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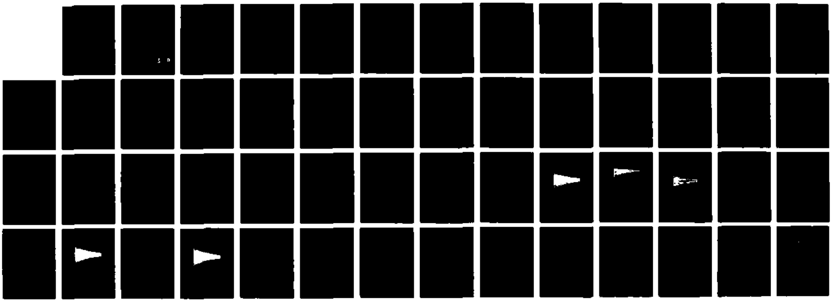

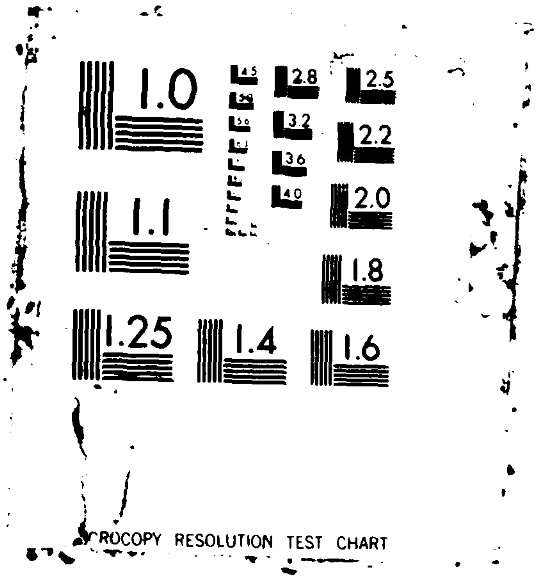
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This report summarizes the results of a one-year investigation of advanced issues related to sensing and control of combustion. The ultimate goal of this work is to develop procedures for optimizing and controlling the operating characteristics of gas turbine combustors. Toward this end, an extensive literature survey has been conducted, and model-based and heuristic-based control schemes have been proposed.

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1 Summary

This report summarizes the results of a one-year investigation of advanced issues related to sensing and control of combustion. The ultimate goal of this work is to develop procedures for optimizing and controlling the operating characteristics of gas turbine combustors. Toward this end, an extensive literature survey has been conducted, and model-based and heuristic-based control schemes have been proposed.

2 Introduction

Improvements in combustion processes have naturally led to a need to develop strategies for sensing, control, and ultimately optimization. Of particular interest is the sensing, control, and optimization of the combustion processes occurring in the combustors of gas turbine engines. The goals of these strategies might be to minimize heat flux transfer to critical areas of the combustor, drive the system away from instability limits, minimize undesirable by-products, and maximize the efficiency and operating range of the engine. At present, the approach to this optimization problem is generally static in nature and usually involves solving for the geometry of the combustor. It is of practical interest to consider the use of active systems to control combustion dynamics in order to modify and improve performance.

The control of combustion processes such as the ones occurring in aircraft gas turbines is of great importance in commercial and military applications. A combustion control system might be designed to modify such quantities as temperature profiles, pressure distributions, and species concentrations in the flow field. The control system would necessarily have available a set of variables that can be manipulated to alter the combustor's performance. These might include fuel concentration, inlet conditions, and combustor geometry. The first step in designing combustion control systems is the development of models for combustion phenomena.

The flow field in a gas turbine is turbulent and is governed by partial differential equations for conservation of mass, momentum and energy. Models of combustors are typically described by nonlinear, mixed hyperbolic-parabolic equations for such variables as velocity and pressure distributions, temperature profile, and species concentrations. If a reasonably accurate model of combustors existed, it could be used to develop control strategies for distributed parameter systems.

Unfortunately, existing models cannot accurately predict the behavior of an actual system, mainly because of a lack of knowledge of combustion chemistry and turbulence. Except for simple hydrocarbons such as methane, ethane, and propane, the reaction rates of the species participating in the chemical reactions are not accurately known. Also not known are the initial and boundary conditions and the thermophysical properties at high temperatures. Therefore, models of gas turbine combustors have been found to predict only to within a factor of two or three the available experimental data [Ramos, 1981]. Thus, it is necessary to investigate control strategies and design procedures that can be robust to relatively gross model errors.

2.1 Research Objective

The long-range objective of this research program is to develop a theory for the control of the distributed combustion processes that occur in a gas turbine engine. This theory is essential for improving the performance of combustion processes by (i) minimizing heat flux to critical sections, (ii) controlling operation near limits of combustion instability, and (iii) maximizing efficiency and range. The development of such a "controlled combustion theory" represents a multi-year effort involving the investigation of a hierarchy of problems of varying complexity.

The research reported in this report summarizes the results of one year's activities in which two different control approaches that are appropriate for simplified combustion models have been explored. Subsequently, studies need to address issues such as the optimal location and number of sensors and actuators required to control these simplified models, the inclusion of more advanced combustion models, and hardware-related concerns such as the design of rugged sensors and actuators.

2.2 Scope

This report consists of three main parts. First, it summarizes the results of an extensive literature survey on the control of distributed parameter systems and combustion, in particular. Second, it develops a model-based strategy for control of simplified combustion processes. Third, it develops a heuristic-based approach for combustion control that looks promising as a direction of future research work in this area.

3 Literature Survey

The control of systems governed by partial differential equations has significant practical importance. These systems are called *distributed-parameter systems* (DPS) because of the spatial as well as temporal dependence of the system variables; they contrast with lumped-parameter systems which are governed by ordinary differential equations. DPS are dynamical systems modeled in an infinite dimensional state space [Wang, 1964].

The control of DPS represents a mathematical challenge which has received considerable attention in the last decade [Lions, 1971, Aziz *et al.*, 1977, Tzafestas, 1982a, Ahmed and Teo, 1981, Banks and Pritchard, 1978] mostly from mathematicians (rather than engineers). The theory of DPS control has been formulated within the framework of functional analysis [Lions, 1971, Wang, 1964, Balas, 1982, 1983] and substantial theoretical progress has been made. However, practical application of the theory has significantly lagged behind the theoretical development. The main applications have been in heat exchangers in which parabolic equations [Lausterer and Eitelberg, 1982] or first-order hyperbolic equations [Kanoh, 1982] have been considered. Mainly, these applications are concerned with linear partial differential equations (PDE). In contrast, the combustion processes which occur in a gas turbine combustor are governed by nonlinearly coupled, mixed hyperbolic-parabolic equations.

The research reported in this report is concerned with the development of an appropriate theoretical framework and a practical implementation scheme to control the dynamic combustion processes which occur in gas turbine engines. The approach should, however, be applicable to other processes governed by nonlinear field equations.

3.1 Background

This section summarizes the principal work that has been conducted in the field of DPS control. This summary is not intended to be complete; rather, it reflects what we consider to be the most important references in the control of DPS.

The control of DPS is concerned with equations which are of the hyperbolic, parabolic, and elliptic type. The control can be applied to the boundary or to the interior. McGlothin [1974] developed a modal control model for a class of DPS governed by a one-dimensional parabolic equation and derived necessary and sufficient conditions for complete controllability of the system when the control is affected through mixed boundary conditions. Sakawa [1974] also derived necessary and sufficient conditions for the controllability of DPS with control functions appearing in the coefficients of the PDE as well as in the boundary conditions. Kastenberg [1974] studied the stability of nonlinear parabolic PDEs and described an example of temperature control. Numerical solution of nonlinear DPS governed by parabolic PDEs has been described by Holliday and Storey [1973].

The optimal control of a nuclear reactor which is governed by a linear parabolic PDE has been studied by Chandhuri [1972] who solved the optimal control problem by means of space and space-time discretizations and concluded that for nuclear reactor systems, which are widely distributed in space, approximate lumped model systems are not accurate because of discretization errors. A similar approach was followed by Yu [1971] who approximated a semilinear parabolic PDE by a finite set of nonlinear ordinary differential equations to determine the domain of asymptotic stability of the equations.

Seinfeld and Lapidus [1968] have described two techniques based on a direct search of the performance index and a method of steepest descent to examine the optimal control of DPS governed by hyperbolic and parabolic equations. Johnson [1973] solved a system of N hyperbolic PDEs with control exerted by N forcing functions which are distributed in space and time. The forcing functions were determined such that the system state lies in a given target at a certain time and the control energy is minimum. The problem was solved by means of the method of characteristics. Seinfeld *et al.* [1970] developed a nonlinear filter for nonlinear hyperbolic equations in which observations are made continuously at a finite number of discrete locations. The method of weighted residuals has been applied to hyperbolic equations so that their control is reduced to that of lumped-parameter systems by Parkin and Zahradnik [1971].

DPS control has been studied by means of truncated power series in space whose coefficients are a function of time [Breitholtz and Ovarnstrom, 1982]. The PDEs are integrated in space over a finite number of spatial regions so that a system of ordinary differential equations results. Linearization and lumping approximations were made by Viswanadham *et al.* [1979] to reduce a nonlinear DPS model to a set of ordinary differential equations. They also developed a state space model with three control parameters and two disturbance inputs. Necessary and sufficient conditions for optimality have been derived by Wu and Teo [1983] for boundary value problems of the parabolic type in which the controls are assumed to act through the forcing terms and through the initial and boundary conditions. Klamka [1983] determined the observability, controllability, and stability of a class of self-adjoint parabolic DPS using Hermite polynomials. Seidman [1983] employed semigroup methods to reduce a semilinear problem to a linear one, investigated the possibility of implementing feedback control systems through the boundary conditions, and showed that it is possible to use feedback for which the observer consists of a finite number of sensors for the one-dimensional heat transfer equation. Barbu [1981] obtained necessary conditions for optimality in DPS governed by semilinear and variational parabolic inequalities. Huntley [1979] studied the optimal control of a parabolic DPS by means of a matrix Ricatti formulation and showed that the spurious oscillations in the optimal control system result from the method employed to approximate the integrals. A stationary variational formulation of the necessary conditions for optimality of parabolic DPS with mixed boundary conditions was obtained by Meric [1979] who also employed a finite element method with elements in space and time to solve the optimal control problem.

The "optimum" location of the controllers for a parabolic DPS was deduced by

Martin [1978] by minimizing a quadratic cost functional. Amouroux and Babary [1978] determined the best pointwise location of actuators for a parabolic DPS by minimizing an energy criterion. Di Pillo and Grippo [1980] employed a multiplier method also known as augmented Lagrangian or penalty shifting method to the optimal control of linear parabolic DPS and assessed its convergence in Hilbert space. Kobayashi [1978] studied the controllability of a parabolic DPS. The optimal control of the system was determined by minimizing a functional which measures the distance between the terminal state and a given state. A regularization method was introduced to obtain the optimal control sequence since controllability is not sufficient for the existence of the functional. Linear programming has been used by Huang and Yang [1970] to study the optimal control of a heat exchanger with internal heat generation. Semi-infinite programming techniques have been employed by Glashoff and Gustafson [1976] to compute the optimal control of a class of one-dimensional heat-diffusion equations. The authors employed the Ritz technique.

In addition to the methods employed to solve the control equations mentioned above, *i.e.,* Ritz, Galerkin, dynamic programming, linear programming, semi-infinite programming and finite-element methods, the method of lines which transforms the system of PDEs into a system of ordinary differential equations by discretizing the space while keeping continuous the time variable has also been used [Graney, 1983, Ramos, 1983]. Other methods which have been employed include complete orthogonal Jacobi polynomials [Spalding, 1982], modal approximations [Gould, 1966], Green's functions, invariant-imbedding techniques, methods of characteristics, finite-difference algorithms, power series expansions and Monte Carlo methods [Tzafestas, 1982b]. In addition, finite element techniques [Carotenuto and Raiconi, 1982] and singular perturbation methods have also been employed [Van Harten, 1983].

The background provided in the previous paragraphs suggests that the theory of DPS is relatively well developed for linear systems of parabolic and hyperbolic equations. It also shows that some numerical work has been done to study the stability, controllability and optimal control of equations of the parabolic (heat diffusion) or hyperbolic (heat exchangers) type. However, very little work has been performed on the control of mixed hyperbolic-parabolic equations, and on the control of non-linearly coupled equations such as the ones of combustion theory [Williams, 1985].

In this research effort, we have taken first steps toward the development of a theory for the control of DPS governed by nonlinearly coupled mixed hyperbolic-parabolic equations such as the ones that govern the combustion processes in a gas turbine engine. The development of this theory has proceeded by examining simplified combustion problems including one-dimensional combustion models.

It is emphasized here that the control of the combustion processes in a gas turbine combustor requires first the development of a control theory for nonlinearly coupled, mixed hyperbolic-parabolic equations. Such a theory does not exist at the present time.

4 Modeling

4.1 Governing Equations

The equations governing chemically reacting flows can be found in Williams [1985]. These equations can be simplified by neglecting Soret and Dufour effects, body forces, bulk viscosity, radiation and pressure-gradient diffusion. If the flow is assumed one-dimensional, the pressure is constant, the specific heats of all the species are assumed constant and equal, and Fick's law is used for the species diffusion, the following system is obtained in cartesian coordinates

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \tag{1}$$

$$p = \text{constant} = p_o \tag{2}$$

$$p = \rho \tilde{R} T \sum_{i=1}^{N} \frac{Y_i}{W_i} \tag{3}$$

$$\frac{\partial}{\partial t}(\rho Y_i) + \frac{\partial}{\partial x}(\rho u Y_i) = \frac{\partial}{\partial x}\left(\rho D \frac{\partial Y_i}{\partial x}\right) + \dot{\omega}_i \,, \; i = 1, \ldots, N-1 \tag{4}$$

$$Y_N = 1 - \sum_{i=1}^{N-1} Y_i \tag{5}$$

$$\rho C_p \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right] = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) - \sum_{i=1}^{N} \dot{\omega}_i h_i^{\circ} \tag{6}$$

where ρ is the density; t is time; x is the spatial coordinate; u is the fluid velocity; p is the pressure; T is the temperature; $\tilde{R}$ is the universal gas constant; Y_i and W_i are the mass fraction and molecular weight of species i , respectively; N is the number of species; $\dot{\omega}_i$ and h_i° are the reaction rate and enthalpy of formation of species i , respectively; D is the diffusion coefficient; C_p is the specific heat at constant pressure; and, k is the thermal conductivity. Assuming that the Lewis number ($Le = k/\rho D C_p$) is unity, and introducing the mapping [Spalding, 1956; Ramos, 1986a]:

$$(t, x) \rightarrow (\tau, \psi) \tag{7}$$

where

$$\tau = t \tag{8}$$

$$\frac{\partial \psi}{\partial x} = \rho \tag{9}$$

$$\frac{\partial \psi}{\partial t} = \int_{-\infty}^{x} \frac{\partial \rho}{\partial t} \, dx = - \int_{-\infty}^{x} \frac{\partial}{\partial x} (\rho u) \, dx = \dot{m}_o - \rho u \tag{10}$$

$$\dot{m}_o = \rho u \quad \text{at} \quad x = 0 \tag{11}$$

and substituting into equations (4) and (6), the following system results

$$\frac{\partial Y_i}{\partial \tau} + \dot{m}_o \frac{\partial Y_i}{\partial \psi} = a \frac{\partial^2 Y_i}{\partial \psi^2} + \frac{\dot{\omega}_i}{\rho} \tag{12}$$

$$\frac{\partial T}{\partial \tau} + \dot{m}_o \frac{\partial T}{\partial \psi} = a \frac{\partial^2 T}{\partial \psi^2} - \sum_{i=1}^{N} \frac{h_i^{\circ}}{\rho} \frac{\dot{\omega}_i}{C_p} \tag{13}$$

Furthermore, if a one-step irreversible chemical reaction is used

Fuel (species 1) + Oxidizer (species 2) $\rightarrow$ Products (14)

Equations (12) and (13) can be written as

$$\frac{\partial Y_i}{\partial \tau} + \dot{m}_o \frac{\partial Y_i}{\partial \psi} = a \frac{\partial^2 Y_i}{\partial \psi^2} + \frac{\omega_1}{\rho} \tag{15}$$

$$\frac{\partial T}{\partial \tau} + \dot{m}_o \frac{\partial T}{\partial \psi} = \alpha \frac{\partial^2 T}{\partial \psi^2} + Q \frac{\dot{\omega}_1}{\rho C_p} \tag{16}$$

where $\alpha = \rho^2 D$ was assumed constant and Q is the heat of combustion.

Equations (15) and (16) can be written as

$$\frac{\partial Y_{sz}}{\partial \tau} + \dot{m}_o \frac{\partial Y_{sz}}{\partial \psi} = \alpha \frac{\partial^2 Y_{sz}}{\partial \psi^2} \tag{17}$$

where

$$Y_{sz} = \frac{C_p T}{Q} - Y_1 \tag{18}$$

is a Shvab-Zel'dovich variable.

We will assume that Y_{sz} = constant. This assumption permits us to relate Y_1 to T . For a second-order reaction, *i.e.,*

$$\dot{\omega}_1 = - A \exp\left(- \frac{E}{RT} \right) Y_1 Y_2 \frac{\rho^2}{W_2} \tag{19}$$

$\dot{\omega}_1$ is only a function of T .

In equation (19) A is the pre-exponential factor and E is the activation energy of the reaction. Substitution of equation (19) into equation (16) yields

$$\frac{\partial T}{\partial \tau} + \dot{m}_o \frac{\partial T}{\partial \psi} = \alpha \frac{\partial^2 T}{\partial \psi^2} + F(T) \tag{20}$$

where

$$F(T) = \frac{Q}{\rho C_p} \dot{\omega}_1 \tag{21}$$

Equation (20) can be non-dimensionalized by introducing

$$\Theta = \frac{T-T_u}{T_b-T_u} \ , \ x^* = \frac{X}{k} C_p \dot{m}_o \ ; \quad \psi^* = \frac{\psi}{\rho_u k} C_p \dot{m}_o \tag{22}$$

$$t^* = \tau \frac{u_o}{k} C_p \dot{m}_o \tag{23}$$

This results in

$$\frac{\partial \Theta}{\partial t^*} + \frac{\partial \Theta}{\partial \psi^*} = \frac{\partial^2 \Theta}{\partial \psi^{*2}} + G(\Theta) \tag{24}$$

where

$$G(\Theta) = F(T) \frac{\rho_u k}{m^2 \; {}_{(o)}C_p \; (T_b - T_u)} \tag{25}$$

is a highly nonlinear function of Θ .

$G(\Theta)$ can be approximated as [Spalding, 1957].

$$G(\Theta) = a\ \Theta^n\ (b - \Theta)^{\ell} \tag{26}$$

where a, b, n and ℓ are constants. The case $b = n = \ell = 1$ corresponds to Fisher's equation [Fisher, 1937, Reitz, 1981].

We will consider two reaction rate terms:

$$G(\Theta) = \Theta(1 - \Theta) \tag{27}$$

in Section 5 and

$$G(\Theta) = \Theta^2(1 - \Theta) \tag{28}$$

in Section 6. For convenience we will introduce V (= 1) in equation (24) and replace Θ , t^* and ψ^* b , T , t and x , respectively. Thus, equation (24) will be written as

$$\frac{\partial T}{\partial t} + V \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} + G(T) \tag{30}$$

where

$$G(T) = T(1-T) \qquad \text{or} \qquad G = T^2(1-T) \tag{31}$$

Equation (30) is subjected to the following boundary conditions

$$T(-\infty, t) = 1 \quad , \quad T(\infty, t) = 0 \tag{32}$$

Note that the reaction terms, *i.e.*, equation (31), are identically equal to zero in the burned and unburned gases. Therefore, no cold boundary difficulty exists [Williams, 1985] and the problem is well-posed. In chemically reacting flows, the reaction rate is small (but not zero) at the cold (unburned gas) boundary, the problem is ill-posed and can only be treated by means of time-dependent numerical techniques [Aldushin, *et al.*, 1981; Zel'dovich, *et al.*, 1985] to achieve the steady state. However, it can be shown that the reaction rate at the cold boundary is many orders of magnitude smaller than that at the flame front and, asymptotically, is exponential small [Clavin and Linan, 1984]. Equations (30) and (31) have been the subject of numerous analytical and numerical studies [McKean, 1975; Murray, 1977; Fife, 1979; Ramos, 1983, 1985, 1986b].

5 Lumped Control Strategy

5.1 Introduction

We consider here equation (30) *i.e.,*

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} - V\frac{\partial T}{\partial x} + T(1-T) \qquad (33)$$

where $G(T) = T(1 - T)$ and $x \in [\ 0\ ,\ L\ ]$ subject to the boundary conditions:

$$T(x=0) = 0\ ;\ T(x=L) = 1 \qquad (34)$$

The solution to equation (33) subject to (34) at a given time is a monotonically increasing function in the spacial variable as illustrated in Figure 1. A positional error, e , can be defined by:

$$e = x^* - x_s \qquad (35)$$

where x^* is the x-coordinate of a well-defined (moving) point on the temperature profile and x_s is the desired location of that point. For instance, the point x^* might be defined as the location of the point where $T(x^*) = 0.5$ or the inflection point, where $\partial^2T(x^*)/\partial x^2 = 0$. The point x^* must be defined such that it exists and is unique at all times. It is assumed that the coefficient V in equation (33) can be manipulated to alter the solution, and thus alter the error, e . Changing V alters both the shape of the profile and the rate at which the point x^* moves. Expresssed in functional form:

$$\frac{dx^*}{dt} = f_1(V,t) \qquad (36)$$

or, equivalently:

$$\frac{de}{dt} = f_2(V,t) \qquad (37)$$

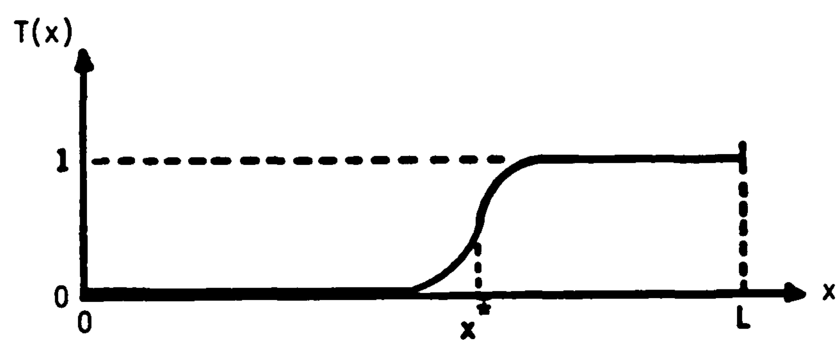

Figure 1: Solution to Equation (33) Subject to Conditions (34).

It is also assumed that the temperature distribution, $T(x)$, in equation (33) can be sensed at discrete points, T_i :

$$T_i = T(x_i) \; ; \; i = 1, \ldots N \tag{38}$$

where N is the number of sensors. These temperature measurements are further assumed to be related to the error:

$$T_i = f_{3i}(e,t) \tag{39}$$

Even without explicit knowledge of the functions f_{3i} , the existence of equation (39) is implicitly assumed when a measurement of the form (38) is used to control the error, e . If the relationship implied by equation (39) does not exist, a controller based on these measurements cannot control the error.

We next begin discussion of the control problem, the goal of which is to drive the error defined by equation (35) to zero and to regulate it there.

5.2 Definition of the Control Problem

Given the above definitions, the following control problem can be posed: Find a relationship from the sensor measurements, T_i , to the control, V , such that the well-defined point is driven to a desired location, $x^* \rightarrow x_s$ (or equivalently, $e \rightarrow 0$). That is, we seek a function, f_4 :

$$V = f_4(T_1 , \ldots , T_N , t) \tag{40}$$

such that $e \rightarrow 0$ subject to:

$$\frac{de}{dt} = f_2\{f_4(T_1 , \ldots , T_N , t)\} \tag{41}$$

or, equivalently:

$$\frac{de}{dt} = f_5(e,t) \tag{42}$$

Equation (42) indicates the dependence of e on itself. The problem thus defined represents a classical or lumped closed-loop control problem. The objective of the problem is to find a function f_4 such that the function f_5 is at least stable. In addition, it

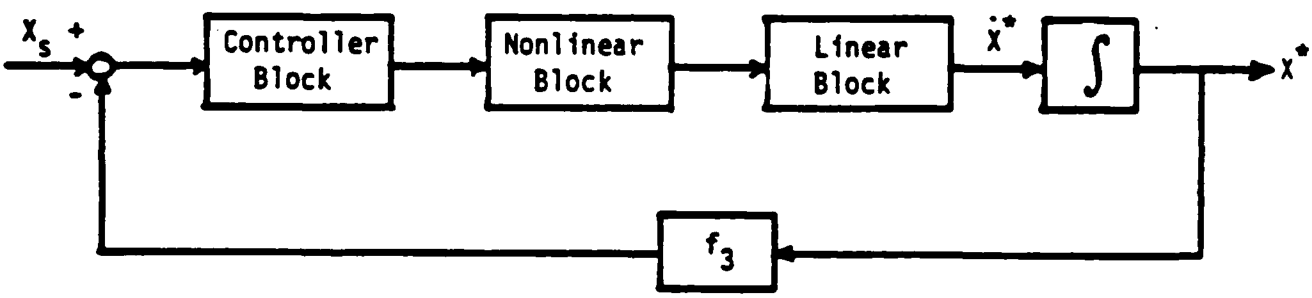

Figure 2: Closed-Loop Control System.

may be desired to require equation (42) to satisfy certain constraints or to minimize a functional that includes the control V.

5.3 Synthesis of a Controller

The control problem defined in equation (41) is *not* a distributed parameter problem. Equation (41) is an ordinary differential equation; the distributed system of equation (33) is implicit in the functions f_2 and f_4. This apparent simplification is a result of the way in which the error, e , and the sensor measurements, T_i , were defined. The simplification is not a result of approximating the partial differential equation (33). An evaluation of these functions still requires a solution of (33). Once these functions have been obtained, a design procedure can be developed that treats the system as being nonlinear and non-autonomous (time-varying).

The structure of the problem is shown in Figure 2. The control objective is to regulate a point on the temperature profile that can be identified with a well-defined spacial coordinate, x^* , at a desired location.

One method of designing a system like that illustrated in Figure 2 is to apply a stability criterion and find a range of controller gains that leads to a stable closed-loop system. A satisfactory stability criterion might be the off-axis circle criterion [Hedrick and Paynter, 1980]. This criterion yields a sufficient condition for stability of a control loop with a single nonlinearity in terms of a minimum and a maximum gain. If the nonlinearity is bounded by these two gains, the closed-loop system will be stable. To apply the criterion, it is necessary to transform the control system shown in Figure 2 to the system shown in Figure 3. Figure 3 represents a system with simple proportional control. The off-axis circle criterion requires an approximation of the shape of the Nyquist plot of the linear block in Figure 3. Since there is a free integration in the loop of Figure 2, the linear block in Figure 3 must also contain a free integration. It is also known that the steady state response to a constant value of the control, V , is a constant value of x^* . Therefore, the linear block cannot contain another free integration. This information is enough to specify the low frequency part of the Nyquist diagram. The high frequency

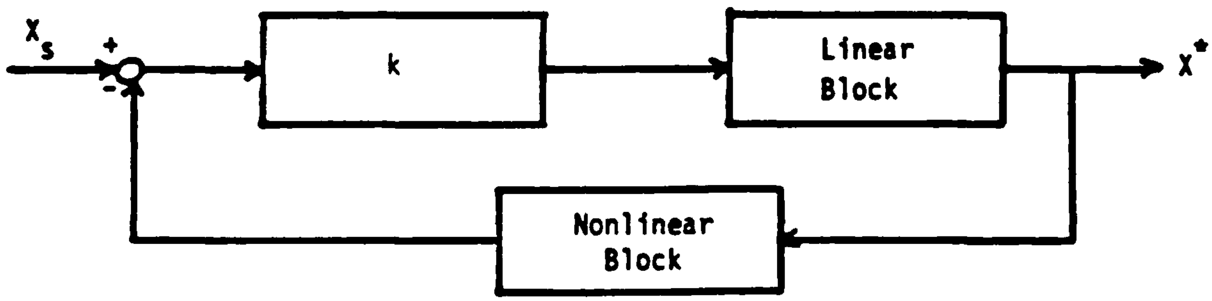

Figure 3: Closed-Loop System for Stability Analysis.

portion of the curve can be approximated from the requirements of the Hurwitz criterion for physical systems. Figure 4 illustrates two possible curves that satisfy these requirements. Note that the shape of these curves was determined deductively and not analytically. However, an analytical approach is possible. Application of the off-axis circle criterion to the Nyquist curves in Figure 4 suggests that the gain, k, that can be tolerated by the system of Figure 3 is bounded from above and is not bounded from below. Numerical simulations suggest that the upper bound on k is quite high. In order to compensate for the fact that the value of V is not known exactly, the proportional control scheme must be augmented by an integrator. The following controller is proposed:

$$V = k_1(T_1 + T_2 - 1) + k_2 \int (T_1 + T_2 - 1)dt \tag{43}$$

Since the stability criterion used is a sufficient one for proportional control, the gains should be selected such that k_2 is small and $k_1 \gg k_2$. This is effectively proportion-plus-integral (PI) control.

5.4 Comparison With Alternate Control Scheme

The control law (43) was compared with a control law proposed by Ramos (1984).

$$\frac{dV}{dt} = k_3(T_1 + T_2 - 1)\, V \tag{44}$$

The controllers given by equations (43) and (44) were compared with respect to their abilities to move from an initial state to a prescribed equilibrium state, $x^* \rightarrow x_s$ where $T(x^*) = 0.5$. The relative settling times are known to be directly related to the relative stability of the two controlled systems [Ogata, 1970]. The method used to solve equation (33) was a fourth-order accurate method of lines approximation, and the differential equations were integrated by a means of a fourth-order Runge-Kutta method.

The results from the first simulation with controller (44) are shown in Figures 5 and 6. The following values of gain and sensor locations were used: $x(T_1) = 45$, $x(T_2) = 55$, $k_3 = 0.1$. With a large <u>initial error</u>, the controller is ineffective as can be seen in Figure 5. The system with controller (43) responded so quickly that the resolution with the number of points recorded (100 points separated by 5 units of time) was not enough for

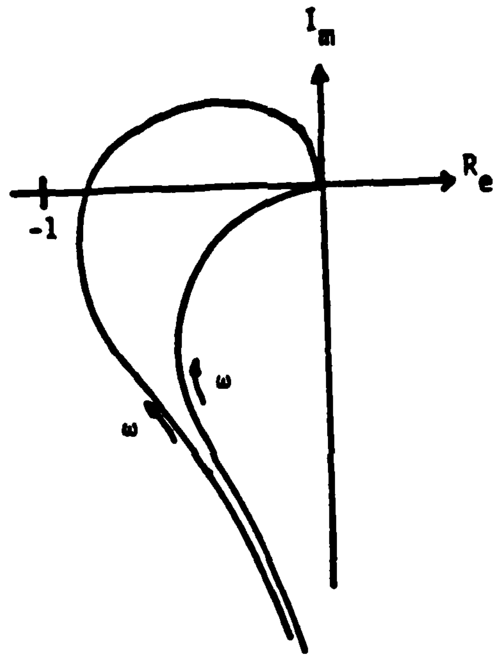

Figure 4: Nyquist Curves for the System of Figure 3.

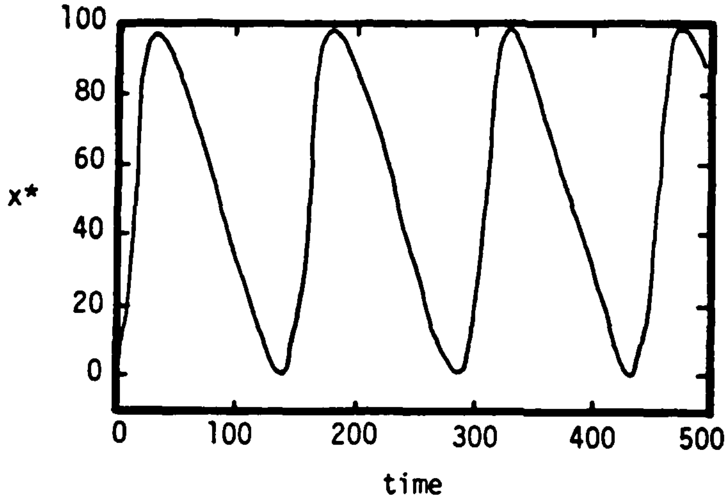

Figure 5: Location of the x* Point as a Function of Time.

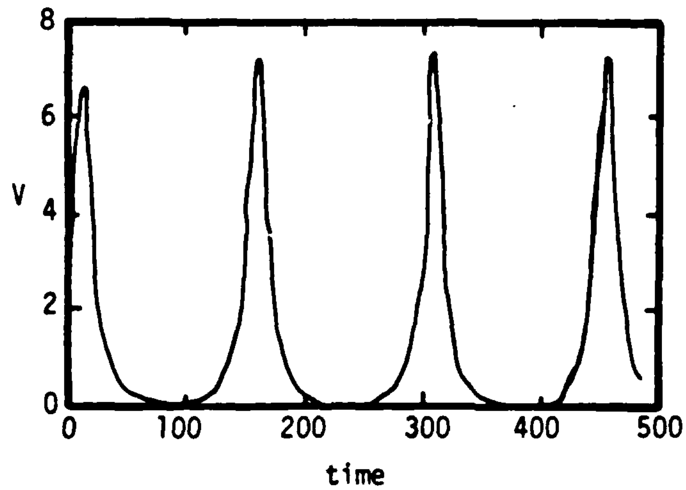

Figure 6: Control Signal as a Function of Time.

the transient to be observed and the results are not shown. Figure 5 also shows the comparatively slow response of the system when $dx^*/dt < 0$. The time required for x^* to go from its minimum value $x^* \sim 0$ to its maximum value $x^* \sim 100$ is about one-fourth of the time required for the return. The reason for this can be seen in the plot of the control signal shown in Figure 6. Because the control signal, V, is very small when the temperature profile passes the $x^* = x_s$ point in the negative direction, the effective time-constant is very small, which results in the slow response. Because V is much larger when the profile crosses in the other direction, the response is much quicker. Without a proper justification, this is an undesirable property.

Figures 7 and 8, and Figures 9 and 10 illustrate a more telling comparison. Here the sensors have been moved closer to the initial value of x^*. The following gains and sensor locations were used with control law (44): $x(T_1) = 7.5$, $x(T_2) = 12.5$, $k_3 = 0.1$. Results from this simulation are shown in the first pair of figures. Figures 9 and 10 are the results of a simulation with control law (43), with the same sensor locations and the following gains: $k_1 = 3.0$, $k_2 = 0.1$. For the purposes of comparison, these gains were selected so that the maximum values of the control would be about the same for the two control laws. At $t \sim 50$, Figure 9 shows the system with PI control nearly at steady-state. The system controlled by equation (44) is still far from the desired equilibrium point. The temperature responses at the desired location, $x_s = 10$, are plotted for these two simulations in Figures 11 and 12. Results from another simulation are presented in Figures 13 and 14. In these simulations the gain of the proportional part of control law (43) was set to $k_1 = 50$. The settling time with these parameters is over an order of magnitude faster than with the lower value. It is also several orders of magnitude faster than the response with control law (44).

A more concrete comparison can be made by defining a performance index. The following quadratic cost function was chosen:

$$J = \Delta e^2 + \Delta V^2 \tag{45}$$

where

$$\Delta e^2 = \int_0^T (x^* - x_s)^2 dt \tag{46}$$

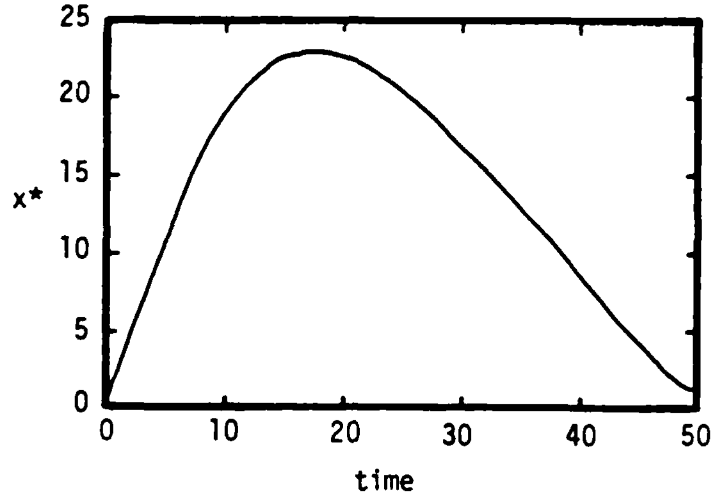

Figure 7: Location of the x^* Point as a Function of Time.

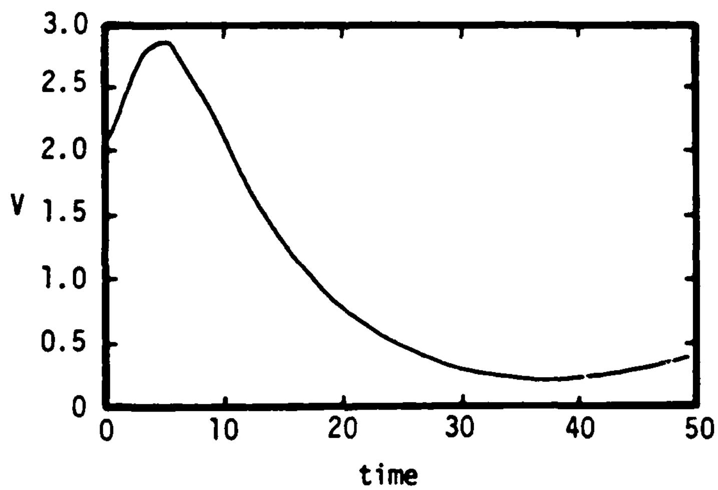

Figure 8: Control Parameter, V, as a Function of Time.

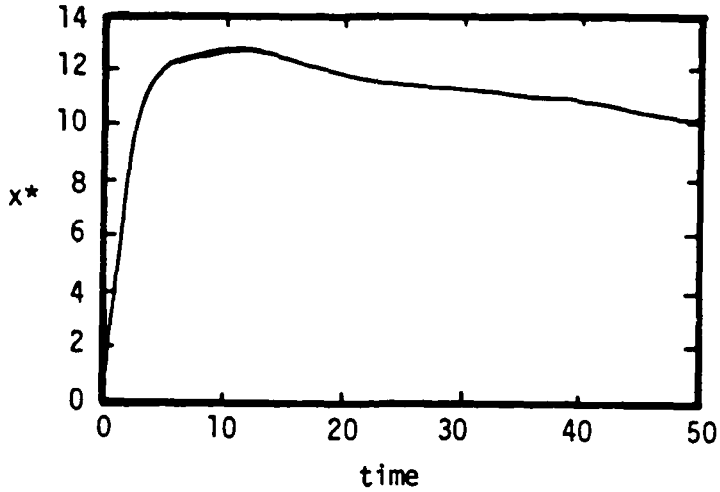

Figure 9: Location of the x^* Point as a Function of Time.

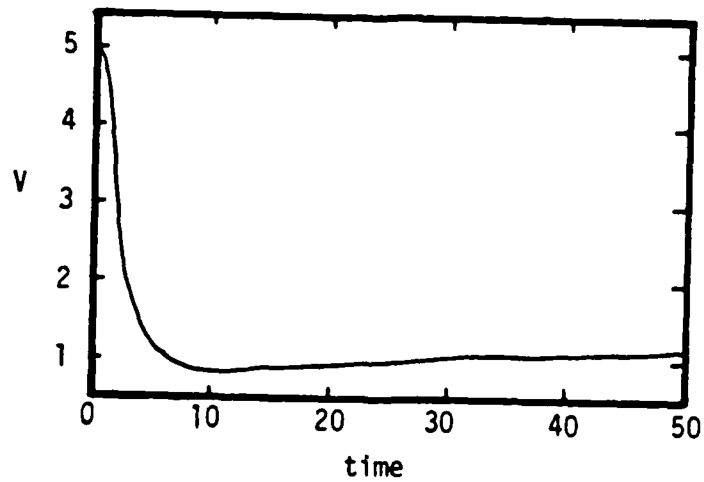

Figure 10: Control Parameter, V, as a Function of Time

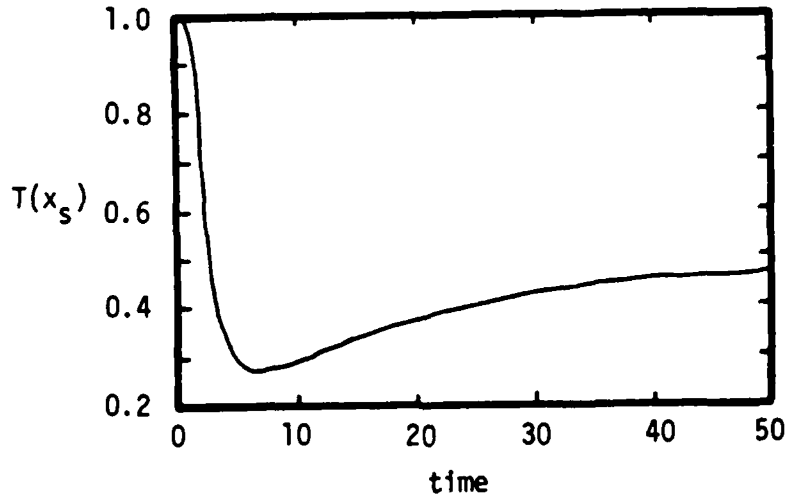

Figure 11: Temperature Response at x_s with Control Law (43).

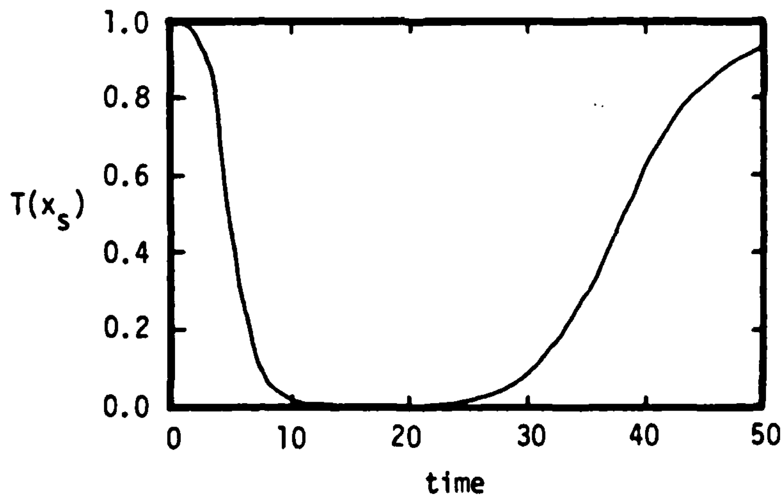

Figure 12: Temperature Response at x_s with Control Law (44).

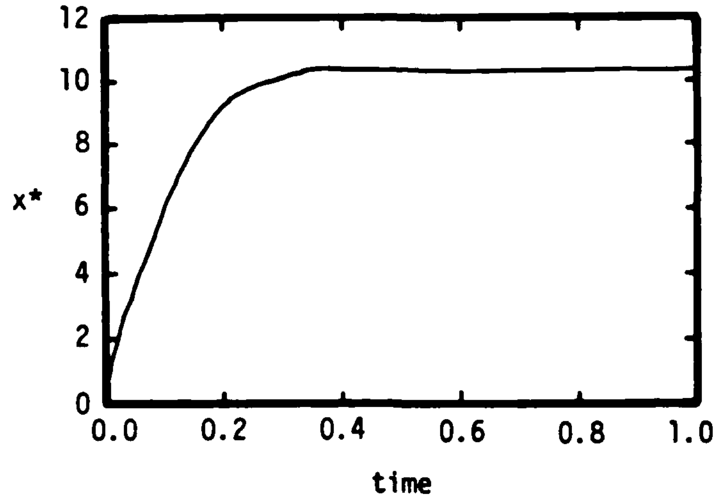

Figure 13: Location of the x^* Point as a Function of Time.

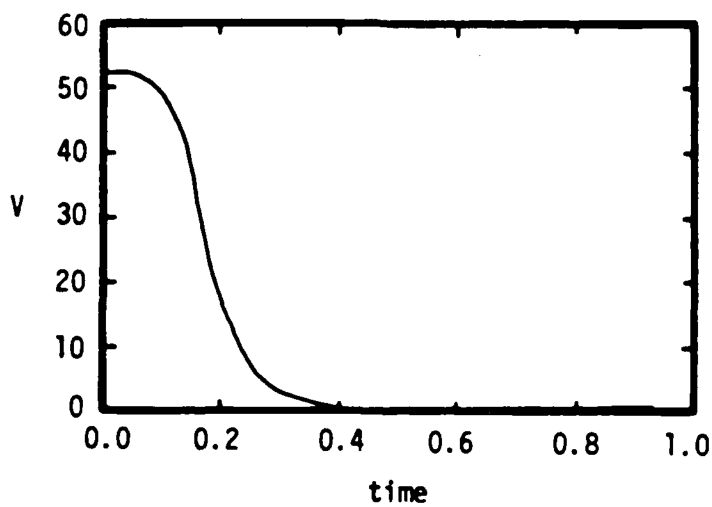

Figure 14: Control Parameter V as a Function of Time

and

$$\Delta V^2 = \int_0^T (V - V_s)^2 dt \tag{47}$$

where V_s is the target value of the control that results in the desired steady-state solution. The interpretation of Δe is an integrated error, and ΔV is the integral of the excess control used over that required for the stabilization of the front.

Table 1 compares the three criterion for the simulations presented in Figures 7 through 14. Notice that the performance of the first PI controller is an order of magnitude better than controller (44). The PI controller with the large proportional gain is over two orders of magnitude better than the controller with the smaller gain. Since the magnitude of the final error was not penalized, the cost for the system controlled by equation (44) is actually higher.

Table 1. Values of Integrated Cost Functions.

Control Law	J	Δe^2	ΔV^2
Equation (44)	6026	5975	51
PI-Control (43), $k_1 = 3$	821	786	35
PI-Control (43), $k_1 = 50$	10	0	10

5.5 Discussion of Lumped Control

Several comments can be made regarding the control laws given by equations (43) and (44). The use of control law (43) suggests itself logically from the requirements of zero steady-state error and fast response. Controller (43) can be shown to satisfy a sector-gain constraint stability criterion. It provides better regulation of the error, Δe^2 , and is more efficient in its use of control effort, ΔV^2 . Equation (44) is heuristically-based, and has been motivated by the desire to drive the system error to zero. It requires a finite time to drive the system to zero steady-state error.

Thus, a control law, equation (43), has been suggested that appears to control the system in a desired way. The gains, k_1 and k_2 can be selected by simulating the system behavior and identifying the set of gains that yields optimum performance. It is important to recognize that the control problem discussed here represents a very simplified problem. The solution based on simulation does not provide insight nor understanding into the structure of the control problem. The next part of this project report develops a heuristic-based design procedure for this type of problem. After such a design procedure has been developed, then more difficult and realistic distributed parameter problems can be approached.

6 Heuristic Control Strategy

We consider here equation (30) with $G(T) = T^2(1-T)$, *i.e.*,

$$T_t + VT_x = T_{xx} + T^2\,(1-T) \tag{48}$$

subject to

$$T(-\infty,t) = 1 \tag{49}$$

$$T(\infty,t) = 0 \tag{50}$$

In steady state, equation (48) can be reduced to the following system of ordinary differential equations

$$T_x = \theta \tag{51}$$

$$\theta_x = V\theta - T^2\,(1-T) \tag{52}$$

which has the following critical points

$$(T,\theta) = (0,0) \tag{53}$$

and

$$(T,\theta) = (1,0) \tag{54}$$

Linearizing equation (48) around $(T,\theta) = (0,0)$ the following system results

$$T'_x = \theta' \tag{55}$$

$$\theta'_x = V\theta' \tag{56}$$

where the primes denote perturbations around the steady state. Equations (55) and (56) indicate that $(T,\theta) = (0,0)$ is a stable (unstable) node for $V < 0$ $(V > 0)$.

Linearizing equation (52) around $(T,\theta) = (1,0)$ we obtain

$$T'_x = \theta' \tag{57}$$

$$\theta'_x = V\theta' + T' \tag{58}$$

The eigenvalues of the matrix given by equations (57) and (58) are

$$\lambda = \left[V \pm (V^2 + 4) \right]^{1/2} /2 \tag{59}$$

Therefore, the critical point $(T,\theta) = (1,0)$ is a saddle-point for any value of V. For positive values of V there exists a unique trajectory emanating from the unstable node $(T,\theta) = (0,0)$ and passing through the critical point $(T,\theta) = (1,0)$. Travelling wave solutions of the form

$$T = \Psi(x-Vt) \tag{60}$$

are then only possible for a particular value of V. Note that in our case $V = 1$.

Integration of the steady form of equation (48) yields

$$V = \int_{-\infty}^{\infty} T^2\,(1-T)\,dx \tag{61}$$

where the conditions

$$T_x(-\infty,t) = T_x(\infty,t) = 0 \tag{62}$$

have been used.

If $0 \leq T \leq 1$, equation (61) yields a positive value of V and corresponds to the unique trajectory which connects the unstable node and the saddle point. Furthermore, if $0 \leq T \leq 1$

$$\Psi'(z) > 0 \tag{63}$$

for finite values of $z = x - Vt$ because

$$\Psi(-\infty) = 0 \qquad \text{and} \qquad \Psi(\infty) = 1 \tag{64}$$

Therefore, the front is a monotonically increasing function (Fife, 1979) and since $G(T) \in C^1$ [0,1], there is a one-to-one correspondence between the travelling wave front and the positive solution of equation (52) subject to

$$\theta(0) = \theta(1) = 0 \tag{65}$$

Evidently, equation (65) is identical to equation (62).

Equation (48) can be solved numerically by means of finite-difference or finite element methods for different values of V (Ramos, 1985). However, it can be shown that its analytic solution is

$$T(x,t) = \Psi(z) = 1/\left[1 + \exp V(z-Vt)\right] \tag{66}$$

where $V = 2^{-1/2}$.

In this section, equation (48) is used to devise a control strategy on the value of V so that the wave front can be placed between two stationary sensors located at x_1 and x_2. The temperatures measured by these sensors are denoted by $T_1(t)$ and $T_2(t)$, respectively. The value of V may be physically interpreted as the air-fuel velocity which in steady state should be equal to the wave front velocity, *i.e.*, equal to $2^{-1/2}$.

The following control strategy is proposed. Suppose a gas turbine combustor where fuel and air are injected at a velocity V and two sensors are located at x_1 and x_2. Based on the temperatures T_1 and T_2 measured by the sensors, a control strategy is derived so that the flame is driven towards a position located between the sensors.

Although the analytic steady state solution of equation (48) is known, *cf.* equation (66), it will not be used in establishing the control law. The reason is that in practical combustion problems, the value of V depends on the reaction rates of the different species participating in the reaction and these are not well known except for very simple reactants. Furthermore, in a gas turbine combustor there will be uncertainties on the thermophysical properties, inlet and boundary conditions, *etc.*, in addition to errors in the modelling of turbulence and errors in the numerical scheme employed to solve the governing equations.

We thus consider equation (48) subject to equations (49) and (50) and develop a *heuristic control strategy* to vary the value of the air–fuel velocity V so that the flame is located between the two sensors located at x_1 and x_2. The strategy uses the fact that T is monotonic, $0 \leq T \leq 1$ as follows. The wave front location is arbitrarily defined as the temperature at which

$$T(x,t) = 0.5 \tag{67}$$

i.e., the average of the temperatures at the upstream and downstream boundaries. This temperature is referred to as $T_w = 0.5$. Because of the monotonicity of T(x,t) it is known that *if* $x_1 < x_2$ the following possibilities exist

1. $T_w < T_1 < T_2$
2. $T_1 < T_w < T_2$
3. $T_1 < T_2 < T_w$

Possibility 1 corresponds to a wave front located upstream of sensor 1. Therefore, V should be increased to move the front somewhere between the two sensors.

Possibility 2 does not require any control strategy as the front is already located between the two sensors. If the value of V does not correspond to the exact steady state wave speed, the front may be moving toward either sensor 1 or sensor 2.

Possibility 3 indicates that the front is located to the right of the second sensor. Therefore, the value of V should be decreased so that possibility 2 is satisfied.

A simple control strategy which accounts for these possibilities is as follows:

$$\frac{dV}{dt} = \frac{V}{\tau}\left(T_1 - T_w\right) \qquad \text{if} \qquad T_1 > T_w \tag{68}$$

$$\frac{dV}{dt} = \frac{V}{\tau}\left(T_2 - T_w\right) \qquad \text{if} \qquad T_2 < T_w \tag{69}$$

where τ is a non-dimensional time constant. Equations (68) and (69) can be integrated to yield

$$V(t) = V(t_i) + \frac{1}{\tau}\int_{t_i}^{t} \left(T_1 - T_w\right)\, dt \tag{70}$$

and

$$V(t) = V(t_o) + \frac{1}{\tau} \int_{t_o}^{t} \left(T_2 - T_w\right) dt \tag{71}$$

respectively, where t_i and t_o denote the initial times at which equations (68) and (69) are employed.

In the calculations reported here equations (70) and (71) were not used; equations (68) and (69) were discretized as follows

$$V^{n+1} = V^n / [\,1 - \Delta t (T_1^n - T_w)/\tau\,] \tag{72}$$

and

$$V^{n+1} = V^n / [\,1 - \Delta t (T_2^n - T_w)/\tau\,] \tag{73}$$

Note that if either $T_1^n = T_w$ or $T_2^n = T_w$, $V^{n+1} = V^n$, where n and n+1 correspond to $n\Delta t$ and $(n+1)\Delta t$ and Δt is the time step.

The control strategy represented by equations (68) and (69) is a simple proportional control where the temperatures measured by the two sensors are employed to vary the value of V in equation (1). V is, of course, assumed positive, *cf.* equation (61). Other control strategies are also possible, *e.g.*, (Ramos, 1984; Dzielski and Nagurka, 1985):

$$\frac{dV}{dt} = \frac{V}{\tau} (T_1 + T_2 - 1) \tag{74}$$

Equation (74) was found to be much inferior to equations (68) and (69) and will not be discussed in this section. Note that equations (68) and (69) do not employ the exact steady state wave front speed, but they do drive the front towards a position located between the two sensors, i.e., this is not a conventional control strategy.

Equation (48) subject to equations (49) and (50) and the control laws of equations (72) and (73) were solved numerically by means of the following time-linearization algorithm (Ramos, 1983)

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} + V^n \frac{T_{i+1}^{n+1} - T_{i-1}^{n+1}}{2\Delta x} = \frac{1}{\Delta x^2}\left[T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1}\right] + S_i^{n+1} \tag{75}$$

where the source term S_i^{n+1} was linearized as

$$S_i^{n+1} = S_i^n + \left(\frac{dS}{dT} \right)_i^n (T_i^{n+1} - T_i^n) \tag{76}$$

where Δx is the grid spacing and i denotes the grid point $x_i = (i-1)\Delta x$, i = 1,2,..., N+1 and N+1 is the number of grid points employed in the calculations; N=450.

The domain $-\infty < x < \infty$ was truncated to $-50 \leq x \leq 400$ and the locations of the truncated domain boundaries were selected so that the wave front is not affected by them. In the calculations reported here $\Delta x = 1$ and $\Delta t = 0.5$, and the values of x_1, x_2 and τ were varied in order to analyze their effects on the numerical results and time required by the front to reach a position located between the two sensors. The initial value of V was also varied; however, only results for V(0) = 1 are presented here.

The initial value of the temperature profile corresponds to the exact solution, *i.e.,* equation (66) with t = 0 and $V = 2^{-1/2}$.

1 Heuristic Control Results

In Figures 15-17 the temperatures measured by sensors 1 and 2 and the value of V are shown as a function of time for $x_1 = 44$, $x_2 = 49$ and $\tau = 5$. Figures 18-20 show similar results for the same sensors locations and $\tau = 25$, whereas in Figures 21-23, $x_1 = 19$, $x_2 = 49$ and $\tau = 5$.

Figures 15 and 16 indicate that initially the wave front was located to the left of sensor 1 but it moved with time to some position located between the two sensors. These figures indicate that the T_1 and T_2 profiles show high frequency oscillations which are caused by the change in control strategy given by equations (68) and (69). These oscillations are also due to the steepness of the wave front.

The wave front speed as a function of time is shown in Figure 17. This figure indicates that when the wave front is located to the left of sensor 1, the wave velocity is increased according to equation (68). This velocity increase steepens the temperature

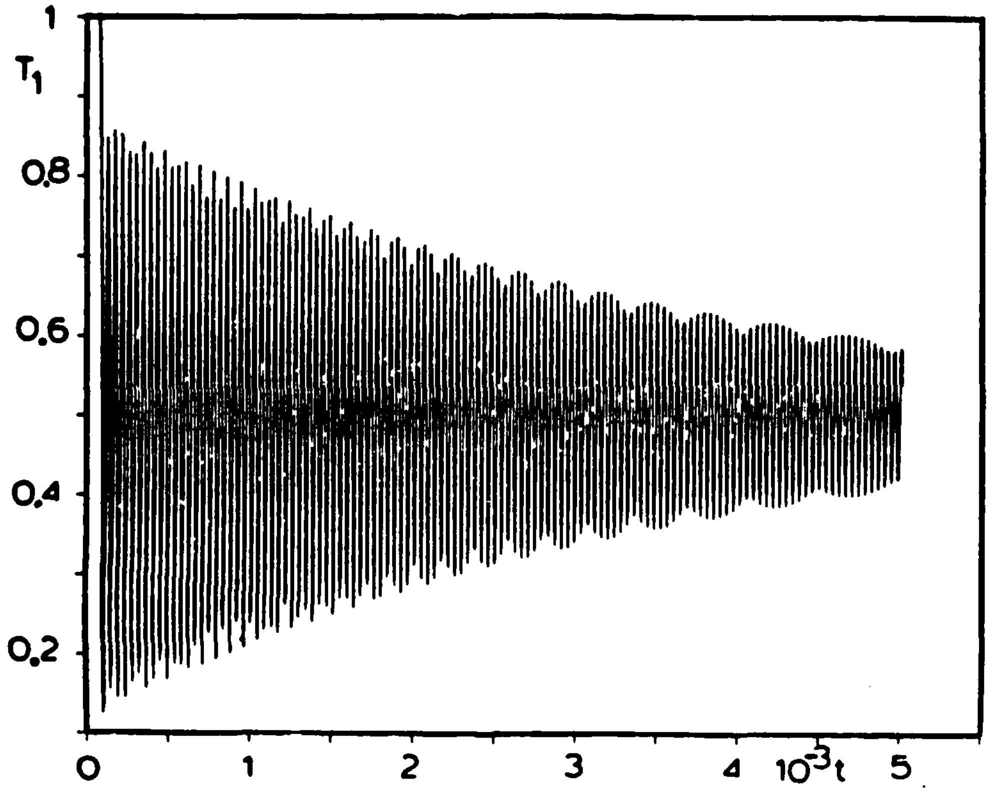

Figure 15: Temperature measured by the first sensor as a function of time (x_1 = 44, x_2 = 49, τ = 5).

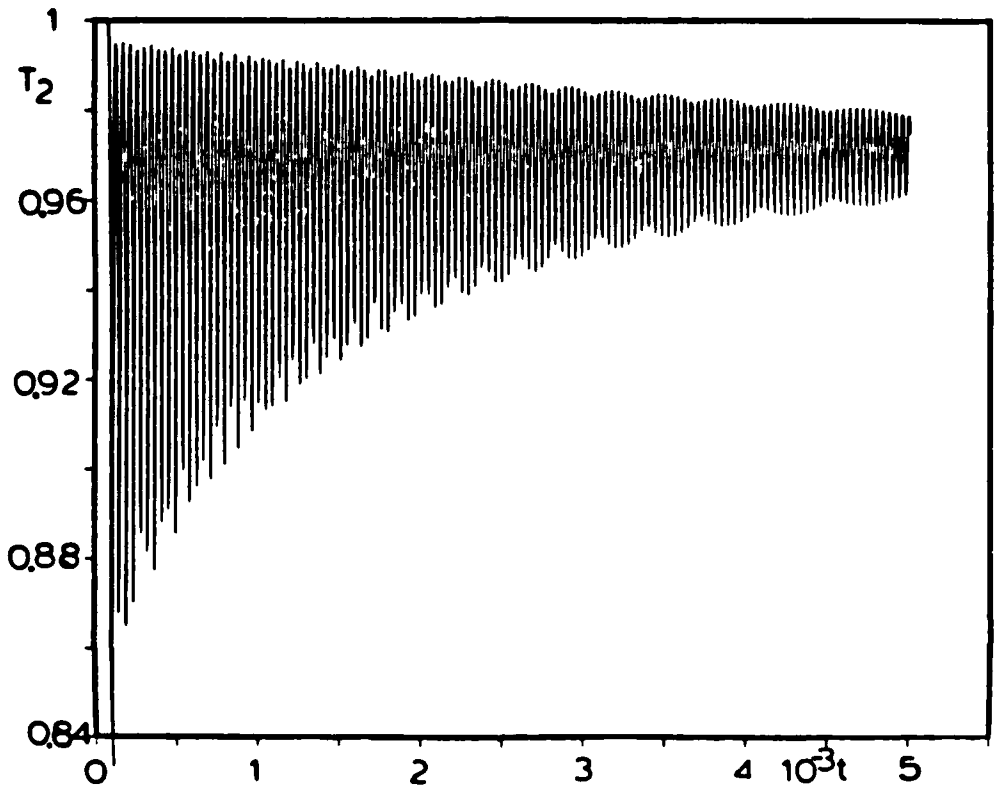

Figure 16: Temperature measured by the second sensor as a function of time (x_1 = 44, x_2 = 49, τ = 5).

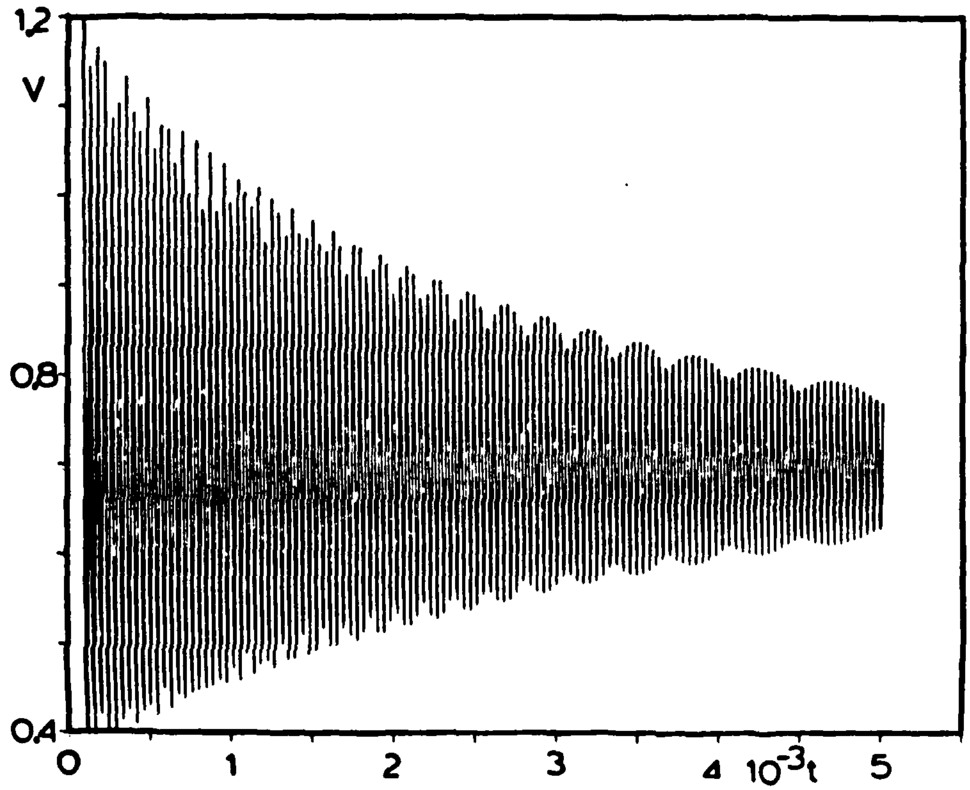

Figure 17: Wave front speed as a function of time (x_1 = 44, x_2 = 49, τ = 5).

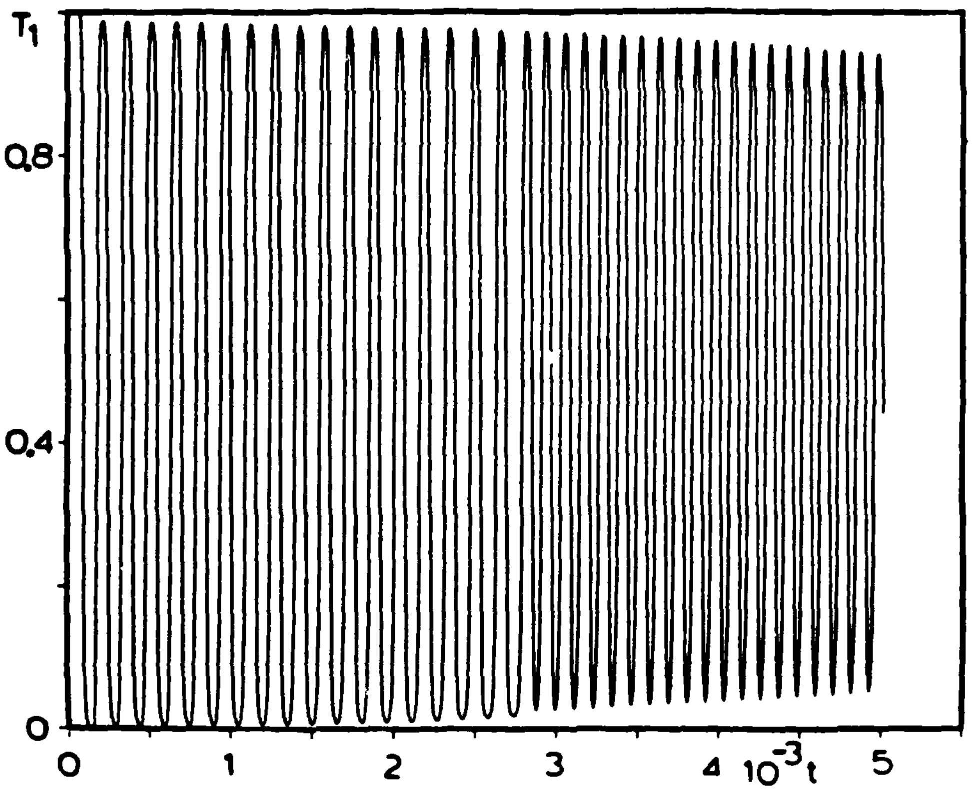

Figure 18: Temperature measured by the first sensor as a function of time (x_1 = 44, x_2 = 49, τ = 25).

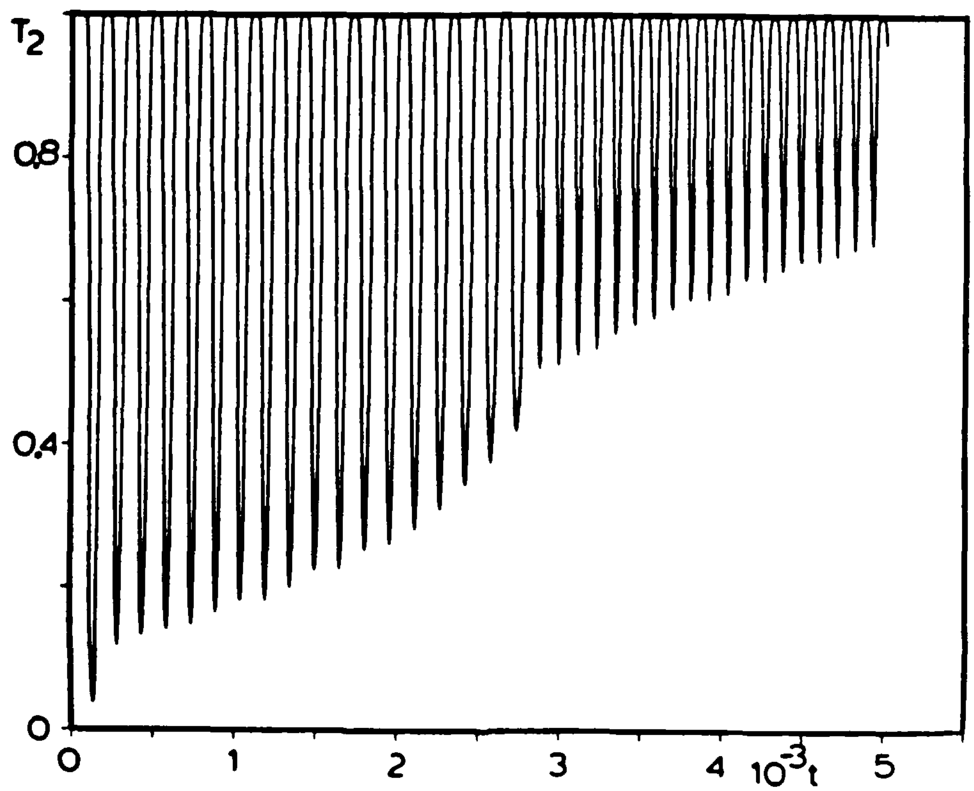

Figure 19: Temperature measured by the second sensor as a function of time (x_1 = 44, x_2 = 49, τ = 25).

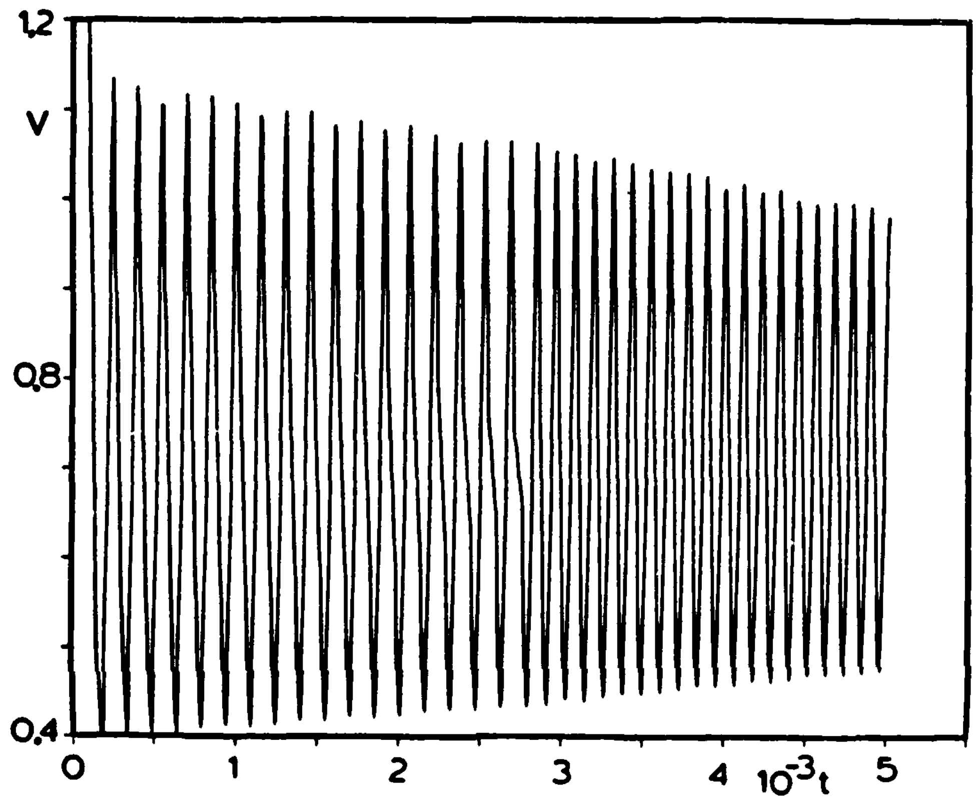

Figure 20: Wave front speed as a function of time (x_1 = 44, x_2 = 49, τ = 25).

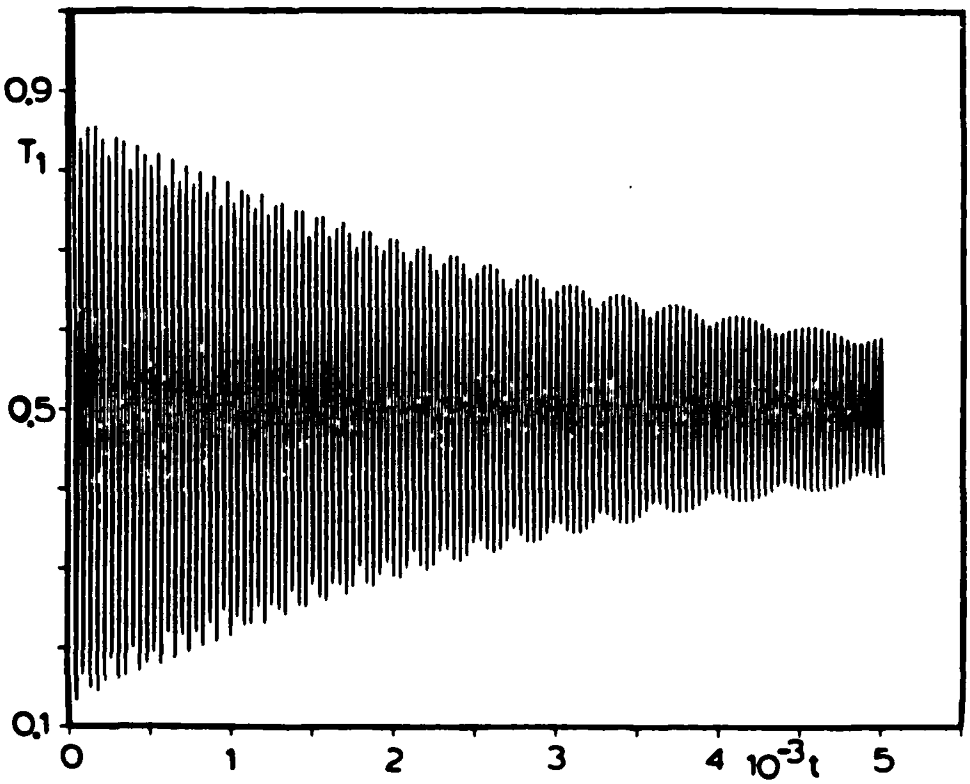

Figure 21: Temperature measured by the first sensor as a function of time (x_1 = 19, x_2 = 49, τ = 5).

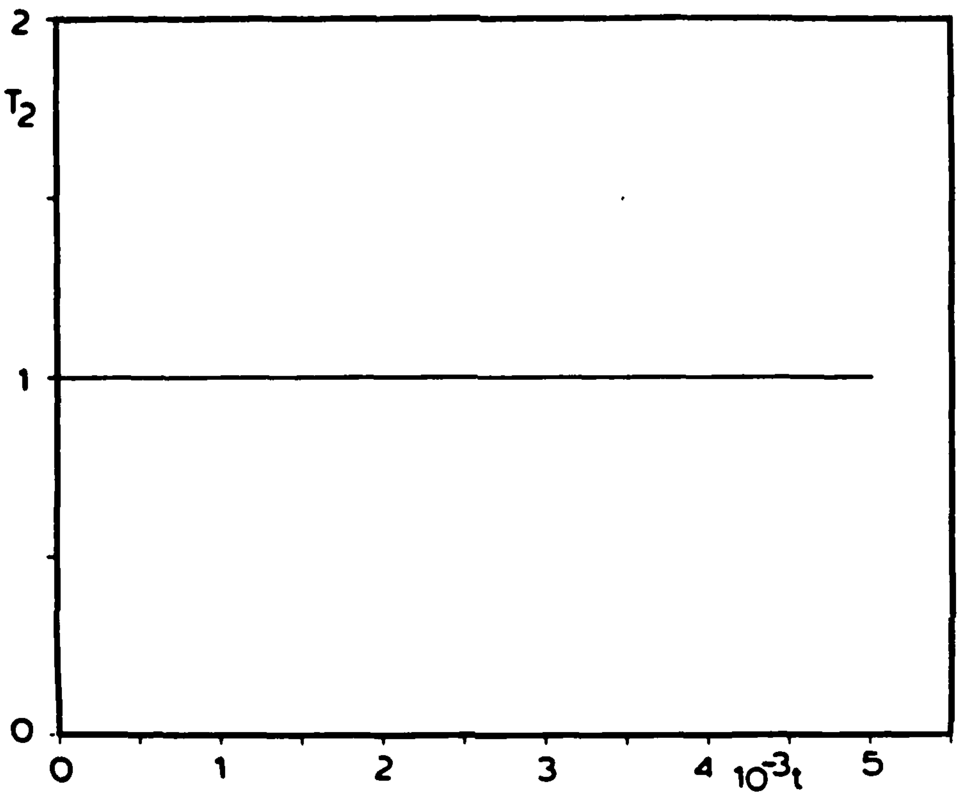

Figure 22: Temperature measured by the second sensor as a function of time (x_1 = 19, x_2 = 49, τ = 5).

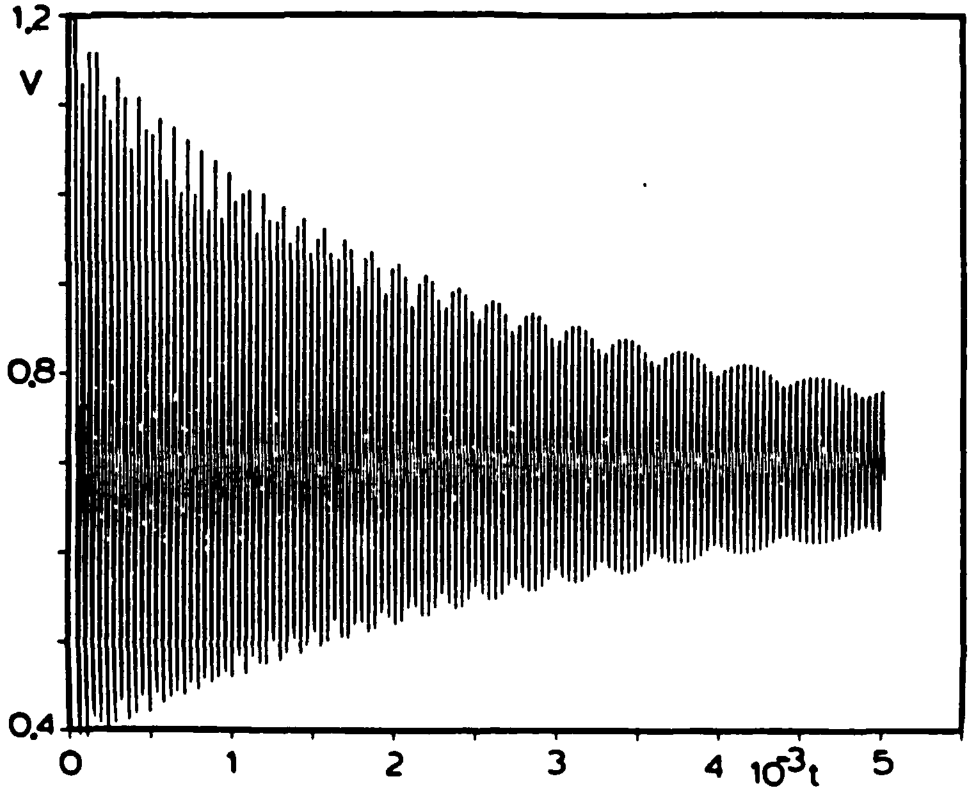

Figure 23: Wave front speed as a function of time (x_1 = 19, x_2 = 49, τ = 5).

gradient at the wave front. On the other hand, when the wave front is located to the right of sensor 2, the velocity is decreased according to equation (69) so that the front moves towards a position located between the two sensors. This velocity decrease smooths the temperature gradient at the wave front.

In steady state conditions there is a balance between diffusion and reaction at the wave front; away from the front, the reaction terms are exponentially small and convection is balanced by diffusion. If the temperature profile at the wave front is steeper (smoother) than the steady state profile, the front velocity is larger (smaller) than its steady state value. Therefore, when the temperature gradients at the wave front are larger (smaller) than their steady state values, the diffusion terms do not balance the reaction terms, and the front continues moving towards sensor 2 (sensor 1). This behavior is similar to the flashback and blowoff phenomena observed in Bunsen burners (Glassman, 1977).

Figures 15–17, as well as Figures 18–23, indicate that the heuristic control strategy given by equations (68) and (69) drives the wave velocity to its steady state value, *i.e.,* $V = 2^{-1/2}$. The high frequency oscillations shown in Figures 15–17 *are superimposed* on the mean temperature responses measured by sensors 1 and 2, and on the mean wave velocity response, respectively. Similar high frequency oscillations are also observed in Figures 18–23.

Figures 18–20 indicate that larger time constants result in slower control processes. Even when the time constant is very large, the control strategy represented by equations (68) and (69) still drives the front to a position located between the two sensors. Note also that the control strategy proposed here also drives V to its steady state value, *i.e.,* $V = 2^{-1/2}$.

Figures 15–17 and 21–23 indicate that, for the same time constant, the distance between the sensors does not seem to play an important role in the control of V. This is shown in the T_1 and V profiles presented in Figures 15 and 17, and 21 and 23, respectively. Figure 22 indicates that for $\tau = 5$, $x_1 = 19$ and $x_2 = 49$ and the initial conditions employed in this simulation, the temperature measured by the second sensor is that of the downstream boundary. This is not surprising as the control is made through equation (68) only.

The results shown in Figures 15-23 indicate that the smaller the time constant the shorter the time required to achieve the control objective. They also indicate that there are circumstances under which the control is only executed by the temperature measured by one of the sensors.

1 Conclusions

The heuristic control strategy proposed in the section above only uses the temperatures measured by two sensors located at two different spatial locations and only involves a minimum knowledge about the temperatures expected in the model equation as well as the temperature monotonicity. It may also be used in more practical combustion phenomena such as those occurring in gas turbines where the temperature or heat transfer losses through the turbine walls may be measured in order to vary the secondary air flow rate or fuel flow rate so that they are within the limits of the material of the wall or turbine blades. In these situations, however, the temperature may not be a monotonically increasing function of the distance along the combustor, but one can always estimate the lower and higher temperatures and define an appropriate heuristic control strategy. In this sense the heuristic control strategy differs in a substantial manner from conventional control problems where the state variables are acted upon by controllers so that a state is achieved, say, in the shortest time. Furthermore, conventional control problems are usually based on reasonable accurate models of the process that it is to be controlled. Reasonable models for the combustion process in gas turbines, for example, do not exist at the present time; these models may predict recirculation zones, temperatures, *etc.*, within a factor of 2 or 3 or those observed experimentally because of inadequate knowledge of the initial and boundary conditions, thermophysical properties, turbulence, reaction rates, *etc.* For these problems which are (within a factor of 2 or 3) from their real life counterparts simple heuristic control strategies such as the one proposed here may prove useful for the design and control of practical combustors.

8 Acknowledgement

The authors are grateful to Mr. J. E. Dzielski for his contributions on lumped-control strategies and for many lively discussions on control of combustion processes.

9 References

Ahmed, N.U., and Teo, K.L., 1981, *Optimal Control of Distributed Parameter Systems*, Elsevier/North Holland, New York.

Aldushin, A.P., Khudyaev, S.I., and Zel'dovich, Ya.B., 1981, "Flame Propagation in the Reacting Gaseous Mixture," *Archivum Combustions*, Vol. 1, pp. 10-21.

Amouroux, M. and Babary, J.P., 1978, "On the Optimal Pointwise Control and Parametric Optimization of Distributed Parameter Systems," *Int. J. Control*, Vol. 28, pp. 789-807.

Aziz, A.K., Wingate, J.W., and Balas, M.J. (Eds.), 1977, *"Control Theory of Systems Governed by Partial Differential Equations,"* Academic Press, New York.

Balas, M.J., 1982, "Toward a More Practical Control Theory for Distributed Parameter Systems," in *Control and Dynamic Systems: Advances in Theory and Applications*, Leondes, C.T. (Ed.), Vol. 18, pp. 361-421, Academic Press, New York.

Balas, M.J., 1983, "The Mathematical Structure of the Feedback Control Problem for Linear Distributed Parameter Systems with Finite-Dimensional Controllers," in *Control Theory for Distributed Parameter Systems and Applications*, Kappel, F., Kunisch, K., and Schappacher, W. (Eds.), Springer-Verlag, New York.

Banks, S., and Pritchard, A. (Eds.), 1978, *"Control of Distributed Parameter Systems,"* Pergamon Press, New York.

Barbu, V., 1981, "Necessary Conditions for Distributed Control Problems Governed by Parabolic Variational Inequalities," *SIAM J. Control and Optimiz.*, Vol. 19, pp. 64-86.

Breitholtz, C., and Ovarnstrom, B., 1982, "A Computer Aided Design Approach to the Development of Simplified Control Models for Nonlinear Distributed Systems," in *Control of Distributed Parameter Systems*, Babary, J.-P. and Le Letty, L. (Eds.), Pergamon Press, New York.

Carotenuto, L., and Raiconi, G., 1982, "The Finite Element Method in Distributed Parameter Control Systems," in *Distributed Parameter Control Systems*, Tzafestas, S.G. (Ed.), pp. 239-271, Pergamon Press, New York.

Chandhuri, S.P., 1972, "Distributed Optimal Control in a Nuclear Reactor," *Int. J. Control*, Vol. 16, pp. 927-937.

Clavin, P., and Liñan, A., 1984, "Theory of Gaseous Combustion," Velarde, M.G. (Ed.), *Nonequilibrium Cooperative Phenomena in Physics and Related Fields*, pp. 291-338, Plenum Press, New York.

Di Pillo, G., and Grippo, 1980, "On the Method of Multipliers for a Class of Distributed Parameter Systems," in *Control of Distributed Parameter Systems 1982*, Banks, S.P., and Pritchard, A.J. (Eds.), pp. 173-180, Pergamon Press, New York.

Dzielski, J.E. and Nagurka, M.L., 1985, "Position Control of a One-Dimensional Flame Front," Report, Department of Mechanial Engineering, Carnegie-Mellon University, Pittsburgh, PA.

Fife, P.C., 1979, *Mathematical Aspects of Reacting and Diffusing Systems*, Springer-Verlag, New York.

Fisher, R.A., 1937, "The Wave of Advance of Advantageous Genes," *Annals of Eugenics*, Vol. 7, pp. 355-369.

Glashoff, K., and Gustafson, S.A., 1976, "Numerical Treatment of a Parabolic Boundary-Value Control Problem (Heat Diffusion)," *J. Optimiz. Theory and App.*, Vol. 19, pp. 645-663.

Glassman, I., 1977, *Combustion*, Academic Press, New York.

Gould, L.A., and Murray-Lasso, M.A., 1966, "On the Model Control of Distributed Systems with Distributed Feedback," *IEEE Trans. Autom. Control.*, Vol. AC-11, pp. 729-737.

Graney, L., 1983, "The Numerical Solution of Differential Equations Arising in Control Theory for Lumped and Distributed Parameter Systems," in *Control Theory for Distributed Parameter Systems and Applications*, Kappel, F., Kunisch, K., and Schappacher, W. (Eds.), pp. 109-126, Springer-Verlag, New York.

Hedrick, J.K., and Paynter, H.M., 1980, *ASME Monograph, Nonlinear System Analysis and Synthesis: Techniques and Applications*, ASME, New York.

Holliday, J.H., and Storey, C., 1973, "Numerical Solution of Certain Nonlinear Distributed Distributed Parameter Optimal Control Systems," *Int. J. Control*, Vol. 18, pp. 817-825.

Huang, H.S., and Yang, W.J., 1970, "Optimum Control of Heat Exchangers with Internal Heat Generation," *App. Science Res.*, Vol. 23, pp. 95-112.

Huntley, J., 1979, "A Note on the Application of the Matrix Riccati Equation to the Optimal Control of Distributed Parameter Systems," *IEEE Trans. Autom. Control*, Vol. AC-24, pp. 487-489.

Johnson, T.L., 1973, "Minimum-Energy Terminal State Control of First Order Hyperbolic Systems in One Spatial Variable Using the Method of Characteristics," *SIAM J. Control*, Vol. 11, pp. 119-129.

Kanoh, H., 1982, "Distributed Parameter Heat Exchangers - Modeling, Dynamics and Control," in *Distributed Parameter Control Systems: Theory and Application*, Tzafestas, S.G. (Ed.), pp. 417-449, Pergamon Press, New York.

Kastenberg, W.E., 1974, "An Asymptotic Stability of Non-Linear Distributed Parameter Energy Systems," *Int. J. Control*, Vol. 19, pp. 73-79.

Klamka, J., 1983, "Controllability, Observability and Stabilizability of Distributed Systems with Infinite Domain," in *Control of Distributed Parameter Systems 1982*, pp. 171-175, Pergamon Press, New York.

Kobayashi, T., 1978, "Some Remarks on Controllability for Distributed Parameter Systems," *SIAM J. Control and Optimiz.*, Vol. 16, pp. 733-742.

Lausterer, G.K., and Eitelberg, E., 1982, "Experimental Application of Distributed-Parameter Control Theory," in *Distributed Parameter Control Systems: Theory and Application*, Tzafestas, S.G. (Ed.), pp. 329-366, Pergamon Press, New York.

Lions, J.L., 1971, *"Optimal Control of Systems Governed by Partial Differential Equations,"* Springer-Verlag, New York.

Martin, J.-C.E., 1978, "Optimal Allocation of Actuators for Distributed-Parameter Systems," *Trans. ASME, J. Dyn. Syst. Meas. and Control*, Vol. 100, pp. 227-228.

McGlothin, G.E., 1974, "A Modal Control Model for Distributed Systems with Application to Boundary Controllability," *Int. J. Control*, Vol. 20, pp. 417-432.

McKean, H.P., 1975, "Application of Brownian Motion to the Equation of Kolmogovov-Retrorskii-Piskunov," *Comm. Pure Appl. Math.*, Vol. XXVIII, pp. 323-334.

Meric, R.A., 1979, "Finite Element Methods for an Optimal Control Problem Governed by a Parabolic Equation," *Int. J. Num. Meth. Eng.*, Vol. 14, pp. 624-628.

Murray, J.D., 1972, *Lectures on Nonlinear Differential Equation Models in Biology*, Oxford University Press, Oxford, UK.

Ogata, K., 1970, *Modern Control Engineering*, Prentice-Hall, Englewood Cliffs, New Jersey.

Parkin, E.S., and Zahradnik, R.L., 1971, "Computation of Near-Optimal Control Policies by Trajectory Approximation: Hyperbolic-Distributed Parameter Systems with Space - Independent Controls," *AIChE J.*, Vol. 17, pp. 409-414.

Ramos, J.I., 1981, "Exhaust Gas Emissions from a Swirl-Stabilized Combustor," *Heat Transfer-Milwaukee*, AIChE Symposium Series No. 208, Vol. 77, pp. 393-399.

Ramos, J.I., 1983, "A Review of Some Numerical Methods for Reaction-Diffusion Equations." *Mathematics and Computers in Simulation*, Vol. XXV, pp. 538-548.

Ramos, J.I., 1984, "Heuristic Control Strategies for One-Dimensional Reaction-Diffusion Equations," Report CO/84/10, Department of Mechanical Engineering, Carnegie-Mellon University, Pittsburgh, PA.

Ramos, J.I., 1985, "Numerical Solution of Reactive-Diffusive Systems: Part 1, Part 2, Part 3," *Int. J. Comp. Math.*, Vol. 18, pp. 43-65, 141-161, 289-309.

Ramos, J.I., 1986a, "Numerical Methods for One-Dimensional Reaction-Diffusion Equations Arising in Combustion Theory," *Annual Review of Numerical Fluid Mechanics and Heat Transfer* (in press).

Ramos, J.I., 1986b, "A Heuristic Control Strategy for Non-Linear Reaction-Diffusion Equations," *International Journal of Control*, Vol. 43, No. 2, pp. 473-483.

Reitz, R.D., 1981, "A Study of Numerical Methods for Reaction-Diffusion Equations," *SIAM J. Sci. Stat. Comp.*, Vol. 2, pp. 95-106.

Sakawa, Y., 1974, "Controllability for Partial Differential Equations of Parabolic Type," *SIAM J. Control*, Vol. 12, pp. 389-400.

Seidman, T.I., 1983, "Stabilization of Nonlinear Parabolic Equations," in *Control of Distributed Parameter Systems 1982*, Barbary, J.-P., and Le Letty, L. (Eds.), pp. 131-134, Pergamon Press, New York.

Seinfeld, J.H., and Lapidus, L., 1968, "Computational Aspects of the Optimal Control of Distributed - Parameter Systems," *Chem. Eng. Science*, Vol. 23, pp. 1461-1484.

Seinfeld, J.H., Gavalas, G.R.. and Hwang, M., 1970, "Nonlinear Filtering in Distributed Parameter Systems," 11th Joint Automatic Control Conference of the American Automatic Control Council, pp. 573-583, ASME, New York.

Spalding, D.B., 1956, "The Theory of Flame Phenomena with Chain Reaction," *Phil. Trans. Roy. Soc. London, Series A*, Vol. 249, pp. 1-25.

Spalding, D.B., 1957, "I. Predicting the Laminar Flame Speed in Gases with Temperature-Explicit Reaction Rates," *Comb. Flame*, Vol. 1, pp. 287-295.

Spalding, G.R., 1982, "Modeling Techniques for Distributed Parameter Systems," in *Control and Dynamic Systems*, Leondes, C.T. (Ed.), Vol. 18, pp. 105-130, Academic Press, New York.

Tzafestas, S.G. (Ed.), 1982a, "*Distributed Parameter Control Systems: Theory and Application*," Pergamon Press, New York.

Tzafestas, S.G., 1982b, "Hybrid Simulation of Distributed Parameter Systems," in *Distributed Parameter Control Systems*, Tzafestas, S.G. (Ed.), pp. 1-55, Pergamon Press, New York.

Van Harten, A., 1983, "Singularly Perturbed Systems of Diffussion Type and Feedback Control," in *Singular Perturbations in Systems and Control*, Ardana, M.D. (Ed.), pp. 309-337, Springer-Verlag, New York.

Viswanadham, N., Patnaik, L.M., and Sarma, I.G., 1979, "Robust Multivariable Controllers for a Tubular Ammonia Reactor," *Trans. ASME, J. Dyn. Syst. Meas. and Control*, Vol. 101, pp. 290-298.

Wang, P.K.C., 1964, "Control of Distributed Parameter Systems," in *Advances in Control Systems: Theory and Applications*, Leondes, C.T. (Ed.), Vol. 1, pp. 75-172, Academic Press, New York.

Williams, F.A., 1985, "*Combustion Theory*," 2nd Edition, The Benjamin/Cummings Publishing Company, Menlo Park, California.

Wu, Z.S., and Teo, K.L., 1983, "Optimal Control Problems Involving Second Boundary Value Problems of Parabolic Type," *SIAM J. Control*, Vol. 21, pp. 729-757.

Yu, D., 1971, "Coupled-Spatial Discretization and Zubov's Method in the Determination of Domain of Asymptotic Stability for a Nonlinear Distributed Parameter System," 12th Joint Automatic Control Conference of the American Control Council, pp. 72-78, IEEE, New York.

Zel'dovidh, Ya.B., Barenblatt, G.I., Librovich, V.B., and Makhviladze, G.M., 1985, "*The Mathematical Theory of Combustion and Explosions*," Plenum Press, New York.

10 Publications Resulting From This Research

1. Ramos, J.I., 1986, "A Heuristic Control Strategy for Non-Linear Reaction-Diffusion Equations," *International Journal of Control*, Vol. 43, No. 2, pp. 473-483.

2. Ramos, J.I., 1986, "Numerical Methods for One-Dimensional Reaction-Diffusion Equations Arising in Combustion Theory," *Annual Review of Numerical Fluid Mechanics and Heat Transfer* (in press).

11 Professional Personnel Associated With This Research

1. M.L. Nagurka, Co-PI.
   PhD, MIT, 1983. Thesis Title: Curving Performance of Rail Passenger Vehicles.

2. J.I. Ramos, Co-PI.
   PhD, Princeton University, 1980. Thesis Title: Axisymmetric Flow Fields in Motored and Firing Spark-Ignition Engines.

3. W.A. Sirignano, Co-PI.
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4. J.E. Dzielski, Graduate Student.
   MS, MIT, 1984. (Note: No degree awarded as a result of this work.)

12 Additional Information

- No conference presentations.
- No consultative or advisory functions to other laboratories or agencies.
- No patents stemming from this research.

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