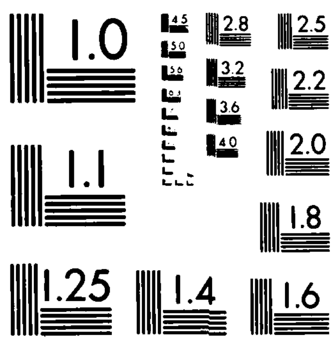
AD-A181 067 NUMERICAL METHODS FOR MATRIX COMPUTATIONS USING ARRAYS OF PROCESSORS(U) STANFORD UNIV CA DEPT OF COMPUTER SCIENCE G H GOLUB 30 APR 87 ARO-20251.1-MA 1/1

UNCLASSIFIED DAAG29-83-K-0124 F/G 12/1 NL

END
7 87
DTIC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

NUMERICAL METHODS FOR MATRIX COMPUTATIONS

USING ARRAYS OF PROCESSORS

FINAL REPORT

GENE H. GOLUB
Principal Investigator

April 30, 1987

U. S. ARMY RESEARCH OFFICE

DAAG29-83-K-0124

STANFORD UNIVERSITY
STANFORD, CA 94305

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ADA181067

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER ARO 20251.1-MA	2. GOVT ACCESSION NO. N/A	3. RECIPIENT'S CATALOG NUMBER N/A
4. TITLE (and Subtitle) Numerical Methods for Matrix Computations Using Arrays of Processors		5. TYPE OF REPORT & PERIOD COVERED Final Report 8/15/83 - 10/15/86
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Gene H. Golub		8. CONTRACT OR GRANT NUMBER(s) DAAG29-83-K-0124
9. PERFORMING ORGANIZATION NAME AND ADDRESS Computer Science Department School of Engineering Stanford University, Stanford, CA 94305		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE April 30, 1987
		13. NUMBER OF PAGES four
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Parallel Processing
Systolic Arrays
Geodetic Computations
Domain Decomposition

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The Final Report describes the scientific activity conducted under this grant. The emphasis of this research has been the solution of sparse matrix problems arising in applications, on parallel and systolic-like architectures.

1. Problem Studied

The basic objective of this project has been to consider a large class of matrix computations with particular emphasis to algorithms which can be implemented on arrays of processors. In particular, we have been interested in methods which are useful for sparse matrix computations. These computations arise in a variety of applications such as the solution of partial differential equations by multigrid methods and in the fitting of geodetic data. Some of the methods developed have already found their use on some of the newly developed architectures (see below).

2. Summary of Important Results

Five reports and papers have been written during the duration of this grant. We describe some of the results given in these reports. A complete list of the reports and papers is given in Section 3 of this report.

The serial multigrid algorithm is a fast and efficient technique for solving elliptic partial differential equations. The algorithm consists of "solving" a series of problems on a hierarchy of grids with different mesh sizes. For many problems, it is possible to prove that its execution time is asymptotically optimal. Not only is it asymptotically optimal but when properly implemented it is competitive with other algorithms on grids of a modest size. Given its success on serial computers, it is natural to consider its performance characteristics on parallel machines.

Primarily, this research considers the mapping of the multigrid algorithm to the distributed memory, message passing hypercube computer. The work illustrates how the topology of the hypercube fits the data flow of the multigrid algorithm, and therefore allows parallels implementations with relatively low communication cost. It has been shown that the multigrid algorithm is an asymptotically optimal parallel algorithm in a certain sense. A timing model for the execution time of a particular implementation was developed and found to accurately model experimental results obtained from runs on the Intel iPSC system. Further, this model was used to explore the influence of machine and algorithm parameters on the efficiency of the method.

One difficulty with the parallel multigrid algorithm is a load balancing problem that creates inefficiency on large processor systems (caused by processors becoming idle on coarse grids). The current research, which is described in [1], is concerned with evaluating the magnitude of this problem and developing new algorithms which do not have these difficulties. One new algorithm exploits idle processors to accelerate the convergency of the basic multigrid method. Additional work is necessary to fully evaluate this algorithm; however, the preliminary analysis and experiments are promising.

The possibilities of systolic-like architectures (n x n grids of relatively simple and small processors) has been demonstrated in performing the direct sparse Cholesky factorization of a positive definite matrix in [5]. These matrices arise in the discretization of elliptic, partial differential equations by finite elements or finite differences. The factorization and backsolve, each require 0(n) parallel floating point multiplications, realizing the theoretical parallel execution times previously determined abstractly, without means of implementation. The algorithm described here has been the basis for the nested dissection program developed for the connection machine; its implementation has been quite successful.

Several algorithms have been developed, which are particularly appropriate for vector architectures. In particular, two problems have been studied which are of a statistical nature: the computation of variances for large data samples and the geodetic data fiting problem.

The problem of computing the variance of a sample of N data points may be difficult for certain data sets, particularly when N is large and the variance is small. In [1], we studied several algorithms and their round-off error bounds. We presented a new algorithm which is highly efficient in a vector environment and which has excellent numerical properties.

In [3], we have described and compared some numerical methods for solving large dimensional linear least squares problems that arise in geodesy and, more specially, from Doppler positioning. The methods that are considered are the direct orthogonal decomposition, and the combination of conjugate gradient type algorithms with projections as well as the exploitation of "Property A". Numerical results are given and the respective advantage of the methods are discussed with respect to such parameters as CPU time, input/output and storage requirements.

Iterative methods are often used for solving the linear systems arising from the approximation to elliptic partial differential equations. The Chebyshev and second-order Richardson methods are classical iterative schemes for solving such systems. We consider in [4], the convergence analysis of these methods when each step of the iteration is carried out inexactly. This has many applications, since a preconditioned iteration requires, at each step, the solution of a linear system which may be solved inexactly using an "inner" iteration. We have also derived an error bound which applies to the general nonsymmetric inexact Chebyshev iteration. In particular, in domain decomposition (or substructuring), it may be desirable to solve the subsystem approximately.

3. Papers and Technical Reports

[1] Chan, Tony F., Golub, Gene H., and LeVeque, Randall J., "Algorithms for Computing the Sample Variance: Analysis and Recommendations", The American Statistician, August 1983, Vol. 37, No.3.

[2] Chan, Tony and Tuminaro, Ray, "Design and Implementation of Parallel Multigrid Algorithms", to appear in the proceedings of the Third Copper Mountain Conference on Multigrid Methods, 1987.

[3] Golub, G. H., Manneback, P., and Toint, PH. L., "A Comparison Between Some Direct and Iterative Methods for Certain Large Scale Geodetic Least Squares Problems", Siam J. Sci. Stat. Comput., Vol. 7, No. 3, July 1986.

[4] Golub, Gene H., and Overton, Michael, "The Convergence of Inexact Chebyshev and Richardson Iterative Methods for Solving Linear Systems", Numerical Analysis Project, Manuscript NA-87-01.

[5] Worley, P. H. and Schreiber, R., "Nested Dissection on a Mesh-Connected Processor Array", in New Computing Environments: Parallel, Vector and Systolic, SIAM, Philadelphia, 1986.

4. Scientific Personnel

Gene H. Golub, Principal Investigator

Robert Schreiber, Associate Investigator

Ray Tuminaro, Student Research Assistant

END

7-81

DTIC