the Delaunay edges that lie on a curvilinear cluster are expected to be squeezed compared to the Delaunay edges extending on the two sides of the curve laterally. If this is the case, then the squeezedness measure will be high, indicating the possibility of the Delaunay edge being on a curve.

All of the properties described above have been used in an algorithm that we are developing for obtaining perceptual segmentation of dot patterns. The segmentation algorithm consists of three phases: (1) identification of different types of structures in the pattern, namely interiors of clusters, borders of clusters, bars, and curves; (2) correction of the erroneous results of the first phase which are a result of local ambiguities; (3) combining these results to obtain a unified segmentation. The Voronoi neighborhood properties were used in the modules of the first phase for obtaining the four different types of structures. These modules are formulated as probabilistic relaxation labeling schemes. The Voronoi neigbhorhood properties described above were used in deriving the expressions for the compatibility coefficients of these modules that are necessary for this formulation.

The correction module takes these results which might have wrong identifications in them because of the insufficiency of local evidence, and tries to eliminate the errors by integrating more global information from multiple sources (modules). It enforces such requirements as border smoothness and a high degree of agreement between the results of the independent modules. This is also formulated as a probabilistic relaxation scheme.

The corrected results of this phase is fed into the combination phase which combines the outputs to obtain a unified segmentation. The combination starts with the interior regions of the clusters and looks only at the border segments adjacent to these interior regions. If there are any incomplete borders, it extends these borders and tries to combine them with other border segments such that the resulting closed border is the smoothest. The eventual goal is to have all the borders around interior regions to be closed, smooth curves.

After this step it is possible to regard the groups of clusters obtained in this fashion as tokens — or dots — to be further grouped. In this manner it is possible to obtain a hierarchy of grouping at different levels of resolution.

## 5. CONCLUSION

Analysis of dot patterns requires a sound notion of the local environment of a point. We have used the Voronoi neighborhoods of points to characterize the local structure because of the many intuitively appealing properties of such neighborhoods [1]. We have described various geometric properties of the Voronoi polygons and the Voronoi tessellation that provide useful information about the geometric structure of the dot patterns. We have used these properties in our research in designing a unified segmentation algorithm for processing unrestricted dot patterns. The results we have obtained have shown that the Voronoi tessellation and the geometric properties of the Voronoi polygons described in this paper can be used successfully to infer information about the geometric structure of dot patterns.

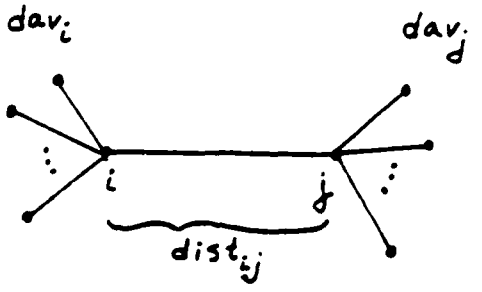


Figure 5 — The distance measure for the Delaunay edge $(i, j)$, $dist_{ij}$, is defined in terms of the length of the Delaunay edge $(i, j)$, $d$, and the average Delaunay edge lengths on its two endpoints, $dav_i$ and $dav_j$. Let $D = min(dav_i, dav_j)$. Then $dist_{ij} = 1 - D/d$ if $d > D$, and $dist_{ij} = 0$ otherwise.

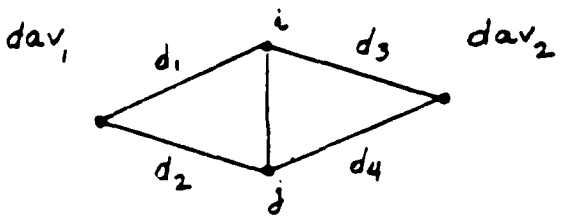


Figure 6 — Squeezedness measure, $sq_{ij}$, is defined in terms of the length of the Delaunay edge $(i, j)$, $d$, and the average Delaunay edge lengths on its two sides laterally, $dav_1$ and $dav_2$. Let $D = min(dav_1, dav_2)$. Then $sq_{ij} = 1 - d/D$ if $d < D$, and $sq_{ij} = 0$ otherwise.

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