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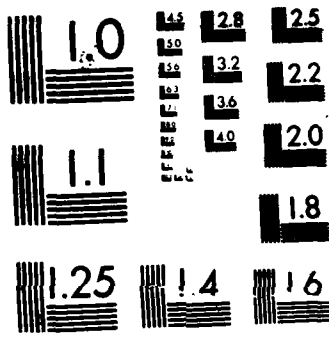
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Research Report CCS 565

USING DATA ENVELOPMENT ANALYSIS TO MEASURE
THE EFFICIENCY OF NOT-FOR-PROFIT ORGANIZATION
A CRITICAL EVALUATION-COMMENT

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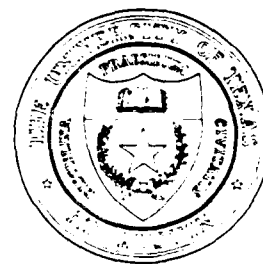THE EFFICIENCY OF NOT-FOR-PROFIT ORGANIZATIONS:
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ABSTRACT

This note responds to Nunamaker (1985) who supposedly deals with deficiencies in Data Envelopment Analysis (DEA) as an approach for (i) measuring efficiencies of not-for-profit entities identified as Decision Making Units (DMUs) and (ii) locating sources and amounts of inefficiencies in each of the inputs used and in each of the outputs produced by each DMU. Corrections and comments are offered with references supplied for interested readers who wish to examine more detailed treatments of the topics covered.

KEYWORDS .'

Data Envelopment Analysis
CCR Ratio Model
Pareto-Koopmans Efficiency ,

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Nunamaker's (1985) critical evaluation of DEA is erroneous
in its interpretation of the concept of Pareto-Koopmans efficiency
as introduced by Charnes, Cooper and Rhodes (1978) for use in DEA.
This turns out to seriously affect what Nunamaker has to say and is
accompanied by other inadequacies and misleading characterizations
of DEA.

In this comment we focus on Nunamaker's interpretations of
the Pareto-Koopmans efficiency concept for which we use the
following DEA model:

$$\min \quad \theta \qquad\qquad -\epsilon \sum_{i=1}^{m} s_i^- \quad -\epsilon \sum_{r=1}^{s} s_r^+$$

subject to

$$(1) \qquad \theta x_{io} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- \qquad , i=1,\ldots,m,$$

$$y_{ro} = \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+, \quad r=1, \ldots, s,$$

where the variables to be determined are $\lambda_j \geq 0$, $j=1, \ldots, n$; $s_i^- \geq 0$,
$i=1, \ldots, m$; $s_r^+ \geq 0$, $r=1,\ldots,s$; and θ is not constrained in sign. This
model, apart from some changes in the symbols, is the same as
version (2) in Nunamaker. It is obtained from the CCR ratio form of
Charnes, Cooper and Rhodes (1978) by using the theory of fractional
programming as given in Charnes and Cooper (1962).

The above formulation is the one that originally gave rise
to the name Data Envelopment Analysis.[1] The reasons underlying the

choice of this name are seen by noting that the above optimization envelops from above the observed output values y_{ro}, r=1, ..., s, for the decision making unit (DMU) being evaluated, and it also envelops its input values x_{io}, i=1, ..., m, from below, with the latter also being adjusted by the "scale" or "intensity" variable θ.

Denoting optimal values by a star, Nunamaker proceeds on the assumption that $\theta^*=1$ is necessary and sufficient for Pareto-Koopmans efficiency and this leads him to believe that this concept contains a "subtle yet important weakness...as used by DEA." However, as noted on p. 433 of Charnes, Cooper and Rhodes (1978) there are two conditions which must be fulfilled for Pareto-Koopmans efficiency: (i) $\theta^*=1$ and (ii) the slack variables s_i^{-*} and s_r^{+*} in (1) must all be zero. In short, $\theta^*=1$ is a necessary but not a sufficient condition.

To see how the presence of slack enters into the Pareto-Koopmans efficiency condition, let it be supposed that an optimum is achieved with $\theta^*=1$ but some $s_i^{-*}>0$. Such a solution means that it is possible to replace the observed $\hat{x}_{io}$ with a new $\hat{x}_{io} = x_{io}-s_i^{-*} < x_{io}$ and it is therefore possible to reduce this observed x_{io} input value without disturbing any other value in any of the constraints. Hence DMU_o cannot be characterized as Pareto-Koopmans efficient even with $\theta^*=1$.

Similar remarks apply to the presence of non-zero output slack. Indeed, it is possible to have an optimum in which both output and input slacks may be non-zero, in which case outputs may be augmented and inputs may be decreased simultaneously. Inefficiency in the sense of Pareto-Koopmans optimality would then be present in both outputs and inputs.

The two conditions for efficiency noted above may be simultaneously represented by rewriting the functional in (1) in the following form:

$$(2) \qquad h_o = \theta - \epsilon \sum_{i=1}^{m} s_i^- - \epsilon \sum_{r=1}^{s} s_r^+$$

and then setting the objective as min h_o. Defining $\epsilon > 0$ as a non-Archimedean infinitesimal in this expression for the functional provides access to the Non-Archimedean Efficiency theorem as given in Charnes and Cooper (1985),[2/] which we now summarize by denoting optimal values with a star and writing

$$(3) \qquad \min \ h_o = h_o^* = \theta^* - \epsilon \sum_{i=1}^{m} s_i^{-*} - \epsilon \sum_{r=1}^{s} s_r^{+*} \leq 1,$$

with $h_o^* = 1$ if and only if DMU_o is efficient. Note that $h_o^* = 1$ implies $\theta^* = 1$. On the other hand it is possible to have $\theta^* = 1$ and $h_o^* < 1$, in which case efficiency is not achieved because some of the optimal slack values are not zero.

Nunamaker's central proposition is that when a DMU has been accorded efficient status by DEA, then introducing a new variable will never alter this previously achieved efficient status. This is not correct, as will now be shown by means of the following simple example:

$$\min h_o = \theta \qquad -\epsilon s_1^+ \qquad -\epsilon s_1^-$$

subject to

(4)
$$1 = \lambda_1 + \lambda_2 - s_1^+$$

$$\theta = \lambda_1 + \lambda_2 \qquad + s_1^-$$

$$\lambda_1, \lambda_2 s_1^+, s_1^- \geq 0 .$$

In this example we are considering 2 DMUs which we identify via subscripts as DMU_1 and DMU_2, each of which uses a single input in unit amount, as shown in the second constraint of (4), to produce a single output in a unit amount, as represented in the first constraint. Their output and input values being the same, DMU_1 and DMU_2 are evidently both efficient with

(5)
$$\min h_o = h_o^* = \theta^* = 1$$

and all slack at zero value when evaluating either DMU_1 or DMU_2.

Now suppose another variable is added with data as exhibited in the third constraint in the following model where DMU_1 is being evaluated:

$$\min \ h_o' = \theta \qquad -\epsilon s_1^+ -\epsilon s_1^- -\epsilon s_2^-$$

subject to

(6)
$$1 = \lambda_1 + \lambda_2 - s_1^+$$

$$\theta = \lambda_1 + \lambda_2 \qquad + s_1^-$$

$$2\theta = \lambda_1 + \lambda_2 \qquad + s_2^-$$

$$\lambda_1, \ \lambda_2, \ s_1^+, \ s_1^-, \ s_2^- \geq 0 .$$

Evidently $\lambda_1 = 1$ and $\lambda_2 = 1$ will continue to satisfy the constraints with $s_2^- = 1$ required to supplement the choice of $\lambda_2 = 1$ in the latter case. These are not alternate optima, however, since the choice of $\lambda_1 = 1$ does not maximize the slacks, and hence it does not minimize h_o'. The latter is achieved only for $\lambda_2^* = 1$, $s_2^{-*} = 1$ and $\theta^* = 1$ so that:

$$(7) \qquad \min h_o' = h_o'^* = \theta^* - \varepsilon s_2^{-*} < h_o^* = 1 \ .$$

Hence, contradicting Nunamaker's central proposition, DMU_1 has lost its efficient status when the data for this new variable are introduced in the course of moving from (4) to (6).

As Nunamaker notes, it is possible for a DMU to be rated as efficient with $h_o^* = 1$ when one of its inputs is "small" and another of its inputs is "very large." This is a requisite of the concept of Pareto-Koopmans efficiency which, as used in DEA, avoids the need for assigning a priori systems of weights to reflect the relative importance of different inputs (as in index numbers). DEA also avoids the need for explicitly stipulating the functional forms that are supposed to relate the variables to each other (as in regression systems). Furthermore, the DEA efficiency ratio is independent of the units of measurement of inputs and outputs, as shown in Charnes and Cooper (1980) and Charnes and Cooper (1985). Thus, by changing these units one can get some small and some large as desired for any purpose without affecting the value of this efficiency rating.

To be noted is that the values of the slack variables in the constraints for (1) are stated in whatever metric is regarded as natural or convenient for identifying these variables as potential sources of inefficiencies. The non-Archimedean elements appear only in the functional. As already

noted, the θ^* value in the functional is mathematically independent of the units of measurement used. The slack variables in the functional may also be made independent of the unit of measurement by following the route used in Charnes, Cooper, Golany, Seiford and Stutz (1985) and dividing each s_i^- and s_r^+ in the functional by its corresponding x_{io} or y_{ro}. These adjustments in the functional do not alter the statement of the Non-Archimedean Efficiency Theorem as given in (3), above, and their use does not preclude continuing with whatever metrics are regarded as natural or convenient for the slack values in the constraints.

Nunamaker's discussion of DEA is also inadequate in that it omits any reference to the values of the dual variables which can be used to guide still further tradeoff adjustments. The possible use of optimal dual variable values for the purposes of effecting tradeoffs has been repeatedly noted, starting with the article by Charnes, Cooper and Rhodes (1978) (which Nunamaker cites) and has been carried into further development in subsequent articles such as Charnes, Cooper, Golany, Seiford and Stutz (1985). Thus while preserving the advantages of using the concept of Pareto-Koopmans efficiency in the manner noted in the preceding paragraphs, DEA is also able to delineate the efficient production frontier and to provide information to effect whatever tradeoffs may be desired by movement along these frontiers. Note, however, that movement along such efficiency frontiers requires making tradeoffs in which additional reductions in some resources or augmentations in some outputs may be obtained provided one is willing to increase other resources or diminish other outputs. Using such tradeoff possibilities may involve departures from the conditions of Pareto-Koopmans efficiency but DEA nevertheless provides such information to guide these tradeoffs, if wanted, and also to identify

efficiency frontiers which, in piecewise linear fashion, indicate
where these tradeoffs are to be considered.

Extensions for evaluating returns to scale possibilities are
available via the formulations in Banker, Charnes and Cooper (1984).
Returns to scale possibilities -- which may differ from one output to
another, as well as from one DMU to another -- may also be dealt with
via the concept of Most Productive Scale Size introduced by Banker
(1984). Using this concept in their DEA approach to data from North
Carolina hospitals, Banker, Conrad and Straus (1986) identified both
increasing and decreasing returns to scale as being present in
individual hospitals along with technical inefficiencies which had
all been "averaged out" in an earlier econometric study using a
translog version of "flexible functional form" regressions.

Further extensions of DEA continue to be made in contemporary
research but we doubt that any of them will be up to handling the
"creative accounting, political lobbying, [bogus] alterations of
input/output mixes," etc., which Nunamaker fears may be induced
by DEA. Difficulties like these and incentives for their use are
present, however, in every other known system of comparative
evaluation -- including the theory of competitive markets, in which
economic theory must assume an absence of force and fraud to obtain
the desirable results from the exchanges that such markets are
supposed to produce. The remedy for these difficulties lies in
using a multiplicity of controls including the extensive use of

audits noted on p. 57 of Nunamaker. DEA can also add to the repertory of such controls and, in fact, it can be used to guide and control audit processes along lines like those described by Thomas (1986) in his evaluation of the uses of DEA and other analytic tools for the Public Utility Commission of Texas in guiding its legislatively mandated audits of managerial efficiency.

FOOTNOTES

1/ The multiplicative form given in Charnes, Cooper, Seiford and Stutz
(1983) and the additive form given in Charnes, Cooper, Golany, Seiford
and Stutz (1985) provide alternative models which also utilize the
Data Envelopment Analysis principle.

2/ These $\varepsilon > 0$ values may be thought of as reciprocals of the non-Archimedean
"large" constants usually symbolized as M and associated with the use of
artificial variables in linear programming so that one can alternately
represent (2) in the form

$$\hat{h}_o = M\theta - \sum_{i=1}^{m} s_i^- - \sum_{r=1}^{s} s_r^+ .$$

In any case, methods for treating such non-Archimedean constructs via
ordinary simplex calculations are given on pp. 196 ff. in Chapter VI of
Charnes and Cooper (1961).

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This note responds to Nunamaker (1985) who supposedly deals with deficiencies in Data Envelopment Analysis (DEA) as an approach for (i) measuring efficiencies of not-for-profit entities identified as Decision Making Units (DMUs) and (ii) locating sources and amounts of inefficiencies in each of the inputs used and in each of the outputs produced by each DMU. Corrections and comments are offered with references supplied for interested readers who wish to examine more detailed treatments of the topics covered.

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