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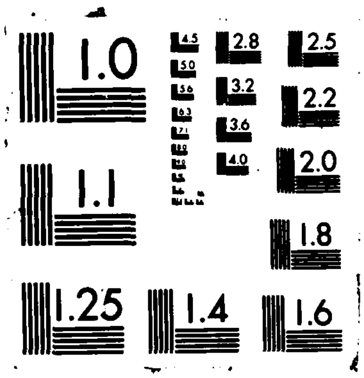
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A STUDY ON MODULATION/DEMODULATION FOR A SATELLITE ANALOG TELEMETRY SYSTEM

by

Chen Yiyuan

HUMAN TRANSLATION

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A Study on Modulation/Demodulation for a Satellite Analog Telemetry System /61

Chen Yiyuan

Abstract

The analog telemetry subcarrier channel for satellites employs the FSK modulation scheme. The primary information transmitted is the time of the pulse front. However, the time at which a pulse appears varies. Therefore, it is similar to the PPM-FM modulation scheme. In the demodulation process, the bandwidth is increased to improve the time resolution. However, increasing bandwidth will lower the signal-to-noise ratio of the channel and the probability of making an error is greatly increased. Hence, a suitable bandwidth must be selected.

This paper discusses the principle and advantages of the cross level detection method. It is better than the fixed level detection method in time resolution and decision failure probability.

I. Presentation of the Problem

Satellite analog telemetry is treated as a subcarrier modulation on the downward carrier of the overall microwave

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system. Within a period of 1-1.5 seconds, nine different signal pulses must be transmitted. When there are no signals, the subcarrier is transmitting at frequency f_0. When transmitting various signal pulses, the subcarrier transmits at frequencies f_1, f_2, and f_3. In order to distinguish these different signal pulses, three different signal pulse widths are employed. The key in signal transmission is to ensure the accuracy of time resolution for these pulse fronts (leading or trailing fronts). Therefore, the FSK modulation is similar to the PPM-FM or PDM-FM modulation scheme. It is required that the RMS of the random time error be less than 0.28 millisecond. The probability of misjudging a pulse signal must be less than 10^{-3}.

Based on the accuracy requirement, if the conventional fixed voltage method is used to determine the front of the signal pulse, then analog telemetry takes a large portion of the transmission power. The power distribution for various transmitting channels on a satellite is very tight. To this end, there is a need to study various demodulation schemes in depth in order to search for a modulation or demodulation scheme that can meet the accuracy and misjudgement requirements when the signal-to-noise ratio of the input subcarrier is low.

II. FSK Signal Passing Through an Ideal Band Filter

Because the interval between pulses is relatively long, the pulse width is narrow relative to the time interval. Therefore, it can be treated with a frequency modulation scheme for a single pulse. The FSK signal modulated from a signal pulse is

$$S(t)=\begin{cases}\cos\omega_1 t & -\frac{\tau}{2}\leqslant t\leqslant\frac{\tau}{2}\\ \cos\omega_2 t\end{cases} \tag{1}$$

Let us introduce the gate function g(t) /62

$$g(t)=\begin{cases}1 & -\frac{\tau}{2}\leqslant t\leqslant\frac{\tau}{2}\\ 0\end{cases} \tag{2}$$

then

$$S(t)=\begin{cases}S_1(t)=g(t)\cos\omega_1 t\\ S_2(t)=\overline{g(t)}\cos\omega_2 t\end{cases}$$

$$g(t)\longleftrightarrow G(\omega)=\tau S_a\left(\frac{\omega\tau}{2}\right) \tag{3}$$

where

$$S_a\left(\frac{\omega\tau}{2}\right)=\frac{\sin\frac{\omega\tau}{2}}{\omega\tau/2}$$

The patterns for g(t) and S(t) are shown in Figure 1.

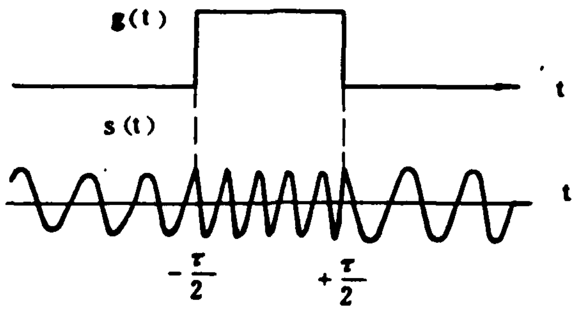

Figure 1 FSK Modulation of a Single Pulse

From the modulation theorem

$$S_1(\omega)=\frac{1}{2}[G(\omega+\omega_1)+G(\omega-\omega_1)]$$

$$=\frac{\tau}{2}\left[S_a\left(\frac{\omega+\omega_1}{3}\tau\right)+S_a\left(\frac{\omega-\omega_1}{2}\tau\right)\right] \quad (4)$$

$$\overline{g(t)}\longleftrightarrow\overline{G(\omega)}=2\pi\delta(\omega)-\tau S_a\left(\frac{\omega\tau}{2}\right)$$

$$S_2(\omega)=\pi\delta(\omega+\omega_2)+\pi\delta(\omega-\omega_2)-\frac{\tau}{2}\left[S_a\left(\frac{\omega+\omega_2}{2}\tau\right)+S_a\left(\frac{\omega-\omega_2}{2}\tau\right)\right] \quad (5)$$

$$S(\omega)=S_1(\omega)+S_2(\omega)$$

Based on equations (4) and (5), we get

$$S(\omega)=\frac{\tau}{2}\left[S_a\left(\frac{\omega+\omega_1}{2}\tau\right)+S_a\left(\frac{\omega-\omega_1}{2}\tau\right)-S_a\left(\frac{\omega+\omega_2}{2}\tau\right)\right.$$

$$\left.-S_a\left(\frac{\omega-\omega_2}{2}\tau\right)\right]+\pi\delta(\omega+\omega_2)+\pi\delta(\omega-\omega_2) \quad (6)$$

Hence, based on equation (6) we know the absolute value of the frequency spectrum of the FSK modulation of a single pulse is as shown in Figure 2. /63

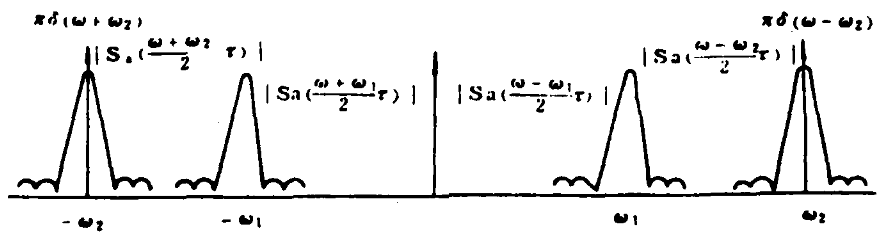

Figure 2 Frequency Spectrum of the FSK Modulation of a Single Pulse

If $\omega_1 - \omega_2 \gg 2\pi/\tau$, then the overlap of the frequency spectrum of $S_1(\omega)$ and $S_2(\omega)$ can be neglected. When band filters centered at ω_1 and ω_2 are used, if the filter bandwidth is

$$2\Delta\omega \ll \omega_1 - \omega_2$$

then the demodulation of the signal $S_1(t)=g(t)\cos\omega_1 t$ can be treated in the same way as in single frequency ASK demodulation. This will not cause any apparent signal distortion.

The signal output of $S_1(t)$ from the band filter (not considering the phase shift) is

$$S_1'(t) = \frac{K}{\pi}\left\{ Si\left[\Delta\omega\left(t+\frac{\tau}{2}\right)\right] - Si\left[\Delta\omega\left(t-\frac{\tau}{2}\right)\right]\right\}\cos\omega_1 t \qquad (7)$$

where $Si[x] = \int_0^x \frac{\sin x}{x}\,dx$ is a sine integral.

Therefore, the signal output $S'(t)$ of the original signal $S_1(t)$ from the band filter is still a $\cos\omega_1 t$ signal. However, its amplitude is modulated by the following function

$$\frac{K}{\pi}\left\{ Si\left[\Delta\omega\left(t+\frac{\tau}{2}\right)\right] - Si\left[\Delta\omega\left(t-\frac{\tau}{2}\right)\right]\right\}$$

This modulating function is the output response of a rectangular pulse g(t) whose width is τ passing through a low pass filter with a bandwidth $\Delta\omega$. Therefore, the output wave form of the FSK signal mentioned above, after going through a band filter with a center frequency ω_1 and a bandwidth $2\Delta\omega$ and passing through an ideal envelope detector, would be similar to the output from an ideal low pass filter after a rectangular pulse passes through it.

Similarly, after the signal $S_2(t)$ passes through the band filter centered at ω_2, its output is

$$S_2'(t)=\frac{K}{\pi}\left\{\frac{\pi}{2}-Si\left[\Delta\omega\left(t+\frac{\tau}{2}\right)\right]+Si\left[\Delta\omega\left(t-\frac{\tau}{2}\right)\right]\right\}\cos\omega_2 t \tag{8}$$

III. Demodulation Scheme Using Half Height as the Criterion

In the conventional ASK demodulation scheme, the half height of the pulse after the envelope detector is used as the voltage criterion, as shown in Figure 3. Based on the analysis in the previous section, after going through a band filter whose center frequency is ω_1 and bandwidth is $2\Delta\omega$ and passing through an envelope detector with a bandwidth $\Delta\omega$, the output of the signal is

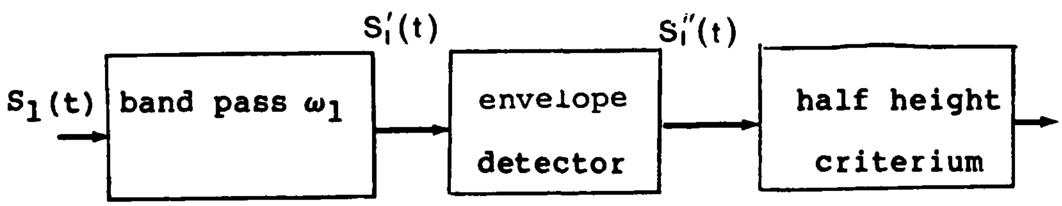

Figure 3 ASK Demodulation

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$$S_1''(t)=\frac{K}{\pi}\left\{Si\left[\Delta\omega\left(t+\frac{\tau}{2}\right)\right]-Si\left[\Delta\omega\left(t-\frac{\tau}{2}\right)\right]\right\} \tag{9}$$

If $\Delta\omega \gg 2\pi/\tau$, then the interaction between the leading front response and trailing front response can be neglected. The accuracy of time determination at half height is proportional to the slope of the rising or falling edge of the pulse envelope at half height. On the Si(x) curve, the maximum rising slope occurs at x=0. The slope is 1; i.e.

$$\frac{d}{dx}\mathrm{Si}(x)\Big|_{max} = \frac{\sin x}{x}\Big|_{x=0} = 1,$$

The response curve of a unity step function passing through a low pass filter is

$$\frac{K}{\pi}\left[\frac{\pi}{2} + \mathrm{Si}(\Delta\omega t)\right]$$

The maximum slope should be at $\Delta\omega t=0$, which is located at t=0. The magnitude of the slope is related to the parameter on the horizontal axis. When $\Delta\omega t$ is used as the horizontal axis, the maximum slope is 1. When t is used as the horizontal axis, the maximum slope is

$$\mathrm{tg}\,\theta\Big|_{max} = \frac{K\Delta\omega}{\pi} \qquad (10)$$

Figure 4 shows this response curve.

If the bandwidth of low pass filter is F_g, the optional value should be at one half of the bandwidth of the band filter $2\Delta\omega$. Therefore, after substituting $2\pi F_g = \Delta\omega$ into equation (10), the maximum slope (i.e., the slope at half height) of the signal pulse edge is $2\sqrt{2S}\ F_S$. Due to the superpositioning of the RMS value of the random noise level σ_N, the time determination accuracy is affected. The RMS value of the error in time, σ_N, is determined by the slope at which the determination is made and the value of σ_N. The slope at half height is $tg\theta=2\sqrt{2S}\ F_S$. Based on the geometry shown in Figure 5 we get

$$t_N=\frac{\sigma_N}{tg\,\theta}=\frac{\sigma_N}{2\sqrt{2S}\,F_s} \quad (12)$$

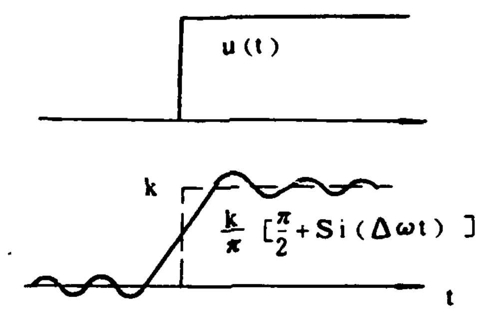

Figure 4 Low Pass Filter Response to Unity Step Function

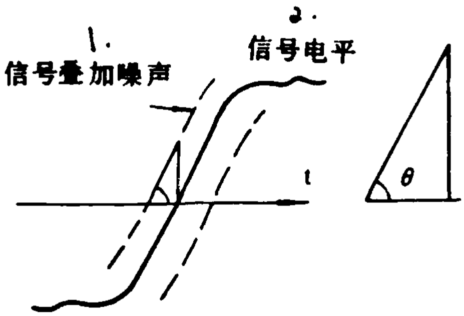

Figure 5 Effect of Noise Superpositioning on Time Determination

1. signal with noise on top
2. signal level

Let $\sigma_N = \sqrt{N}$ where N is the random noise power of the subcarrier channel. Then, equation (12) can be re-written as

$$t_N = \frac{1}{2\sqrt{2F_s}}\sqrt{\frac{N}{S}} \tag{13}$$

Let $N=\phi_0 . 2F_s$ where ϕ_0 is noise power spectrum density demodulated by the carrier receiver of the central microwave system. Then, equation (13) can be re-written as:

$$t_N = \frac{1}{2\sqrt{F_s}}\sqrt{\frac{\phi_0}{S}} \tag{14}$$

Based on equation (14), when the subcarrier signal to noise /65
spectrum density S/ϕ is fixed, t_N can be reduced by increasing F_s. Can we increase F_s to the extent possible to improve the time

accuracy? Increasing F_S is to enlarge the bandwidth of the output low pass filter for envelope demodulation and the bandwidth of the input band filter, which will lower the apparent signal-to-noise ratio $S/\phi_0 F_S$. Thus, the noise pulse will frequently exceed the level of determination. Or, the signal level might be lower than the level of determination because of the superposition of noise. Consequently, the probability of making an error is greatly increased.

IV. Probability of Error in Using Fixed Level for Determination

When the spacing between the different frequencies of the FSK signal is much larger as compared to the bandwidth of the band filter, the signal entering the band filter centered at ω_C can be treated as

$$S_C(t)=A_C\cos\omega_C t$$

The Gaussian noise going through this finite bandwidth is

$$\begin{aligned} n(t) &= x_C(t)\cos\omega_C t - x_S(t)\sin\omega_C t \\ &= V(t)\cos[\omega_C t + \phi(t)] \end{aligned} \quad (15)$$

$x_C(t)$ and $x_S(t)$ are Gaussian distributions. Compared to $\cos\omega_C t$, they are slowly varying functions. Furthermore

$$\overline{n^2(t)} = \overline{x_C^2(t)} + \overline{x_S^2(t)} = \sigma_N^2 = N \quad (16)$$

$$\left.\begin{aligned} p(x_C) &= \frac{1}{\sqrt{2\pi}\,\sigma_N} e^{-x_C^2/2\sigma_N^2} \\ p(x_S) &= \frac{1}{\sqrt{2\pi}\,\sigma_N} e^{-x_S^2/2\sigma_N^2} \end{aligned}\right\} \tag{17}$$

The noise envelope V(t) obeys the following distribution:

$$p[V(t)] = \begin{cases} \dfrac{V(t)}{\sigma_N^2} e^{-V^2(t)/2\sigma_N^2} & \text{when } V(t) > 0 \\ 0 & \text{when } V(t) \leq 0 \end{cases} \tag{18}$$

Let us assume that the demodulated pulse height is A_C and the half height level for determination is $A_C/2$. When there is no signal, the output noise level is V(t). According to equation (18), the probability of V(t) exceeding the level of determination P_{es} is

$$P_{es} = \frac{T_S}{T} \int_{A_C/2}^{\infty} p(V)dV \tag{19}$$

where T_S is the time without any signal within the period T.

When the input of the band filter contains signal plus noise, the probability that the superimposed level R(t) is lower the level of determination is

$$P_{em} = \frac{T_m}{T} \int_0^{A_C/2} \frac{R(t)}{\sigma_N^2} e^{-[R^2(t)+A_C^2]/2\sigma_N^2} I_0\left[\frac{R(t)A_C}{\sigma_N^2}\right] dR(t) \tag{20}$$

where R(t) is the mixed envelope of the demodulated signal plus noise.

$$I_0(x)=\frac{1}{2\pi}\int_0^{2\pi} exp[x\cos(\theta+\theta_0)]d\theta$$

is a zero order modified Bessel function. T_m is the time period with signal present. Under larger signal to noise ratios, $A_C \gg \sigma_N$, then the probability of the envelope distribution of signal and noise is approaching a Gaussian distribution. Its approximation is

$$P[R(t)]=\frac{1}{\sqrt{2\pi}\,\sigma_N}e^{-[R(t)-A_C]^2/2\sigma_N^2} \tag{21}$$

$$P_{em}=\frac{T_m}{T}\int_0^{A_C/2}\frac{1}{\sqrt{2\pi}\,\sigma_N}e^{-[R(t)-A_C]^2/2\sigma_N^2}\,dR(t) \tag{22}$$ /66

Because signal pulses are not present most of the time, the probability of error due to the fact that the noise level exceeds the criterium is high when there is no signal around. If we adopt a method to measure the width (i.e. to determine whether the leading and trailing edge of a signal is within a specific width range), then many false signals can be eliminated. If the tolerance of width is ΔD, then we will miss some signals if ΔD is too small. If ΔD is too large, then the error (false alarm) analyzed above will go up.

When the width method is employed, we can consider that T_m

and T_s take up half of the determination time T each. This is approximately assumed based on the signal pulse width and on the magnitude of the width error ΔD used in practice (ΔD should also be related to the bandwidth F_s). Therefore, $T_s \approx T_m \approx T/2$. Thus

$$P_{es} = \frac{1}{2}\int_{A_C/2}^{\infty} \frac{V(t)}{\sigma_N^2} e^{-V^2(t)/2\sigma_N^2} dV(t) = \frac{1}{2} e^{-A_C^2/8\sigma_N^2} \quad (23)$$

Based on the fact that the subcarrier signal to noise ratio is $(C/N)_i = A_C^2/2\sigma_N^2$, then

$$P_{es} = \frac{1}{2} e^{-\frac{1}{4}(C/N)_i} \quad (24)$$

Similarly,

$$P_{em} = \frac{1}{2}\int_{0}^{A_C/2} \frac{1}{\sqrt{2\pi}\sigma_N} e^{-[R(t)-A_c]^2/2\sigma_N^2} dR(t)$$

We get

$$P_{em} = \frac{1}{2}[1 - 2\Phi(A_C/2\sigma_N)]$$

or

$$P_{em} = \frac{1}{2}\left\{1 - 2\Phi\left[\frac{1}{2}\sqrt{(C/N)_i}\right]\right\} \quad (25)$$

where $\Phi(x)$ is the probability integral. Thus, we can derive Table 1.

Table 1

$(C/N)_i$	15dB	13dB	10dB
P_{es}	1.8×10^{-4}	3.4×10^{-3}	4×10^{-2}
P_{em}	5×10^{-5}	8×10^{-4}	1.3×10^{-2}
P_e	2.3×10^{-4}	4.2×10^{-3}	5.3×10^{-2}

When F_s is 400Hz and the subcarrier input signal to noise ratio spectrum density is S/ϕ_0=44dBHz, the error in time determination is

$$\tau_N=\frac{1}{2\sqrt{F_s}}\sqrt{\phi_0/S}\cong 0.16\text{ms}$$

Since the misjudgement made at the pulse edge due to intense noise interference is identical at the leading edge and at the trailing edge, the probability of making an error in determining a signal pulse should be $2P_e$. When S/ϕ_0 is 44dBHz, the signal pulse misjudgement probability is 4.6×10^{-4}. When S/ϕ_0 is 42dBHz, i.e. $(C/N)_i$=13dB, τ_N=0.2ms. Although the time accuracy is satisfied, the misreading probability will increase to 8.4×10^{-3}. It is required in section I that there is no more than one misjudgement in one thousand signal pulses. Therefore, in the demodulation scheme, the S/ϕ_0 of the analog telemetry subcarrier of the microwave receiver will have to be 43dBHz to meet both accuracy and reading error requirements.

It must be pointed out that if there is no width criterium, /67
due to the fact that there are no signal pulses most of the time

(the signal pulses occupy less than one tenth of the total time), $T_s \cong T$ and $P_e \cong P_{es}$. Thus, as T increases, P_e also goes up significantly. For instance, when S/ϕ_0=44dBHz (equivalent to $(S/N)_i$=15dB), if the single event determination time is the unity pulse response time $1/F_s$=2.5 msec, the probability of misreading is 1.8×10^{-4}. When the time criterium used is 2.5 seconds (equivalent to the entire signal pulse spectrum from the satellite as it spins at 24 revolutions per minute), $P_e = 1.8 \times 10^{-1}$ which is too high. After a width criterium is adopted, those misreadings which do not meet the width requirement will be eliminated. The only functional time for determination is limited to the duration when the signal pulse is present and the time period before and after the pulse. By doing so, the probability of making a wrong reading is drastically reduced.

V. Effect of Signal Level Variation on Fixed Level Detection

When the half height fixed level detection scheme is used, if the apparent pulse level varies, there will be relatively large system error τ_s, as shown in Figure 6. The figure shows that the projected pulse amplitude is A. Therefore, the half height level of detection is A/2. However, when the pulse amplitude varies to rA, if A/2 is still used as the criterium, a system error τ_s will arise based on the geometry shown in Figure 6.

$$\tau_s \cong \frac{r-1}{2rF_s} \qquad (26)$$

From equation (26), we can understand that the variation of the subcarrier signal level from the central carrier receiver will cause some determination error. If we specify that τ_s must be less than 0.2 msec, then 1.16>r>0.84. The voltage level must vary within a range less than 0.32A; i.e. less than -5dB. If the detection level and the signal level are correlated, i.e. the detection level is made to vary with the signal level, it is possible to minimize τ_s based on theory. However, it imposes complicated demands on the equipment which is usually inappropriate to adopt in practice. The error in judgement caused by the variation of the input signal is one of the disadvantages of using the half height fixed level detection scheme.

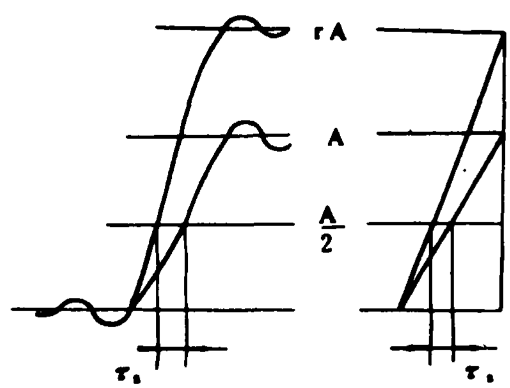

Figure 6 Effect of Pulse Signal Amplitude Variation

VI. Cross-level Detection Scheme in FSK Demodulation

The FSK modulation scheme is used on the transmission end in the analog telemetry of International Communication Satellite No. 4. However, on the receiving end, the ASK scheme is adopted. Of course, this is the simplest way. This scheme, together with half height detection, requires a high subcarrier signal to noise spectrum density ratio to simultaneously meet the time accuracy and decision failure probability requirements. In addition, it is also influenced by the variation of the subcarrier signal amplitude. Figure 7 shows the principle of the cross-level detection scheme. The outputs on the two envelope detectors are detected in

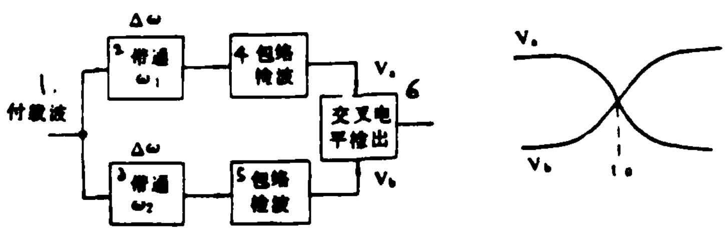

Figure 7 Principle of Cross-level Detection

1. subcarrier
2. bandwidth 1
3. bandwidth 2
4. envelope detection
5. envelope detection
6. cross-level detection

the voltage level detection circuit. When the levels change from /68
$V_a > V_b$ to $V_b > V_a$, the time at which $V_a = V_b$ is the time signal to be transmitted. This is cross-level detection.

If V_a and V_b are true signal levels $\bar{V}_a$ and $\bar{V}_b$ with random noise variables added, because it is assumed that near the crossover point

$$V_a \gg \sigma_N \quad V_b \gg \sigma_N$$

therefore, V_a and V_b can be considered as random signals in Gaussian distribution centered around $\bar{V}_a$ and $\bar{V}_b$, respectively. Let us assume that $\bar{V}_a$ and $\bar{V}_b$ cross over at t_0 when there is no noise, as shown in Figure 8. Assuming the effect of noise on V_a is $\bar{V}_a - \sigma$ and the effect on V_b is $\bar{V}_b + \sigma$, then the intersect is located at t_1. On the other hand, if the effect on V_a is $\bar{V}_a + \sigma$ and that on V_b is $\bar{V}_b - \sigma$, then the intersect is located at t_2. If the superimposed noise is within the range $\pm\sigma$, then the cross over point ought to be in the region between t_1 and t_2. This situation is explained in Figure 8. Based on the analysis in Section IV, this range between t_1 and t_2 is the margin of error in the detection of time (RMS value) when using the ASK demodulation scheme (half height criterium).

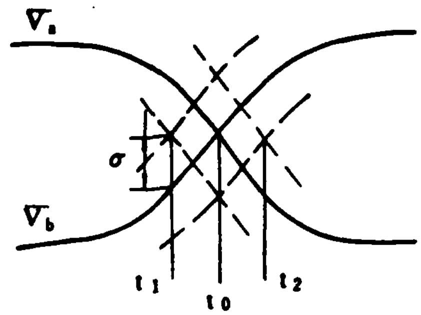

Figure 8 Effect of Noise Superposition on Cross-level Detection

In order to calculate the probability that the intersect falls in the range between t_1 and t_2, let us separately calculate the probability that $V_a > V_b$ when $t = t_1$, i.e. $P_{t=t_1}(V_a > V_b)$, and the probability that $V_b > V_a$ when $t = t_2$, i.e. $P_{t=t_2}(V_b > V_a)$. The conditions to have a cross over point within t_1, t_2 are determined by the conditions to switch $V_a > V_b$ to $V_b > V_a$. The probability of having both events happening in t_1, t_2 is the product of the two probabilities mentioned above.

$$P_{t_1,t_2} = P_{t=t_1}(V_a > V_b) \cdot P_{t=t_2}(V_b > V_a) \qquad (27)$$

where

$$P_{t=t_1}(V_a > V_b) = \int_{-\infty}^{\infty} dV_a \int_{-\infty}^{V_a} p(V_a, V_b)\, dV_b \qquad (28)$$

$p(V_a, V_b)$ is the probability density distribution of a two-dimensional random variable. When $\sigma << \overline{V}_a, \overline{V}_b$, we can consider that V_a is a Gaussian distribution centered around $\overline{V}_a$ and V_b is a Gaussian distribution centered around $\overline{V}_b$. At $t=t_1$, $\overline{V}_a$ is larger than $\overline{V}_b$ by 2σ. Therefore, $P_{t=t_1}(V_a>V_b)$ represents the probability of having an intersect on the right side of the line $t=t_1$ in Figure 9 (i.e. $t>t_1$). If $V_b>V_a$ when $t=t_1$, it indicates that an intersect has already been created on the left side prior to t_1.

$$p(V_a, V_b)=\frac{1}{2\pi\sigma^2}e^{-\frac{(V_a-\overline{V}_a)^2+(V_b-\overline{V}_b)^2}{2\sigma^2}} \tag{29}$$

Let us change the coordinates. Let $u_1=V_a-\overline{V}_a$, $u_2=V_b-\overline{V}_b$

$$P_{t=t_1}(V_a>V_b)=\int_{-\infty}^{\infty}du_1\int_{-\infty}^{u_1+\overline{V}_a-\overline{V}_b}\frac{1}{2\pi\sigma^2}e^{-\frac{u_1^2+u_2^2}{2\sigma^2}}du_2 \tag{30}$$

At $t=t_1$, $\overline{V}_a-\overline{V}_b=2\sigma$.

σ is the RMS value of the noise voltage level. After normalization, $x=\frac{u_1}{\sigma}$ $\quad y=\frac{u_2}{\sigma}$

$$p(V_a>V_b)=\int_{-\infty}^{\infty}dx\int_{-\infty}^{x+2}\frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}}dy$$

$$=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx\int_{-\infty}^{x+2}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}dy$$

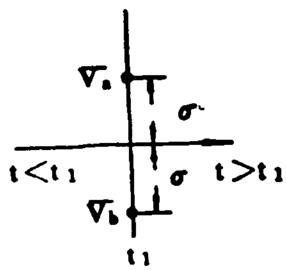

Figure 9 When $\overline{V}_a$ and $\overline{V}_b$ Differ by 2σ

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$$=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}[erf(x+2)]dx \qquad (31)$$

This integral is solved numerically by using a computer. When $\overline{V}_a-\overline{V}_b=2\sigma$, on the right side of $t=t_1$ we get

$$P(V_a > V_b)=0.92 \qquad t\geq t_1$$

Similarly, the probability of having an intersect on the left side of $t=t_2$ is

$$P(V_b > V_a)=0.92 \qquad t\leq t_2$$

From Figure 10 we can see that the probability of having an intersect within the range t_1, t_2 is

$$P_{t_1,t_2}(\text{with intersect})=0.84$$

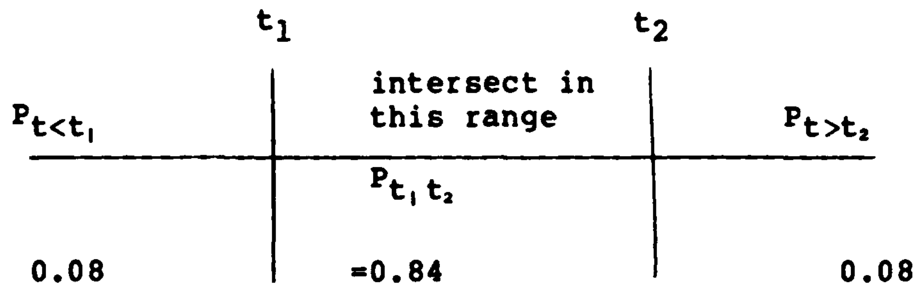

Figure 10 Probability of Cross-level Detection

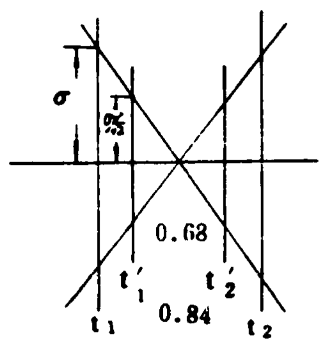

Figure 11 Probability of Cross-level Detection at Various Noise RMS Values

In fixed level detection (see Figure 5), due to the superposition of noise on top of signal, the random level can be treated as a Gaussian distribution centered around $\bar{V}$ and its RMS value is σ. The determining time can be considered to be within the range t_1, t_2, where $t_1=t_0-t_N$ and $t_2=t_0+t_N$. t_N is the RMS value of the time error. The probability that the determining point falls in the range of t_1, t_2 is also the probability that the random level V falls into the range $(A/2-\sigma)$ and $(A/2+\sigma)$ at time t_0. This probability should be 0.68 based on a Gaussian distribution. Therefore

$$P_{t_1,t_2} \text{ fixed level} = 0.68$$

Thus, we can see that the probability for the cross-level detection point to fall in the time span between t_1 and t_2 is different from that in the fixed level scheme. The probability for the point to fall within t_1, t_2 in the fixed level scheme is smaller. This shows that the accuracy of cross-level detection is higher than that of fixed level detection.

Let the RMS value of the cross-level detection noise remain to be σ. However, let us define another point t'_1 at which $\bar{V}_a-\bar{V}_b=2\,\sigma/\sqrt{2}$. Let us define another point t'_2 where $\bar{V}_b-\bar{V}_a=2\,\sigma/\sqrt{2}$. Similarly, we can calculate that the probability of having a cross over point within t'_1, t'_2 is

$$P t'_1 t'_2 \text{ cross-level} \cong 0.68$$

Thus, the time error (RMS value) caused by cross-level detection is equal to that of the fixed level method as the noise level is reduced to $\sigma/\sqrt{2}$ (comparing at the same probability). Therefore, the following conclusion is reached. When the same time fluctuation accuracy is attained, the subcarrier signal to noise ratio required by cross-level detection is 3dB lower than that required by fixed level detection.

VII. Probability of Error of the Cross-level Detection Scheme

Cross-level detection will cause decision failure as well. Cross over point may appear at a time it should not appear. Based on Figure 7, when the subcarrier at frequency ω_1 is /70
received, the two branches of output from the envelope detectors are

$$\left.\begin{aligned} V_a(t) &= \sqrt{[A_c + x(t)]^2 + y^2(t)} \\ V_b(t) &= \sqrt{x^2(t) + y^2(t)} \end{aligned}\right\} \quad (32)$$

Based on the earlier analysis, the probability distributions of V_a and V_b are

$$\left.\begin{aligned} p(V_a) &= \frac{V_a}{\sigma^2} I_0\left(\frac{A_c V_a}{\sigma^2}\right) e^{-(V_a^2 + A_c^2)/2\sigma^2} & 0 \leq V_a \leq \infty \\ p(V_b) &= \frac{V_b}{\sigma^2} e^{-V_b^2/2\sigma^2} & 0 \leq V_b \leq \infty \end{aligned}\right\} \quad (33)$$

Hence

$$P(V_b > V_a) = \int_0^\infty p(V_a)\left[\int_{V_a}^\infty p(V_b)\,dV_b\right] dV_a$$

where

$$\int_{V_a}^{\infty} p(V_b)dV_b=\int_{V_a}^{\infty} \frac{V_b}{\sigma^2} e^{-V_b^2/2\sigma^2}\, dV_b=e^{-V_a^2/2\sigma^2} \tag{34}$$

According to equation (34)

$$P(V_b>V_a)=\int_0^{\infty}\frac{V_a}{\sigma^2}I_0\left(\frac{A_C V_a}{\sigma^2}\right)e^{-(2V_a^2+A_C^2)/2\sigma^2}dV_a \tag{35}$$

Let

$$t=\frac{\sqrt{2}V_a}{\sigma},\quad a=\frac{A_C}{\sqrt{2}\sigma}$$

Hence

$$P(V_b>V_a)=\frac{1}{2}e^{-a^2/2}\int_{\beta}^{\infty} tI_0(at)e^{-(t^2+a^2)/2}dt \tag{36}$$

where

$$\int_{\beta}^{\infty} tI_0(at)e^{-(t^2+a^2)/2}dt=Q(a,\beta)$$

which is called the Marcum Q function. We already know

$$Q(a,o)=1,\qquad Q(o,\beta)=e^{-\beta^2/2}$$

Therefore,

$$P(V_b > V_a) = \frac{1}{2}e^{-a^2/2} = \frac{1}{2}e^{-A_C^2/4\sigma^2} = \frac{1}{2}e^{-\frac{1}{2}(C/N)i} \quad (37)$$

As compared to the fixed level detection formula (25), the probability of decision failure with the cross-level scheme, $P(V_b > V_a)$, is obviously much less. The comparison is shown in Table 2.

Table 2

$(C/N)_i$	15dB	13dB	10dB
P_e	2.3×10^{-4}	4.2×10^{-3}	5.3×10^{-2}
$P(V_b > V_a)$	8.8×10^{-8}	4.5×10^{-5}	6.7×10^{-3}

VIII. Conclusions

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Based on this analysis, the cross-level detection scheme has a higher accuracy and lower misjudgement probability. In addition, its detection time will not be affected by subcarrier signal level changes. Therefore, in this modulation work with special requirements, the cross-level detection scheme is far more superior.

Reference

Chen Yiyuan and Lin Zhenkai: ((Principle of Information Transmission in Telemetry)), Defense Publishing Co., 1980.

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