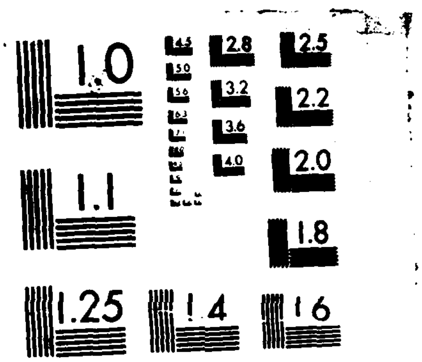
AD-A180 175 DELAY DISTRIBUTION ANALYSIS OF WINDOW RANDOM-ACCESS ALGORITHMS (U) VIRGINIA UNIV CHARLOTTESVILLE DEPT OF ELECTRICAL ENGINEERING L GEORGIADIS ET AL 29 APR 87 1/1

UNCLASSIFIED UVA/525415/EE87/105 N00014-87-K-0742 F/G 20/3 NL



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A Technical Report
Contract No. N00014-86-K-0742
September 1, 1986 - August 31, 1988

DELAY DISTRIBUTION ANALYSIS OF WINDOW RANDOM-ACCESS ALGORITHMS

Submitted to:

Office of Naval Research
800 N. Quincy Street
Arlington, Virginia 22217-5000

Attention: Dr. R. N. Madan
Code 1114SE

Submitted by:

L. Georgiadis
Research Assistant Professor

M. Paterakis
Graduate Research Assistant

P. Kazakos
Professor

Report No. UVA/525415/EE87/105

April 1987

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22901

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Copy No. ____

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

ADA180175

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS NONE
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) UVA/525415/EE87/105	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION University of Virginia Dept. of Electrical Engr. 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Office of Naval Research Resident Representative
6c. ADDRESS (City, State, and ZIP Code) Thornton Hall Charlottesville, VA 22901	7b. ADDRESS (City, State, and ZIP Code) Joseph Henry Building, Room 623 2100 Pennsylvania Avenue, N.W. Washington, DC 20037
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Dept. of Navy Office of Naval Research 8b. OFFICE SYMBOL (If applicable) N00014	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER 00014-87-K-0742
8c. ADDRESS (City, State, and ZIP Code) 800 North Quincy Street Arlington, VA 22217-5000	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. | PROJECT NO. | TASK NO. | WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
Delay Distribution Analysis of Window Random-Access Algorithms

12. PERSONAL AUTHOR(S)
L. Georgiadis, M. Paterakis, P. Kazakos

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical	FROM 9/1/86 TO 8/31/88	April 29, 1987	29

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A method for delay distribution analysis of Window Random-Access algorithms is presented. The window size is allowed to vary during the operation of the algorithm. It is shown that the quantities of interest in the computation of the delay distribution, can be related to the solution of appropriate infinite systems of linear equations. Once the constants and the coefficients of the unknowns of a system are determined, bounds on the solution can be developed by applying previously developed methodologies. The method is applied to the delay distribution analysis of the Capetanakis Window Random-Access algorithm and the Part-and-Try algorithm, both under binary feedback.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. R. N. Madan	22b. TELEPHONE (Include Area Code) (202) 696-4217	22c. OFFICE SYMBOL N00014

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

Unclassified

1. INTRODUCTION

Window Rândom-Access algorithms constitute an important class of Multiple-Access algorithms; they are distributive and attain high throughputs and low delays by controlling the number of simultaneously transmitting users. The throughput analysis of algorithms in this class, is a relatively easy task. The delay analysis, however, presents difficulties, due mainly to the presence of variable window sizes and the complicated state space that some of these algorithms create. This restrictions prohibits the application of results from standard queueing theory in the delay analysis.

Many attempts for the delay analysis have been made. In [1], the class of algorithms with constant window size was considered, and upper bounds on the expected delays were developed. In [2], [3], methods for the computation of bounds on the moments of the delays were presented. A method for the computation of delay distribution for constant window algorithms appears in [4]. The method in [4] relies on a clever decomposition of the delay process, which allows the application of results from standard queueing theory. It is not possible, however, to extend the techniques in [4] to the case of variable window size. The computation of the delay distribution for variable window size algorithms remains an open problem. One possible approach is to compute bounds on the moments of the delays as in [2] or [3], which can then be used for an approximate evaluation of the delay distribution. This approach, however, is not computationally practical.

In this paper, we show that the methodology employed in [3], can be extended to provide bounds on the distribution of the delays. The quantities of interest are related to the solution of a denumerable system of <u>linear</u> equations. Methods for the computation of the constant terms and the coefficients of the unknowns of the system are developed. The methodology is applied to the delay distribution analysis of both the Capetanakis Window-Access algorithm with binary feedback and the Part-and-try algorithm with binary feedback. It can also be applied directly to other Window Random-Access algorithms with different feedback. An interesting result of the analysis is that as the arrival rate increases, the tales of the distribution become longer, but the median grows much slower than the expected delay.

2. MODEL SPECIFICATION

We consider a single slotted channel that is being accessed by a number of independent packet transmitting users. The length of a packet is equal to the length of a slot, and packet transmission may start only at the beginning of a slot. Simultaneous transmission of more than one packets in the same slot, results in complete loss of the information included in the involved packets. The latter event is referred to as a "collision" event. At the end of each slot, all users receive a feedback that provides some information about the chanell activity in that slot. Common types of feedback are the binary C-NC (collision versus noncollision) feedback, and the ternary 0-1-C (empty versus success versus collision) feedback. To resolve the collision, the users follow the rules of a Random-Access algorithm. The algorithm is implemented by each user in a distributed fashion, using only the available feedback. The cumulative packet generating process is assumed to be Poisson with rate λ packets per slot.

We assume that a Window Random-Access algorithm is employed, whose basic operating characteristics are the following (see Figure 1): Suppose that at the beginning of slot v all packet that arrived before time $t_v<v$ have been successfully transmitted, and there is no information concerning the packets that may have arrived in the interval $[t_v,v)$, (i.e., the distribution of the interarrival times of the packets in $[t_v,v)$ is the same as the one assumed originally). The beginning of such a slot v is called a *Conflict Resolution Point (CRP)*. The time difference $d_v = v-t_v$ is referred to as the *lag at v*. In slot v, the users that generated packets in the interval $[t_v, t_v+\tau_v)$, where $\tau_v = \min(d_v,\Delta)$, are allowed to transmit; Δ is a parameter to be properly chosen for throughput maximization. After a random number of slots and following the rules of the algorithm, another CRP, v', is reached, with a corresponding $t_{v'} > t_v$. All the packets that have been generated in the interval $[t_v, t_{v'})$, have been successfully transmitted in the interval $[v,v')$. The intervals $[v,v')$, $[t_v, t_v+\tau_v)$, $[t_v, t_{v'})$ are called *conflict resolution interval, transmitted interval,* and *resolved interval,* respectively. The length of τ_v, is called the *window size at time v* . Clearly, the window size varies with time, and its maximum size is Δ. Note also, that the length of the conflict resolution interval is one, if and only if there are at most one packets in the transmitted interval.

Algorithms that operate as described above, are the Capetanakis Window Random-Access algorithm [5] and the Part-and-Try algorithm, [5], [6], under either binary C-NC feedback, or ternary 0-1-C feedback.

3. STEADY STATE DELAY DISTRIBUTION ANALYSIS

Let packets be labeled 1,2,3,... according to the order of their arrival instants. The delay D_n experienced by the n-th packet is defined as the time difference between its arrival and the end of its successful transmission. We will be interested in evaluating the steady state distribution of D_n, when it exists.

Let v_i ; i≥1 be the sequence of successive CRPs and let d_i be the lag at v_i. The sequence d_i ; i≥1 is a Markov chain with state space F. For most of the existing Window Random-Access algorithms, F is a denumerable subset of the interval $[1,\infty)$. Let $T_1=1$, $d_1=1$, and define T_{i+1}, as the first slot after T_i, at which $d_{T_{i+1}} = 1$. From the description of the algorithm it can be seen that the induced delay process probabilistically restarts itself at the beginning of each slot T_i, i=1,2,... The interval $[T_i,T_{i+1})$ will be referred to as the *i-th session*. Note that the sessions have lengths that are i.i.d. random variables.

Let R_i, i=1,2,..., denote the number of packets successfully transmitted in the interval $(0,T_i]$; (note that R_i also represents the number of packets arrived during the interval $[0, T_i-1)$, since T_i is a CRP at which the lag is unity). Then, $C_i=R_{i+1}-R_i$; i≥1, is the number of packets successfully transmitted in the interval $(T_i,T_{i+1}]$ - these are the packets that arrived during the interval $[T_i-1, T_{i+1}-1)$. The sequence R_i; i≥1, is a renewal process, since C_i, i≥1, is a sequence of nonnegative i.i.d. random variables. Furthermore, the delay process D_n, n≥1, is regenerative with respect to the renewal process R_i, i≥1, with regeneration cycle C_1.

Let

$$I_n(s)=\begin{cases} 1 & \text{if } D_n \le s \\ 0 & \text{otherwise} \end{cases}$$

From the regenerative theorem [3], we conclude that if $C=E(C_1) < \infty$, then

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N} I_n(s) = \lim_{N\to\infty}\frac{1}{N}E(\sum_{n=1}^{N} I_n(s)) = \frac{E(\sum_{n=1}^{C_1} I_n(s))}{C} \tag{1}$$

In addition, since $P(C_1=1) > 0$, the distribution of C_1 is aperiodic and there exists a proper random variable D_∞, such that the sequence D_n; n=1,2,... converges in distribution to D_∞. D_∞ represents the steady state delay induced by the algorithm and its distribution satisfies the equality

$$P(D_\infty \le s) = \frac{E(\sum_{n=1}^{C_1} I_n(s))}{C} \tag{2}$$

From (2) we observe that the steady state distribution of the delays can be determined by computing the quantities of the right hand side of the equality. In [3] it was shown that the finiteness and the computation of C is related to the existence and the computation of an appropriate solution to an infinite system of linear equations. In this section we will show that the same is true for the quantity $E(\sum_{n=1}^{C_1} I_n(s))$.

The following definitions will be used in the sequel.

l	: Length of a conflict resolution interval
δ	: Length of a resolved interval
τ	: Window size.
$E(X/\tau)$	: Expected value of the random variable X, given that the window size is τ.

$p(x,r/\tau)$: The probability that the conflict resolution interval has length x and the resolved interval has length r, given that the window size is τ.

$p(x/\tau)$: The probability that the conflict resolution interval has length x, given that the window length is τ.

h_d : Number of slots needed to reach a CRP with lag 1 given that the current lag is equal to d, $d \varepsilon F$.

$k_d(s)$: Number of successfully transmitted packets with delay less than s, in the interval h_d.

$m_{\tau,d}(s)$: Number of successfully transmitted packets with delay less than s during a conflict resolution interval, given that the window size is τ and the lag is d.

$n_\tau(s)$: Number of successfully transmitted packets with delay less than s, during a conflict resolution interval, given that the window size is τ *and the lag is* τ. That is, $n_\tau(s)=m_{\tau,\tau}(s)$.

Let us also define,

$$K_d(s) = E(k_d(s))$$

$$M_{\tau,d}(s) = E(m_{\tau,d}(s))$$

$$N_\tau(s) = E(n_\tau(s))$$

$$H_d = E(h_d)$$

Note that by definition,

$$K_1(s) = E(\sum_{n=1}^{C_1} I_n(s))$$

Also,

$$C=\lambda H_1$$

Therefore, the determination of $K_1(s)$ and H_1, will permit the computation of the steady-state distribution of the delays.

Consider the arrangement of Fig. 1. The delay D of the successfully transmitted packet 1, can be decomposed as follows:

$$D=\theta_1+d-\tau+\theta_2$$

Therefore,

$$D \le s \text{ iff } \theta_1+\theta_2 \le s-d+\tau \tag{3}$$

But $\theta_1+\theta_2$ is statistically identical to the delay that the successfully transmitted packet experiences if the transmitted interval is τ *and the lag is* τ. The last observation shows that $m_{\tau,d}(s)$ is identically distributed with $n_\tau(s-d+\tau)$. Observe now that

$$k_d(s)=\begin{cases} m_{\tau,d}(s) & \text{if } d_{v'}=1 \\ m_{\tau,d}(s)+k_{d'}(s) & \text{if } d_{v'}=d' \ne 1 \end{cases} \tag{4}$$

and that

$$d_{v'}=d_v-\delta+l, \quad \tau=\begin{cases} d_v & \text{if } d_v \le \Delta \\ \Delta & \text{if } d_v > \Delta \end{cases} \tag{5}$$

From (4) and (5) we conclude that

$$K_d(s)=M_{d,d}(s)+\sum_{\substack{r,x \\ x\ne 1}} K_{d-r+x}(s)p(x,r/d) \quad \text{if } 1\le d\le \Delta,\ d\varepsilon F \tag{6a}$$

$$K_d(s)=M_{\Delta,d}(s)+\sum_{r,x} K_{d-r+x}(s)p(x,r/\Delta) \quad \text{if } d>\Delta,\ d\varepsilon F \tag{6b}$$

Since $m_{\tau,d}(s)$ is identicaly distributed with $n_\tau(s-d+\tau)$, equations (6a) and (6b) become,

$$K_d(s)=N_d(s)+\sum_{\substack{r,x \\ x\ne 1}} K_{d-r+x}(s)p(x,r/d) \quad \text{if } 1\le d\le \Delta,\ d\varepsilon F \tag{7a}$$

$$K_d(s) = N_\Delta(s-d+\Delta) + \sum_{r,x} K_{d-r+x}(s)p(x,r/\Delta) \quad \text{if } d>\Delta,\ d\varepsilon F \tag{7b}$$

Equations (7a), (7b) comprise a denumerable system of linear equations. Of interest to us is the element $K_1(s)$ of a particular solution of this system. The methodology developed in [3] can be used for the study of system (7). Note that the coefficients of the unknowns are independent of s. This observation represents a computational advantage when the solution to (7) is approximated by the solution of appropriate finite linear system of equations [3]. In this case, the approximate solution can be represented in the form,

$$\mathbf{K}(s) = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{N}(s)$$

where $\mathbf{A}$ is a square matrix whose elements are independent of s. The matrix $(\mathbf{I}-\mathbf{A})^{-1}$ can be computed once, and then used for the computation of the approximate solution for various values of s.

We now proceed in the development of an initial upper bound on the solution of system (7). Following the methodology in [3], such a bound will be the sequence $K_d^0(s) = \gamma_u(s)d+\zeta_u(s)$, if $\gamma_u(s)$, $\zeta_u(s)$, can be determined so that the following inequalities are satisfied

$$K_d^0(s) \geq N_d(s) + \sum_{\substack{r,x \\ x\neq 1}} K_{d-r+x}^0(s)p(x,r/d) = K_d^1(s) \quad \text{if } 1\leq d\leq\Delta,\ d\varepsilon F \tag{8a}$$

$$K_d^0(s) \geq N_\Delta(s-d+\Delta) + \sum_{r,x} K_{d-r+x}^0(s)p(x,r/\Delta) = K_d^1(s) \quad \text{if } d>\Delta,\ d\varepsilon F \tag{8b}$$

Substituting $K_d^0(s)$ in the right hand side of inequalities (8), it can be easily seen that if $d\varepsilon F$,

$$K_d^1(s) = K_d^0(s) + N_d(s) + \gamma_u(s)(E(l/d) - E(\delta/d) - (1+\lambda d)e^{-\lambda d}) - \zeta_u(s)(1+\lambda d)e^{-\lambda d} \quad \text{if } 1\leq d\leq\Delta \tag{9a}$$

$$K_d^1(s) = K_d^0(s) + N_\Delta(s-d+\Delta) - \gamma_u(s)(E(\delta/\Delta)-E(l/\Delta)) \quad \text{if } d>\Delta \tag{9b}$$

Observe now that $N_\Delta(s)$ is an increasing function of s. Therefore, from (9b) we conclude that,

$$K_d^1(s) \le K_d^0(s) + N_\Delta(s) - \gamma_u(s)(E(\delta/\Delta) - E(l/\Delta)) \quad \text{if } d>\Delta \tag{10}$$

From (10) we conclude that if $E(l/\Delta) < E(\delta/\Delta)$, the condition for stability of the system, inequalities (9b) are satisfied if

$$\gamma_u(s) = \frac{N_\Delta(s)}{E(\delta/\Delta) - E(l/\Delta)} \tag{11}$$

With this value of $\gamma_u(s)$, it can be seen that inequalities (9a) are satisfied if

$$\zeta_u(s) = \max\{-\gamma_u(s), \sup_{1 \le d \le \Delta} (\psi(d))\} \tag{12}$$

where

$$\psi(d) = \frac{N_d(s) + \gamma_u(s)[E(l/d) - E(\delta/d) - (1+\lambda d)e^{-\lambda d}]}{(1+\lambda d)e^{-\lambda d}} \tag{13}$$

From the above discussion we conclude that the solution to system (7) satisfies the inequalities

$$K_d(s) \le \gamma_u(s)d + \zeta_u(s), \ d \varepsilon F \tag{14a}$$

where $\gamma_u(s)$, $\zeta_u(s)$, are given by equations (11), (12) respectively. The uniqueness of the solution is guaranteed by the same techniques as in [3]. If we use a similar method for the development of a lower bound, we find that

$$\gamma_l(s)d + \zeta_l(s) = K_d(s), \ d \varepsilon F \tag{14b}$$

where

$$\gamma_l(s) = 0 \quad \text{and} \quad \zeta_l(s) = \inf_{1 \le d \le \Delta} \{(N_d(s)/((1+\lambda d)e^{-\lambda d})\}$$

Bounds on H_1 are given in [3], formulas (18), (19).

As is explained in [3], the bounds (14) can be used to further improve the bounds on $K_1(s)$. To proceed further, however, the computation of the quantities $E(l/d)$, $E(\delta/d)$, $p(x,r/d)$, and $N_d(s)$ is necessary. In section 3, we present a method for the computation of $N_d(s)$ for the Capetanakis dynamic algorithm, and then proceed in the computation of tight upper and lower bounds on the quantities of interest. In section 4, we present a method for the computation of the quantities $N_d(s)$ and $p(x,r/d)$ for the Part-and-Try algorithm, and then we develop bounds on the distribution of the delays. Due to the complicated state space of the latter algorithm, the development of tight bounds for high input rates becomes computationally cumbersome.

3. THE CAPETANAKIS WINDOW RANDOM-ACCESS ALGORITHM WITH C-NC FEEDBACK

A complete description of the algorithmic rules can be found in [5]. In this algorithm, the resolved interval is always equal to the window size i.e., $\tau \equiv \delta$. This results in a significant simplification of the state space F, when the maximum window size Δ is a rational number. The restriction of Δ to the rational numbers facilitates the development of tight bounds for the delay distribution and does not represent any disadvantage in practice. Moreover, if m packets are involved in a conflict, the conflict resolution process depends only on m and not on the window length or the generation time of each of the packets. The last property facilitates the development of efficient methods for the computation of the quantities of interest.

Since $\tau \equiv \delta$, we have that, $E(\delta/d) = d$ and

$$p(x,r/d) = \begin{cases} p(x/d) & \text{if } r=d \\ 0 & \text{otherwise} \end{cases}$$

Formulas for the computation of $E(l/d)$, $p(x/d)$, can be developed by following the reasoning in [5]. In the next section we develop a method by which the quantities $N_d(s)$ can be computed.

3.1 The Computation of $N_d(s)$.

Let M be the number of packets in the window d (see Fig 2). Then,

$$N_d(s) = E(n_d(s)) = E(E(n_d(s) / M)) = \sum_{m=1}^{\infty} E(n_d(s) / M = m) e^{-\lambda d} \frac{(\lambda d)^m}{m!} \tag{15}$$

Let

$$J_{i,m}(s) = \begin{cases} 1 & \text{if the delay of the i-th packet in d is at most s} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

The "i-th packet" in (16), is the i-th packet in a random enumeration of the m packets (randomly chosen packet). Then

$$n_d(s) = \sum_{i=1}^{m} J_{i,m}(s) \tag{17}$$

Given M, the generation time of each of the M packets is uniformly distributed in the window d and independent of the generation time of the rest of the packets. From this observation and the definition in (16) we conclude that the random variables $J_{i,m}(s)$, $1 \le i \le m$, are identically distributed (although not independent). Therefore, from (17) we conclude that

$$E(n_d(s) / M = m) = mE(J_{1,m}(s) / M = m) \tag{18}$$

Let θ_1, (θ_2), be the delay of the first randomly chosen packet before (after) the initialization of the collision resolution process i.e. until (after) time v (Fig. 2). Since θ_1 is uniformly distributed in [0, d), we have that

$$E(J_{1,m}(s) / M = m) = \frac{1}{d}\int_0^d E(J_{1,m}(s) / M = m, \theta_1 = \theta)\, d\theta \tag{19}$$

Since the conflict resolution process is independent of the packet generation time in a window, we conclude that given m, the random variables θ_1, θ_2, are independent. Therefore,

$$E(J_1(s) / M = m, \theta_1 = \theta) = p(\theta_1 + \theta_2 \le s / M = m, \theta_1 = \theta) = p(\theta_2 \le s-\theta \;/ M = m) \tag{20}$$

and

$$E(J_{1,m}(s) / M = m) = \frac{1}{d}\int_0^d p(\theta_2 \le s-\theta \;/ M = m)\, d\theta \tag{21}$$

Observe now, that θ_2 takes only positive integer values. Let

$$P_q^{(m)} = p(\theta_2 = q / M = m)\;;\; q=1,2,3,...$$

then, [1,2]

$$p(\theta_2 \le s-\theta \;/ M = m) = \sum_{q=1}^{\lfloor s-\theta \rfloor} P_q^{(m)} \tag{22}$$

Substituting (22) in (21), we conclude that

$$E(J_{1,m}(s) / M=m) = \frac{1}{d}\int_0^d \sum_{q=1}^{\lfloor s-\theta \rfloor} P_q^{(m)}\, d\theta \tag{23}$$

---

[1] $\lfloor a \rfloor$ denotes the integer part of a.

[2] We adopt the notation $\sum_{q=n}^{m} a_q = 0$ if n>m

Let $e=d-s+\lfloor s \rfloor$. Then, since $P_q^{(m)}$ is independent of θ, (23) can be written as follows:

$$E(J_{1,m}(s)/M=m) = \frac{1}{d}\left((s-\lfloor s \rfloor)\sum_{q=1}^{\lfloor s \rfloor} P_q^{(m)} + \sum_{q=1}^{\lfloor s \rfloor -1} P_q^{(m)} + \ldots + \sum_{q=1}^{\lfloor s \rfloor - \lfloor e \rfloor} P_q^{(m)} + (e-\lfloor e \rfloor) \sum_{q=1}^{\lfloor s \rfloor - \lfloor e \rfloor -1} P_q^{(m)}\right) \quad (24)$$

From (15), (18) and (24), we observe that for the computation of $N_d(s)$, only the quantities $P_q^{(m)}$ are needed. In Appendix 1, we provide recursive formulas for the computation of the quantities $P_q^{(m)}$. It is worth noticing that formulas (15), (18) and (24), are valid for any window Random-Access algorithm that has the properties described in the first paragraph of section 3.

3.2 Development of bounds on $K_1(s)$, H_1

For the development of bounds on the delays we chose $\Delta=2.5$ As a result, the state space F becomes simply,

$$F = \{1, 1.5, 2., 2.5, \ldots\}$$

For the algorithm considered in this section, the maximum throughput is achieved for $\Delta=2.67$ [5]. The reduction in throughput due to the choice $\Delta=2.5$ is insignificant (less than .1%).

Since $\delta \equiv \tau$, equations (7a), (7b), become:

$$K_d(s) = N_d(s) + \sum_{\substack{x \\ x \neq 1}} K_x(s) p(x/d) \quad \text{if } d=1, 1.5, 2, 2.5 \quad (25a)$$

$$K_d(s) = N_\Delta(s-d+\Delta) + \sum_{x} K_{d-\Delta+x}(s) p(x/\Delta) \quad \text{if } d=3, 3.5, 4, \ldots \quad (25b)$$

For the development of bounds on $K_1(s)$ we followed the method of truncation of the infinite system (25) [3]. Specifically, in system (25), we replaced the unknowns $K_d(s)$, for $d>40$ with the upper bounds in (14a). The substitution results in a finite system of equations whose solution is an upper bound to the solution of (25)for d=1, 1.5, 2,...,40.. For the development of lower bounds, we replaced the unknowns $K_d(s)$, d>40 with the lower bound in (14b). The same methodology is employed for the development of bounds on H_1. The resulting upper and lower bounds on the distribution of the delays differ by .01 in the worst case. In figure 3, we provide the distribution of the delays for various values of the arrival rate. An important observation is that as the arrival rate increases, the tales of the distribution become longer, but the median grows much slower than the expected delays.

4. THE PART-AND-TRY ALGORITHM WITH C-NC FEEDBACK

A detailed description of the algorithm can be found in [6]. The algorithm has throughput .45, operates under binary feedback and it is not blocked in the presence of feedback errors. The techniques used in the present section, can be easily applied to the analysis of the algorithm under ternary feedback.

4.1 The computation of $N_d(s)$.

Let M be the number of packets in the window d (see Fig 4.). As in section 3.1, formula (15) holds. Since not all the packets in a window are successfully transmitted, however, we need to modify the definition of $J_{i,m}(s)$ in (16). Let

$$J_{i,m}(s) = \begin{cases} 1 & \text{if the i-th randomly chosen packet in d is sucessfully transmitted and its delay is at most s.} \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

then, as in (18),

$$E(n_d(s) / M=m) = mE(J_{1,m}(s) / M=m) \tag{27}$$

and

$$E(J_{1,m}(s) / M=m) = \frac{1}{d} \int_0^d E(J_{1,m}(s) / M=m, \theta_1=\theta)d\theta \qquad (28)$$

Again, θ_2 takes integer values. However, θ_1 and θ_2 are not independent. To proceed further, we define the events

$A_{i,m}(s)$={ the i–th randomly chosen packet is successfully transmitted and $\theta_2 \leq s$.}

$a_{i,m}(q)$ = {the i–th randomly chosen packet is successfully transmitted and $\theta_2=q$, q=1,2,...}

Since θ takes only integer values, we have that

$$A_{i,m}(s) = \bigcup_{q=1}^{\lfloor s \rfloor} a_{i,m}(q) \qquad (29)$$

From (28) and (29) we conclude that

$$E(J_{1,m}(s) / M=m) = \frac{1}{d} \int_0^d p(A_{1,m}(s-\theta) / M=m, \theta_1=\theta)d\theta \qquad (30)$$

$$= \frac{1}{d} \int_0^d \sum_{q=1}^{\lfloor s-\theta \rfloor} p(a_{i,m}(q) / M=m, \theta_1=\theta)d\theta$$

Observe now that $p(a_{1,m}(q) / M=m, \theta_1=\theta)$ depends on θ through the ratio $\phi=\theta/d$. Let us define

$$f_1(q,m,\phi) = p(a_{i,m}(q) / M=m, \theta_1=\phi d) \; ; \; \phi=\frac{\theta}{d} \; \varepsilon \; [0,1) \qquad (31)$$

then, the following equations hold:

$$f_1(0,m,\phi) = 0, \text{ for } m=1,2,..., \text{ and } \phi \varepsilon [0,1) \qquad (32a)$$

$$f_1(1,m,\phi) = 0, \text{ for } m=2,3,.., \text{ and } \phi \varepsilon [0,1) \qquad (32b)$$

$$f_1(q,1,\phi) = \begin{cases} 1 & \text{if } q=1 \\ 0 & \text{otherwise} \end{cases} ; \quad \phi \varepsilon [0,1) \qquad (32c)$$

$$
f_1(q,m,\phi)\underset{m\geq 2}{=}\begin{cases} \sum_{j=0}^{m-1} f_1(q-1,j+1,2\phi-1)\begin{bmatrix} m-1 \\ j \end{bmatrix} 2^{(-(m-1))} & .5\leq\phi<1 \\ f_1(q-2,m,2\phi)2^{-(m-1)} + f_1(q-2,m-1,2\phi)(m-1)2^{(-(m-1))} & 0\leq\phi<.5 \end{cases} \tag{32d}
$$

Formulas (32a), (32b), (32c), are obvious. Let us explain formula (32d). Assume that $.5\leq\phi<1$. Then, the packet under consideration (packet 1 in fig 4), lies in the left hand half (l.h.h.) of the window. Let θ_2' be the delay of packet 1, after the first collision. Then,

$$
\theta_2 = \theta_2'+1 \tag{33}
$$

The location of packet 1 in the l.h.h. of the window, is,

$$
\theta_1' = \theta_1-d/2 \tag{34}
$$

Therefore, the new ratio becomes

$$
\phi' = \frac{\theta_1'}{d/2} = 2\phi-1 \tag{35}
$$

Let $G_{j,m-1}$ be the set

$G_{j,m-1}$={j of the m−1 packets (other than packet 1), are located in the l.h.h. of the window.}

Note that $p(G_{j,m-1}) = \begin{bmatrix} m-1 \\ j \end{bmatrix} 2^{-(m-1)}$. From (33), (34) and (35), we derive formula (32d) for $.5\leq\phi\leq1$ by conditioning on $G_{j,m-1}$. If $0\leq\phi<.5$, packet 1 is located in the right hand half (r.h.h.) of the window, In this case, we note that if in the l.h.h. of the window there are more than one packets, then packet 1 *is not transmitted* during the collision resolution process. By conditioning again on the $G_{j,m-1}$, and taking into account the last observation, we derive equation (32d) for $0\leq\phi<.5$.

From equations (32) we conclude by induction the following property for the function $f_1(q,m,\phi)$:

Property 1. For fixed q and m, the function $f_1(q,m,\phi)$ is simple (i.e., it takes a finite number of values), and left continuous. The jumps of the function occur at the points $0, 2^{-(q-1)}, \ldots, k2^{-(q-1)}, \ldots, 1-2^{-(q-1)}$.

Taking into account Property 1, we can compute $E(J_{1,m}(s))$, using formula (30).

4.2 The Computation of p(x,r/d).

The method of conditioning on the number of packets in a window (applied in sections 3.1 and 4.1), does not seem to lead to easily computable recursive formulas. In this section, we present an alternative methodology that results in simple recursive formulas for the computation of p(x,r/d), and provides insight into the structure of these probabilities.

Observe first that p(x,r/d) depends on r through the ratio s=r/d. Let

$$f_2(x,s,d) = p(l=x,\ \delta=sd,\ M\geq 2\ /\ d)$$

Conditioning on the events {M=0}, {M=1} and {M≥2}, and observing that if M=0,1 then $l=1$ and $\delta=d$, we conclude that

$$p(x,r/d) = \delta_K(x-1)\delta_K((s/d)-1)e^{-(\lambda d)}(1+\lambda d) + f_2(x,s,d) \tag{36}$$

where

$$\delta_K(y) = \begin{cases} 1 & \text{if } y=0 \\ 0 & \text{otherwise} \end{cases}$$

The function $f_2(x,s,d)$ satisfies the following recursive formulas

$$f_2(1,s,d) = f_2(2,s,d) = 0, \quad 0<s\le 1 \tag{37a}$$

$$f_2(x,s,d) = \begin{cases} f_2(x-1,2s,d/2) & 0\le s\le .5 \\ f_2(x-2,2s-1,d/2)e^{-(\lambda d/2)}(1+\lambda d/2) & .5<s\le 1 \\ \quad + \delta_K(x-3)\delta_K(s-1)e^{-(\lambda d)}(\lambda d/2)^2 & \end{cases} \tag{37b}$$

We derive equation (37a) for $0\le s\le .5$ The case $.5<s\le 1$ can be derived by a similar reasoning. Let M' be the number of packets in the l.h.h of the window. Let l' be the number of slots during a collision resolution process *after* the first collision. Then,

$$p(l=x,\ \delta=sd,\ M\ge 2\ /\ d) = p(l'+1=x,\ \delta=sd,\ M\ge 2,\ M'\ge 2\ /\ d) + p(l=x,\ \delta=sd,\ M\ge 2,\ M'\le 1\ /\ d)$$

But if $M'\le 1$, then $s>.5$. Therefore $p(l=x, \delta=sd, M\ge 2, M'\le 1 / d) = 0$ if $0.\le s\le .5$. It remains to observe that $p(l'+1=x, \delta=sd, M\ge 2, M'\ge 2 / d) = p(l'=x-1, \delta=2(sd/2), M'\ge 2 / d)$, and that if $M'\ge 2$, the collision resolution process after the first collision is statistically identical to a collision resolution process that started with a window of size d/2.

From (37a), (37b) we conclude by induction that the function $f_2(x,s,d)$ has the following property:

Property 2 For fixed $x\ge 3$, and for any d, s takes a finite number r_x of values. The sequence r_x satisfies the following recursions: $r_3=r_4=1$, $r_x=r_{x-1}+r_{x-2}$; $x\ge 5$. Let A_x be the set of values of s for given x. Then,[3]

$$A_3=\{1\},\ A_4=\{1/2\},\quad A_x = (.5A_{x-1})\cup(.5A_{x-2}+.5)\ ;\ x\ge 5$$

The values of $f_2(x,s,d)$ for various x and s, can be computed from (37b). The

[3] We use the notation $aA+b = \{y : y=ax+b,\ x\in A\}$.

probabilities p(x,r/d), are easily computed from (36).

4.3 The Development of bounds on $K_1(s), H_1$.

The state space F for the Part-and-Try algorithm is a dense subset of $[1, \infty)$. This property complicates the development of finite systems of linear equations whose solution will provide upper or lower bounds to the quantities $K_1(s)$, H_1. Taking advantage of the structure of the probabilities p(x,r/d) (see section 4.2), however, we can proceed as follows: Let B be a finite subset of F, that includes the state d=1. We develop the following finite system of linear equations:

$$Y_d = N_d(s) + \sum_{(d-r+x)\varepsilon B^c} (\gamma_u(s)(d-r+x) + \zeta_u(s))p(x,r/d) + \sum_{\substack{(d-r+x)\varepsilon B \\ x\neq 1}} Y_{d-r+x}p(x,r/d) \quad \text{if } 1 \le d \le \Delta,\ d\varepsilon B \tag{38a}$$

$$Y_d = N_\Delta(s-d+\Delta) + \sum_{(d-r+x)\varepsilon B^c} (\gamma_u(s)(d-r+x) + \zeta_u(s))p(x,r/d) + \sum_{(d-r+x)\varepsilon B} Y_{d-r+x}p(x,r/\Delta) \quad \text{if } d > \Delta,\ d\varepsilon B \tag{38b}$$

$\gamma_u(s)$ and $\zeta_u(s)$ are determined from (11), (12). Due to property 2 of section 4.2, it is simple to find for a given $d\varepsilon B$, the values of x and r such that $(d-r+x)\ \varepsilon\ B$. The summation over the infinite set B^c, can be computed in terms of $E(l/d)$, $E(\delta/d)$, $\gamma_u(s)$, $\zeta_u(s)$, and the probabilities p(x,r/d); $(d-r+x)\ \varepsilon\ B$. The solution Y_d; $d\varepsilon B$ of system (38), is an upper bound to the solution $K_d(s)$; $d\varepsilon B$ of system (7) [3]. Since $1\varepsilon B$, we can determine a bound on $K_1(s)$. Similarly, lower bounds on $K_1(s)$, and upper and lower bounds on H_1 can be developed. For the computations we used the set $B = \{1, 1.125, \ldots, 1+(k/8), \ldots, 9\}$. The resulting bounds on the delay distribution are presented in Fig 5 for $\lambda = .1, .2, .3$. For higher arrival rates, the bounds are not tight (see Table 1) and although they can be improved by enlarging the set B, the computations become cumbersome.

5. CONCLUSIONS

We developed a method for the computation of bounds on the delay distribution of Window Random-Access algorithms. The method has been applied to the delay distribution analysis of the Capetanakis Window Random-Access algorithm and the Part-and-Try algorithm both under binary C-NC feedback. The bounds developed for the Capetanakis algorithm, are tight for all arrival rates within the stability region of the algorithm. For the Part-and-Try algorithm, however, the bounds are satisfactory for relatively low arrival rates. The computational difficulty in obtaining tight bounds for the latter algorithm, is due to its complicated state space. The techniques can be easily applied to other Window Random-Access algorithms whose operating characteristics are as described in Section 2.

APPENDIX

Here, we provide recursive formulas for the computation of $P_q^{(m)}$. Clearly,

$$P_q^{(m)} = 0;\ q \le 0, \qquad P_q^{(1)} = \begin{cases} 1 & \text{if } q=1 \\ 0 & \text{otherwise} \end{cases} \tag{A.1}$$

Let l_n be the number of slots needed for the resolution of multiplicity $n \ge 0$ conflict. Then, from the operation of the algorithm we conclude that

$$\underset{m\ge 2}{P_q^{(m)}} = \begin{cases} P_{q-1}^{(n+1)} & \text{with prob. } \dfrac{1}{2}\dbinom{m-1}{n}\dfrac{1}{2^{m-1}} \\ P_{q-1-l}^{(m-n)} & \text{with prob. } \dfrac{1}{2}\dbinom{m-1}{n}\dfrac{1}{2^{m-1}}\, p(l_n = l) \end{cases} \tag{A.2}$$

The upper part of (A.2) is derived by considering the event that the packet under consideration, together with n of the rest m-1 packets retransmit immediately after the initial collision. The lower part of (A.2) is derived by considering the event that the packet under consideration does not transmit immediately, while the n of the rest m-1 packets retransmit immediately after the initial collision and it takes l number of slots to resolve a collision of multiplicity n. The probabilities $p(l_n=l)$ can be computed by similar reasoning. Averaging in (A.2), we finally have the following recursive formulas for $P_q^{(m)}$ for $m \ge 2$.

$$P_q^{(m)} = 2^{-m} \sum_{n=0}^{m-1} P_{q-1}^{(n+1)} \binom{m-1}{n} + 2^{-m} \sum_{n=0}^{m-1} \sum_{l=1}^{q-2} \binom{m-1}{n} P_{q-1-l}^{(m-n)}\, p(l_n=l) \tag{A.3}$$

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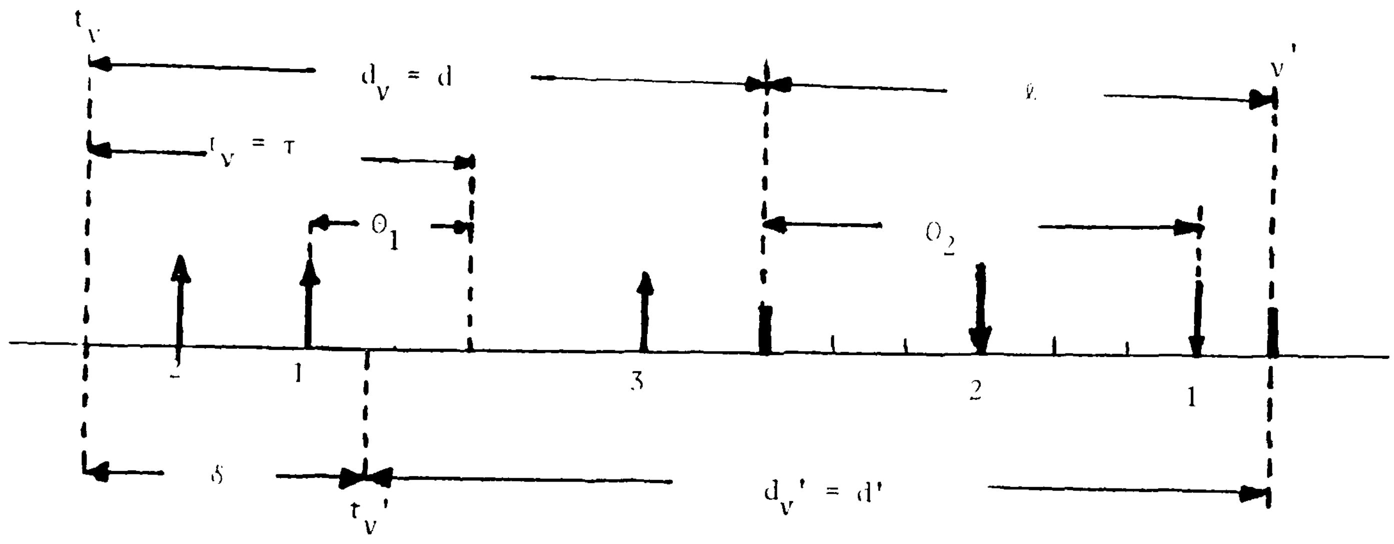


↑ :generation instant of the j-th randomly chosen packet
j

↓ :successful transmission of the j-th randomly chosen packet
j

Figure 1. Illustration of relationships among certain random variables related to the operation of window Random-Access algorithm.

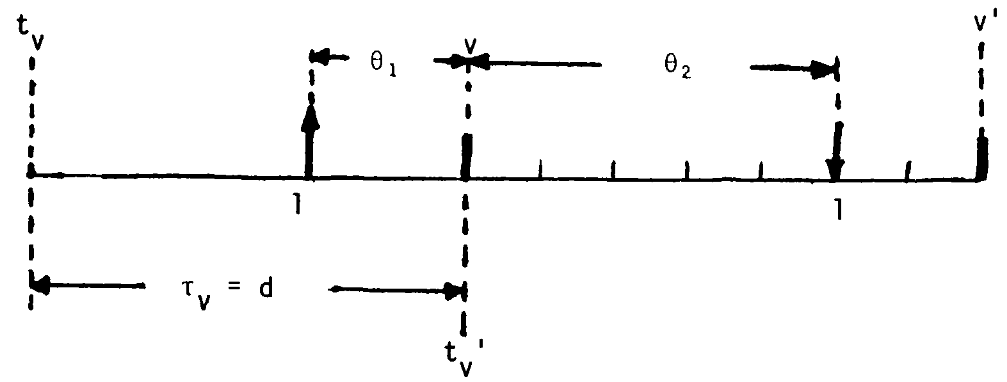


Figure 2.

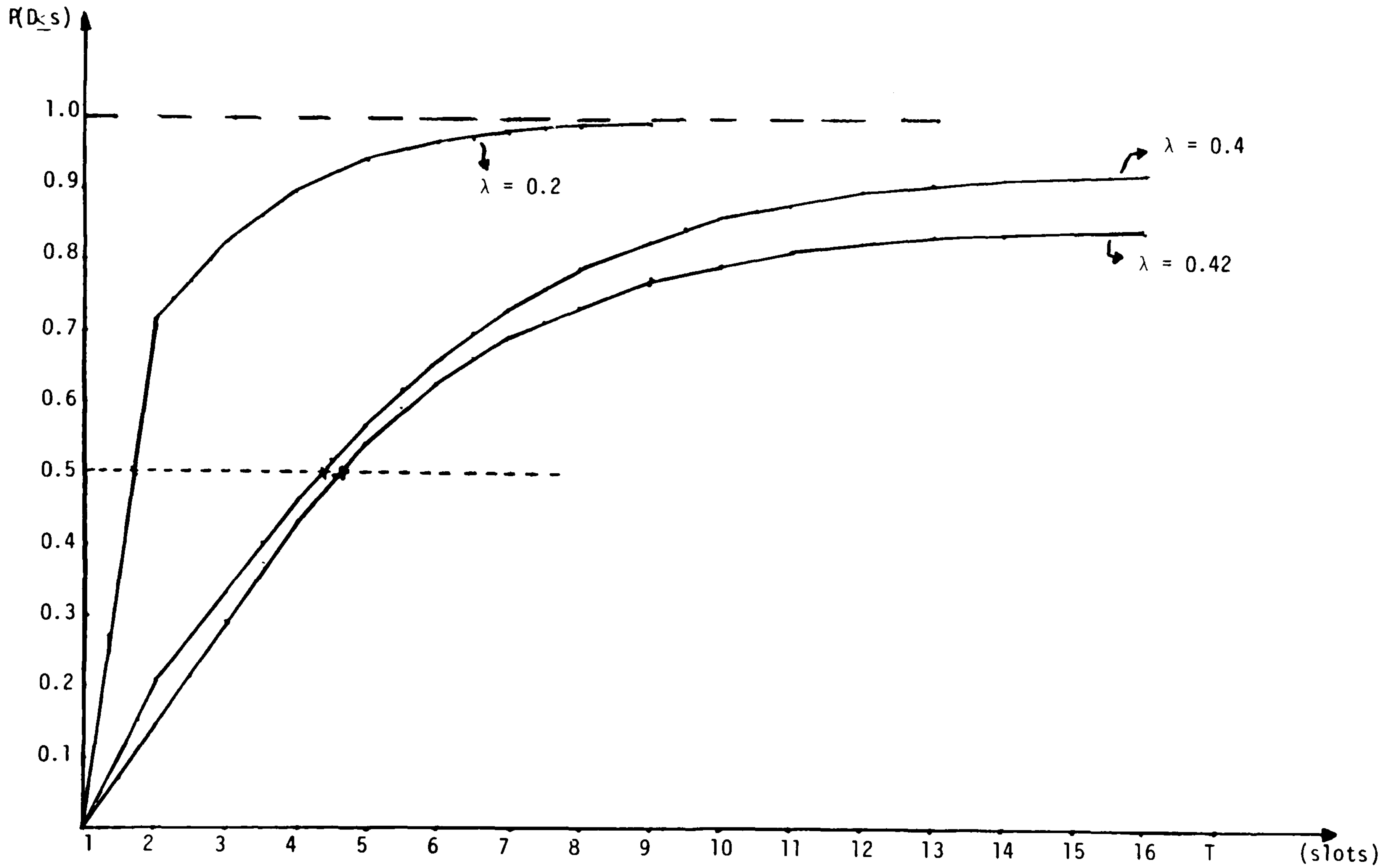
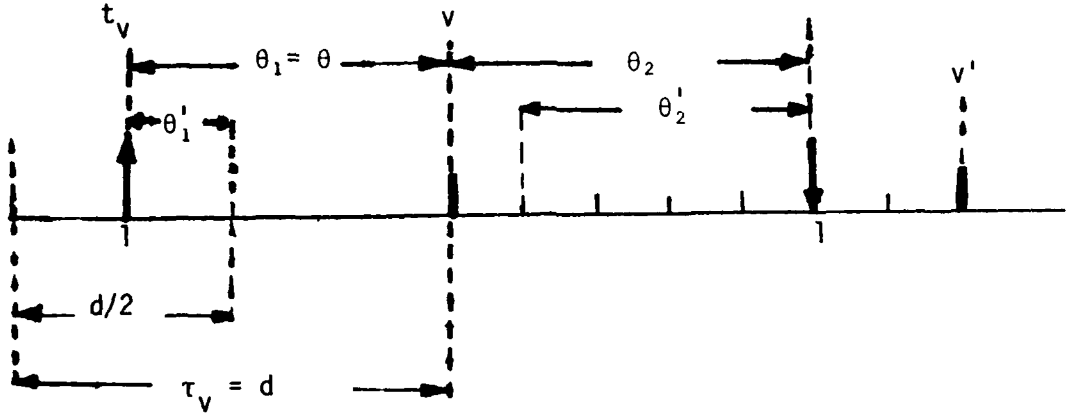


Figure 3.

Delay distribution for the Capetanakis Window RAA.



$$\phi = \theta_1/d$$

$$\phi' = \theta_1' \,/\, (d/2) = 2\phi - 1$$

Figure 4.

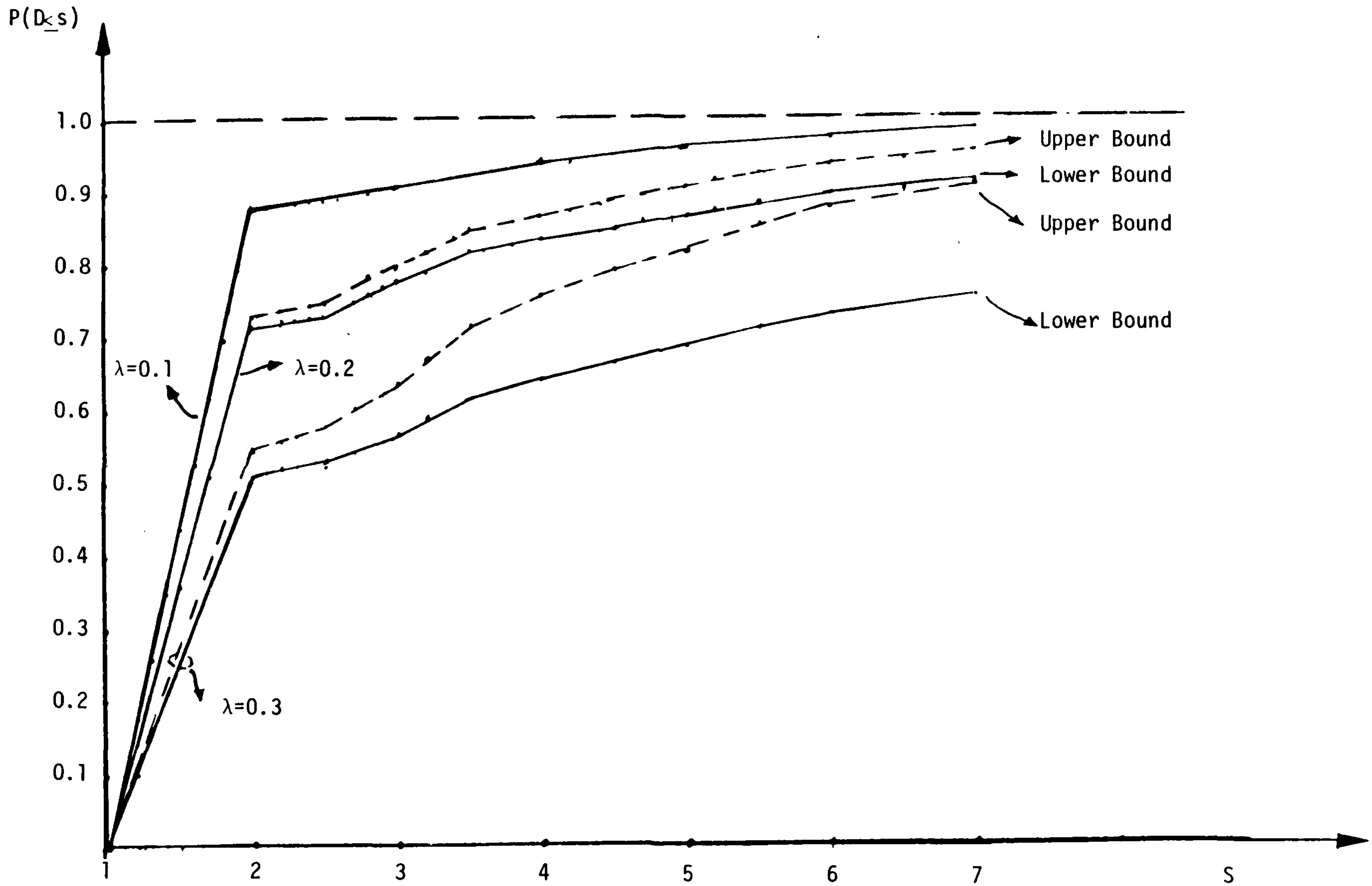


Figure 5.
Delay distribution for the Part-and-Try algorithm with binary C-NC feedback

	$P(D \leqslant s)$	
s	Upper Bound	Lower Bound
1.5	.103	.105
2.0	.208	.305
2.5	.216	.360
3.0	.243	.434
3.5	.271	.547
4.0	.286	.616
4.5	.300	.672
5.0	.316	.708
6.0	.344	.815
7.0	.365	.875

Table 1
Bounds on the delay distribution for the Part-and-Try algorithm with binary C-NC feedback, for $\lambda = 0.4$.

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