AD-A179 626 IMPERFECT REPAIRS FOR MULTI-UNIT SYSTEMS AND DESCRIPTION OF LIFELENGTHS B (U) ARIZONA UNIV TUCSON DEPT OF MATHEMATICS M SHAKED ET AL 27 MAR 87 1/1

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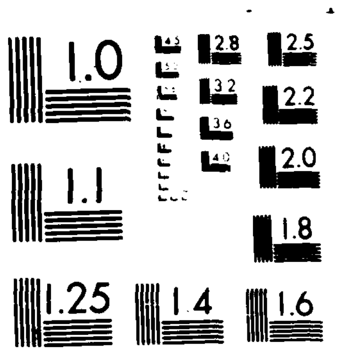

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Work progressed on two major tasks: (1) Modeling for multi-unit systems with units which undergo imperfect repairs before eventual fatal failure; and (2) Modeling multivariate life distributions through multivariate conditional hazard rate functions. In the first task, the paper "Multivariate imperfect repair", to appear in Operations Research deals with probalistic properties of four proposed models. In the second task several papers have been written regarding properties of conditional hazard rate functions.

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Annual Report for period 6-1-84 through 5-31-85

Grant No: AFOSR-8[redacted]-0205

Title: Imperfect repair for multi-unit systems

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Principal Investigators: Moshe Shaked

J. George Shanthikumar

1. Introduction.

In the original proposal (control number 83-NM-365) we proposed to work on the following two major tasks:

Task 1. Modeling for multi-unit systems with units which undergo imperfect repairs before eventual fatal failure.

Task 2. A search for optimal maintenance, replacement and procurement policies for multi-unit system under imperfect repair.

When we were trying to model multi-unit imperfect repairs we realized that we were really developing a novel approach to reliability theory. The main thrust of this approach is to model dynamically the joint behavior of random lifetimes as time progresses. Thus we found it necessary to work on the following task:

Task 3. Modeling multivariate life distributions through the so called multivariate conditional hazard rate functions.

During the period of the year, which is covered in this interim report, we worked mainly on Tasks 1 and 3. A brief summary of our results is given below.

2. Multi-unit imperfect repair.

The model that we have proposed to study was the one in which, when a device fails, an effort is made to repair it (and the repair is minimal in the sense that, if the repair is successful then the repair puts the device back into a working condition, but without making it as good as new). The main difficulty that one is confronted with while studying this model is the possible complex stochastic effects of the successful or unsuccessful repair of one unit on the residual lives of the other units. In order to avoid as much complexity as possible we first introduced and studied the following simple model:

Model 1. n items start to function at (the same) time. Upon failure an item undergoes a repair. With some probability p the repair is unsuccessful and the item is scrapped. With probability $1-p$ the repair is successful and minimal.

In this simple model it is assumed that when a device fails and is succesfully (minimally) repaired, the other devices "do not know" about the failure and repair. This assumption enabled us to write explicitly the joint probability distribution of the lives of the surviving devices as is detailed in [1].

In some applications, Model 1 is too simplistic. In our first generalization of Model 1 we allowed the probability p to depend on the number of devices which are still functioning (for example, when the distribution of the cost of minimal repair depends on the number of components which have already failed):

Model 2. n items start to function at (the same) time. Upon failure an item undergoes a repair. If i items $(i = 0,1, \ldots, n - 1)$ have already been scraped then, with some probability p_{i+1} the repair is unsuccessful and the item is scrapped, and with probability $1 - p_{i+1}$ the repair is successful and minimal.

For both models we obtained in [1] the joint density of the lifelengths of the devices under imperfect repair.

In Models 1 and 2 it is implicitly assumed that no two devices can fail at the same time. If there is a positive probability that two items (or more) can fail at the same time then (at least) two interpretations of Model 1 are possible:

Model 3. Same as Model 1 but with the additional postulation that if two or more items fail at the same time then each of them, independently of the others, is successfully minimally repaired with with probability $1-p$ and is scrapped with probability p.

Model 4. Same as Model 1 but with the additional postulation that if two or more items fail at the same time then, with probability $1-p$ all the failed items are successfully minimally repaired and with probability p all the failed items are scrapped.

Model 3 is applicable when each of the failed items undergoes imperfect repair independently of the others (for example if there are different repair facilities for different kinds of items). Model 4 is applicable, for example, when a shock which potentially could kill some items simultaneously is effective only with probability p and is ineffective (because of some kind of shield, say) with probability $1-p$.

Reference [1] deals with the probabilistic properties of the joint distribution functons which result from Models 1 - 4.

3. Multivariate conditional hazard rates and total hazards.

While studying the various models of multi-unit imperfect repairs discussed above, we noticed that every (absolutely continuous) multivariate distribution function can be characterized by a set of the so called multivariate conditional hazard rate functions. Intuitively, these functions describe the instantaneous failure rate of the surviving components conditioned on the complete knowledge of the identities and the failure times of the failed components. These functions determine the multivariate distribution function and vice versa.

In the setting of multivariate imperfect repair these functions are very useful. It turns out that in Model 1, e.g., one can get the hazard rate functions of the times to failure under imperfect repair by multiplying by p the hazard rate functions of the original lifetimes (i.e., when no repairs are performed).

But the conditional hazard rate functions are useful for many other applications. They can be used to identify positively dependent lifetimes, stochastically ordered lifetimes, jointly distributed lifetimes which satisfy various multivariate IFR and NBU properties and so on. These properties of the conditional hazard rate functions are discribed in [2] and [3].

The conditional hazard rate functions may depend not only on the past of the important components but also on some other factors. In particular, in some applications the hazard rate and the repair rate of a repairable component may depend on previous repair times and failure times of other components. Such a model is discussed in [4].

4. Related research.

Apart from the research described above, other ongoing research in reliability theory has been supported under the current grant.

Various notions of multivariate NBU (new better than used) and IFRA (increasing failure rate average) are studied in [5]. These notions are used in [6] and [7] to study properties of first passage times of stochastic processes of importance in reliability theory.

In [8] properties of a special subset of the set of cut sets of a network is used to develop bounds for network reliability. In [9] algorithms are developed to compute the lifetime distributions of consecutive-k-out-of-n systems with exchangeable lifetimes. IN [10] and [11] simple bounds, on reliabilities of systems which use "second hand" components, are obtained. Also the choice of the best "second hand" component is described there.

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