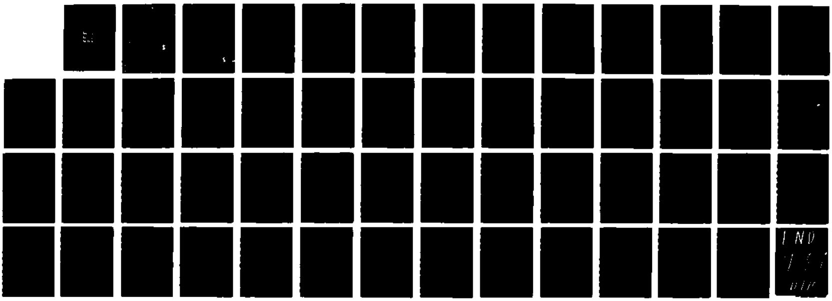
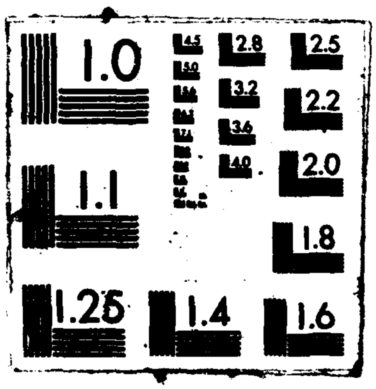
AD-A181 135 APPLICATION OF SOLAR SEISMOLOGY TO STUDY OF THE CONVECTION ZONE(U) ARIZONA UNIV TUCSON DEPT OF PHYSICS H A HILL 22 APR 87 AFOSR-TR-87-0593 AFOSR-82-0019 1/1

UNCLASSIFIED F/G 3/2 NL




1.0
4.5
5.0
5.6
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

UNCLASSIFIED

AD-A181 135

DTIC FILE COPY

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 87-0593
6a. NAME OF PERFORMING ORGANIZATION: University of Arizona; 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: AFOSR
6c. ADDRESS (City, State and ZIP Code): College of Arts & Sci, Faculty of Science, Dept of Physics, Bldg 81, Tucson, Arizona 85721	7b. ADDRESS (City, State and ZIP Code): Bldg 410, Bolling AFB, DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR; 8b. OFFICE SYMBOL (If applicable): NP	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-82-0019
8c. ADDRESS (City, State and ZIP Code): same as 7b.	10. SOURCE OF FUNDING NOS.: PROGRAM ELEMENT NO. 61102F; PROJECT NO. 2311; TASK NO. A1; WORK UNIT NO.

11. TITLE (Include Security Classification): "APPLICATION OF SOLAR SEISMOLOGY TO STUDY OF THE CONVECTION ZONE" (U)

12. PERSONAL AUTHOR(S): Dr. Henry Hill

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
FINAL	FROM 81/10/01 TO 86/12/	87/04/22	50

16. SUPPLEMENTARY NOTATION

DTIC ELECTE JUN 8 1987 S A

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	Sun, Oscillations, SCLERA

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

In the 1970's an observational technique for the study of solar oscillations with periods longer than 5 min was developed at SCLERS. The technique yields information about the temperature eigenfunction of an oscillation at the extreme solar limb. The primary goal of the observational program for 1982-1986 was to use this technique to systematically increase the data base for the stydd of solar oscillations. Because of the complexity of the eigenfrequency spectrum and because of the relatively small signals produced by the oscillations, the analysis of the observations for identification and classification of modes of oscillations has proved to be very difficult. The main objective of the data analysis program for 1982-1086 was to develop analysis techniques which would lead to mode classifications based on the SCLERA differential radious observations and on the combination of the differential radius observations with differential velocity observations. The goal of the theoretical program has been the inversion of the multiplet fine structure obtained in the data analysis program in order first to test if for internal consistency and second infer information about the radial and latitudinal dependence of the internal rotation of the sun.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☒ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION: UNALCSSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL: DR, HENRY R. RADOSKI	22b. TELEPHONE NUMBER (Include Area Code): (202)767-4906; 22c. OFFICE SYMBOL: NP

DD FORM 1473, 83 APR   EDITION OF 1 JAN 73 IS OBSOLETE.   UNCLASSIFIED

87 5 21 306

SECURITY CLASSIFICATION OF THIS PAGE

AFOSR-TR- 87-0593

STUDY OF SOLAR OSCILLATIONS

Final Report

Contract Number: AFOSR-82-0019
Period: 1/10/81 through 12/31/86
Submitted by: Dr. Henry A. Hill
Date: April 22, 1987

1. Summary of Solar Seismology Work Performed During 1982-1986 and Supported by AFOSR Contract 82-0019

1.1 Scope of the 1982-1986 Research Program

In the 1970s an observational technique for the study of solar oscillations with periods longer than 5 min was developed at SCLERA. The technique yields information about the temperature eigenfunction of an oscillation at the extreme solar limb. The primary goal of the observational program for 1982-1986 was to use this technique to systematically increase the data base for the study of solar oscillations. Because of the complexity of the eigenfrequency spectrum and because of the relatively small signals produced by the oscillations, the analysis of the observations for identification and classification of modes of oscillations has proved to be very difficult. The main objective of the data analysis program for 1982-1986 was to develop analysis techniques which would lead to mode classifications based on the SCLERA differential radius observations and on the combination of the differential radius observations with differential velocity observations. The goal of the theoretical program has been the inversion of the multiplet fine structure obtained in the data analysis program in order first to test it for internal consistency and second to infer information about the radial and latitudinal dependence of the internal rotation of the Sun.

The results of these programs appear in a series of works and have led to the publication of a sizable amount of data; the development of the analysis programs for the classification of modes; the classification of ≈ 1209 resolved modes belonging to 166 multiplets, including acoustic, f-, and gravity modes; tests of mode classification programs; inferred information on the internal rotation of the Sun; and evidence of mode coupling which may be relevant to the solar neutrino paradox. These results are reviewed in the following sections.

1.2 Observations and Publication Schedule of Observational Data

1.2.1 Schedule of Observations

The 1983 and 1984 Observations

A series of observations were made in 1983 and 1984 for which the observational technique and telescope apparatus were very similar to those used in the 1979 SCLERA differential radius observations (cf. Bos and Hill 1983). In this work, data was obtained for the solar latitudes -45°, 0°, 45°, and 90°. Data was acquired at 64-second intervals for each of these four latitudes, and the entire 256-second cycle was repeated continuously throughout each observing day. The set of data was obtained between May 1983 and March 1984. From 6/1/83 to 7/20/83 data was taken on 47 days at an average of 8.8 hours per day. From 10/10/83 to 3/21/84 data was taken on 29 days at an average of 5.8 hours per day.

The 1985 Observations

The observations made in 1985 were obtained with a new observational technique described in Section 1.7. With this technique, solar oscillations are detected as perturbations in the radiation intensity at disk center as a function of λ for $0.5 \leq \lambda \leq 1.7$ µ. Observations were made on 35 days spanning a 70-day period from March 23 to May 31, 1985 at an average of 8.3 hours per day.

The 1986 Observations

The observational technique and telescope apparatus for the 1986 observations were also very similar to those used in the 1979 differential radius observations (cf. Bos and Hill 1983) with emphasis placed on equatorial observations. The telescope was continuously manned during the spring and early summer observing season of 1986. Although the particular observing season proved to be exceedingly cloudy, over 100 hours of data were obtained.

1.2.2 Publication Schedule of SCLERA Observational Data

Data from SCLERA observations have been presented in a series of works for the 1978 and 1979 observations. The average of 13 daily power spectra with a 30 μHz resolution is shown in Figure 1 of Caudell et al. (1980) and also in Figure 10 of Caudell (1980) for the 1978 observations. Also, for the 1978 observations, the diagrams of phase versus time for 12 peaks in the power spectrum are shown in Figures 3, 4, and 5 of Caudell et al. (1980) and also in Figures 15, 16, and 17 of Caudell (1980). For the 1979 observations, the list of works containing data is more extensive and is given in Table 1. The data in these works, much of it available as early as 1982, permits certain of the results to be independently tested such as was done for the 1978 observations. In that test, Gough (1980) independently examined the phase diagrams for the 1978 observations and concurred in the conclusions of Caudell et al. (1980). The first data from the 1983 and 1985 observations to be published are scheduled to be available in 1986 (Yi 1986 and Oglesby 1986, respectively).

Table 1
SCLERA Publications of Data to Date for the 1979 Observations

Source	Fig. Number	Resolution	Type of Power Spectra[c]	Frequency Range
Bos (1982)	4.1	30 μHz	P_1	0 - 3.2 mHz
	4.2	"	P_2	"
	4.3	"	P_4	"
	4.4	"	P_3	"
	4.5	0.28 μHz	P_1	240 - 272 μHz
	4.6	"	P_2	"
	4.7	0.28 μHz	P_1	420 - 452 μHz
	4.8	"	P_2	"
	4.9	"	P_4	"
	4.10	"	P_3	"
	4.11	0.28 μHz	P_1	550 - 582 μHz
	4.12	"	P_2	"
	4.13	0.28 μHz	P_1	420 - 452 μHz
	6.3	0.28 μHz	P_1	235 - 285 μHz
Hill, Bos, & Goode (1982)	1	0.28 μHz	P_1	235 - 285 μHz
Bos & Hill (1983)	3[a]	0.28 μHz	P_1	450 - 482 μHz
Hill (1984a)	1[b]	0.28 μHz	P_1	235 - 285 μHz
Hill (1985)	1	0.28 μHz	P_1	2492 - 2502 μHz
	2	"	P_2	3227 - 3237 μHz
	3	"	P_2	3909 - 3919 μHz
	4	"	P_2	3209 - 3225 μHz
	5	"	P_2	3616 - 3632 μHz
	6	"	P_2	3890 - 3906 μHz
Hill, Tash, & Padin (1986)	1	0.28 μHz	P_1	100 - 110 μHz
Hill & Czarnowski (1986)	3	0.28 μHz	P_1	0 - 20 μHz
	4	"	P_2	"

[a] Same as Fig. 6.3 of Bos (1982) and Fig. 1 of Hill (1984a).
[b] Same as Fig. 6.3 of Bos (1982) and Fig. 1 of Hill, Bos, and Goode (1982).
[c] Definition of P_i nomenclature same as used by Hill (1984b).

1.3 The SCLERA Mode Classification Program

The objective of the mode classification program developed at SCLERA is the classification of low to intermediate degree acoustic and gravity modes with low to intermediate radial orders. This program is based primarily on the differential radius observations from SCLERA supplemented with various types of Doppler shift and total irradiance observations from other observatories. These observations provide the following categories of information for use in mode classification: (1) the multiplet fine structure of the eigenfrequency spectrum expected for a slowly rotating axisymmetric system, (2) the symmetry properties of the eigenfunction, (3) the parities of ℓ and m, (4) the magnitude of ℓ, (5) the (exp $im\phi$)-dependence of the eigenfunction, and (6) the θ-dependence of the eigenfunction given by Y_ℓ^m. In addition, a theoretical eigenfrequency spectrum from a standard solar model is used in the classification of the radial order.

The subsections below review how the above information is obtained and/or applied to mode classification. It is apparent that this rather extensive list may permit not only the classification of modes but also the implementation of a number of tests of the accuracy of a given set of mode classifications. The availability of a set of independent tests has become a hallmark of the SCLERA mode classification program. These tests are in addition to those described in Section 1.5 concerned with the reproducibility of results from one year to another.

1.3.1 Symmetry Properties of Eigenfunctions

The determination of the symmetry properties of the eigenfunctions has been one of the most important exercises of the last few years. First, if the Sun were not an axisymmetric system as seen by the normal modes, the complexity of the multiplet fine structure could considerably complicate the mode classifications. Thus the information obtained on the eigenfunction symmetry properties was first used to determine the type of multiplet fine structure to be expected (see Bos and Hill 1983).

With the determination of the symmetry properties of the system, two analysis techniques based on the inferred symmetry properties of the eigenfunctions have been successfully implemented. The first technique uses the symmetry properties to generate power spectra which contain essentially the signals for modes with only one combination of the parities of ℓ and m. The four different combinations of the parities of ℓ and m, respectively, are (even, even), (even, odd), (odd, even), and (odd, odd). This technique leads to an important reduction in the density of peaks in the complex power spectra and permits the immediate classification of the associated parities of ℓ and m with a relatively high probability of being correct. Examples of the successful use of this technique are found in the works of Bos and Hill (1983), Hill, Bos, and Goode (1982), Hill (1984b), and Hill (1985a,b).

The second technique uses the symmetry properties to test for the correctness of a particular mode classification. If the mode classifications for multiplets are correct, the signals in the complex power spectra due to every other member of a multiplet should be confined to one power spectrum while the signals due to the remaining set should be confined to a second power spectrum. Examples of this technique are found in the same works as listed in the previous paragraph. It should be noted that the second technique permits the determination of what fraction of the time incorrect peak classifications are made due to coincidental alignment of the frequency of one mode with that of an alias of a second mode (cf. Section IV<u>g</u> of Hill 1984b and Section IV<u>e</u> of Hill 1985b).

1.3.2 Frequency Patterns of Multiplet Fine Structure

The determination of the symmetry properties of the eigenfunctions as those arising from an axisymmetric system (Bos and Hill 1983, Hill 1984b) furnished an extremely important foundation for defining the basic properties of the multiplet fine structure. In particular, a slowly rotating axisymmetric Sun should lead to a multiplet fine structure that to a first approximation is uniformly spaced in frequency. This frequency pattern is often referred to as a Zeeman pattern, and its simplicity has played an important role in the SCLERA mode classification program. Examples of the value of this technique are found in the first five subsections of Section 1.4.

1.3.3 Azimuthal Dependence of Eigenfunctions

The azimuthal dependence (exp $im\phi$) of an eigenfunction expected for an axisymmetric system is the basis of an important technique developed by Hill (1984b) and extended in the work of Hill, Alexander, and Caudell (1985). The information on the m-dependence is contained in the observable D_{ij}, which is a functional of (exp $im\phi$). Formally, D_{ij} is essentially the ratio of the finite Fourier transform of the sin $m\phi$ component of (exp $im\phi$) to the finite Fourier transform of the cos $m\phi$ component of (exp $im\phi$). The general properties of D_{ij} are that it is a quasi-periodic function of m and is antisymmetric for $m \leftrightarrow -m$. An example of the D_{ij} is shown in Figure 1, which was obtained in the work on the low-order, low-degree acoustic modes (cf. Section 1.4.2).

The functional D_{ij} has been important in the determination of the $m = 0$ member of a multiplet because of its symmetry properties about $m = 0$. It has also been an extremely valuable part of the SCLERA mode classification program because it furnishes a highly statistically significant test of a particular set of mode classifications. Examples of its application are found in the first five subsections of Section 1.4.

1.3.4 Zeros of $\overline{Y_\ell^m}$

A second technique has been introduced to determine the $m = 0$ mode and the ℓ value of a multiplet for modes with $|m| \geq 20$. This technique is based on the properties of $\overline{Y_\ell^m}$ where $\overline{Y_\ell^m}$ is the average of Y_ℓ^m over the detector geometry used by Bos (1982) (see Bos and Hill 1983). The relationship between ℓ and the values of m, m_0, for which $\overline{Y_\ell^m} = 0$, are shown in Table 2 for even intermediate values of ℓ.

The first successful use of these properties with respect to zeros was in the work on phase-locked modes (cf. Section 1.4.5). For such modes, the zeros of $\overline{Y_\ell^m}$ introduce 180° phase shifts in the plots of phases of the modes versus m.

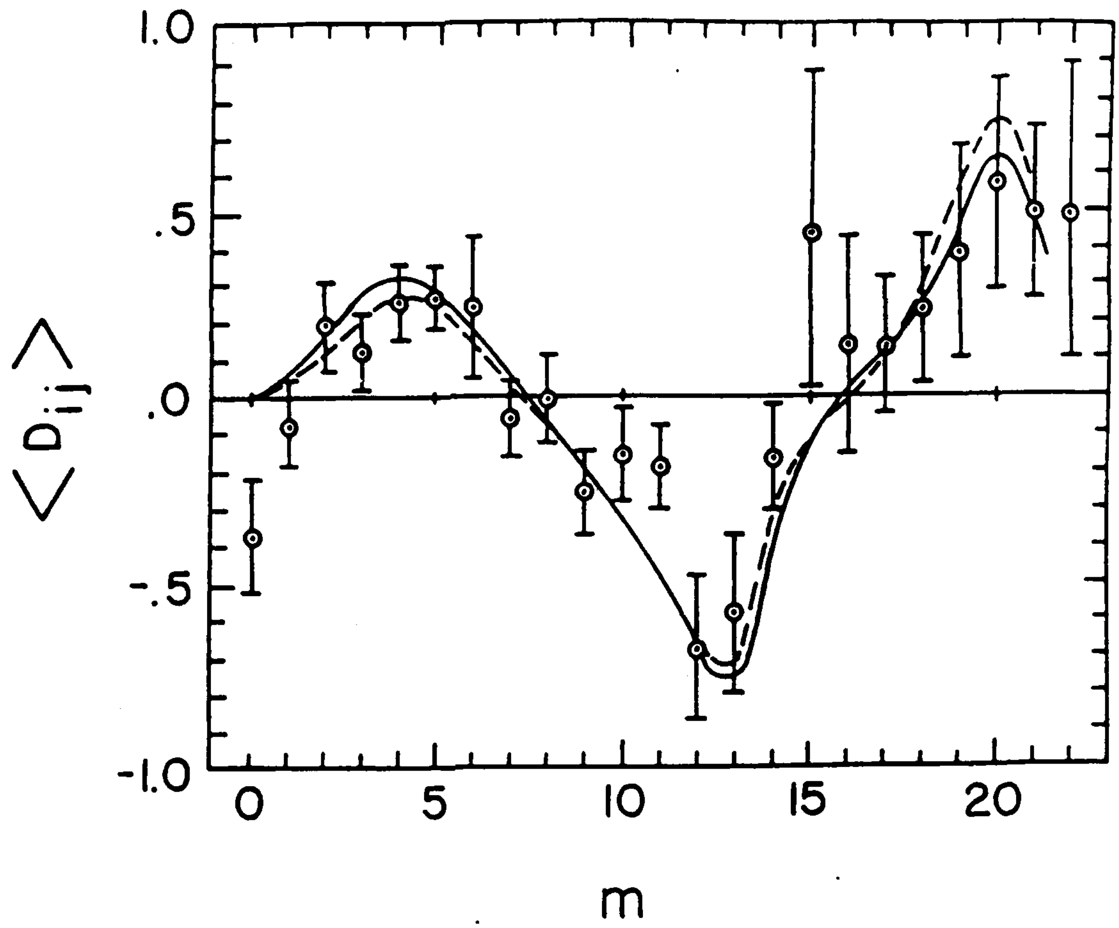


Fig 1 The $e^{im\phi}$ dependence of the eigenfunctions as represented by $\langle D_{ij} \rangle$. The function D_{ij} is defined in terms of the ratio of the finite Fourier transforms of the sin $m\phi$ and cos $m\phi$ dependent terms of the oscillatory component of the radiative intensity $\mathcal{J}'$. The modes included in this analysis by Hill (1984b) are the $p_{1,4}, \ldots, p_{1,22}$; $p_{2,3}, \ldots, p_{2,10}$; and $p_{3,2}$, $p_{3,3}$, $p_{3,4}$. The solid curve is the predicted $\langle D_{ij} \rangle$ using the inferred I'_o obtained by Hill, Alexander, and Caudell (1985). The dashed curve is the predicted $\langle D_{ij} \rangle$ for a theoretically derived I'.

Table 2

Values of m, m_0, for $\overline{Y_\ell^m} = 0$

ℓ	m_0
26	9
28	13
30	17
32	21
34	25

1.3.5 The Ratio of Amplitudes from Differential Velocity and Differential Radius Observations

The gravity mode spectrum is much more complex than the low-order, low-degree acoustic mode spectrum. Evidence for the detection of gravity modes was presented by Hill and Caudell (1979) and by Bos and Hill (1983), but the classification of the modes has been hampered by the complexity of the gravity mode spectrum. Hill and Caudell (1979) were able to estimate the magnitude of ℓ only. Hill, Bos, and Goode (1982) identified a subset of two gravity mode multiplets, but were only able to place a lower limit on ℓ using just the differential radius observations. However, the complexity can be managed with the additional information gained on the degree of a mode as a result of the positive results obtained by Hill (1985b) in a test of the Hill, Tash, and Padin (1986) apparent-velocity hypothesis. In this circumstance, the observed constraints on ℓ and m are decoupled.

The development of the apparent-velocity hypothesis described in Section 1.6.2 arose out of a comparison of results obtained with different observing techniques. This hypothesis interprets the velocity signal obtained in differential Doppler shift studies of long-period oscillations as an apparent velocity due to the combined effect of the surface rotation of the Sun and a perturbation in the radiation intensity produced by a normal mode of oscillation. The ratio of this velocity signal V to the observed amplitude of differential radius amplitude Δr_1 can be written according to Hill, Tash, and Padin (1986) as

$$V\left[\frac{m\Delta r_1}{w(m)}\right]^{-1} = -(i)^m \alpha_s \frac{1}{\ell(\ell+1)} \cdot \frac{\overline{Y_\ell^m}(\pi/2,\ 0)}{\overline{Y_m^\ell}(\pi/2,\ 0)} \cdot \int_0^{\pi/2} f[g_0' - 1.067 g_2' \sin^2\theta]\, P_\ell' \sin^2\theta \cos\theta \, d\theta, \qquad (1)$$

Where α_s is a proportionality constant for the symmetric line contribution, $\overline{Y_\ell^m}(\pi/2,0)$ is the average of Y_ℓ^m over the detector geometry used by Bos and Hill (1983), g_0' and g_2' are the effective first and third derivatives with respect to wavelength of $\mathcal{J}'$ in the spectral line, f is the spatial filter function of the velocity detector, and w(m) is the spatial filter function for the differential radius observations given by Hill, Alexander, and Caudell (1985).

The ratio of m/w(m) is only weakly dependent on m for $2 \leq |m| \leq 10$ so that the ratio $V/[m\Delta r_1/w(m]$ is to a good approximation only a function of ℓ. Examples of what might be expected are shown in Figure 2. Thus, information is available on the magnitude of ℓ by combining the results of differential velocity and differential radius observations. An example of the application of this to mode classification is found in the work of Hill (1985b) described in Section 1.4.4.

1.3.6 The Ratio of Amplitudes from Total Irradiance and Differential Radius Observations

A technique has been developed to obtain detailed information on the limb-darkening properties of the oscillatory component of the total radiation intensity associated with an oscillation. This amplitude of the total radiation intensity $\mathcal{J}'$ can be written as

$$\mathcal{J}' = I'(\mu) Y_\ell^m(\theta,\phi) \qquad (2)$$

where μ is the cosine of the heliocentric angle between the observer and the direction defined by (θ,ϕ) and $I'(\mu)$ is only weakly dependent on ℓ. In particular, it is the limb-darkening properties of $I'(\mu)$ that are of

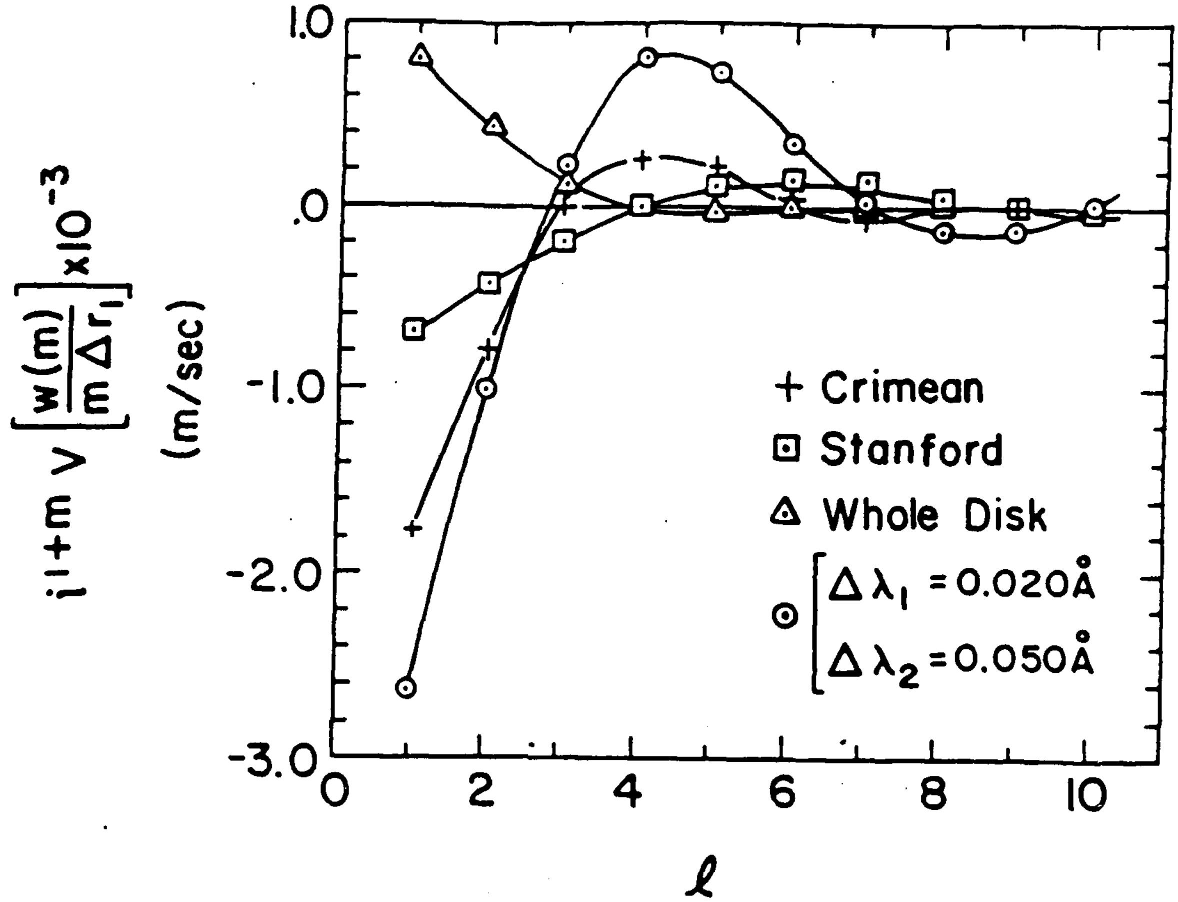


Fig 2 The predicted ratio of the symmetric line apparent velocity signal to the differential radius signal for different velocity detector configurations. The predicted apparent velocity is based on the properties of the FeI 5124 transition. Two curves are for the differential velocity detector configurations used at the Crimean and Stanford observatories. The predicted apparent velocity for the whole disk observations assumed spectrometer specifications which are the same as those of the Stanford differential velocity observations. The remaining curve is for a disk annulus configuration used in the Crimean observations, but with spectrometer slits centered at ± 0.035 Å off line center with slit widths of 0.030 Å.

interest here. With Equation (2), the ratio of the fractional change in total irradiance S'_F/S_F to the differential radius amplitude Δr_1 can be written as

$$[S'_F/S_F][\frac{\Delta r_1}{w(m)}]^{-1} = Q \frac{\int_0^\pi P_\ell^m g'_F \sin\theta\cos\theta d\theta}{\int_0^\pi g_F \sin\theta\cos\theta d\theta} , \quad (3)$$

where $I'(\mu)$ has been expressed as

$$I'(\mu) = I'_0 g'_F (\mu) \quad (4)$$

(see Hill and Kroll 1986). The factors Q and I'_0 are constants. The parameter w(m) is the factor that relates the observed signal Δr_1 to the fractional change in the radiation intensity at the extreme equatorial limb and is given by Hill, Alexander, and Caudell (1985).

The information on the spatial properties of g'_F is contained in the Fredholm integral equations of the first kind given by Equation (3). This information is obtained by the inversion of integral equations for a given set of ℓ.

This technique has been successfully employed by Hill and Kroll (1986) in the inversion of a set of six integral equations for $\ell = 1,\ldots,5$. The resulting g'_F is shown in Figure 3.

1.4 Summary of Modes Classified and Observationally Derived Eigenfrequency Diagram

1.4.1 Low-Degree 5 min Oscillations[2]

Individually resolved low-degree 5 min modes have been identified using the differential radius observations from SCLERA augmented with the information on the eigenfrequency spectrum obtained from Doppler shift and total irradiance observations (Claverie et al. 1981; Grec, Fossat, and Pomerantz 1983; Duvall and Harvey 1983; and Woodard and Hudson 1983). In

2. This is a synopsis of Hill (1985a).

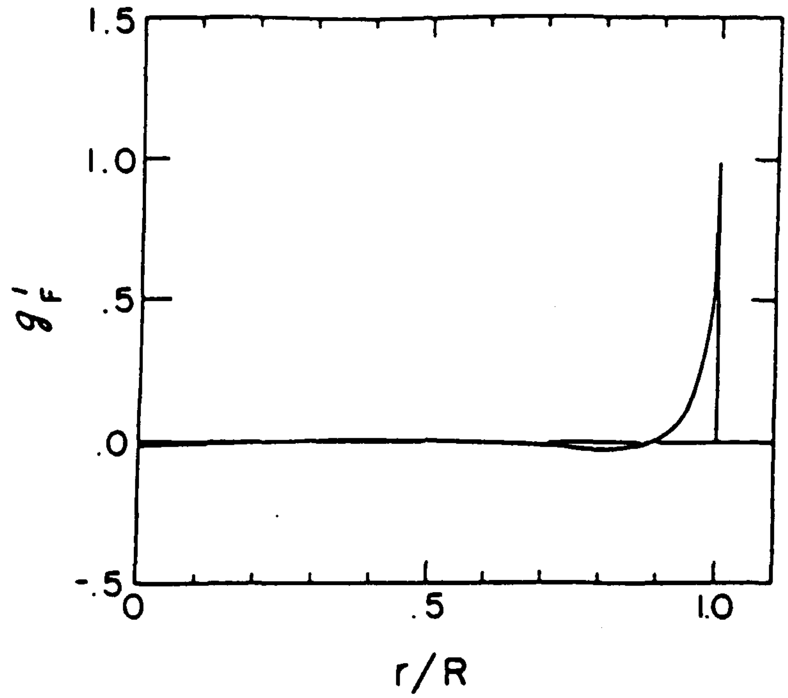


Fig 3 The radial dependence of g'_F obtained by Hill and Kroll (1986) in the inversion of integral Equations of the type given by Equation 3. The observations used were the differential radius observations of 1979, the total irradiance observations of Woodard and Hudson (1983), and the mode classifications by Hill (1985b). The value of ℓ indicated in this analysis are $\ell = 1,\ldots,5$.

particular, independent works on the 5 min oscillations were used to furnish measures of the $m = 0$ eigenfrequencies for the low-degree 5 min modes as a starting point in classifying the peaks in the power spectra of the 1979 differential radius observations. With this information, the $\ell = 0$ and $\ell = 1$, $m = 0$ singlets were identified first, followed by the $\ell = 1$, $m = \pm 1$ doublets. Once the multiplet fine structure was obtained for the $\ell = 1$ multiplet, the analysis was extended to include the $\ell = 2$ multiplet. This procedure was repeated until multiplets with $\ell = 6$ had been classified.

A total of 184 modes were classified belonging to 83 multiplets. The multiplets are indicated in Figure 6 (cf. Section 1.4.6). It should be noted that, in this analysis, consideration was given to the complexities introduced in the power spectra when the mode coherence time is less than the length of the observing period. Also, the symmetry properties of the signals associated with the classified modes furnished a statistically significant test of the classifications.

1.4.2 Low-order, Low-degree Acoustic Modes[3]

The first major mode classification project at SCLERA was concerned with the low-degree acoustic modes for frequencies just above the asymptotic gravity mode limit (see Figure 6 of Section 1.4.6). The reduced complexity of the eigenfrequency diagram in this region and the intrinsic line widths for these modes of ≤ 0.1 μHz were the two factors that dictated this frequency region for study in the first analysis. It was in this project that the fundamental features of the SCLERA mode classification program were developed, as outlined in Section 1.3.

Identification of multiplets was implemented as an iterative process. Initially, the Zeeman-like frequency pattern was sought for a multiplet where the pattern was most easily discernible at statistically significant levels and the associated D_{ij} (see Section 1.3.3) exhibited physically plausible properties. In the multiplet subsequently classified as $p_{1,20}$, a relatively large fraction of the maximum number of peaks possible, $2\ell + 1$, was tentatively identified. The measures of the D_{ij} were used for

3. This is a synopsis of Hill (1984).

identifying the $m = 0$ member of the multiplet. With the identification of the $m = 0$ mode, the classification of n was made by comparing the observed $m = 0$ eigenfrequency to that predicted theoretically with the standard solar model of Saio (1982). When the most likely location of the $m = 0$ mode was defined and the multiplet classified according to n, the frequency pattern of the multiplet was fit, using a least-squares analysis, to a power series cubic in m written as

$$\nu_{n\ell m} = \nu_{n\ell} + \nu'_{n\ell}m + \nu''_{n\ell}m^2/2 + \nu'''_{n\ell}m^3/3! \qquad (5)$$

where $\nu_{n\ell m}$ is the eigenfrequency of the mode specified by (n,ℓ,m). The actual final selection of members of the multiplet was based upon the deviation, $\Delta\nu$, between the frequency of a peak under consideration and the frequency determined by the polynomial fit. The maximum deviation accepted was ± 0.06 μHz, approximately twice the standard deviation of $\Delta\nu$ (see Section IVc of Hill 1984b).

After the analysis of the above multiplet was completed, identification of contiguous multiplets in ℓ was undertaken. Information about the rotational splitting effects linear, quadratic, and cubic in m and about the $m = 0$ mode eigenfrequency obtained from the least-squares analysis of the multiplet fine structure was used in conjunction with the theoretical eigenfrequency spectrum of Saio's (1982) model to predict the parameters of multiplets contiguous in ℓ. With the inclusion of the predicted values of these parameters, the process for the previous multiplet was repeated, adding one constraint: the composite list of observed multiplet parameters must be internally consistent.

The process was reiterated until candidates for $n = 1$ multiplets were identified for $\ell = 4,\ldots,22$, for $n = 2$ multiplets for $\ell = 3,\ldots,10$, and for $n = 3$ multiplets for $\ell = 2$, 3, and 4. A total of 293 modes were classified belonging to 30 multiplets. These multiplets are included in Figure 6 of Section 1.4.6.

1.4.3 Intermediate-Degree f-modes[4]

The mode classification project for the low-order, low-degree acoustic modes took advantage of the minimum in the complexity of the eigenfrequency spectrum found just above the asymptotic g-mode limit (see Section 1.4.2). For this same frequency region, the intermediate-degree f-modes ($n = 0$) are expected based on the theoretical eigenfrequency spectrum of a standard solar model. These modes are of particular interest because they can play an important role in studying the outer ~ 10%, by radius, of the solar interior.

Preliminary evidence of intermediate-degree f-modes was found in a test of the validity of a uniformly rotating convection zone model by Hill, Rosenwald, and Rabaey (1985). In the subsequent work of Hill (1986b), which extended the analysis of Hill, Rosenwald, and Rabaey (1985), the multiplet fine structure for the intermediate-degree f-mode multiplets of angular degree $\ell = 33$ was identified. With this finding as a starting point in a mode classification program similar to that used by Hill (1984b), the study of the intermediate-degree f-modes in the 450-650 μHz region of the spectrum was initiated.

In the classification of the $\ell = 33$ multiplet, the frequencies of modes were fit to Equation (5). The final selection of members of the multiplet was based upon the deviation, $\Delta\nu$, between the frequency of a peak under consideration and the frequency determined by the polynomial fit. After the analysis of this multiplet was completed, the analysis of contiguous multiplets in ℓ was undertaken. Information about the rotational splitting effects linear, quadratic, and cubic in m and about the $m = 0$ mode eigenfrequency obtained from the least-squares analysis of the multiplet frequency pattern was used in conjunction with the theoretical eigenfrequency spectrum of Saio's (1982) model to predict the parameters of the multiplets contiguous in ℓ. With the inclusion of these parameters, the process for the first multiplet was repeated with one added constraint: the composite list of observed multiplets must be internally consistent as measured by the following criteria. These criteria are: (1) the modes

---

4. This is a synopsis of the work by Rabaey and Hill (1986)

exhibit an almost uniformly spaced, Zeeman-like frequency pattern; (2) the modes possess minimum quadratic and cubic terms in the departure from the uniform spacing; (3) the linear, quadratic and cubic terms are consistent with those found for neighboring multiplets in ℓ; (4) the frequency of the axisymmetric mode is consistent with the frequencies of corresponding modes in neighboring multiplets; and (5) the classification of multiplets identified by criteria (1)-(4) is confirmed by the observed horizontal spatial properties of the oscillations as measured by D_{ij}. As noted in Section 1.3.3, D_{ij} is a function of m which allows the confirmation of the $m = 0$ mode of the multiplet.

With this process a set of 18 multiplets and a total of nearly 340 modes were identified and classified with ℓ values from 19 to 36 (Rabaey and Hill 1986.) The resulting $m = 0$ eigenfrequencies $\nu_{0,\ell}$ are indicated in Figure 6 of Section 1.4.6. In Figure 4 the results of the linear term in m, $\nu'_{0,\ell}$, which is due to rotational splitting effects, are shown. The error bars shown are formal errors based upon the least-squares analysis. Since the rotational splitting kernels for the intermediate-degree f-modes are well localized in the outer 10% of the Sun, one can easily verify that these values of $\nu'_{0,\ell}$ are consistent with the rotational curve obtained by Hill, Rosenwald, and Rabaey (1985). In Figure 5 a comparison is made of $N_{0,\ell}/N_{max}$ and p where $N_{0,\ell}$ is the number of modes identified for a given ℓ, N_{max} is the maximum number expected for that multiplet and p is the probability of finding one or more peaks from a random distribution within ± 0.065 μHz of $\nu_{0,\ell,m}$ obtained in the polynomial fits (see Hill 1984b). The values of p range from 0.301 to 0.346 and the values of $N_{0\ell}/N_{max}$ range from a low of 0.321 to a maximum of 0.469. Computing $\bar{d}$, the weighted average of the difference between the observed ratio $N_{0,\ell}/N_{max}$ and p, we find $\bar{d} = 0.088 \pm 0.016$. This mean difference is a 5.4 standard deviation result. Using this number, we can compute the probability that the 18 Zeeman-like frequency patterns were obtained from a randomly distributed set of peaks in frequency. This probability is less than 10^{-6}. This test (which is a sufficient-condition test) further strengthens the case that multiplets have been identified in the power spectra.

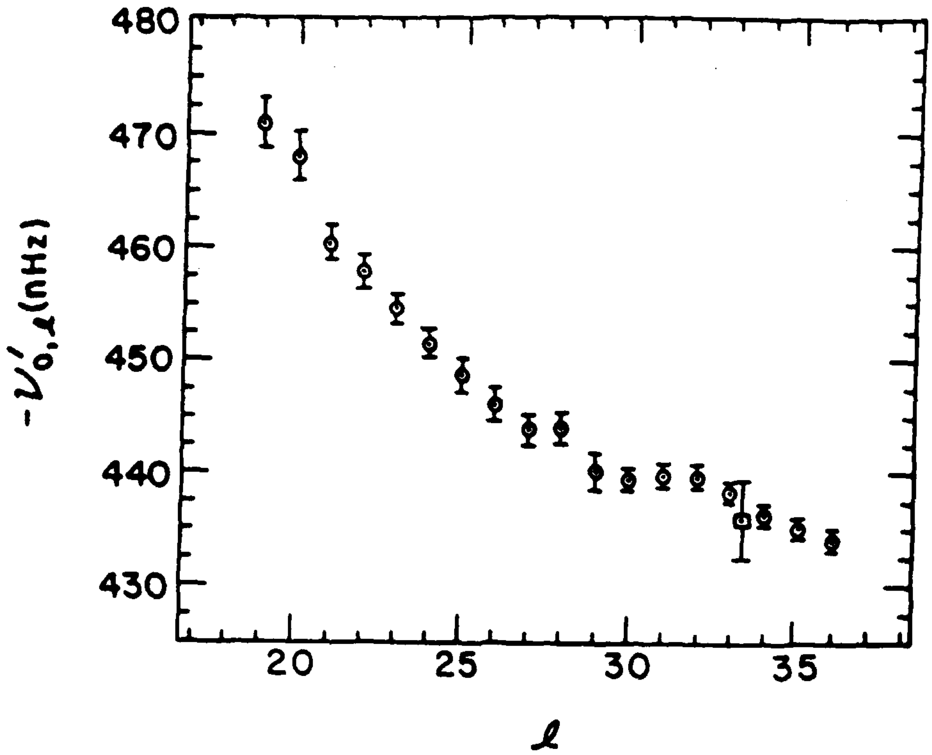


Fig 4 The observed rotational splitting linear in m shown as function of ℓ for intermediate degree f-modes (Rabaey and Hill 1986). Note that the rotational splittings are negative. The point plotted with the square box is the value of $\nu'_{0\ell}$ found for ℓ = 33 by Hill (1986b).

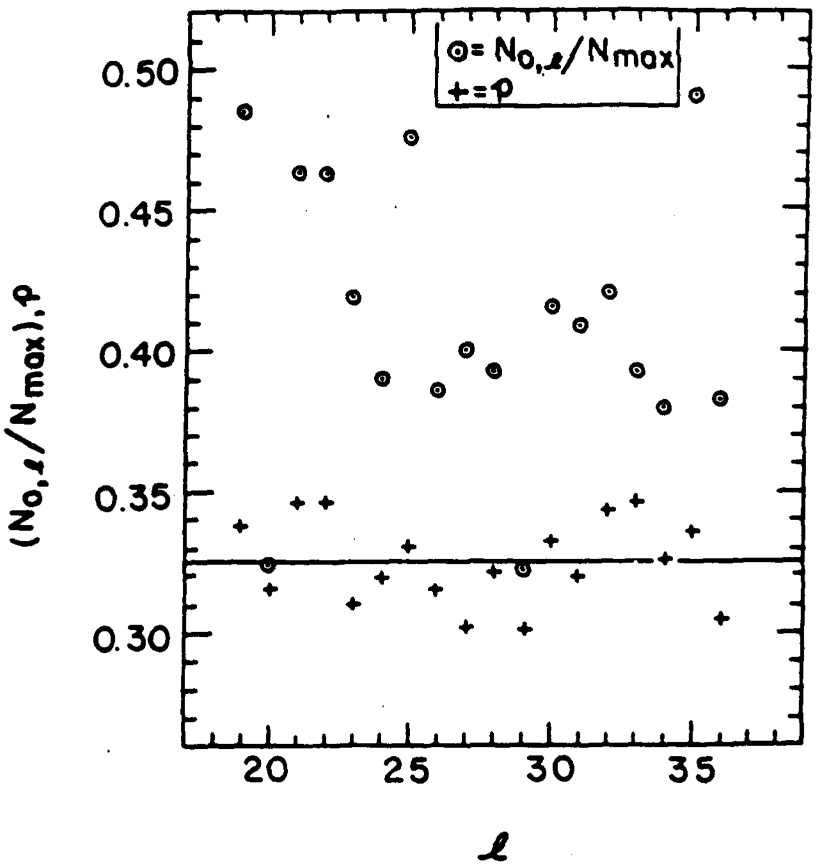


Fig 5 A comparison of the observed ratio $N_{0\ell}/N_{max}$ (circles) and the probability p (+) as a function of ℓ for intermediate degree f-modes (Rabaey and Hill 1986). The quantity $N_{0\ell}$ is the number of peaks classified in a given multiplet and N_{max} is the maximum number possible for that multiplet. The probability p is the chance of finding one or more peaks from a random distribution with ± 0.065 μHz of the frequency $\nu_{0\ell m}$ given by Equation 5. The solid line is the average value of p in this region of the frequency spectrum.

1.4.4 Low-Degree Gravity Modes[5]

The amplitudes of peaks in the power spectra obtained with the differential Doppler studies based on the Crimean observations (Kotov et al. 1983) and with the 1979 differential radius observations of SCLERA have been examined for evidence of low-degree gravity modes with $m = 0$ eigenfrequencies between 77.9 μHz and 132.2 μHz. In this analysis, the families of peaks in the respective power spectra are classified as rotationally split multiplets when (1) they exhibit a Zeeman-like frequency pattern, (2) they possess the minimum quadratic and cubic terms in m in the departure from uniform spacing, (3) the linear, quadratic, and cubic terms are consistent with those found for neighboring multiplets in n and ℓ, (4) the frequency of the axisymmetric mode is consistent with the frequencies of corresponding modes in neighboring multiplets, (5) the ratio of the differential velocity and differential radius amplitudes is confined to a unique value for each value of ℓ (see Section 1.3.5), and (6) the classification of multiplets identified by criteria (1)-(5) is confirmed by the observed horizontal spatial properties being characterized by m (see Section 1.3.3).

The internal rotation of the Sun implied by the rotational splitting results of Hill, Bos, and Goode (1982) and Hill (1984b, 1985a) was used as a starting point in the outlined search for identification and classification of internal gravity modes. The initial value assumed for the rotational splitting between contiguous modes of a gravity mode multiplet was taken as $-2.9[1 - 1/\ell(\ell + 1)]$ μHz. This approximate expression for the ℓ-dependence is based on the theoretical properties found, for example, in the work of Gough (1981). Should sets of peaks be found with a self-consistent set of Zeeman-like frequency patterns and if all of the other properties considered in the search with regard to ℓ and m are satisfied, the search for other solutions need not be carried out. This is because the observed spatial properties are expected to establish at a statistically significant level the uniqueness of a given identification and classification.

---

5. This is a summary of the work by Hill (1985b).

The result of this analysis was the classification of 31 gravity mode multiplets with $1 \le \ell \le 5$. Of a maximum possible number of 235 modes belonging to the 31 multiplets, 152 were classified. These multiplets are included in the tabulations of Section 1.4.6.

1.4.5 Intermediate-Degree Gravity Modes[6]

The first evidence of the excitation of intermediate-degree solar gravity modes was obtained in the 1970s by Hill and Caudell (1979). They found evidence which indicated the excitation of a number of the gravity modes with frequencies near 250 μHz and 370 μHz and $20 < \ell < 40$. These results were obtained using the 1973 solar diameter observations. With the availability of the 1979 differential radius observations, with their improved signal-to-noise ratio, and the development of the SCLERA mode classification program, four gravity mode multiplets have been identified with $\nu_{n,\ell} \approx 350$ μHz and of intermediate degree ($\ell \approx 30$). The preliminary results have been reported by Hill (1986c).

The criteria used in the identification of intermediate-degree g-mode multiplets were quite similar to those used by Hill (1984b) in the study of low-order, low-degree acoustic modes. The principal differences have to do with the starting values of $\nu'_{n,\ell}$ used in the search for multiplets and the method used to determine the $m = 0$ mode and the degree of the multiplet. For this work, the values of $\nu'_{n,\ell}$ obtained by Hill, Bos, and Goode (1982) and Hill (1985b) were used as the initial values assumed in a search for evidence of multiplets. The difference with regard to the assignment of m and ℓ arose because of the limited number of multiplets included in the current analysis, the complexity of the g-mode spectrum, and the reduced effectiveness of D_{ij} (cf. Section 1.3.3) in the study of intermediate-degree g-modes. Its reduced effectiveness in this study is due in part to the closeness of $\nu'_{n,\ell}$ to 4/d (see Hill, Bos, and Goode 1982; Hill 1985b) and in part to the effect of atmospheric seeing on the D_{ij} for $m > 20$ (see Section IIIa of Hill, Alexander, and Caudell 1985).

---

6. Synopsis of work by Hill (1986c)

The technique used to identify the m = 0 mode and to determine the value of ℓ for a given multiplet was based in that analysis on the properties of $\overline{Y}_\ell^m$, where $\overline{Y}_\ell^m$ is the average of Y_ℓ^m over the detector geometry used by Bos (1982) (see Bos and Hill 1983), as discussed in Section 1.3.4. The tentative assignments of n, ℓ, m, and $\nu_{n,\ell}$ for the multiplets for which mode-coupling evidence was found are given in Table 3. The standard solar model of Saio (1982) was used in the assignments of n.

Table 3

Phase-locked Gravity Mode Multiplets
(tentative assignments)

Classification	$\nu_{n,\ell}$ (μHz)
$g_{14,28}$	341.81
$g_{15,30}$	341.43
$g_{13,30}$ or $g_{15,34}$	≈ 350
(ℓ ≈ 32)	≈ 350

1.4.6 Observationally Derived Eigenfrequency Diagram

It is quite common in the study of the solar 5 min oscillations to display the eigenfrequencies in a k - ω diagram. In practice, this corresponds to plotting the eigenfrequency as a function of the wavenumber k which is $\propto [\ell(\ell+1)]^{1/2}/R$. Such information, traditionally based on Doppler shift observations, is typically confined to frequencies between 2 mHz and 4 mHz.

The mode classification program at SCLERA has made it possible to extend the observed k - ω diagram from 2 mHz to 80 μHz for low- and intermediate-degree acoustic, f-, and gravity modes. This mode classification program used the differential radius observations from

SCLERA, the differential velocity and infrared observations from the Crimean Astrophysical Observatory for long period oscillations, a series of velocity observations of the 5 min oscillations, and the total irradiance observations.

The results of this mode classification program are summarized in Table 4, which shows the mode type, order, degree, and subsections of Section 1.4 where the analysis is presented. The domain of the results are also indicated in Figure 6 in a $\nu_{n,\ell}$ - ℓ diagram analogous to the k - ω diagram discussed above. To date, 166 multiplets have been identified and ≈ 1209 resolved modes classified. The observed m = 0 eingenfrequencies are found to be ≈ 10 μHz above the theoretical eigenfrequencies for the acoustic and f-modes and to be ≈ 1 μHz above the theoretical eigenfrequencies for the low-degree gravity modes.

Table 4

Mode Type	Subsection	Order n	Degree ℓ	Number of Multiplets	Number of Modes
acoustic	1.4.1	$12\leq n\leq 27$	$0\leq \ell\leq 6$	83	184
acoustic	1.4.2	1,2,3	$2\leq \ell\leq 22$	30	293
f	1.4.3	0	$19\leq \ell\leq 36$	18	340
gravity	1.4.4	$5\leq n\leq 29$	$1\leq \ell\leq 5$	31	152
gravity	1.4.5	n = 15	$28\leq \ell\leq 34$	4	240

1.5 Tests

The tests of the SCLERA observations and of the results from the multiplet classification program fall into two classes: comparisons of results based on two different years of SCLERA observations, and comparisons of SCLERA observations with those based on observations obtained at different observatories using different observing techniques.

There are four sets of SCLERA observations that have been analyzed during the last five years. The observations are those made in 1978 by Caudell (1980), in 1979 by Bos (1982), in 1983 by Yi and Czarnowski (1986), and in 1985 by Oglesby (1986). The 1979 observations have been analyzed by

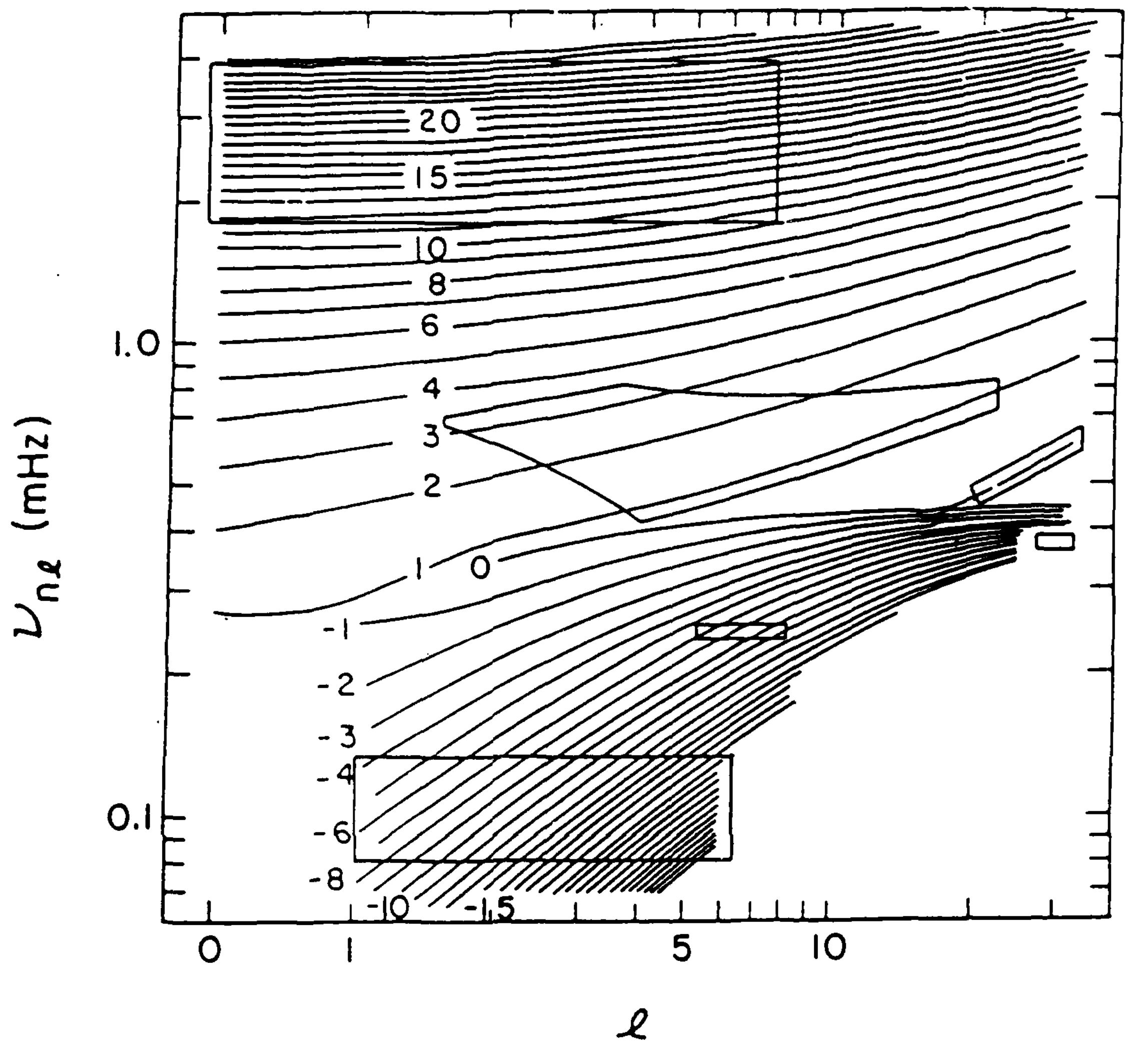


Fig 6 The theoretical m = 0 eigenfrequency spectrum based on the standard solar model of Saio (1982). The enclosed areas represent the domain of the mode classifications that have been obtained in the SCLERA mode classification program (see Table 4). The acoustic and gravity modes are indicated by positive and negative radial order numbers, respectively.

Hill (1984b) and 30 low-order, low-degree acoustic mode multiplets classified (cf. Section 1.4.2). The 1978, 1983, and 1985 observations have been compared to the 1979 observations by testing for signals in these former three sets of observations at the eigenfrequencies $\nu_{n\ell m}$ derived from the 1979 observations. Statistically significant positive results have been obtained in each of the three comparisons. These findings, reviewed in Sections 1.5.1, 1.5.2, and 1.5.3, respectively, are the results of particularly stringent tests of the quality of the data and the accuracy of the mode classification program.

Results based on the 1979 SCLERA observations are also compared with results obtained elsewhere: with infrared observations made at the Crimean Astrophysical Observatory, with the total irradiance observations made on the Solar Maximum Mission (SMM), and with a series of velocity observations of the 5 min oscillations. Each of these comparisons, also reviewed below, leads to positive results.

1.5.1 Comparison of 1978 and 1979 SCLERA Observations[7]

The 1978 solar diameter observations of Caudell et al. (1980) have been examined for evidence of the low-order, low-degree acoustic modes found by Hill (1984b) in the 1979 observations of Bos and Hill (1983). The power spectrum of the 1978 observations was tested for evidence of (1) a non-random distribution of peaks relative to the eigenfrequency spectrum found by Hill (1984b), (2) spatial symmetry properties of the eigenfunctions for the modes classified by Hill (1984b), and (3) a θ-dependence, specified by angular order m, of the amplitudes of oscillations, a, for these same modes. The characterization of the symmetry properties of the eigenfunctions and the values of m were taken from the work of Hill (1984b). The findings of these three tests confirm the detection of modes and their classifications as given by Hill (1984b).

The first step in the comparison of the 1978 and 1979 observations was to ascertain whether the power spectrum for the independent set of 1978

---

7. This is a synopsis of the work by Hill and Caudell (1985)

observations consisted of a set of peaks randomly distributed in frequency relative to the multiplet eigenfrequencies found by Hill (1984b). The negative result obtained represents a confirmation at the 2.7 σ level of the multiplet classifications of Hill (1984b) (cf. Sections IIIc and IIId of Hill and Caudell 1985). The second step was to determine whether the spectrum for the 1978 observations was consistent with the symmetry properties expected for the eigenfunctions of the multiplet modes classified by Hill (1984b). The spectrum was found to be consistent at the 4.1 σ level. A third independent test substantiates the isomorphism found by Hill (1984b) between the observed square amplitude of the modes and $|m|$. The results of this test support the above confirmations. Further, the positive cross-correlation coefficient obtained between the respective $\langle a^2 \rangle$ establishes, at a 1.8 σ level, further support for the conclusions by Hill (1984b) regarding multiplet detection and classification.

These three results, one at a confidence level of 2.7 σ, a second at 4.1 σ, and a third at 1.8 σ, are all independent. This is apparent from the fact that, in one case, a search was made for peaks at frequency locations where they can be present on symmetry grounds and in the second case, a search was made for peaks at frequency locations where they cannot be present on symmetry grounds. In the third case, a search was made for an isomorphism between the detected amplitude and $|m|$. Thus, when combined, these three independent results present a very strong test of the internal consistency of the multiplet classification program with positive results.

1.5.2 Comparison of 1983 and 1979 SCLERA Observations[8]

The 1983 data was obtained with the same detector used in 1979 at SCLERA. However, in 1983 the detector was rotated periodically so that the edge of the solar disk could be sampled at eight different positions. These eight positions were chosen to lie on four solar diameters which were oriented at -90, -45, 0, and 45 degrees from the projected solar axis.

---

8. This is a synopsis of the work by Yi (1986). Preliminary results have been reported in Yi and Czarnowski (1986).

The 1983 data spans 52 days, from 30 May 1983 to 20 July 1983. The actual analysis used a total of 319.5 hours of data drawn from 44 days.

The detector was rotated to a new diameter every 64 seconds. At each diameter, the solar edge was scanned 8 times. Since the detector has 6 slits arranged in a pattern that is symmetric with respect to the solar diameter (see Bos and Hill 1983), 48 limbs were sampled and recorded each time the detector was rotated to a new position. Thus, for the equatorial position (and each of the other positions) 8 limbs were recorded every 256 seconds.

The techniques used at SCLERA to obtain the FFTD lock-on points for each limb have been extensively discussed elsewhere (cf. Bos and Hill 1983). The spectra analyzed were obtained with a standard FFT program applied to the lock-on points.

Previous work at SCLERA (cf. Hill 1984b) produced a set of solar acoustic mode eigenfrequencies based on the analysis of the 1979 data. The 1983 data were examined for evidence of these modes in order to further test the mode classification program and, should a positive result be obtained, possibly learn something about stability of the frequencies of the low-order, low-degree acoustic modes over a five-year period.

In this analysis, the previously classified $n = 1$ multiplets with $\ell = 4,\ldots,22$ were divided into three groups: 1) $\ell = 4,\ldots,9$; 2) $\ell = 10,\ldots,16$; and 3) $\ell = 17,\ldots,22$. It has been found that slight shifts of the eigenfrequencies based on the 1979 observations match at a statistically significant level a set of peaks in the 1983 spectra. The shifts in frequencies which were found effective are -0.02, 0.03, and 0.05 μHz for groups 1, 2, and 3, respectively. A tolerance of ±0.06 μHz was used to determine whether a peak in a spectrum matched a given eigenfrequency.

A second comparison of the 1983 and 1979 observations was made based on an assumed $(\exp im\phi)$-dependence of the eigenfunctions. The information on the ϕ-dependence is contained in D_{ij}, which is a function of m (cf. Section 1.3.3; Hill 1984b; Hill, Alexander, and Caudell 1985). These were divided into three groups where $|m| = 0,\ldots,7$; $|m| = 8,\ldots,14$; and $|m| = 15,\ldots,22$ and the weighted average $\langle D_{ij}\rangle$ are 0.125 ± 0.043, -0.0079 ± 0.074, and 0.476 ± 0.034 for the three groups, respectively. The $\langle D_{ij}\rangle$'s obtained for the three groups are consistent at a statistically significant level with the corresponding quantities calculated using the 1979 observations.

Also, in previous work at SCLERA (cf. Hill 1984b), it has been shown that another quantity -- R_{ij} -- can be used in determining the orientation of an axis of symmetry in the Sun. Use was made of this quantity in analyzing the 1983 data. A total of 182 acoustic modes with $\ell = 4,\ldots,22$ were identified in the 1983 data. Those with $\ell = 8, 9, 10, 11, 12$ were used to calculate the orientation of the internal axis of symmetry. A total of 29 modes with these ℓ values were found. The angle of the axis of symmetry from the pole, calculated from the R_{ij}'s observed for these modes, is 0.2 ± 0.8 deg.

1.5.3 Comparison of 1985 and 1979 SCLERA Observations[9]

SCLERA has been involved since 1984 in expanding its differential radius observational program with the introduction of a new technique to search for global solar oscillations in the visible to near infrared continuum (cf. Section 1.7). In 1985, data was collected using this new technique.

Two hundred ninety-one hours of data were collected in a 70-day period from 23 March to 31 May 1985. A 200 x 200 arcsec aperture was centered on the solar disk and the intensity at 48 different wavelengths was measured within the spectral range of 0.5 to 1.7 μ. The radiation intensities observed at these wavelengths were Fourier analyzed and the resulting power spectra examined for evidence of the low-order, low-degree acoustic modes identified in the 1979 SCLERA observations by Hill (1984b).

The eigenfrequency spectrum for $n = 1$, $\ell = 7,\ldots,22$ multiplets was divided into the three groups $7 \leq \ell \leq 12$, $13 \leq \ell \leq 17$, and $18 \leq \ell \leq 22$. The eigenfrequencies of the modes belonging to a group were shifted in frequency as a group to search for a maximum coincidence rate between the peaks in the power spectra of the 1985 data and the frequencies of the previously classified modes. A maximum allowable shift was ± 0.10 μHz. Maxima in the coincidence rates were found for the frequency shifts of -0.06, 0.01, and 0.04 μHz, respectively.

---

9. This is a synopsis of work by Oglesby (1986). Preliminary results have been reported in Oglesby (1986).

The mean difference $\bar{d}$ between the observed coincidence rate and the expected rate between two distributions that are randomly distributed with respect to each other was calculated. This yielded a value for $\bar{d}$ for each of the three groups of multiplets of

$$\bar{d} = \begin{cases} 0.214 \pm 0.046; & 7 \le \ell \le 12 \\ 0.145 \pm 0.040; & 13 \le \ell \le 17 \\ 0.132 \pm 0.035; & 18 \le \ell \le 22 \end{cases} \quad . \qquad (6)$$

These values of $\bar{d}$ are 4.7, 3.6, and 3.8 standard deviations, respectively, above zero. The probability of obtaining a particular positive $\bar{d}/\sigma$ where the $\nu_{n\ell}$ of the classified modes for the three bins has been allowed to vary by ±0.10 μHz is estimated to be within an order of magnitude of

$$p \lesssim \begin{cases} 1 \times 10^{-5}; & 7 \le \ell \le 12 \\ 2 \times 10^{-3}; & 13 \le \ell \le 17 \\ 4 \times 10^{-4}; & 18 \le \ell \le 22 \end{cases} \quad . \qquad (7)$$

For the three groups of multiplets described above, the positive results at the 4.7σ, 3.6σ, and 3.8σ level represent a highly statistically significant result which leads to a rejection of the hypothesis that the peaks in the power spectrum of the 1985 data are randomly distributed with respect to the classified eigenfrequency spectrum based on the 1979 observations. This represents a particularly important finding since the confirmation comes from an independent set of observations using a different observational technique.

It was also noted in the analysis that apparently the eigenfrequencies underwent a slight frequency shift dependent on ℓ between 1979 and 1985. Since the multiplets discussed here reside primarily in the convection zone, knowledge of any change of these eigenfrequencies may yield interesting information about the level of variations in the properties of the convection zone over, for example, the solar cycle.

Recently, f-modes have been identified in the 1979 data (cf. Section 1.4.3). An analysis similar to the one above is underway to determine if a correlation for these modes exists between 1979 and 1985. Preliminary results indicate agreement at the 3σ level.

1.5.4 Comparison of Infrared and Differential Radius Observations[10]

The well-documented 160 min period solar oscillation (Kotov et al. 1983) has been classified by Hill, Tash, and Padin (1986) as an internal gravity mode with $\ell = 2$ and $m = 2$ where ℓ is the degree and m the angular order of the eigenfunction. This particular classification was determined by comparing the results obtained on the 160 min period oscillation from differential velocity observations made at the Crimean Astrophysical Observatory and from differential radius observations made at SCLERA. The ℓ and m classification has subsequently been confirmed in the work of Hill (1985b), in which the 160 min period oscillation is further classified as the $m = 2$ member (retrograde) of the $g_{9,2}$ multiplet.

Differential infrared observations of the 160 min period oscillation were made at the Crimean Astrophysical Observatory in 1977 and 1978 by Kotov and Koutchmy (1979) and in 1981 by Kotov, Koutchmy, and Koutchmy (1983). The above m classification for this mode was tested further by comparing these infrared observations with the differential radius observations made at SCLERA in 1979. Such an analysis involves comparison of relative amplitudes and phases of signals obtained at the two observatories. Besides furnishing an additional test to the $g_{9,2,2}$ classification, a comparison like this presents the opportunity to study further the properties of the 160 min period temperature eigenfunction in the photosphere.

For this comparison, predicted infrared signals based on the SCLERA 1979 observations are compared to those observed, assuming $|m| = 2$, the value assigned to the 160 min period oscillation by Hill, Tash, and Padin (1986). The amplitude and phase of the predicted 160 min period signal for the 1977-1978 observations are in good agreement with the signal observed. The predicted amplitude agrees with the amplitude found in the 1981 observations; however, no such agreement is found for the relative phases. On the other hand, the 1977-1978 infrared observations are of a much higher statistical quality than are the 1981 observations. If the 1977-1978 and 1981 observations were weighted according to observing time alone (leaving out the important difference in frequency resolution), the relative weights

10. This is a synopsis of the work by Hill (1986a).

would be 4.9 to 1. Taking this weighting into account, it is concluded that both amplitudes and phases of observed infrared signals are in good agreement with those predicted. Furthermore, the relative phases of the 1977-78 infrared observations and the 1979 differential radius observations are consistent with respect to phase only if $|m| = 2$. We thus have an independent confirmation of the $|m| = 2$ assignment for the 160 min period oscillation made by Hill, Tash, and Padin (1986).

1.5.5 Comparison of Total Irradiance and SCLERA Observations[11]

A power spectrum of total irradiance observations has been presented by Woodard and Hudson (1983) in the frequency range of gravity modes. For the same frequency region, a number of gravity modes have been classified by Hill (1985b) (cf. Section 1.4.4) using differential radius observations and the differential velocity observations of Kotov et al. (1983). A comparison has been made between the total irradiance power spectrum and the classified gravity mode spectrum to test the mode classification program and to test for evidence of gravity modes in the total irradiance observations. In a second test for such evidence, a comparison is made between the amplitudes of signals in the total irradiance and differential radius observations. The first test represents a study of the frequency properties of the observed signals while the second test represents, because of differences in the spatial filter functions of the two different types of observations, a study of the horizontal spatial properties of the observed signals.

The comparison of the classified gravity mode spectrum with the total irradiance power spectrum entailed the examination of the peaks in the total irradiance power spectrum and the frequencies of the classified gravity modes. It was observed that no two or more members of the set of gravity mode frequencies have a common nearest peak in the total irradiance spectrum, and in 10 of the 11 cases, the magnitude of the difference between the gravity mode frequencies and the total irradiance peak frequencies was ≤ 0.08 μHz. The root-mean-square deviation was 0.065 μHz.

---

11. This is a synopsis of the work by Hill and Kroll (1986).

The statistical significance of the frequency association described above was determined by estimating the probability of obtaining the same association between two unrelated sets of computer-generated frequencies. Based on 162,055 sets of such frequencies, it is inferred that there is a probability of

$$p = 0.020 \tag{8}$$

for obtaining the observed frequency alignment and association between the total irradiance spectrum and the relevant portion of Hill's (1985b) classified gravity mode spectrum if the two spectra are unrelated. This small percentage is evidence in support of the correctness of the gravity mode classification of Hill (1985b) and of the importance of the effects of gravity modes in the total irradiance spectrum.

In general, the ratio of amplitudes of an oscillation obtained in two different types of observations will depend on both ℓ and m of the mode, principally because of different detector spatial filter functions. Thus, if the relevant modes of oscillations have not been classified, the comparison of two different types of observations can be very difficult. However, if the gravity mode spectrum classified by Hill (1985b) is correct, it can be used to reduce the ratio of amplitudes obtained in two different types of observations from a two-parameter function to a one-parameter function. Such a reduction in independent variables could make it possible to test further for the correctness of the mode classification program and for the presence of gravity mode signals in the total irradiance observations.

The ratios of the amplitudes of signals for total irradiance observations and differential radius observations were examined for agreement with those expected based on the mode classifications of Hill (1985b). It was found in this analysis that the probability of obtaining the observed correlation of relative amplitudes is

$$p \leq [illegible] \times 10^{-3} \tag{9}$$

if the mode classifications were incorrect and/or if the total irradiance power spectrum contains no evidence of gravity modes.

The results of the second test, combined with the independent findings of the first test, furnish a quite statistically significant result in terms of the correctness of the gravity mode classification in both ℓ and m by Hill (1985b) and a quite statistically significant demonstration that evidence of gravity modes is present in the total irradiance power spectrum.

1.5.6 Test of a Uniformly Rotating Convection Zone Model[12]

Considerable discrepancy exists among the reported determinations of rotational splitting in the eigenfrequency spectrum of the Sun. The values obtained by Claverie et al. (1981) and Woodard and Hudson (1983), and their inconsistency with the results of Hill, Bos, and Goode (1982) and Hill (1985a), are discussed in Hill (1985a). The sharpest discrepancy is found between the results of Duvall and Harvey (1984), Brown (1985), and Libbrecht (1986), obtained for 5 min oscillations, and those of Hill (1984, 1985a,b), obtained for 5 min, ≈ 27 min, and ≈ 160 min oscillations. The works of Duvall and Harvey (1984), Brown (1985), and Libbrecht (1986) use Doppler shift observations, while the three works of Hill use differential velocity observations, made at the Crimean Astrophysical Observatory (Kotov et al. 1983), and differential radius observations made at SCLERA. The resolution of this disagreement must depend upon an understanding of the complex details of the respective data acquisition systems and the assumptions and procedures used in analysis. Although this discrepancy has not yet been resolved, a test has been made by Hill, Rosenwald, and Rabaey (1985) and extended by Hill (1986b) of the model that the internal rotation of the convection zone is correctly given by the rotation curve obtained by Duvall et al. (1984).

It is apparent from the properties of the splitting kernels that the internal rotation curve obtained by Duvall et al. (1984) predicts a synodic rotational splitting of ~ 0.425 μHz for the low-order, low-degree acoustic mode and intermediate-degree f-mode multiplets with eigenfrequencies between 450 μHz and 600 μHz. This feature of the Duvall et al. (1984) curve, which is common to all essentially uniformly rotating models of the convection

12. This is a synopsis of the work by Hill (1986b).

zone, makes it particularly easy to test for the correctness of this class of models. Note that, for a sizable fraction of the observed signals, the frequency spacings between power spectra peaks due to these oscillations should be either mod - 0.425 μHz or mod - 0.85 μHz, depending upon the spatial filtering properties of the signal detector. This follows as a direct consequence of the fact that, for oscillations whose coherence times are longer than the observing time span, the shape of the power spectrum for each oscillation detected is determined completely by the observing window function, i.e., it is the same for each of these individual modes of oscillation. A mod - 0.425 μHz or mod - 0.85 μHz characteristic peak spacing is a relatively easy feature to identify.

The results of the search for evidence of uniformly rotating convection zone models are negative. At the 10.6 σ level, no evidence for an essentially uniformly rotating convection zone was found if

$$\langle \Omega_{eq}(r)/2\pi \rangle \leq 0.425 \ \mu\text{Hz} \ , \qquad (10)$$

and at the 4.8 σ level, no evidence was found for an essentially uniformly rotating convection zone if

$$\langle \Omega_{eq}(r)/2\pi \rangle \leq 0.438 \ \mu\text{Hz} \qquad (11)$$

where Ω_{eq} is the rotational speed at the equator.

1.6 Comparison of Results from Different Types of Observations: Work on Resolution of Problems

One of the more important activities at SCLERA has been the search to understand some of the problems that have arisen in comparisons of results from different observations. The problems addressed to date concern the differences in reported multiplet fine structure and in inferred internal rotation, the varied results on the 160 min oscillation, the relatively large scatter in the reported eigenfrequencies of the low-degree five-min oscillations, and the reported detection of intermediate-degree gravity modes. This work, which is summarized in the following sections, has served to more clearly define the features of several of the problems and to introduce interesting hypotheses for some others.

1.6.1 Multiplet Fine Structure and Inferred Internal Rotation

The identification of the multiplet fine structure due to slow internal rotation of the Sun has presented many more problems than anticipated, and as a result the analysis technique of Section 1.3.1 is not as easy to implement as once thought. In particular, there is widespread observational disagreement among the reported determinations of rotational splitting in which there are essentially three different sets of findings (see Hill, Bos, and Goode 1982, Hill 1984b, 1985a,b, and Rabaey and Hill 1986 for one finding; Duvall and Harvey 1984, Brown 1985, and Libbrecht and Zirin 1986 for a second; and Duvall, Harvey, and Pomerantz 1986 for a third). The first set of findings is essentially based on the differential radius observations of typically longer period oscillations, the second set of findings is based on Doppler shift observations of the five min oscillations, and the third set of findings is based on intensity observations within an absorption line of the five min oscillations.

The dissonance of the information about rotational effects and the apparent grouping of the different findings according to observing technique demonstrates the merits of employing different observational techniques in testing the validity of a given interpretation of a set of observations. The need for continued diversity in observational technique for future studies is vital for the resolution of this difficulty and for the further development of solar seismology. It is in this spirit that a number of different analyses have been undertaken with regard to the identification of the proper rotational splitting effects.

The analyses fall into several categories. They are: (1) the determination of the multiplet fine structure for low-degree five min modes, low-degree acoustic modes, intermediate-degree f-modes, and low- and intermediate-degree gravity modes using primarily the differential radius observations from SCLERA based on the SCLERA mode classification program; (2) the incorporation of the observations obtained using different observing techniques into the SCLERA mode classification program; (3) the inferring from these results the implied internal rotation and the determination of whether the observed multiplet fine structure is consistent with a single internal rotation curve for the Sun, (4) the comparison of the results obtained from one year's observation with those of another as a check on the mode classification program, and (5) the introduction of a new observational technique to enlarge the number of different types of observations used to study this problem.

The results from the works that fall into these five categories have not led to a resolution of the rotational splitting problem but have served to more clearly define the nature and extent of the difficulty.

The multiplet fine structure for the low-order, low-degree acoustic modes, intermediate-degree f-modes, and intermediate-degree gravity modes obtained in the analysis of the 1979 differential radius observations is taken from Hill (1984b), Rabaey and Hill (1986), and Hill (1986c). The $m = 0$ eigenfrequency for the modes is discussed in Sections 1.4.2, 1.4.3, and 1.4.5, respectively.

The multiplet fine structure for the five min modes has been obtained in an analysis based on differential radius observations combined with a number of Doppler shift observations and total irradiance observations (Woodard and Hudson 1983, Claverie et al. 1981, Grec, Fossat, and Pomerantz 1983, Duvall and Harvey 1983, and Scherrer et al. 1983).

The multiplet fine structure for the low-degree gravity modes has been obtained in an analysis based on differential radius observations combined with the differential velocity observations made at the Crimean Astrophysical Observatory (Kotov et al. 1983). The basis of this analysis is discussed in Section 1.6.2.

The degree of internal consistency of the multiplet fine structure has been examined in a series of works (Hill et al. 1984, Hill, Rosenwald, and Rabaey 1985, and Hill et al. 1986), in which each succeeding analysis included the input of a larger number of multiplets. In the most recent work (Hill et al. 1986), the multiplet fine structure for 30 low-order, low-degree acoustic modes, for 83 low-degree 5 min multiplets, and for two intermediate-degree gravity mode multiplets was used. These analyses indicated that the multiplet fine structure obtained for the multiplets is consistent with a single internal rotation curve for the Sun. They also indicate that the multiplet fine structure obtained for the intermediate-degree f-modes is consistent with this internal rotation curve.

The efficiency of the multiplet classification program that yielded the multiplet fine structure has also been tested by comparing the power spectra of observations obtained in different years and with different observing techniques. These tests are described in Sections 1.5.1, 1.5.2, 1.5.3, and 1.5.5. It is the first three of these four tests that furnish the most critical test of the multiplet classifications. This is because these tests are concerned with the low-order, low-degree acoustic modes which have a coherence time < 1 y. Consequently, these tests are not seeking to determine if one power spectrum is random or not random relative to another power spectrum (which would not test the mode classifications), but are trying to ascertain if a power spectrum is random or not random relative to the frequency spectrum of the classified modes based on the 1979 observations. As noted in Sections 1.5.1, 1.5.2, and 1.5.3, positive results were obtained in each of the three tests, indicating a high level of proficiency for the mode classification program.

These series of analyses and tests have yielded an observationally based multiplet fine structure that is internally self-consistent. Further, the structure is observed to be stable from one year to another. There is one common feature of the observations used in this analysis that may be relevant to defining the nature of the reported differences in the multiplet fine structure. That feature is that the differential radius observations, the differential velocity observations (see Sections 1.4.4 and 1.6.2), and the total irradiance observations are sensitive primarily to the temperature eigenfunctions of the oscillations.

1.6.2 Apparent-Velocity Hypothesis.[13]

Comparing observations of oscillations detected by Doppler shift measurements with ones detected by solar diameter measurements was first thought to involve only ξ_r/r, the fractional change in the radius and its accompanying horizontal displacement, ξ_h, produced by an oscillation. The only complexity in this case would arise from the radial dependence of ξ_r/r

13. This is taken from the work of Hill, Tash and Padin (1985).

and ξ_h/r and the angular dependence of the oscillation given by the spherical harmonic Y_ℓ^m.

It is now clear, however, that another dimension introduced further complexities. Examination of the observed amplitudes, from both Doppler shift and differential radius measurements, reveals yet another significant internal discrepancy. As noted above, the first such discrepancy led to the recognition that the solar diameter measurements were detecting only shape changes in the limb-darkening function. The second major discrepancy that has been identified by Hill and Tash (1984) and Hill, Tash, and Padin (1986) between observed amplitudes and theory led to a reexamination of the physical mechanism by which the Doppler shift measurements detect the long-period oscillations. It was quickly discovered in this reexamination that the apparent Doppler shift, due to the contribution of a perturbation in the radiation intensity found in the differential radius measurements and the equatorial surface velocity, was large enough to give rise to apparent velocities of fractions of m/sec for the nonradial oscillations, velocities of the same order as those reported in Doppler shift studies (Hill, Tash, and Padin 1986).

The hypothesis has been examined that the primary physical mechanism by which the 160 min long-period solar oscillation is detected via Doppler shifts involves temperature perturbations conjoined with solar surface rotation, and not physical displacement of the solar surface. The formalism has been developed by Hill, Tash and Padin (1986) for computing the apparent Doppler shifts from the temperature perturbations obtained in diameter and differential radius observations. Calculations take into account opacity effects based on a non-LTE analysis of the FeI 5124 transition used in the differential velocity observations. A symmetric line profile is not assumed. The predicted apparent velocities are obtained with uncertainties of +120%, -40%, and, for physically plausible values of ℓ, the predicted values are consistent with observed velocities. The observed velocities for the Crimean and Stanford observations of the 160 min oscillation are 0.58 m/sec and 0.17 m/sec, respectively. If $\ell = 1$, the predicted apparent velocites are 0.343 and 0.13 m/sec, respectively. If $\ell = 2$, the predicted apparent velocites are 0.26 and 0.14 m/sec, respectively. Good agreement is also obtained when comparing phases of $\ell = 2$. The observed phase of the differential radius observations is -82° ± 30°; the predicted values are either 0° or 180° for $\ell = 1$, $|m| = 1$; and ±90° if $\ell = 2$, $|m| = 2$ with an uncertainty of ±18°. Detection of the oscillation by an apparent-velocity shift favors $m \neq 0$, the magnitude of the observed phase is only consistent with $\ell = 2$, and the sign of the relative phase restricts m to be positive for $\ell = 2$. The 160 min oscillation is thus classified as an $\ell = 2$, $m = 2$ mode. The apparent-velocity hypothesis has important implications for gravity mode classification (cf. Sections 1.3.5 and 1.4.4) and for the design of instruments to detect gravity modes.

1.6.3 Deviations from Asymptotic Theory[14]

Examination of the observed differences between eigenfrequencies $\nu_{n+1,\ell}$ and $\nu_{n,\ell}$, $\Delta\nu_{n,\ell}$, reveals significant departures from the differences predicted by asymptotic theory. In particular, the set of $\Delta\nu_{n,0}$ calculated using the $\ell = 0$ eigenfrequency spectra of the 5 min modes obtained by Woodard and Hudson (1983) and Hill (1985a) shows considerable deviation from that predicted by asymptotic theory. The deviations based on the observations of Hill (1985a) from asymptotic theory predictions are approximately a factor of 4 larger than any attributable to errors in frequency measurements. This same factor of 4 is also obtained with the observations of Woodard and Hudson (1983).

The scatter in $\Delta\nu_{n,\ell}$ obtained from Doppler shift observations is even larger. Based on the work of Harvey and Duvall (1984), the scatter in the case for $\ell \geq 1$ is approximately twice the line width or more of the modes of oscillation for $n \sim 20$ and four line widths or more for $n \sim 14$. These results pose what may be a very meaningful question: how can there be such large deviations in velocity observations relative to temperature observations (as inferred from total irradiance and differential radius observations)? This relatively larger scatter appears also to be present in the velocity observations of Libbrecht and Zirin (1986). The large deviations may be particularly significant in light of the reported discrepancies in rotational splitting that are obtained using velocity and differential radius observations.

---

14. This is a synopsis of the work by Hill and Rosenwald (1986).

The extent to which the asymptotic theory predictions are realized depends upon the degree to which the properties of the solar interior meet the conditions required for the applicability of the theory. As Christensen-Dalsgaard et al. (1985) state, "The asymptotic analysis is valid if the scale of variation of the background state is much greater than all oscillation wavelengths considered, and that condition is indeed satisfied throughout most of the interior of any traditional theoretical model of the Sun." The deviations discussed above may be the manifestation of conditions in the solar interior which do not meet the requirements for asymptotic theory to be applicable. Should this be the case, these deviations take on a particular importance: they may offer a diagnostic probe to those regions of the solar interior where a significant fractional change occurs in a characteristic length of the eigenfunction used in the study.

If there is a layer in the Sun that gives rise to a sharp change in local properties, thus generating conditions for which asymptotic theory is not applicable, the effect of this sharp change will depend on its location relative to the location of the nodes of the eigenfunction. The location of a node near a particular radius depends on n and ℓ and appropriate simultaneous shifts in n and ℓ may keep that node location relatively fixed. Thus, if the model is a relatively accurate representation in this case, it should be possible to demonstrate that the observed deviations from asymptotic theory for $0 \leq \ell \leq 6$ are projections of a single distribution onto seven different surfaces.

Guided by this model, the seven different sets of $\Delta\nu_{n,\ell}$ for $\ell = 0,\ldots,6$ were superimposed. In this superposition relative shifts were allowed in frequency, $\Delta\nu$, and $n, \Delta n$, which were proportional to shifts in $\ell, \Delta\ell$. It was discovered empirically that there exists a $\Delta\nu/\Delta\ell$ and a $\Delta n/\Delta\ell$ which show that the seven apparently unordered distributions of $\Delta\nu_{n,\ell}$ are projections of a single parent function. This extraordinary result of this superposition is shown in Figure 7 which was obtained with

$$\Delta\nu/\Delta\ell = -0.35; \; \Delta n/\Delta\ell = 0.42 \quad . \qquad (12)$$

The probability that the parent function was obtained from seven sets of random frequencies would appear to be quite small, $\approx 1.2 \times 10^{-7}$. To

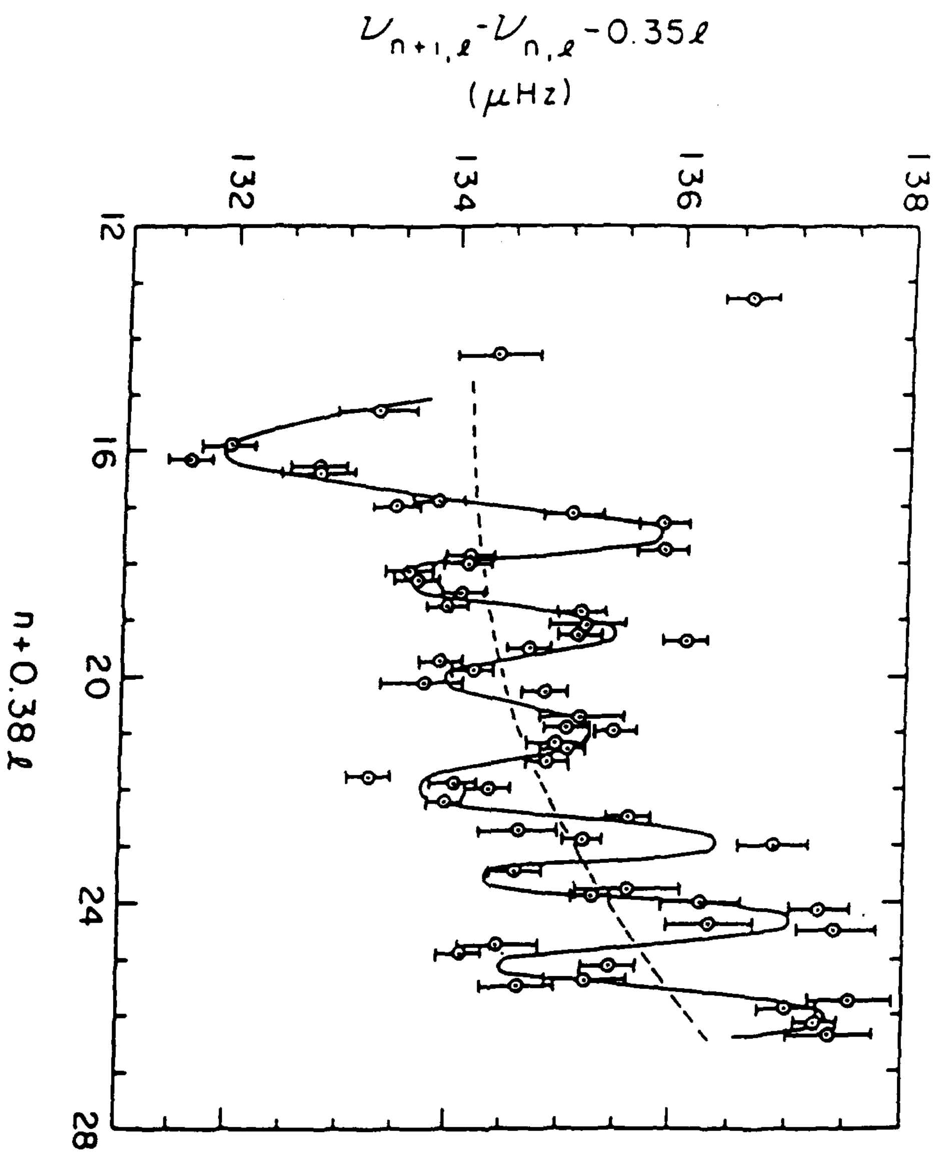


Fig 7 The n-dependence of the superposed spectra of $\Delta\nu_{n,\ell}$ obtained by the superposition of the observed $\Delta\nu_{n,\ell}$ for $\ell = 0,\ldots,6$ with shifts in frequency of $\Delta\nu/\Delta\ell = -0.35\ell$ μHz and shifts in n of $\Delta n/\Delta\ell = 0.420$. The solid curve is an estimate of the quasi-periodic parent function for that domain of $n + 0.42\ell$ where the parent function is generally overdetermined. The dashed curve is an estimate of the average of superposed $\Delta\nu_{n,\ell}$ with the short-wavelength quasi-periodic structure removed.

address the question are the quasiperiodic results shown in Figure 7 physically plausible, a numerical experiment was performed. In this experiment the equilibrium conditions of a standard solar model were locally perturbed in a narrow range in radius and the effect of the localized perturbation on the eigenfrequencies calculated numerically. Excellent agreement was found between the observations and the predictions of the numerical experiment. This agreement between observation and theory indicates that the type of mechanism leading to the observations is understood and that it is appropriate to use the results of the theoretical analysis to infer the location of the non-applicability of asymptotic theory. It is inferred that the effective radius of this location is

$$r/R = 0.757 \pm 0.002 \quad . \tag{13}$$

The closeness of this value to what is currently thought to be the base of the convection zone suggests that the non-applicability of the asymptotic theory is connected with the transition region near the inner boundary of the convection zone.

In summary, evidence has been obtained which indicates that another important diagnostic tool is available for work in solar seismology. This probe is sensitive to those interior regions of the sun where the conditions for the applicability of asymptotic theory for low-degree, 5 min oscillations are not met. Should these findings be confirmed in future work, studies of the type described here may examine the temporal stability as well as the spatial properties of the equilibrium conditions of these regions where the scale of the change in the equilibrium state of the Sun is of the order of the wavelength of the oscillations used in the study.

1.6.4 Implications of Phase-locked Gravity Modes[15]

The work of Hill (1986c) presents evidence of phase locking for four intermediate degree gravity modes (cf. Section 1.4.5). The evidence of phase locking of a single set of gravity modes leads to a modification of

15. This is a synopsis of Hill (1986c).

the neutrino production rates predicted by a standard solar model with no solar oscillations present. The evidence of two or more different sets of phase-locked gravity modes should lead to a periodic modulation in time of the neutrino production rates. The periods of modulation are determined by the sets of properties of the phase-locked gravity modes, while the amplitudes of modulation are determined by both the properties of the modes and the details of the neutrino production. Two facts -- that a modulation is a natural consequence of phase locking and that the periods can be derived from solar seismological studies -- place the mode-locking hypothesis in a unique position relative to the many other suggestions that have been offered for resolving the solar neutrino paradox: a periodic modulation of the neutrino production rates is not a natural consequence of most if not all of these other suggestions, and the relevance of the phase-locking hypothesis can be tested. Such a test can be made by looking for a correlation between the predicted periodic behavior of the neutrino production rates and their observed fluctuations.

The presence of the relatively large temperature eigenfunctions of phase-locked gravity modes for $0.05 \leq r/R \leq 0.45$ and the associated energy transport of the modes are both expected to alter the neutrino production rates. These two mechanisms lead to a modification of the effective temperature profile defined for a given process in the Sun. It is anticipated in particular that the former will be the more important process for the modification of the production rate of the neutrino from the decay of ^{8}B.

For a set of phase-locked modes belonging to the multiplet $g_{n,\ell}$, there exists a rotating coordinate system with a rotational speed $\Omega = -2\pi\nu'_{n,\ell}$ where each mode of the set has the same frequency of oscillation. In this rotating coordinate system, there will be one or more regions where the temperature eigenfunctions for the members of the set of modes are all in phase. These regions of constructive interference thus all rotate at $-2\pi\nu'_{n,\ell}$, the same rate as the rotating coordinate system. For a second set of phase-locked modes, an identical set of properties exists except for one: the respective rotating coordinate systems will in general have slightly different rotation rates because the respective $\nu'_{n,\ell}$ are slightly different.

It is the slightly different rotation rates of the respective regions of constructive interference that are of interest here. Periodically, the regions of constructive interference belonging to two different sets of phase-locked modes will pass through each other. Because of the importance of the higher-order processes indicated by phase locking, this passage of two different phase-locked regions of oscillations will lead to enhanced neutrino production rates. The periodicity of the enhanced neutrino production is obtained directly from the observed $\nu'_{n,\ell}$ for the phase-locked modes. The results of the analysis indicate a typical period for this periodicity of ≈ 2 y.

Although the analysis of the 1979 observations for the properties of the intermediate-degree gravity modes is not yet completed, the statistical significance of the evidence for the detection of phase-locked modes is sufficiently high to warrant certain preliminary observations. First, the presence of the phase-locked gravity modes may be important to the resolution of the solar neutrino paradox. More importantly, should the mode-locking hypothesis be correct in part, it can be subjected to a direct observational test, an opportunity which places this hypothesis in a unique position with respect to the other suggestions for the resolution of the paradox. This test, based on the observed presence of several sets of phase-locked gravity modes which should lead to a periodicity in the neutrino production rates, will consist of looking for predicted periodicities in, for example, the neutrino capture rates recorded at the Homestake Gold Mine (Rowley, Cleveland, and Davis 1985). The predicted periodicities would be based on the 1979 solar seismological observations.

It should be noted that a positive result from the test described above would have an impact on several areas of research. First, of course, such a result would confirm the presence of phase-locked gravity modes. Second, a positive result would provide the opportunity to infer in a quantitative manner the net contribution of the phase-locked modes to the solar neutrino paradox relative to other possible mechanisms such as neutrino oscillations. Third, and most important, the confirmation of the mode-locking hypothesis would demonstrate the value of solar seismology for future studies of solar neutrinos through the phase-sensitive detection of modulation in the neutrino production rates.

1.7 Development of a New Observational Technique at SCLERA[16]

The development of a new observational technique was motivated by two different interests. First, an instrument that could detect the temperature eigenfunction of gravity modes on the solar disk would augment the differential radius observations made at the extreme limb and would be very valuable in the study of gravity modes. Second, the influx of discrepant results as discussed in Section 1.6.1 clearly indicates the desire for the development of new observational techniques that may help resolve the interpretational problems we currently have.

A technique to detect solar oscillations through the Eulerian perturbation of the radiation intensity $\mathcal{J}'$, has been developed and tested at SCLERA. In 1985 $\mathcal{J}'$ was measured at 48 different wavelengths within the spectral range of 0.5 to 1.7 microns (cf. Section 1.2.1). Since $\mathcal{J}'$ has a wavelength dependence which is different from that associated with changes in the transparency of the earth's atmosphere, the contribution of $\mathcal{J}'$ to the observed signal can be identified. The contributions were Fourier analyzed and the resulting power spectrum indicated to a high degree of statistical significance that global solar oscillations had been detected (cf Section 1.5.3). These results indicate a promising future for the use of this technique in solar seismology studies.

1.8 Computer Facilities for Data Analysis and Theoretical Calculations

The decision was made in 1982 to establish a dedicated computer system capable of meeting SCLERA's off-line computer requirements for the five years 1983-88. This decision was based on projected real costs of using existing facilities at the University and the recognition that these projected costs, if realized, would constitute too heavy a burden on the program. During 1980-81 most of the off-line computer work was done at the University Computer Center; this work was funded by a grant from the University of Arizona in the amount of $25,000. During 1981-82, SCLERA's computer requirements increased by more than 100%. The total cost during

16. This is a synopsis of the work by Oglesby (1986).

this period was also met primarily by grants from the University. The computer requirements for 1983-88 were projected to increase linearly with time at the rate experienced in 1981 and 1982. However, the University instituted in 1984 a major reduction in its computing grants. This made it necessary to find an alternative to using the University Computer Center, an alternative which would meet our computing requirements at a cost that could be supported by the program. That alternative was the decision to establish a dedicated computer system capable of meeting SCLERA's off-line computer requirements for a five year period. The five year plan consisted of starting with a system capable of meeting the 1983-84 requirements followed with periodic upgrades of the system as the program needs developed.

The computer that was put on-line (January 1983) is a Prime 250-II. This initial system was configured with a one Mbyte memory, a 64 Mbyte cartridge module disk, a magnetic tape, a medium-speed line printer/plotter console, five user terminals, and one remote dial-up telephone modem.

The projected increase in computer requirements for 1984 proved to be accurate and the first upgrade of the off-line computer system was made in March of 1984. This upgrade involved increasing the memory to two Mbytes and adding a 300 Mbyte Winchester Disk Drive.

The data analysis has required extensive use of computers because of the complexity of the eigenfrequency spectrum and because of the relatively small signals associated with solar oscillations. The theoretical calculations have required extensive use of computers because the properties of solar models are being tested at an unprecedented level of accuracy in astrophysics. This complexity of the data analysis and theoretical calculations continues to increase as projected in 1982 as more and improved observations are made. In 1985, the capacity of the Prime 250-II system saturated; this was projected to pose serious limitations on the potential of the 1986 program. In March of 1986 the Prime 250-II system was replaced by a Celerity 1200 computer. The configuration and capacity of the new system is discussed in Section 3. The Celerity 1200 with upgrading should meet the off-line computer requirements for several years with effective low operation costs.

1.9 References

Bos, R. J. 1982, Ph. D. Thesis, University of Arizona.

Bos, R. J. and Hill, H. A. 1983 Solar Phys. 82, 89.

Brown, T. M. 1985 Nature 317, 591.

Caudell, T. P. 1980 Ph.D. Thesis, University of Arizona.

Caudell, T. P., et al. 1980 Lecture Notes in Physics No. 125 (Springer-Verlag, Berlin), p. 206.

Christensen-Dalsgaard, J., Duvall, T. L., Gough, D. O., Harvey, J. W., and Rhodes, E. J., Jr. 1985, Nature 315, 378.

Claverie, A., et al. 1981 Nature, 293, 443.

Duvall, T. L., Jr. and Harvey, J. W. 1983 Nature, 302, 24.

_____. 1984 Nature, 310, 191.

Duvall, T. L., Harvey, J. W., and Pomerantz, M. A. 1986, Nature, submitted.

Duvall, T. L., Jr., et al. 1984 Nature 310, 22.

Gough, D. O. 1980, in Nonradial and Nonlinear Stellar Pulsation, Lecture Notes in Physics No. 125, ed. H. A. Hill and W. A. Dziembowski (Berlin: Springer-Verlag), p. 273.

_____. 1981, MNRAS 196,731.

Grec, G., Fossat, E., and Pomerantz, M. A. 1983, Solar Phys., 82, 55.

Harvey, J. W., and Duvall, T. L., Jr. 1984, in Solar Seismology from Space: A Conference at Snowmass, Colorado (JPL Pub. 84-84), p. 165.

Hill, H. A. 1984a, Int. Jour. of Theo. Phys. 23, 683.

_____. 1984b, "The Observed Low Degree Acoustic Mode Eigenfrequency Spectrum and Its Properties," SCLERA Monograph Series in Astrophysics, no. 1.

_____. 1985a,

_____. 1985b,

_____. 1986a, Ap. J. 303, 489-494.

_____. 1986b, Submitted for publication to Publication of the Astronomical Society of the Pacific, Feb. 3, 1986.

_____. 1986c, "Evidence of Gravity-mode Coupling and its Relevance to Solar Neutrino Production Rates," to be published in Neutrino '86; The 12th

International Conference on Neutrino Physics and Astrophysics, Sendai, Japan, June 3-8, (1986).

Hill, H. A. and Caudell, T. P. 1979, Mon. Not. Ast. Soc. 186, 327.

_____. 1985, Ap. J. 299, 517.

Hill, H. A. and Czarnowski, W. M. 1986 "Comparison of Differential Velocity and Differential Radius Observations of Solar Oscillations for Evidence of Gravity Modes," submitted to Astronomy and Astrophysics.

Hill, H. A. and Kroll, R. 1986, submitted for publication to Journal of Geophysical Research.

Hill, H. A. and Rosenwald, R. D. 1986, "Multiplet Classification of Intermediate Solar f-modes and Associated Rotational Splitting," poster paper 56:02 for the 167th Meeting of the American Astronomical Society, Jan. 5-9, 1986, Houston, Texas.

Hill, H. A. and Tash, J. 1984, Bull. of the Am. Ast. Soc. 16, 1000.

Hill, H. A., Alexander, and Caudell, T. P. 1985, "The Horizontal Spatial Properties of the Eulerian Perturbation of Radiation Intensity Produced by the Low-order, Low-degree Acoustic Modes of the Sun," SCLERA Monograph Series in Astrophysics, no. 2.

Hill, H. A., Bos, R. J. and Goode, P. R. 1982, Phys. Rev. Letts. 49, 1794.

Hill, H. A., Rosenwald, R. D., and Rabaey, G. F. 1985, "The Sun's Gravitational Quadrupole Moment Inferred from the Fine Structure of the Acoustic and Gravity Normal Mode Spectra of the Sun," in Proceedings of the Marcel Grossmann Meetings on the Recent Developments of General Relativity, Rome Italy, June 17-21.

Hill, H. A., Tash, J. and Padin, C. 1986, Ap. J. 304, 560.

Hill, H. A., et al. 1984, Solar Terrestrial Physics: Proceedings of Second Indo-US Workshop on Solar Terrestrial Physics, pp. 443-450.

Hill, H. A., et al. 1986, Ap. J. 310, in press.

Kotov, V. and Koutchmy, S. 1979 Uspechi Riz. Nauk (USSR), 128, 730.

Kotov, V. A., Koutchmy, S. and Koutchmy, O. 1983, Solar Phys. 82, 21.

Kotov, V. A., Severny, A. B., Tsap. T. T., Moiseev. I. G., Efanov, E. A., and Nesterov, N. S. 1983, Solar Phys. 82, 9.

Libbrecht, K. G. 1986, Nature 319, 753.

Libbrecht, K. G. and Zirin, H. 1986, Astrophys. J., in press.

Oglesby, P.H. 1986 Ph. D. Thesis, University of Arizona.

Rabaey, G. F. and Hill, H. A. 1986, "The Intermediate-degree F-mode

Multiplets of the Sun," Stellar Pulsation: A Memorial to John P. Cox, Aug. 11-15, Los Alamos, New Mexico.

Rowley, J. K., Cleveland, B. T., and Davis, R., Jr. 1985, in Solar Neutrinos and Neutrino Astronomy (AIP Conf. Proc. No. 126), M. L. Cherry, K. Lande, and W. A. Fowler, eds. (American Institute of Physics, New York), p. 1.

Saio, H. 1982, private communication.

Scherrer, et al. 1983 Solar Phys. 82, 75.

Woodard, M., and Hudson, H. S., 1983, Solar Phys. 82, 67.

Yi, L., 1986 Ph. D. Thesis, University of Arizona.

Yi, L., and Czarnowski, W. M. 1986, "Comparison of 1983 and 1979 SCLERA Observations," Stellar Pulsation: A Memorial to John P. Cox, Aug. 11-15, 1986, Los Alamos, New Mexico.

END

7-81

DTIC