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**Relativistic Self Focusing of Short Pulse
Radiation Beams in Plasmas**

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RELATIVISTIC SELF FOCUSING OF SHORT PULSE RADIATION BEAMS IN PLASMAS

The interaction of intense radiation beams with plasmas plays an important role in such areas as laser driven fusion, ionospheric processes and pulsed radiation. Recently the possibility of employing intense laser beams to accelerate electrons to ultra-high energies has also stimulated interest in radiation self-focusing in plasmas.¹⁻¹⁴ One such laser driven acceleration scheme is the laser beat wave accelerator. This is a collective acceleration scheme which utilizes a large amplitude plasma wave with phase velocity slightly less than the velocity of light to accelerate charged particles. The large amplitude plasma wave is excited by the nonlinear coupling of two intense laser beams propagating through the plasma. In this process the two laser beams with frequencies ω_1, ω_2 and corresponding wave numbers k_1, k_2 couple through the plasma to produce a ponderomotive wave with frequency $\omega_1 - \omega_2$ and wave number $k_1 - k_2$. If $\omega_1 - \omega_2 = \omega_{pe}$, the plasma wave will initially grow linearly in time. If the laser frequencies are much greater than the ambient plasma frequency, ω_{pe} , then the phase velocity of the ponderomotive wave is nearly equal to the group velocity of the laser wave. Electrons which are either injected into the plasma or part of the thermal tail of the plasma distribution can be accelerated by the large gradients associated with the plasma wave.

In this paper the self-focusing properties of a plasma on an intense electromagnetic beam are studied.¹⁵⁻²² The propagation of an intense radiation beam in a plasma can result in self-focusing by creating a density depression in the plasma as well as by increasing the electron mass by relativistic effects. The density depression is due to transverse ponderomotive forces which tend to expel plasma from high field regions. The radial ponderomotive force on the electrons is larger than on the ions by the

The electrons forced out of the radiation beam region set up an electrostatic restoring force which, on a slower time scale, causes the ions to be pulled. The charge separation which sets up the electrostatic restoring force extends over a Debye length which is assumed small compared to the radiation beam radius. This density depression creates a local increase in the effective index of refraction and acts as an optical guide for the radiation beam. In addition to this self-focusing mechanism a further reduction in the plasma frequency occurs in regions of high field intensity due to the relativistic mass increase of the electrons in the presence of the radiation beam. The self-focusing due to the density depression occurs on a longer time scale than does the relativistic mass increase self-focusing effect. When the ponderomotive pressure is much less than the plasma kinetic pressure the time scale for the density depression to occur is $\tau_g = R/C_g$, where R is the radiation beam radius and C_g is the ion acoustic speed. Since R is assumed much greater than the Debye length, the characteristic time is greater than the ion time scale ($\tau_g \gg \omega_{pi}^{-1}$). On the other hand, the time scale for the relativistic mass increase is $\tau_R = \omega^{-1}$ where ω is the radiation frequency and is assumed much greater than the electron plasma frequency.

Previous research on self-focusing was primarily concerned with the ponderomotive self-focusing effect.¹⁵⁻¹⁷ Max¹⁵ presented steady-state solutions for the radiation beam structure due to ponderomotive self-focusing in a nonrelativistic plasma. In this treatment it was assumed that the plasma had attained equilibrium with the radiation field and, hence, the plasma density was represented by a Boltzman-type response, $n = \exp(e\phi_p/T_e)$, where ϕ_p is the ponderomotive potential and T_e is the electron temperature. Felber¹⁶ performed a similar treatment of ponderomotive self-focusing with the inclusion of relativistic effects and, again, the assumption of an equilibrium

Maxwell-type density distribution was used. Such treatments, using an equilibrium density response, are invalid for short radiation pulses, $\tau < \tau_D$. On such short time scales, the plasma ions can not respond to the ponderomotive force.

The problem of relativistic self-focusing alone, valid for short times before the ion density responds to ponderomotive force, has been considered by Max et al.,¹⁸ Sprangle and Tang¹⁹ and Schmidt and Horton.²⁰ Numerical simulations have also been attempted for arbitrary times including the combined effects of relativistic, ponderomotive and thermal self-focusing.^{21,22} This problem of self-focusing in the transition period during which the plasma profile begins to respond to ponderomotive forces and thermal heating will be addressed in a later paper.

The analysis presented here will be concerned only with relativistic self-focusing on a time scale sufficiently short so that the plasma density profile does not evolve significantly under the influence of the radiation beam. This implies that the pulse length of the radiation beam, τ_L , must be short compared to τ_D and, of course, long compared to a radiation period, τ_R .

The relativistic self-focusing effect is analyzed for a helically polarized beam propagating in the z direction. The beam is assumed to be axially symmetric with respect to the z axis and has a profile which is only a function of r, z and t. An equation which describes the envelope of the radiation beam is derived. The envelope equation includes diffraction effects as well as relativistic plasma effects and has a form similar to a particle beam envelope equation.^{23,24} In the present radiation beam envelope equation, diffraction effects are manifested through a term which is equivalent to beam emittance in the particle beam envelope equation.

Mathematical Formulations

Consider a helically polarized radiation beam propagating within a cold collisionless plasma. The vector potential of the field is taken to have the form

$$A_{\perp}(r, z, t) = A(r, z) (\cos(kz - \omega t) \hat{e}_x - \sin(kz - \omega t) \hat{e}_y), \quad (1)$$

where the amplitude $A(r, z)$ is a slowly varying function of r and z , and the frequency ω is assumed to be much greater than the effective plasma frequency. The approximate local dispersion relation associated with the field in (1) is

$$\omega = ck + (c^2 k_{\perp}^2 + \omega_p^2(r, z)) / 2ck, \quad (2)$$

where $k_{\perp} = (k_x^2 + k_y^2)^{1/2}$ is the transverse wave number, $k_{\perp}^2 \ll k^2$ and $\omega_p^2(r, z)$ is the effective background plasma frequency. For purposes of this discussion the effective plasma frequency is written in the form

$$\omega_p^2(r, z) = \frac{n(r, z)}{n_0} \frac{\omega_{p0}^2}{\gamma_{\perp}(r, z)}, \quad (3)$$

where $\omega_{p0} = (4\pi |e|^2 n_0 / m_0)^{1/2}$ is the ambient plasma frequency, $n(r, z)$ is the modified electron density of the plasma due to the excited plasma wave, $\gamma_{\perp}(r, z) = (1 + a^2(r, z))^{1/2}$ is the relativistic mass factor and $a(r, z) = |e|A(r, z) / m_0 c^2$ is the normalized laser field amplitude. The relativistic factor γ_{\perp} arises from the plasma electrons' relativistic mass change due to their transverse oscillations in the radiation field.

Using the ray equations from geometric optics, the transverse motion of the electromagnetic rays are given by

$$\frac{d}{dt} \vec{r}_{\perp} = \frac{\partial \omega}{\partial \vec{k}_{\perp}} \quad (4a)$$

$$\frac{d}{dt} \vec{k}_{\perp} = - \frac{\partial \omega}{\partial \vec{r}_{\perp}}, \quad (4b)$$

where $\vec{r}_{\perp} = \bar{x}(t)\hat{e}_x + \bar{y}(t)\hat{e}_y$ is the transverse position of a ray, $\vec{k}_{\perp} = k_x(t)\hat{e}_x + k_y(t)\hat{e}_y$ is the transverse wave number of a ray and ω is given by (2). Substituting (2) into (4a) and (4b) yields the transverse ray equations

$$\frac{d^2 \bar{x}}{dt^2} + \Omega^2(\bar{r}, \bar{z}) \bar{x} = 0, \quad (5a)$$

$$\frac{d^2 \bar{y}}{dt^2} + \Omega^2(\bar{r}, \bar{z}) \bar{y} = 0, \quad (5b)$$

where

$$\Omega^2(r, z) = \frac{1}{2k_r^2} \frac{\partial \omega_p^2(r, z)}{\partial r}, \quad (5c)$$

and

$$\frac{\partial \omega_p^2}{\partial r} = \omega_{po}^2 \frac{\partial}{\partial r} \frac{n(r, z)/n_o}{(1+a^2(r, z))^{1/2}}. \quad (5d)$$

Equations (5a,b) describe the transverse motion of the rays and assume that the rays travel nearly parallel to the z axis, i.e., $|dx/dt|, |dy/dt| \ll v_g = c$ where v_g is the group velocity.

Derivation of Radiation Beam Envelope Equation

In this section an envelope equation is derived which describes the transverse dynamics of the radiation beam envelope as it propagates through a plasma. The derivation is similar to that used to obtain a particle beam envelope equation²⁴ in the sense that the ray equations in (5a,b) have a form which is similar to particle orbits in the paraxial approximation.

To this end, various moments are taken of the ray equations given by (5). Multiplying (5a) by \bar{x} and $d\bar{x}/dt$, (5b) by \bar{y} and $d\bar{y}/dt$ and combining, yields the following virial and energy equations

$$\frac{1}{2} \frac{d^2 \bar{r}^2}{dt^2} - \bar{v}^2 + \Omega^2(\bar{r}, \bar{z}) \bar{r}^2 = 0, \quad (6a)$$

$$\frac{d\bar{v}^2}{dt^2} + \Omega^2(\bar{r}, \bar{z}) \frac{d\bar{r}^2}{dt} = 0, \quad (6b)$$

where $\bar{r}^2 = \bar{x}^2 + \bar{y}^2$ and $\bar{v}^2 = (d\bar{x}/dt)^2 + (d\bar{y}/dt)^2$. Substituting (6a) into (6b) yields

$$\frac{1}{2} \frac{d^3 \bar{r}^2}{dt^3} + \frac{d}{dt} (\Omega^2 \bar{r}^2) + \Omega^2 \frac{d\bar{r}^2}{dt} = 0. \quad (7)$$

To obtain the envelope equation, consider a thin slice of the beam lying in the transverse. The transverse segment of the beam is assumed to consist of N individual rays traveling through it. The mean square radius of the radiation beam envelope is then defined as

$$R^2(\bar{z}, t) = \langle \bar{r}^2 \rangle = \frac{1}{N} \sum_{i=1}^N \bar{r}_i^2 \quad (8)$$

where \bar{r}_i is the radial position of the i th ray and $\langle \quad \rangle$ denotes the averaging operation. Mathematically, this averaging can be written in terms of an integral over the distribution function of initial ray positions at the initial time $t_0, f(\bar{r}_0, t_0)$.

$$\langle Q(\bar{r}) \rangle = \int_0^{\infty} d\bar{r}_0 \bar{r}_0 f(\bar{r}_0, t_0) Q(\bar{r}), \quad (9)$$

where $Q(\bar{r})$ is a quantity dependent on the ray position $\bar{r} = \bar{r}(\bar{r}_0, t)$.

Performing this average over all the rays on (7) and using (8) gives

$$\frac{1}{2} \frac{d^3 R^2}{dt^3} + \frac{d}{dt} \langle \Omega^2 \bar{r}^2 \rangle + \langle \Omega^2 \frac{d\bar{r}^2}{dt} \rangle = 0. \quad (10)$$

In order to express the averages in (10) in a more manageable form, the individual radial ray velocity is written as a sum of a mean velocity term and a residual velocity term. That is the individual ray velocity is written as

$$\bar{v} = \frac{\bar{r}}{R} \frac{dR}{dt} + \delta \bar{v}, \quad (11)$$

where $(\bar{r}/R)dR/dt$ is defined as the mean radial velocity at the ray position \bar{r} and $\delta \bar{v}_r$ is the residual radial velocity. Substituting (11) into the last term in (10) yields

$$\langle \Omega^2 \frac{d\bar{r}^2}{dt} \rangle = 2 \langle \Omega^2 \bar{r}^2 \rangle \frac{dR/dt}{R} + 2 \langle \Omega^2 \bar{r} \delta \bar{v}_r \rangle. \quad (12)$$

To further simplify (12), notice that

$$\begin{aligned} \frac{1}{2} \frac{d}{dt} R^2 &= \frac{1}{2} \frac{d}{dt} \langle r^{-2} \rangle \\ &= \langle \dot{r} \rangle = R \frac{dR}{dt} + \langle \dot{r} \delta \dot{v} \rangle. \end{aligned} \quad (13)$$

Hence, $\langle \dot{r} \delta \dot{v} \rangle = 0$. Similarly, assuming $\langle r^{-n} \rangle = c_n R^n$ where c_n is a constant, then

$$\begin{aligned} \frac{1}{n} \frac{d}{dt} R^n &= \frac{1}{n} \frac{d}{dt} \langle r^{-n} \rangle \\ &= \langle r^{-n-1} \dot{v} \rangle = c_n R^{n-1} \frac{dR}{dt} + \langle r^{-n-1} \delta \dot{v} \rangle. \end{aligned} \quad (14)$$

Hence, the conclusion is reached that $\langle r^{-m} \delta \dot{v} \rangle = 0$ for any positive integer m .

Assuming that $\Omega^2(r)$ can be expanded in a power series of r implies that $\langle \Omega^2 r \delta \dot{v} \rangle = 0$. Thus, (12) becomes

$$\langle \Omega^2 \frac{dr^{-2}}{dt} \rangle = 2 \langle \Omega^2 r^{-2} \rangle \frac{1}{R} \frac{dR}{dt}. \quad (15)$$

Substituting (15) into (10) yields

$$\frac{1}{2} \frac{d^3 R^2}{dt^3} + \frac{d}{dt} \langle \Omega^2 r^{-2} \rangle + 2 \langle \Omega^2 r^{-2} \rangle \frac{dR/dt}{R} = 0. \quad (16)$$

Since the last two terms in (16) combine to give $R^{-2} d(R^2 \langle \Omega^2 r^{-2} \rangle) / dt$, (16) can be written as

$$\frac{d}{dt} \left[R^2 \frac{d^2 R^2}{dt^2} - \frac{1}{2} \left(\frac{dR^2}{dt} \right)^2 + 2R^2 \langle \Omega^2 r^{-2} \rangle \right] = 0. \quad (17)$$

Integrating (17) and taking the constant of integration to be $2c^2 \epsilon^2$ gives

$$\frac{d^2 R}{dz^2} + \frac{\langle K^2 r^2 \rangle}{R} - \frac{\epsilon^2}{R^3} = 0, \quad (18)$$

where $K^2 = \Omega^2/c^2$ and $z = ct$.

Equation (18) describes the evolution of the radiation beam envelope, R , as function of z . Diffraction effects which result in a spreading of the beam are contained in the constant of integration term ϵ . The form of the envelope equation for the radiation beam, (18), is similar to the usual particle beam envelope equation. The second term in (18) can be either focusing or defocusing depending on the sign of Ω^2 . In the present situation, because of the relativistic transverse oscillations of the background plasma electrons, this term results in focusing of the radiation beam. The third term in (18) is always a defocusing term and is due to diffraction effects.

The diffraction constant ϵ^2 can be evaluated for the special case of a Gaussian ray distribution. Using the definition in (9) and requiring $\langle 1 \rangle = 1$ and $\langle r^2 \rangle = R^2$ then implies

$$f(\bar{r}_0, t_0) = \frac{2}{R^2} e^{-\bar{r}_0^2/R^2}. \quad (19)$$

Since the density of rays is proportional to the intensity of the radiation field, this implies that $f(\bar{r}_0, t_0)$ is proportional to $a^2(\bar{r}_0, t)$. In standard notation, the vector potential of a Gaussian radiation field is written as

$$a(r, z) = \frac{a_0 R_{s0}}{R_s(z)} e^{-r^2/R_s^2}, \quad (20)$$

where $R_s(z)$ is the spot size, R_{s0} the minimum spot size and a_0 is the normalized peak radiation amplitude. In a vacuum, $R_s(z)$ is given by

$$R_s(z) = R_{s0} \left[1 + \frac{(z-z_0)^2}{z_R^2} \right]^{1/2}, \quad (21)$$

where $z_R = kR_{s0}^2/2$ is the Rayleigh length²⁵ and z_0 is the axial location of the minimum spot size.

Since $f(\vec{r}_0, t_0)$ is proportional to $a^2(\vec{r}_0, t)$, then by comparing (19) and (20) implies that the mean-square radial ray position is related to the spot size by $R^2 = R_s^2/2$. Rewriting the envelope equation (18) in terms of the spot size R_s gives

$$\frac{d^2 R_s}{dz^2} + \frac{2\langle \vec{r}^2 K^2 \rangle}{R_s} - \frac{\epsilon_0}{R_s^3} = 0. \quad (22)$$

The integration constant ϵ_0 can now be evaluated by using the vacuum solution (21). In vacuum, $\langle \vec{r}^2 K^2 \rangle = 0$ and, hence, $\epsilon_0^2 = 4/k^2$. Notice that $\epsilon_0 = \lambda/\pi = R_{s0} \theta_d$, where $\lambda = 2\pi/k$ and $\theta_d = \lambda/(\pi R_{s0})$ is the well-known diffraction angle.²⁵

Solution of Envelope Equation

In order to solve the envelope equation (22) it is necessary first to evaluate the average of the quantity $\vec{r}^2 K^2$, where

$$\vec{r}^2 K^2 = \frac{\omega_{p0}^2}{2c^2 k^2} \vec{r} \frac{\partial}{\partial \vec{r}} \frac{n(\vec{r}, \vec{z})/n_0}{(1 + a^2(\vec{r}, \vec{z}))^{1/2}}, \quad (23)$$

where $\vec{r} = \vec{r}(t_0, t)$ and $\vec{z} = ct + \vec{z}_0$.

Recall that the averaging can be written in terms of the distribution of initial ray positions $f(\vec{r}_0, t_0)$ as in (9). Notice that it is possible to define a Eulerian ray distribution function at position r and time t as follows:

$$f(r,t) = \int_0^{\infty} d\vec{r}_0 f(\vec{r}_0, t) \delta(r - \vec{r}(\vec{r}_0, t)). \quad (24)$$

Consider now the average of the quantity $Q(r)$ defined by

$$\begin{aligned} \langle Q(r) \rangle &= \int_0^{\infty} dr r f(r,t) Q(r) \\ &= \int_0^{\infty} dr r \int_0^{\infty} d\vec{r}_0 f(\vec{r}_0, t) \delta(r - \vec{r}(\vec{r}_0, t)) Q(r) \\ &= \int_0^{\infty} d\vec{r}_0 \vec{r}_0 f(\vec{r}_0, t) Q(\vec{r}) \\ &= \langle Q(\vec{r}) \rangle. \end{aligned} \quad (25)$$

Hence, the averaging $\langle Q(\vec{r}) \rangle$ can be replaced by an integration over the Eulerian spatial coordinate r providing the distribution $f(r,t)$ is known.

Since there is no hope in determining an analytic expression for the distribution $f(r,t)$, in order to obtain an estimate of $\langle r^2 k^2 \rangle$, it will be assumed that the radiation beam retains a Gaussian profile as it propagates through the plasma. Thus, $f(r,t)$ will be approximated as

$$f(r,z) = \frac{4}{a_0^2 R_{s0}^2} a^2(r,z), \quad (26)$$

where $z = ct + z_0$ and $a(r,z)$ is given by (20). Notice that $\langle 1 \rangle = 1$ and $\langle r^2 \rangle = R_s^2/2$. Using this expression, then $\langle r^2 k^2 \rangle$ becomes

$$\begin{aligned} \langle r^2 k^2 \rangle &= 2 \left(\frac{\omega_{p0}}{ck a_0 R_{s0}} \right)^2 \int_0^{\infty} dr r^2 a^2(r,z) \frac{\partial}{\partial r} \frac{n(r,z)/n_0}{(1+a^2(r,z))^{1/2}} \\ &= - \left(\frac{\omega_{p0}}{ck} \right)^2 \int_0^{\infty} ds (1-s) e^{-s} \frac{n(s)/n_0}{(1+x^{-2} e^{-s})^{1/2}}, \end{aligned} \quad (27)$$

where $s = R^2/R_s^2$ and $x = R_s/(R_{so} a_o)$.

For the case of a constant density background plasma, $n(r) = n_o$ for $r < R_p$ and $n(r) = 0$ for $r > R_p$, then (27) can be evaluated analytically. For this case, letting $y = e^{-s}$, then

$$\begin{aligned} \langle r^2 k^2 \rangle &= \frac{4\omega_{po}^2}{c^2 k^2} \int_1^{y_p} \frac{dy(1+\ln y)}{(1+x^{-2}y)^{1/2}} \\ &= \frac{4\omega_{po}^2}{c^2 k^2} x^2 \left\{ h(x) - g(x)(1-\ln y_p) - \ln\left[\frac{1+h}{1+g}\right] \right\}, \end{aligned} \quad (28)$$

where $h = [(1+x^{-2})^{1/2}-1]/2$, $g = [(1+x^{-2}y_p)^{1/2}-1]/2$ and $y_p = \exp(-2R_p^2/R_s^2)$.

It is insightful to write the envelope equation in terms of an effective particle located at $R_s(z)$ moving in a potential $V(R_s)$. Equation (22) becomes

$$\frac{d^2 R_s}{dz^2} = - \frac{\partial V(R_s)}{\partial R_s}, \quad (29)$$

where

$$\frac{\partial V}{\partial R_s} = \frac{8\omega_{po}^2}{c^2 k^2} \frac{R_s}{a_o^2 R_{so}^2} \left[h(x) - g(x)(1-\ln y_p) - \ln\left(\frac{1+h}{1+g}\right) \right] - \frac{4}{k^2 R_s^3}, \quad (30)$$

with $x = R_s/(a_o R_{so})$ and $y_p = \exp(-2R_p^2/R_s^2)$.

Envelope Behavior

To analyze the behavior of the radiation envelope, it is most convenient to use the normalized envelope radius $x = R_s/(a_o R_{so})$. In the limit $R_p \rightarrow \infty$, then (30) can be written as

$$\frac{d^2 x}{dt^2} = - v_o \frac{\partial}{\partial x} v, \quad (31)$$

where

$$-\frac{\partial}{\partial x} V = x^{-3} - 16\alpha x \left[\left((1 + x^{-2})^{1/2} - 1 \right) + 2 \ln 2 \right. \\ \left. - 2 \ln \left((1 + x^{-2})^{1/2} + 1 \right) \right], \quad (32)$$

with

$$V_0 = \left(2c / (k R_{s0}^2 a_0^2) \right)^2 \text{ and } \alpha = \left(\omega_{p0} a_0 R_{s0} / (4c) \right)^2.$$

The above equation describes the position of a particle $x(t)$ moving in the effective potential $V(x, \alpha)$. In the expression for $\partial V / \partial x$, the first term on the right of (32) represents vacuum diffraction whereas the term proportional to α represents the relativistic self-focusing of the plasma.

It is interesting to note that the shape of the potential $V(x)$ depends only on the parameter α . As will be shown below, α can be written as the laser power over the critical power. For $\alpha > 1$, the potential $V(x)$ has a minimum and bounded oscillatory solutions for $x(t)$ are possible.

Expanding $V(x)$ for large x (small a_0) gives

$$\frac{\partial V}{\partial x} = x^{-3} - \alpha x^{-3}. \quad (33)$$

For $\alpha = 1$, there are no net diffraction or focusing forces (to leading order) and a matched radiation beam is possible. In this limit ($x \gg 1$), the envelope diffracts for $\alpha < 1$ and focuses for $\alpha > 1$. Since the total radiation power is given by $P = (m_e c^2 \omega a_0 R_{s0})^2 / (8c^2)$, the parameter α can be written as $\alpha = P / P_{\text{crit}}$, where the critical power is given by

$$P_{\text{crit}} = 2c \left(\frac{m_e c^2}{e} \right)^2 \frac{\omega^2}{\omega_{p0}^2} = 17.4 \times 10^9 \frac{\omega^2}{\omega_{p0}^2} \text{ Watts.} \quad (34)$$

Equation (32), $a/v_{pp} = 20$ gives $P_{crit} = 6 \times 10^{12} W$.

Equation (33) indicates that for $x \gg 1$, and $\alpha > 1$, then the radiation will focus until $x = 1$ and (33) is no longer valid. Expanding (32) for small $x \ll 1$ gives

$$-\frac{\partial V}{\partial x} = x^{-3} - 16\alpha. \quad (35)$$

Equation (35) states that for a fixed $\alpha > 1$, focusing will continue until the diffraction term dominates when $x^3 < 1/(16\alpha)$. Hence, at a sufficiently small x , the envelope will be reflected back out towards its original width. When this occurs the envelope radius x will either oscillate between its minimum value at reflection and its original value, or it will continue to diffract indefinitely, depending on the envelope's initial "velocity" (the initial slope in $x(t)$). For a given $\alpha > 1$ and an initial $x \gg 1$, if the initial slope dx/dt is sufficiently small, then $x(t)$ will oscillate between its initial value and its value at reflection. If dx/dt is initially large, $x(t)$ will initially decrease to its minimum value at reflection and then increase indefinitely.

The exact shape of $V(x)$ for a given α must be determined numerically. In general, if $\alpha > 1$ then there will exist a finite well whose minimum occurs at x_f . As α increases, the depth of this well increases, the well becomes narrower and the location of the minimum x_f decreases. The potential $V(x)$ is plotted in Fig. 1 for $\alpha = 5.0$ and Fig. 2 for $\alpha = 1.2$. Notice that $x_f = 0.4$ for $\alpha = 5.0$ and $x_f = 1.8$ for $\alpha = 1.2$.

Recall from the vacuum solution that the minimum of R_s/R_{s0} in vacuum is unity. Hence, the minimum for x in vacuum is $x = 1/a_0$. Ideally, it is possible to have a nonoscillating envelope of constant radius if the radiation

enters the plasma at $t = t_0$ with $x(t_0) = x_f$ and $dx/dt = 0$. This corresponds to the radiation entering the plasma precisely at the bottom of the potential well with zero velocity. In vacuum, $dx/dt = 0$ at the minimum spot size $R_s = R_{s0}$ and, hence, $x = 1/a_0$. In order to achieve a nonoscillating solution, it is required that $x(t_0) = x_f = 1/a_0$. Hence, for $\alpha = 5.0$ ($\alpha = 1.2$) a nonoscillating matched beam would be possible for $a_0 = 2.5$ ($a_0 = 0.55$) provided the radiation envelope enters the plasma with zero slope, $dR_s/dz = 0$. Since current technology limits $a_0 \leq 1$, operation in this constant envelope mode is possible only for α slightly greater than unity.

In practice, it may not be possible to achieve a radiation beam which enters a plasma with $dR_s/dz = 0$ and $x = x_f$. However, it may be possible to operate near $x = x_f$ with a small initial dx/dt . In this case, when $x = x_f + \delta x$ with δx small, the envelope equation (31) can be expanded to give

$$\frac{d^2}{dt^2} \delta x = -\omega_{os}^2 \delta x, \quad (36)$$

where

$$\omega_{os}^2 = -\frac{4V_0}{x_f} \left[1 - 4\alpha x_f^3 \frac{(\sqrt{1+x_f^2} - x_f)}{(1+x_f^2 + x_f\sqrt{1+x_f^2})} \right]. \quad (37)$$

For the parameters $\alpha = 1.2$, $a_0 = 0.55$, $R_{s0} = 0.1$ cm and $\omega/\omega_{p0} = 10$, this gives $x_f = 1.8$, $\lambda_{os} = 2\pi c/\omega_{os} = 43.5$ cm and $\omega_{os}^2/\omega_{p0}^2 = 3 \times 10^{-6}$. The Rayleigh length for this case is $Z_R = 4.0$ cm.

Figures 3-6 show numerical results for the envelope behavior for a test case where $\alpha = 1.2$, $R_{s0} = 0.1$ cm and $\omega/\omega_{p0} = 10.0$. The critical power is given by $P_{crit} = 17 \times 10^{11}$ W, the well minimum occurs at $x_f = 1.8$ and the oscillating wavelength in the well is $\lambda_{os} = 44$ cm. Figure 3 shows operation at $a_0 = 1/x_f = 0.55$ and with zero initial slope, $dR_s/dz = 0$. This is the condition for a nonoscillating matched beam. Here $P = 2.1 \times 10^{12}$ W,

$\omega_{po} = 2.4 \times 10^{12} \text{sec}^{-1}$, $\omega = 2.4 \times 10^{13} \text{sec}^{-1}$ and the Rayleigh length is $z_R = 4.0 \text{ cm}$. Figure 4 shows a case with a_0 slightly less than $1/x_f = 0.55$ with zero initial slope, while Fig. 5 shows a case with $a_0 = 0.55$ and a small initial slope. In both these cases, the envelope shows a small amplitude oscillation whose wavelength is given by $\lambda = \lambda_{os} = 44 \text{ cm}$. Figure 6 presents a case where $a_0 \ll 0.55$ (initially, $\hat{x} \gg 1$) and with a significant initial slope. Here the initial slope is determined by specifying z_0 where $R_s(z) = R_{s0}(1 + z_0^2/z_R^2)^{1/2}$ before entering the plasma at $z = 0$. For this case, the radiation initially focuses down to a value significantly less than R_{s0} ("over-focusing") and then is reflected and diffracts indefinitely.

When larger laser powers are used (larger values of α) the results are qualitatively similar except the potential well is now deeper, narrower and has a minimum occurring at a smaller value of x_f . For example, when $\alpha = 5$, $R_{s0} = .1 \text{ cm}$ and $\omega/\omega_{po} = 10$, then $x_f = 0.41$ and $\lambda_{os} = 8.9 \text{ cm}$. A nonoscillating matched beam requires $a_0 = 2.5$ and no initial slope. For $a_0 = 2.5$, this gives $P_{crit} = 1.7 \times 10^{12} \text{ W}$, $P = 8.7 \times 10^{12} \text{ W}$, $\omega_{po} = 1.1 \times 10^{12} \text{ sec}^{-1}$, $\omega = 1.1 \times 10^{13} \text{ sec}^{-1}$ and $z_R = 1.8 \text{ cm}$.

Numerical Simulation

In order to confirm the above results, the wave equation for the radiation is solved numerically. For a plasma with constant density, and a driving current that results from the relativistic electron oscillations, the wave equation can be written as

$$\left[\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + 2i \frac{\omega}{c} \frac{\partial}{\partial z} \right] a(r, z) = \frac{\omega_{po}^2}{c^2} \frac{a(r, z)}{(1+a^2)^{1/2}}, \quad (38)$$

where the slowly varying amplitude approximation was used. This equation is solved numerically with 64×64 transverse Fourier modes.

The initial transverse radiation profile was a Gaussian with zero wavefront curvature. As the radiation propagated through the plasma, the profile developed finite but small amplitude "skirts" indicating higher order transverse Gaussian modes are involved. The $1/e$ radius (at which the amplitude is $1/e$ that on axis) is chosen to be the measure of the radiation envelope. This is plotted as a function of z in Fig. 7 for the case when the initial normalized envelope $x = R_s / (R_{s0} a_0)$ is somewhat greater than that needed for a matched beam. Figure 7 indicates that the envelope oscillates about the predicted matched beam radius. The oscillatory behavior is no longer simply harmonic, as implied by the above theory, and the oscillation wavelength is approximately twice the predicted value. Notice that after the initial transient, the maximum of the envelope oscillation is only about $2/3$ of the initial radius of the radiation. This discrepancy is probably due to the fact that the profile deviates from its initially Gaussian shape. For the case where the value of the initial normalized envelope x approaches that required for a matched beam, the oscillation amplitude becomes very small and the oscillatory behavior becomes more erratic.

In general, the numerical simulations support the conclusion that the radiation envelope oscillates about the value required for matched beam propagation.

Conclusions

The above results indicate that relativistic electron quiver motion leads to enhanced focusing of radiation beams. Physically, relativistic quivering of the electrons leads to an effective decrease in the plasma density via the electron plasma frequency, $\omega_{pe}^2 = n(r,z)\omega_{p0}^2/(n_0\gamma_{\perp})$ where $\gamma_{\perp}^2 = 1 + a^2(r,z)$. A radiation field $a^2(r,z)$ peaked on axis at $r = 0$ will produce an index of refraction profile $\eta(r,z)$ peaked on axis such that $\partial\eta/\partial r < 0$, where $\eta = ck/\omega = (1 - \omega_p^2(r,z)/\omega^2)^{1/2}$. Since $\partial\eta/\partial r < 0$, this implies focusing of the radiation beam.

The effects of relativistic self-focusing on the development of the radiation envelope $R_g(z)$ are best understood through the analogy of a single particle with orbit $R_g(z)$ moving in an effective potential $V(R_g)$. As discussed above, the shape of the potential V is determined by a single parameter $\alpha = P/P_{crit}$, where the critical power is given by $P_{crit} = 17 \times 10^9 (\omega/\omega_{p0})^2$ Watts. Provided $\alpha > 1$ (laser power greater than the critical power), then there exists a minimum in the effective potential V located at $x_f = x_f(\alpha)$, where $x = R_g/(R_{g0}a_0)$. As α is increased, the well depth increases and becomes narrower, and the location of the minimum x_f decreases. The existence of such a well implies that bounded solutions for $R_g(z)$ are possible where $R_g(z)$ oscillates between the two reflection points associated with the effective potential. For example, if $\alpha > 1$, a radiation beam entering the plasma with an initial value of x greater than x_f will initially begin to focus (x decreases). When this occurs, two outcomes are possible: i) the normalized envelope $x(z)$ will continue to decrease to some minimum value at which it will be reflected and expand indefinitely, or ii) the envelope $x(z)$ will remain bounded, oscillating indefinitely between the minimum reflection point and its initial value. The occurrence of one or the other of the above two cases depends on the initial "convergence angle" of the

envelope $x(z)$ as it enters the plasma or, more precisely, on the initial slope dR_g/dz . If dR_g/dz is too large then case i) occurs: the envelope is reflected and expands indefinitely. If the initial dR_g/dz is sufficiently small, then case ii) occurs: the envelope becomes trapped in the potential well and oscillates about the minimum x_p . Under a special set of initial conditions, $dR_g/dz = 0$ and $x = x_p$, it is possible to have a matched beam with a constant envelope.

The following approximations were made in this analysis: the mathematical model used in the above calculations was based on the ray equations of geometrical optics. These ray equations then used a dispersion relation for an electromagnetic wave in an unmagnetized plasma in which it was assumed $k^2 \gg k_{\perp}^2$ and $\omega^2 \gg \omega_p^2$. The radiation beam was initially assumed to be Gaussian and, furthermore, it was assumed to remain Gaussian as it propagated through the plasma, $a(r,z) = (a_0 R_{s0}/R_g) \exp(-r^2/R_g^2(z))$. The last approximation, which restricts the analysis to short pulses, assumes that the background plasma density remains constant and does not evolve under the influence of the transverse ponderomotive force. Such an approximation should be valid for short pulse times $\tau < R_0/C_s$.

The results discussed above are in qualitative disagreement with those of Schmidt and Horton,²⁰ who also analyzed relativistic self-focusing for short pulses. They found a similar expression for P_{crit} , beyond which self-focusing occurs. They claimed, however, that for $P > P_{crit}$ the radiation beam would collapse down to some minimum radius and neither oscillate nor reflect. This is similar to the results of Max et al.,¹⁸ in which it was determined that self-focusing occurs when a certain threshold is surpassed. Their analysis, however, did not predict the envelope behavior once self-focusing occurs. The above numerical simulation of the wave equation indicates that the correct

envelope behavior is an oscillation about the value required for a matched beam.

The results of Felber,¹⁶ which included relativistic focusing along with that of the equilibrium ponderomotive effect, are in qualitative agreement with those presented above. Specifically, Felber¹⁶ found that the behavior of the radiation envelope could also be described as a particle in an effective potential. Provided the laser power was sufficiently high, the potential exhibited a minimum and, hence, either oscillating, bounded solutions or reflected, diverging solutions were possible. The critical power in Felber's case was similar to that found by Max,¹⁵ $P_{crit} = 2 \times 10^4 T(\text{ev}) \omega^2 / \omega_p^2$ Watts, which is typically less than that discussed above when the effects of ponderomotive focusing are neglected. The main limitation with the analysis of Felber¹⁶ and Max¹⁵ is that both used an equilibrium density response, $n = \exp(e\phi_p/T)$ where ϕ_p is the ponderomotive potential. Such an equilibrium density response is invalid for the leading segment of the radiation pulse.

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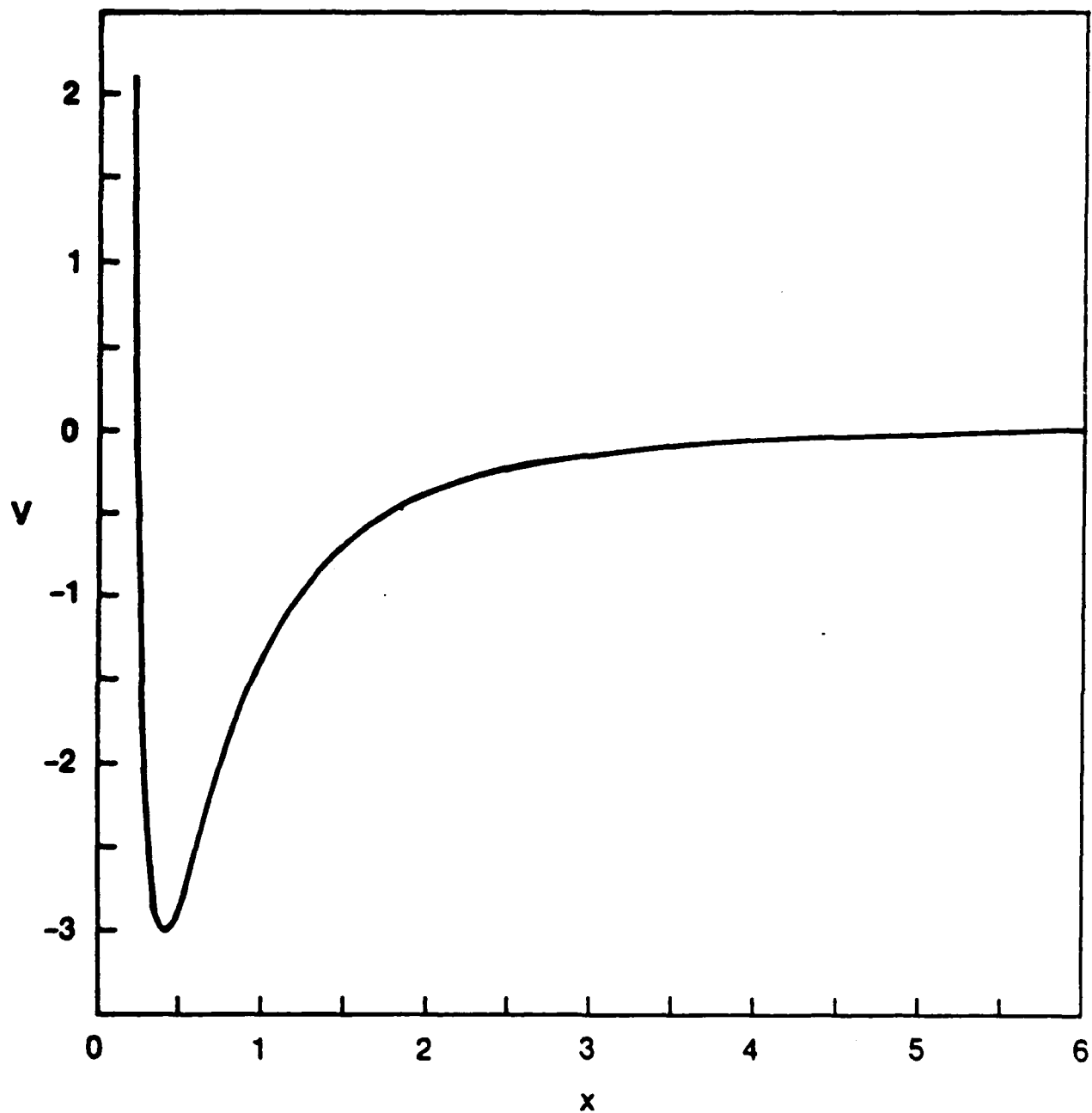


Figure 1. The effective potential $V(x)$ as a function $x = R_s/(R_{s0} a_0)$ for $\alpha = 5.0$. The well minimum occurs at $x_f = 0.4$.

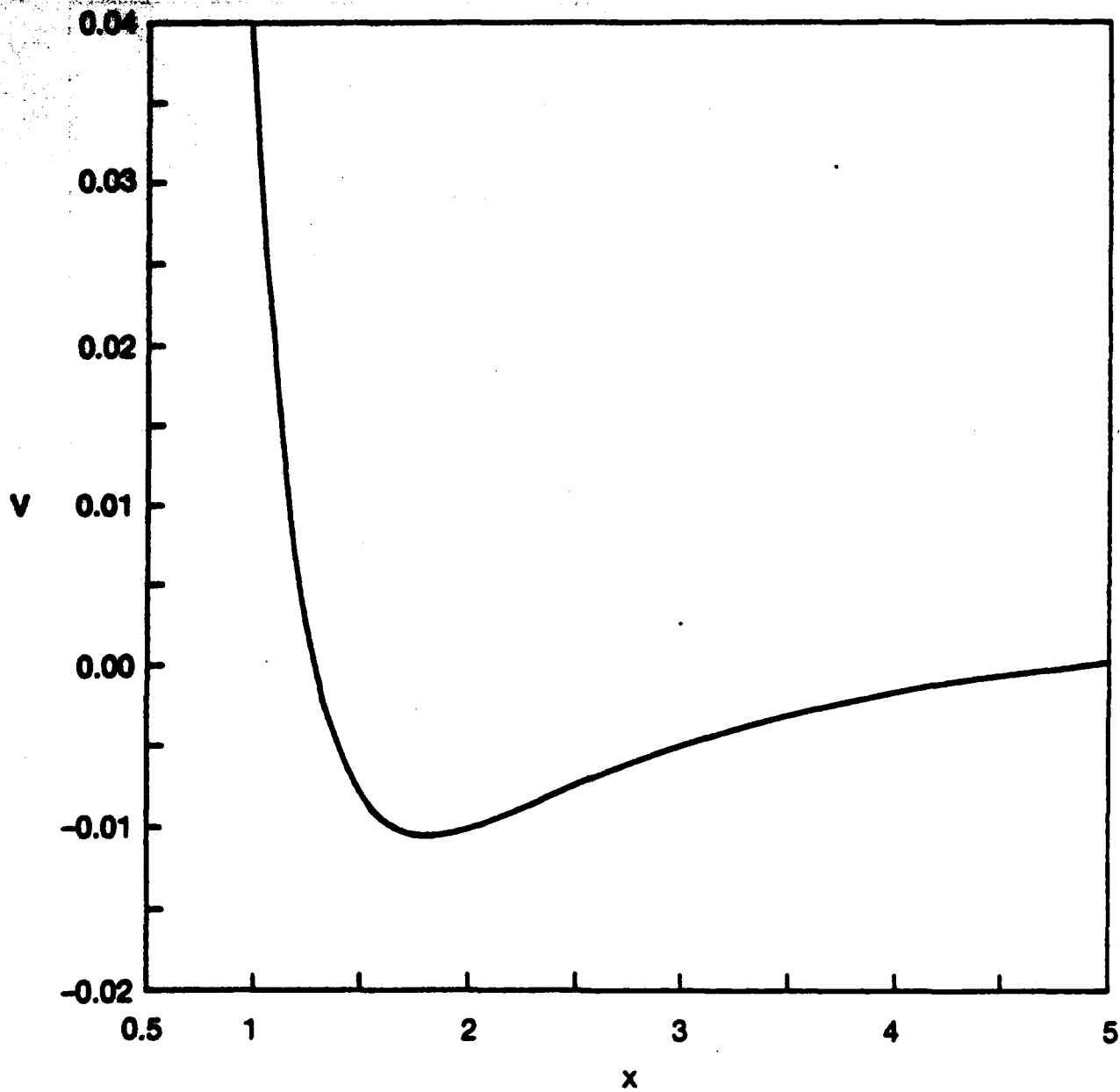


Figure 2. The effective potential $V(x)$ as a function of $x = R_s / (R_{s0} a_0)$ for $\alpha = 1.2$. The well minimum occurs at $x_f = 1.8$.

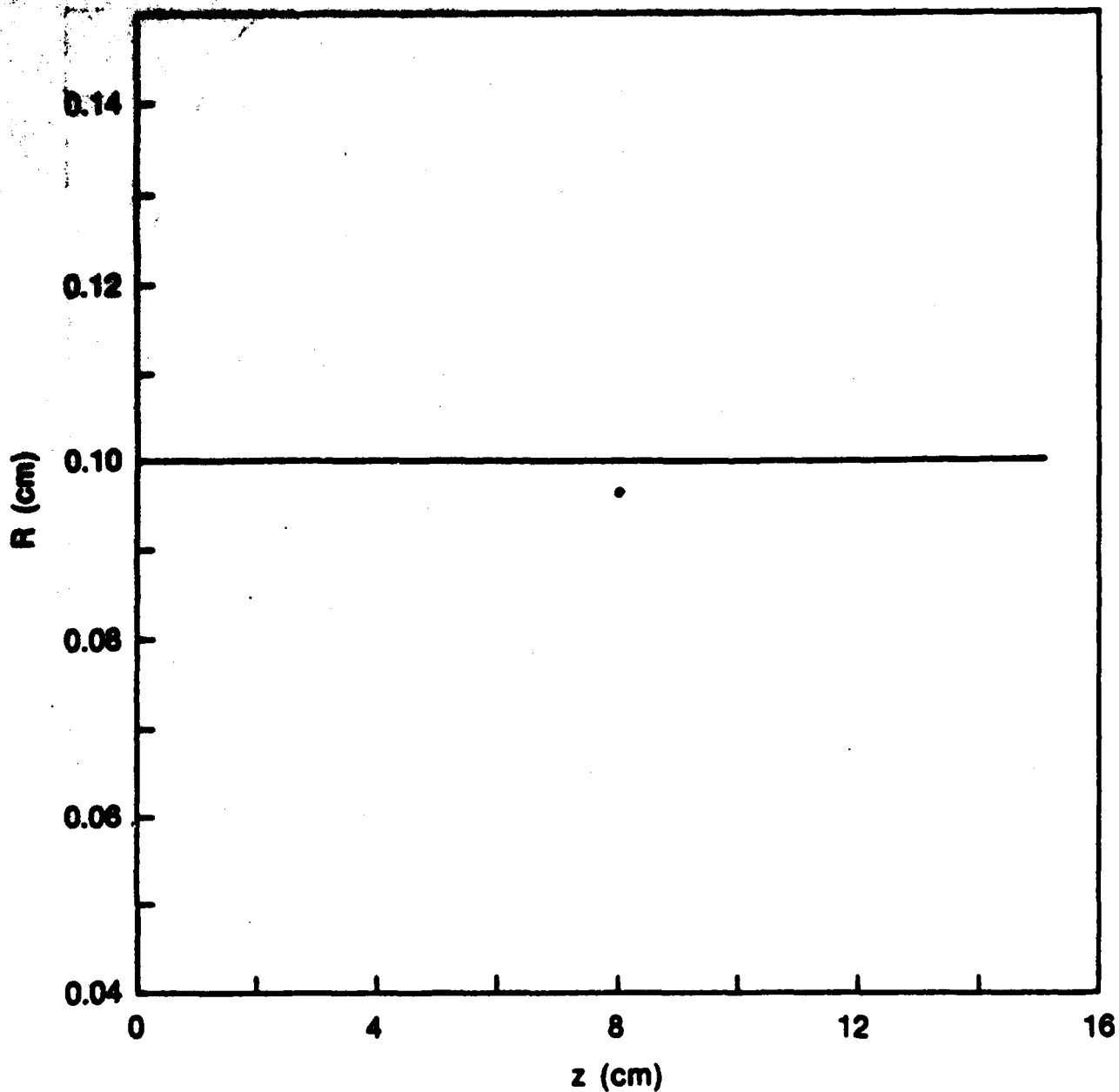


Figure 3. The radiation envelope $R_g(z)$ for the parameters $\alpha = 1.2$, $R_{so} = 0.1$ cm, $\omega/\omega_{po} = 10.0$ and $a_0 = 0.55$. For this run $P_{crit} = 17 \times 10^{11}$ W and $\omega = 2.4 \times 10^{13}$ sec⁻¹. Initially, $x = x_p = 1.8$ and $dx/dz = 0$.

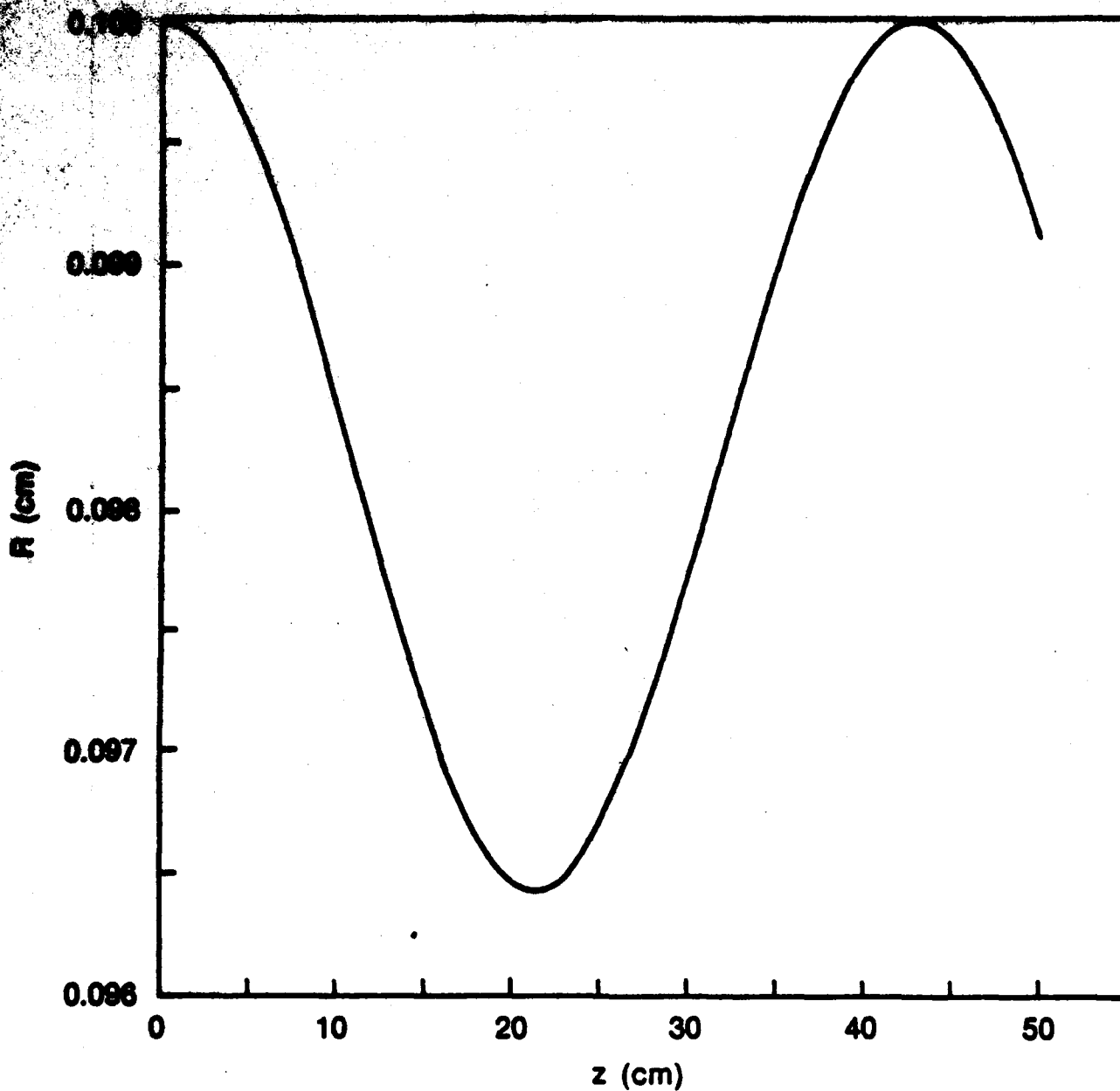


Figure 4. The radiation envelope $R_g(z)$ for the parameters $a = 1.2$, $R_{g0} = 0.1$ cm, $\omega/\omega_{p0} = 10.0$ and $a_0 = 0.54$. For this run $P_{\text{orit}} = 17 \times 10^{11}$ W, $\omega = 2.4 \times 10^{13}$ sec $^{-1}$, $z_R = 4.1$ cm and $\lambda_{\text{os}} = 44$ cm. Initially, $x = 1.02 x_f$ and $dx/dz = 0$.

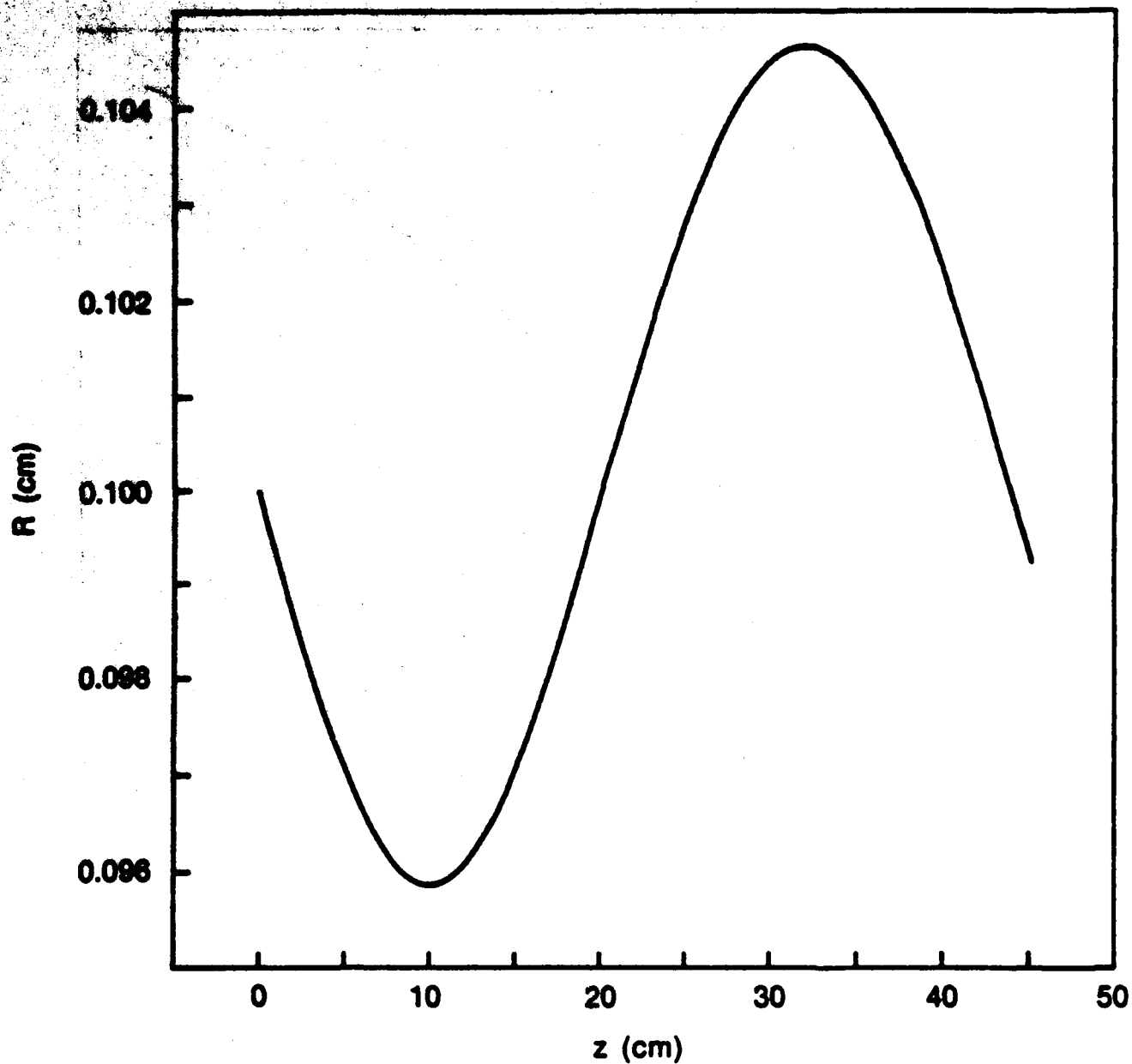


Figure 5. The radiation envelope $R_s(z)$ for the parameters, $\alpha = 1.2$, $R_{s0} = 0.1$ cm, $\omega/\omega_{p0} = 10.0$ and $a_0 = 0.55$. For this run $P_{\text{crit}} = 17 \times 10^{11}$ W, $\omega = 2.4 \times 10^{13}$ sec $^{-1}$, $z_R = 4.0$ cm and $\lambda_{0s} = 44$ cm. Initially, $x = x_f = 1.8$ with a small slope $dx/dz = 0$ ($dR_s/dz = R_{s0} z_0 / z_R^2$, where $z_0 = 0.1$ cm).

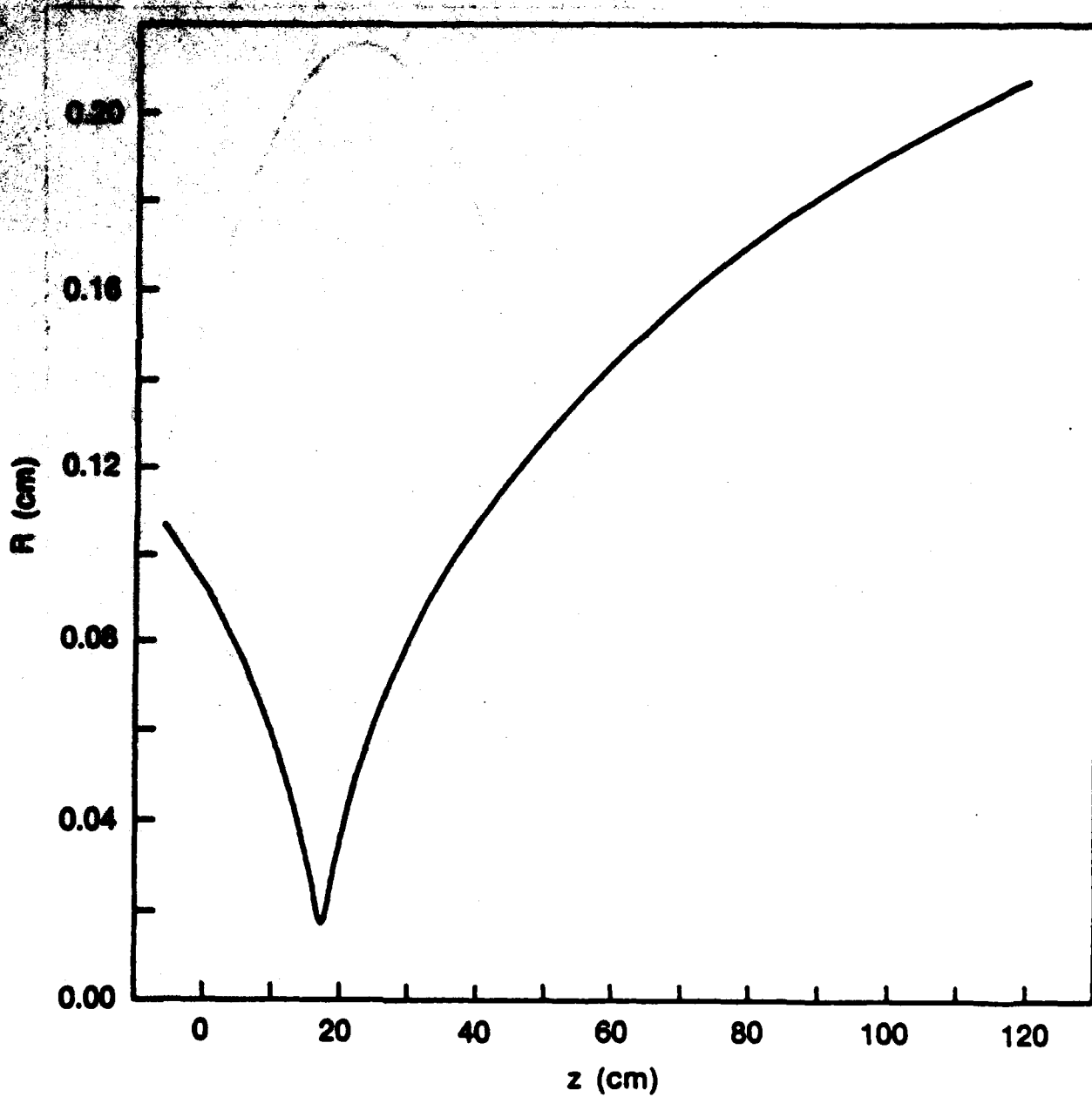


Figure 6. The radiation envelope $R_s(z)$ for the parameters $\alpha = 1.2$, $R_{s0} = 0.1$ cm, $\omega/\omega_{p0} = 10.0$ and $a_0 = 0.14$. For this run $P_{crit} = 17 \times 10^{11}$ W, $\omega = 9.4 \times 10^{13} \text{sec}^{-1}$ and $z_R = 16.0$ cm. Initially, $R_s = R_{s0}(1 + z_0^2/z_R^2)^{1/2}$ with $z_0/z_R = 1/3$.

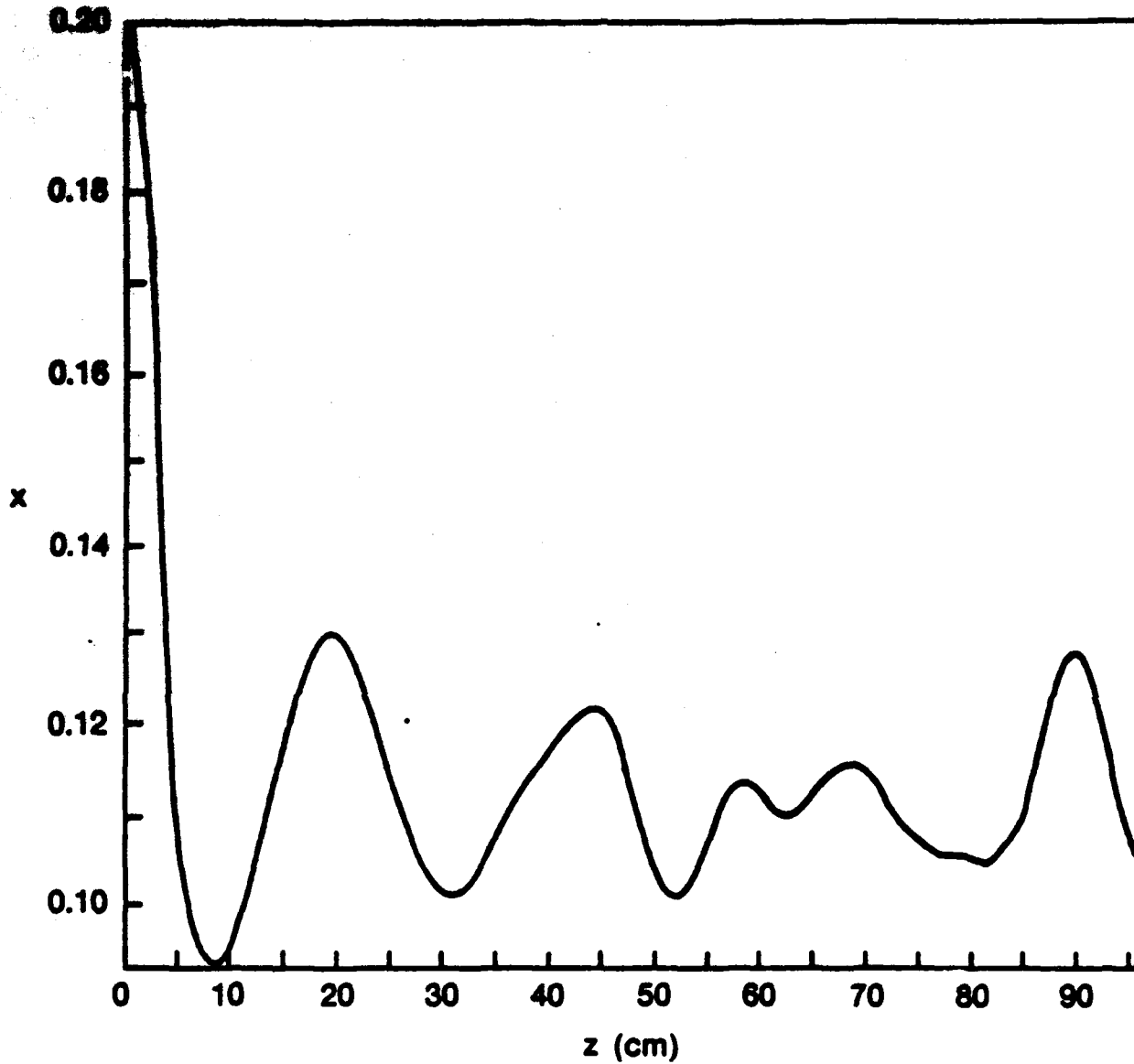


Figure 7. The $1/e$ radial width in terms of $x = R_s / (R_{s0} a_0)$ as a function of z (in cm) as determined by numerical simulation of Eq. (38). For this run, $\alpha = 5$, $\omega/\omega_{p0} = 10$, $a_0 = 2.21$ and $R_{s0} = 0.11$ cm.

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