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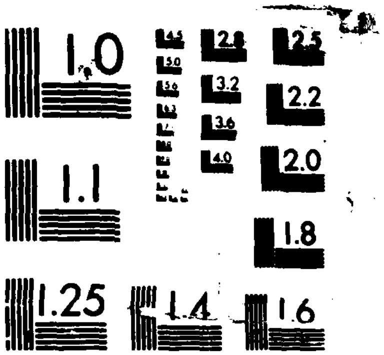

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A COMPUTER BENCHMARK PROGRAM FOR HILBERT MATRIX INVERSION USING IMSL ROUTINES

James A. Theriault

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Approved by A. Mohammed H/APPLIED MATHEMATICS SECTION

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Abstract

In considering potential replacements for computer facilities, such as that for the DREA DECSYSTEM 20/60, comparison standards must be specified. In the comparison standards, two notable characteristics that should be specified are the speed and accuracy of numerical computations. HILBRT is a benchmarking program which may be used for this purpose. It is based on the inversion of finite segments of the infinite Hilbert matrix. The theory, program, and example results are presented.

Résumé

L'examen des systèmes de replacement potentiels pour les systèmes informatiques comme, par exemple, le DECSYSTEM 20/60 du CRDA, exige l'établissement de critères de comparaison. Il est nécessaire de préciser deux caractéristiques importantes dans ces critères de comparaison, c'est-à-dire la vitesse et l'exactitude des calculs numériques. Le programme d'évaluation des performances HLBERT peut servir à cette fin. Il est fondé sur l'inversion de segments finis de la matrice de Hilbert infinie. Le texte expose les principes théoriques, décrit le logiciel et donne un exemple des résultats obtenus.

Contents

1 Introduction

In order to make an objective comparison between computers, one must consider many features. One may consider such things as user friendliness, software availability, hardware availability, speed, and accuracy. This technical communication describes one method of measuring and comparing the features of computer speed and accuracy. The method is based on the inversion of finite segments of the Hilbert matrix.

The Hilbert matrix is a classic matrix in numerical analysis. As the finite segment size increases, the segments become increasingly ill-conditioned. HILBRT is a benchmark program written in ANSI Fortran '77 which inverts finite segments oí the infinite Hilbert matrix using IMSL Version 9.2[IMSL, 1984], then computes accuracy measurements of the solution. Two measures are used. The first is to compute the sup-norm of the difference between the identity matrix and the product of the Hilbert matrix and its IMSL approximated inverse. The second measure is the sup-norm of the difference between the IMSL computed inverse and a computed closed form solution. The results are then written on the file "HIL.DAT." Output consists of the matrix order and both sup-norms for each matrix. These are calculated for matrices starting with order one and increasing by one each time. This is repeated until the IMSL subroutine is unable to find the solution or an order of thirty is reached. Calculations for those matrices of orders one through four are repeated 12000 times. The CPU time required for the repetitions is calculated and written on the output file to allow comparison of systems. The total CPU time required is also included.

In the technical specifications for the replacement of DREA's DECSYSTEM 20/60, the IMSL library is included as an essential requirement so that the use of this library in the benchmark program HILBRT is justified. In essence, the performance characteristics of speed and accuracy depend on the performance of the subroutines in the IMSL library; and, in turn, the performance of the subroutines depends on the computer wordlength and the floating-point format being used. As a result, this benchmark measures speed and accuracy in an IMSL environment.

2 Operational Section

2.1 Identification

PROGRAM HILBRT

A Fortran program to measure computer speed and accuracy. It will be one of many benchmark programs used to assist in the selection of a new DREA central computing facility.

Author: James A. Theriault

2.2 Hardware/Software Environment

HILBRT requires the following software:

- ANSI Fortran '77

- Fortran subroutine LINV2F from IMSL Version 9.2

- Fortran subroutine LEQT2F from IMSL Version 9.2

- Fortran subroutine CPUTIM from DEC 20/60 library. This may be replaced by similar routines on other computers.

HILBRT may be run in batch mode. There is no user input. All output is directed to a named disk file, "HIL.DAT."

2.3 Purpose

The purpose of HILBRT is to benchmark computers in an IMSL environment for the DEC 20/60 replacement. It does so by inverting finite segments of the infinite Hilbert matrix.

2.4 Outline of Working

The finite segment of order n of the Hilbert matrix is given by [Gregory et al, 1969]:

$$A_n = [a_{i,j}^{(n)}] \tag{2.1}$$

where

$$a_{i,j}^{(n)} = \frac{1}{i+j-1}. \tag{2.2}$$

Its inverse may be given in a closed form as shown in section 3.1. The inverse may also be found using various numerical techniques such as the Crout algorithm for Gaussian elimination utilized by the IMSL routine, LEQT2F.

HILBRT uses both methods to compute the inverse, then compares the results for accuracy.

2.5 Input

The program requires no user input.

2.6 Output

The following is the output file, 'HIL.DAT,' from the DREA DEC 20/60.

```
ORDER   NORM OF A*D-I  NORM OF D-B
  1      .00000E+00     .00000E+00
  2      .00000E+00     .23842E-06
  3      .47684E-06     .85831E-04
  4      .76294E-05     .13245E-01
  5      .24414E-03    51.516
  6      .78125E-02    31173.
  7      .18750         .17929E+08

MATRIX SIZE    8 UNABLE TO BE INVERTED.   IMSL ERROR # 129

TIME AFTER   12000 REPETITIONS OF THE FIRST FOUR ORDERS = 4.984
TOTAL CPU TIME = 4.991  MINUTES
```

A refers to the finite segment of the Hilbert matrix. I refers to the identity matrix. D and B refer to the IMSL calculated approximation and the closed form approximation of the inverse respectively.

2.7 Operation

The sequence of monitor instructions for executing HILBRT on the DREA DEC 20/60 is given by

```
@COMPILE DREA:<MAG.THERIAULT>HILBRT.FOR
@LOAD DREA:<MAG.THERIAULT>HILBRT, <LIB>DECLIB/LIB, <LIB>IMSL/LIB
@SAVE
@RUN HILBRT
```

A similar sequence of commands should be available on all benchmarked computers. Care should be taken to ensure that the calculated CPU time does not include time for compile and load operations. *Compiler optimization must not be used.*

HILBRT requires no further user input.

2.8 Restrictions & Limitations

A maximum size for the finite segments of the Hilbert matrix is 30 by 30. This may be changed by modifying the value of NMAX in the parameter statement of the program. It is assumed that the computer can handle Hilbert matrix segments up to order $n = 4$.

2.9 Subroutines Called

The following subroutines and functions are called by HILBRT.

CMPINV - Subroutine to compute inverse of Hilbert matrix and appropriate norms. It has been supplied with the program.

PERM - Real function which computes a permutation. It has been supplied with the program.

CPUTIM - A DEC 20/60 dependent subroutine to return the CPU time in milliseconds since the start of execution. Systems to be compared should have a similar subroutine.

LINV2F - IMSL version 9.2 subroutine to invert matrix. Calls IMSL subroutine LEQT2F to perform inversion.

2.10 Memory Requirements

The following is a list of the files involved and their sizes on the DREA DEC 20. HIL.DAT is the output file. HILBRT.FOR is the Fortran source for HILBRT. HILBRT.REL and HILBRT.EXE are the relocatable and the saved executable versions of HILBRT. IMSL.REL is the IMSL library.

FILE	Characters(7 bit)
DREA:<MAG.THERIAULT>HIL.DAT	457
DREA:<MAG.THERIAULT>HILBRT.FOR	2958

FILE	Words(36 bit)
DREA:<MAG.THERIAULT>HILBRT.EXE	8704
DREA:<MAG.THERIAULT>HILBRT.REL	1427
PS:<LIB>IMSL.REL	400267

2.11 Execution Times

HILBRT requires approximately five minutes of CPU time on the DEC 20/60 to complete an execution. Subroutine CPUTIM is accurate within 10%.

3 Technical Section

3.1 Details of Working

The finite segment of order n of the infinite Hilbert matrix is given by [Gregory et al, 1969]:

$$A_n = [a_{i,j}^{(n)}] \tag{3.1}$$

where

$$a_{i,j}^{(n)} = \frac{1}{i+j-1} \tag{3.2}$$

for $1 \le i \le n$ and $1 \le j \le n$.

Its inverse may be found numerically. It is also available in the following closed form [Gregory et al, 1969].

$$\begin{aligned} B_n &= A_n^{-1} && (3.3) \\ &= [b_{i,j}^{(n)}], && (3.4) \end{aligned}$$

where

$$b_{i,j}^{(n)} = (-1)^{i+j} \frac{(n+i-1)!(n+j-1)!}{(i+j-1)\left((i-1)!(j-1)!\right)^2 (n-i)!(n-j)!}. \tag{3.5}$$

This can be computed for successive values of n by using the recursion,

$$b_{i,j}^{(n+1)} = b_{i,j}^{(n)} \left(\frac{n+i}{n-i+1}\right)\left(\frac{n+j}{n-j+1}\right) \tag{3.6}$$

for $1 \le i \le n$ and $1 \le j \le n$. Additional elements, $b_{i,j}^{(n+1)}$ may be computed as follows:

$$\begin{aligned} b_{n+1,j}^{(n+1)} &= b_{j,n+1}^{(n+1)} && (3.7) \\ &= (-1)^{n+j+1}\left(\frac{(2n+1)!/n!}{(j-1)!(n-j+1)!}\right)\left(\frac{(n+j-1)!/n!}{(j-1)!}\right) && (3.8) \end{aligned}$$

for $1 \le j \le n+1$.

HILBRT uses this and the IMSL subroutine LEQT2F to compute the inverse. The two methods are compared and the results of the comparison are returned as output.

Define the sup-norm, $||C||_\infty = \sup_{i,j}|c_{i,j}| = \max_{i,j}|c_{i,j}|$. Let A_n be defined to be a finite segment of the Hilbert matrix as above. Let D_n be the computed approximation of the inverse using LEQT2F and B_n be the inverse computed using the above closed form.

The first error measure is the sup-norm of the difference between the identity matrix and the product of the Hilbert matrix A_n and its IMSL approximated inverse D_n:

$$\begin{aligned} \epsilon_1^{(n)} &= \|A_n D_n - I_n\|_\infty \tag{3.9}\\ &= \|E\|_\infty \tag{3.10}\\ &= \max_{i,j} |e_{i,j}| \tag{3.11} \end{aligned}$$

where I_n is the identity matrix of order n.

The second error is the sup-norm of the difference between the IMSL approximated inverse D_n and the closed form inverse B_n:

$$\begin{aligned} \epsilon_2^{(n)} &= \|D_n - B_n\|_\infty \tag{3.12}\\ &= \|U\|_\infty \tag{3.13}\\ &= \max_{i,j} |u_{i,j}|. \tag{3.14} \end{aligned}$$

The computation of the inverses and comparisons are repeated 12000 times for matrices of orders one through four. After this the computations continue for increasing n until the IMSL subroutine is unable to compute the inverse or until a maximum order of thirty is reached.

3.2 Flowchart

Figure 3.1 show a flowchart of the program HILBRT.

A DEC 20/60 dependent subroutine, CPUTIM, is utilized to return the CPU time in milliseconds. CPUTIM is called at the start of HILBRT to return the CPU time since the start of the job. It is called again after the first four matrices have been inverted 12000 times to return the time after execution. It is called a final time after all calculations are complete in order to compute the total CPU time. The elapsed times are then calculated in minutes and written on the output file.

A subroutine similar to CPUTIM is usually available on most computers.

3.3 Important Variables

`N`	matrix order
`M`	matrix order (= `N`)
`NMAX`	maximum matrix order
`A(NMAX,NMAX)`	finite segment of Hilbert matrix
`B(NMAX,NMAX)`	inverse of A computed from closed form solution
`D(NMAX,NMAX)`	inverse of A computed by LEQT2F
`NORMI`	$\|\|$ `A D - I` $\|\|_\infty$

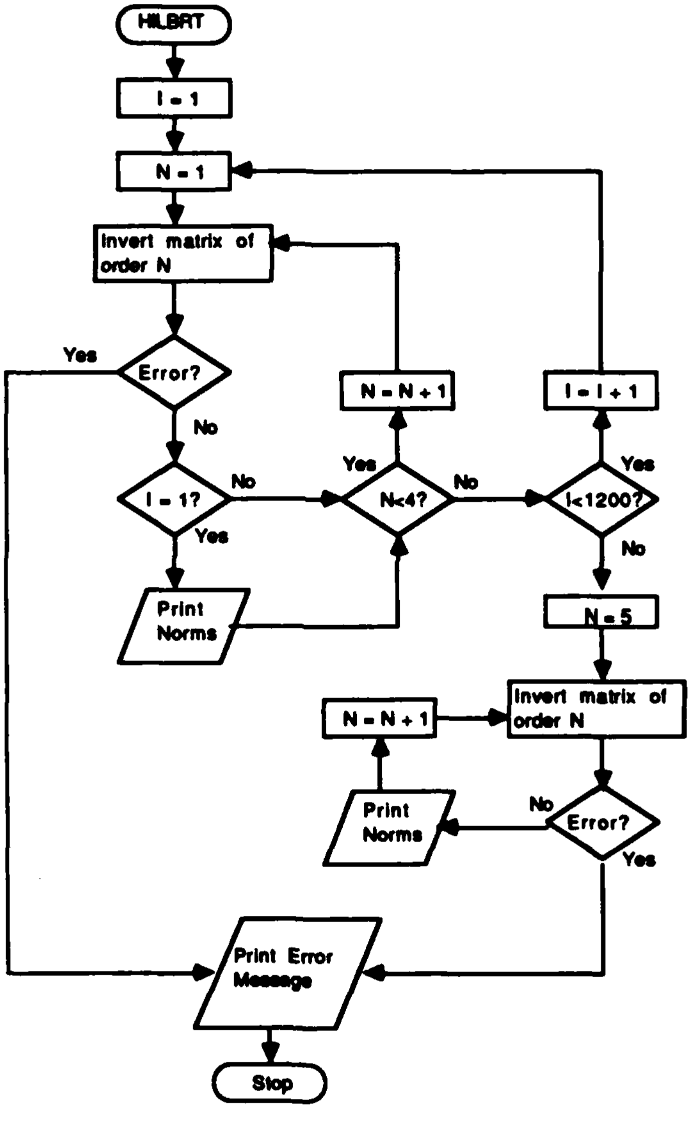

Figure 3.1: Flowchart of HILBRT Program

NORMD	$\|D - B\|_\infty$
ITIM	time at start of job (in msec)
ITIM1	time at end of job (in msec)
ITIM2	time after 12000 repetitions of first 4 matrices (in msec)

4 Summary

HILBRT is a program for benchmarking computers. It has been written in ANSI Fortran '77 with the addition of a call to a DEC 20/60 library subroutine to determine the CPU time. It also requires a number of IMSL Version 9.2 subroutines.

The program creates and inverts finite segments of the infinite Hilbert matrix of orders one through to the largest possible on the particular computer. The program terminates when an IMSL error is detected or when order 30 is completed. The errors in approximating the inverses are written on the disk file "HIL.DAT" along with the time required.

Bibliography

[Gregory et al, 1969] Robert T. Gregory, David L. Karney, *A Collection of Matrices for Testing Computational Algorithms,* John Wiley & sons, Inc., 1969.

[IMSL, 1984] *User's Manual, IMSL Library,* Vol. 2, 1984; International Mathematical & Statistical Libraries, Inc., Houston, Texas.

Appendix A Program Listing

```
      PROGRAM HILBRT
C
C     INVERT FINITE SEGMENT OF INFINITE HILBERT MATRIX
C
C     LAST REVISION: DECEMBER 1, 1986
C     AUTHOR:        J. A. THERIAULT
C
      PARAMETER NMAX=30,IRPT = 12000
      REAL B(NMAX,NMAX),A(NMAX,NMAX),D(NMAX,NMAX),NORMI,NORMD
C     CALL TO DEC20 DEPENDENT SUBROUTINE  (ITIM = TIME FROM START IN MSEC)
      CALL CPUTIM(ITIM)
C
      OPEN(UNIT = 22, FILE = 'HIL.DAT')
      WRITE(22,100)
      DO 8 II = 1,IRPT
         B(1,1) = 1.
         DO 8 N = 1,4
            M = N
            CALL CMPINV(A,B,D,M,NMAX,NORMI,NORMD,IER)
            IF (II.EQ.1) WRITE(22,102) N,NORMI,NORMD
            IF (IER.NE.0) GOTO 9
8     CONTINUE
9     IF (IER.NE.0) THEN
         WRITE(22,105) I
         STOP
      ENDIF
C     CALL TO DEC20 DEPENDENT SUBROUTINE (ITIM2 = TOTAL ELAPSED TIME IN MSEC)
      CALL CPUTIM(ITIM2)
C
      N = 4
11    N = N+1
         M = N
         CALL CMPINV(A,B,D,M,NMAX,NORMI,NORMD,IER)
         IF (IER.NE.0) GOTO 200
         WRITE(22,102) N,NORMI, NORMD
      IF (N.LE.NMAX) GOTO 11
```

```
C
200   WRITE(*,104) N,IER
      WRITE(22,104) N,IER
C     CALL TO DEC20 DEPENDENT SUBROUTINE (ITIM1 = TOTAL ELAPSED TIME IN MSEC)
      CALL CPUTIM(ITIM1)
C
      WRITE(22,106) IRPT,(ITIM2-ITIM)/60000.
      WRITE(22,103) (ITIM1-ITIM)/60000.
100   FORMAT(/ ' ORDER   NORM OF A*D-I  NORM OF D-B')
102   FORMAT (I4,6X,G10.5,4X,G10.5)
103   FORMAT(' TOTAL CPU TIME =',F, '  MINUTES')
104   FORMAT (/' MATRIX SIZE ',I4,' UNABLE TO BE INVERTED.',
     /'    IMSL ERROR #', I4,/)
105   FORMAT(' ERROR WITH ORDER LESS THAN 4', I)
106   FORMAT(' TIME AFTER ',I7,' REPETITIONS OF THE FIRST',
     /' FOUR ORDERS =',F)
      STOP
      END
C
      SUBROUTINE CMPINV(A,B,D,N,NMAX,NORMI,NORMD,IER)
      REAL B(NMAX,NMAX),A(NMAX,NMAX),D(NMAX,NMAX),NORMI,NORMD
      REAL WORK(1000)
C
      NORMI = 0.
      NORMD = 0.
      DO 2 I = 1,N-1
         A(I,N) = 1./(I+N-1.)
2        A(N,I) = A(I,N)
      A(N,N) = 1./(2*N-1.)
      IDGT = 0
      CALL LINV2F(A,N,NMAX,D,IDGT,WORK,IER)
      IF (IER.NE.0) GOTO 8
      DO 4 I = 1,N
         DO 4 J = 1,N
            C = 0.
            DO 3 K = 1,N
3              C = C + A(I,K)*D(K,J)
            IF ((I.NE.J).AND.(ABS(C).GT.NORMI)) NORMI = ABS(C)
            IF ((I.EQ.J).AND.(ABS(C-1.).GT.NORMI)) NORMI=ABS(C-1.)
            IF (ABS(D(I,J)-B(I,J)).GT.NORMD) NORMD=ABS(D(I,J)-B(I,J))
4     CONTINUE
C
      DO 5 I = 1,N
         DO 5 J=1,N
```

```
5          B(I,J) = B(I,J)*((N+J)/(N+1.-J))*((N+I)/(N+1.-I))
      DO 6 J = 1,N+1
         TMP = PERM(J-1,1)
         B(N+1,J) = (-1)**(N+J-1)*(PERM(2*N+1,N)/TMP/PERM(N+1-J,1))
     &                     *(PERM(N+J-1,N)/TMP)
6        B(J,N+1) = B(N+1,J)
8     CONTINUE
C
      RETURN
      END
C
      REAL FUNCTION PERM(M,N)
C     COMPUTE M!/N! WHERE M > N
      F = 1
      DO 10 I = N+1,M
10       F = F * I
      PERM = F
      RETURN
      END
```

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