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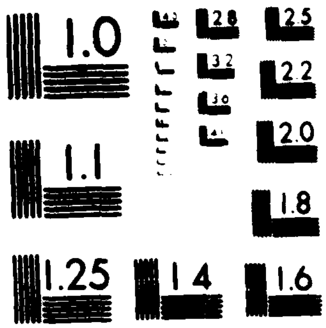

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COMPOSITE VELOCITY PROCEDURES FOR FLOWS WITH PRESSURE INTERACTION

ANNUAL TECHNICAL REPORT

2/1/85 -2/1/86 AFOSR-TR- 87-0357

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ABSTRACT

The composite velocity formulation for the Euler and Reduced Navier-Stokes system of equations has been investigated. For transonic steady flow over airfoils a combination of conservative and non-conservative flux directional systems of difference equations have been developed and applied to obtain full potential, Euler and Reduced Navier-Stokes solutions. the Enquist-Osher procedure for shock capturing has been adapted for these applications to obtain sharp shock waves for both potential and Euler equations. For two-dimensional transient flows and for three-dimensional global relaxation computations, the coupled strongly implicit matrix inversion algorithm has been reformulated in order to allow for time consistency and space marching efficiency, respectively. The algorithm has been tested on several model problems, where the exact solutions are known, for an oscillating airfoil and for the transient behavior on an airfoil, where a large separation region develops or when laminar flow breakdown occurs. The algorithm has also been examined for three-dimensional marching with boundary layer and Reduced Navier-Stokes equations. The results to date have been very encouraging.

A. STATEMENT OF WORK (FROM PROPOSAL) 2/1/85 - 10/31/86

a) A composite velocity/reduced Navier-Stokes (CV/RNS) code will be developed for investigation of flow past lifting airfoils in both subcritical and transonic flow. Full viscous-inviscid interactions shall be evaluated for laminar and turbulent flow conditions. The CV/Euler solutions shall be critically evaluated for accuracy, prediction of entropy and vorticity patterns and variances from the full potential solutions. Comparisons will be made with the CV/RNS solutions to assess the effect of viscous interaction on the inviscid solutions.

b) A time-consistent relaxation procedure will be developed for modeling transient behavior. The procedure will be tested on airfoil geometries with transient boundary conditions or for cases where transient behavior occurs due to large Reynolds number effects.

B. STATUS OF RESEARCH EFFORT 2/1/85 - 2/1/86

As given in the Work Statement the goals and status to date can be summarized as follows:

(1) Application of the Composite Velocity (CV) formulation for the Euler equations and comparison with the full potential solutions. This has essentially reached completion, see write-up, B.1, enclosed.

(2) Composite Velocity/Reduced Navier-Stokes solutions. This has been completed for steady transonic flow over a NACA 0012 at zero incidence, see reference C.3 and reference E.2.

(3) Application of the CV formulation to lifting airfoils. The formulations of (1) and (2) await the development of a grid generation technique that is currently under investigation.

(4) Development of a time-consistent CV relaxation procedure for two-dimensional transient flow. This has been completed, see write-up B.2 and

B. 3. Preliminary tests have also been considered for three-dimensional space marching (a task not scheduled until the option period 11/1/86 - 10/31/87), see write-up B.2.

(5) A new iterative 'direct-solver' formulation to be applied for more efficient computation has also been developed and is discussed in B.4.

Generally speaking, research progress has been quite good and initial solutions for all items in the Work Statement should be completed by 10/31/86.

B.1 COMPOSITE VELOCITY EULER/POTENTIAL SOLUTIONS

Solutions to the Reduced Navier-Stokes equations obtained with the composite velocity system were presented at the 22nd Aerospace Sciences Meeting, January 1986, see reference E.2. It was noted that with the formulation of that time entropy was generated only by viscoscity and that entropy was not generated by shock waves. The following describes a procedure for obtaining Euler solutions for the outer (Euler) flow with accurate entropy generation due to shock waves.

The composite velocity representation may be written as

$$u = \frac{U+1}{h_1}(1+\phi_\xi) = (U+1)u_e \ , \quad v = \frac{\phi_\eta}{h_2} \ . \qquad \text{(1a,b)}$$

The interpretation of the composite velocity terms for inviscid flows varies slightly from that for viscous flows. The ϕ term still represents an irrotational "pseudo" potential function. The U term, however, is no longer associated with the viscous effects but rather it is associated with the effects of rotationality, i.e. vorticity $\Omega = f(U,U_\eta)$.

The following set of Euler equations are obtained for 2-D flow

<u>Continuity</u>

$$[\rho(U+1)(1+\phi_\xi)]_\xi + [\rho\phi_\eta]_\eta = 0 \qquad (2)$$

ξ-Momentum

$$\rho \frac{\partial u}{\partial t} + \frac{1}{D} \left[(\rho h_2 (U^2 + U) u_e^2)_\xi + (\rho h_1 U u_e v)_\eta \right] + \frac{\rho}{h_1} U u_e u_{e_\xi} + \frac{1}{D} \rho u_e v U h_{1_\eta}$$

$$= \frac{\rho T}{h_1} S_\xi - \frac{\rho}{h_1} G_\xi \quad (3)$$

η-Momentum

$$TS_\eta = G_\eta + U \left[\frac{\partial}{\partial \eta} \left(\frac{u_e^2}{2} \right) - \frac{(U+1)}{h_1} u_e^2 h_{1_\eta} \right] \quad (4)$$

where S is entropy and G is a Bernoulli like pressure term. All ξ-derivatives are differenced using backward differences except for ϕ_ξ which is forward differenced. In this manner ϕ is being relaxed; U is marched. This differencing is consistent with Helmholtz's vorticity theorem.

For transonic flows the Enquist-Osher flux biasing scheme has been adapted for the composite velocity system. This scheme consists of defining a new density

$$\bar{\rho}_{i,j} = \rho_{i,j} - \frac{\Delta x}{q_{i,j}} \left((\rho q)_{-\, i,j} - (\rho q)_{-\, i-1,j} \right) \quad (5)$$

where

$$(\rho q)_- = 0.0 \quad M \le 1, \quad (\rho q)_- = \rho q - \rho^* q^* \quad M > 1.$$

Here ρ^* and q^* are the sonic velocity and density. This procedure produces very sharp shocks and guarantees that no expansion shocks can occur. Density shifting is performed only on terms without U in the ξ-derivatives in the continuity equation.

Equations 2-4 are the full Euler equations; the transonic solutions calculated using these equations should contain the rotational effects and the correct entropy rise at the shock wave. If the equations are solved in their present form, however, no entropy is generated at the shock wave and the isentropic, irrotational full potential solution is obtained. If, on the other hand, vorticity is introduced into the flow being solved, this form of the equations will convect this vorticity downstream with no additional generation of vorticity.

The reason this procedure fails to capture rotational effects lies in the form of the ξ-momentum Equation 3. In order to more adequately capture the (Euler) shock wave in transonic flows, the ξ-momentum equation is rewritten in a quasi conservation form

$$\rho \frac{\partial u}{\partial t} + \frac{1}{D}\left[(\rho h_2(U^2+2U)u_e^2)_\xi + (\rho h_1 U u_e v)_\eta\right] + \frac{1}{D}\rho u_e v U h_{1_\eta}$$
$$= -\frac{1}{h_1}(p + \rho u_e^2)_\xi - \frac{1}{h_2}(\rho u_e v)_\eta + \frac{h_{2_\xi}}{D}(\rho v^2 - \rho u_e^2) - 2.0\frac{h_{1_\eta}}{D}(\rho u_e v). \qquad (6)$$

For transonic flow, values of U will be generated at the shock wave. Also, the generation of U values will bring about a corresponding entropy rise at the shock wave.

The modifications to the ξ-momentum equation are easy to impliment in the numerical solution procedure. The right hand side of Equation 3 is evaluated at the previous iteration level. The $(p + \rho u_e^2)_\xi$ term may be evaluated using a two point backward difference and a second order central difference is used for the $(\rho u_e v)_\eta$ term. All the other terms are evaluated at the (i,j) location.

While this form of the equations produces the required entropy rise at the shock wave, it also produces spurious entropy in the regions of high gradients at the leading and trailing edges of the airfoil. Recalling that the nonconservation form produces no entropy, but only convects vorticity introduced into the system, this form of the equations will be used everywhere except in the shock region, where the quasi conservation form of the equations is used. The shock region is defined as the region from the peak Mach number to the point where the change in pressure is less than one percent. This produces a solution procedure with the desirable characteristic that no entropy is produced except in the shock wave region and this entropy is convected accurately downstream.

One result for the pressure coefficient along a NACA0012 airfoil for a freestream Mach number of 0.85 is presented in Figure B1.1. As expected the Euler shock is weaker and lies upstream of the potential shock. The Mach number ahead of the shock has a value of 1.3395. The calculated value of the Mach number after the shock is 0.7683 and the entropy generated has a value of S=0.0270. These values compare well with the values M_2=0.7664 and S=0.0283 obtained from the shock jump conditions, see Figure B1.2. There is a small jump in the value of entropy at the trailing edge and in the shock structure. However, the former effect is localized to the trailing edge region and arises due to the singularity in the present transformation. The latter overshoot is physical and typical of the entropy behavior in the shock structure. This procedure is now being applied for three-dimensional and other two-dimensional geometries for the reduced Navier-Stokes equations.

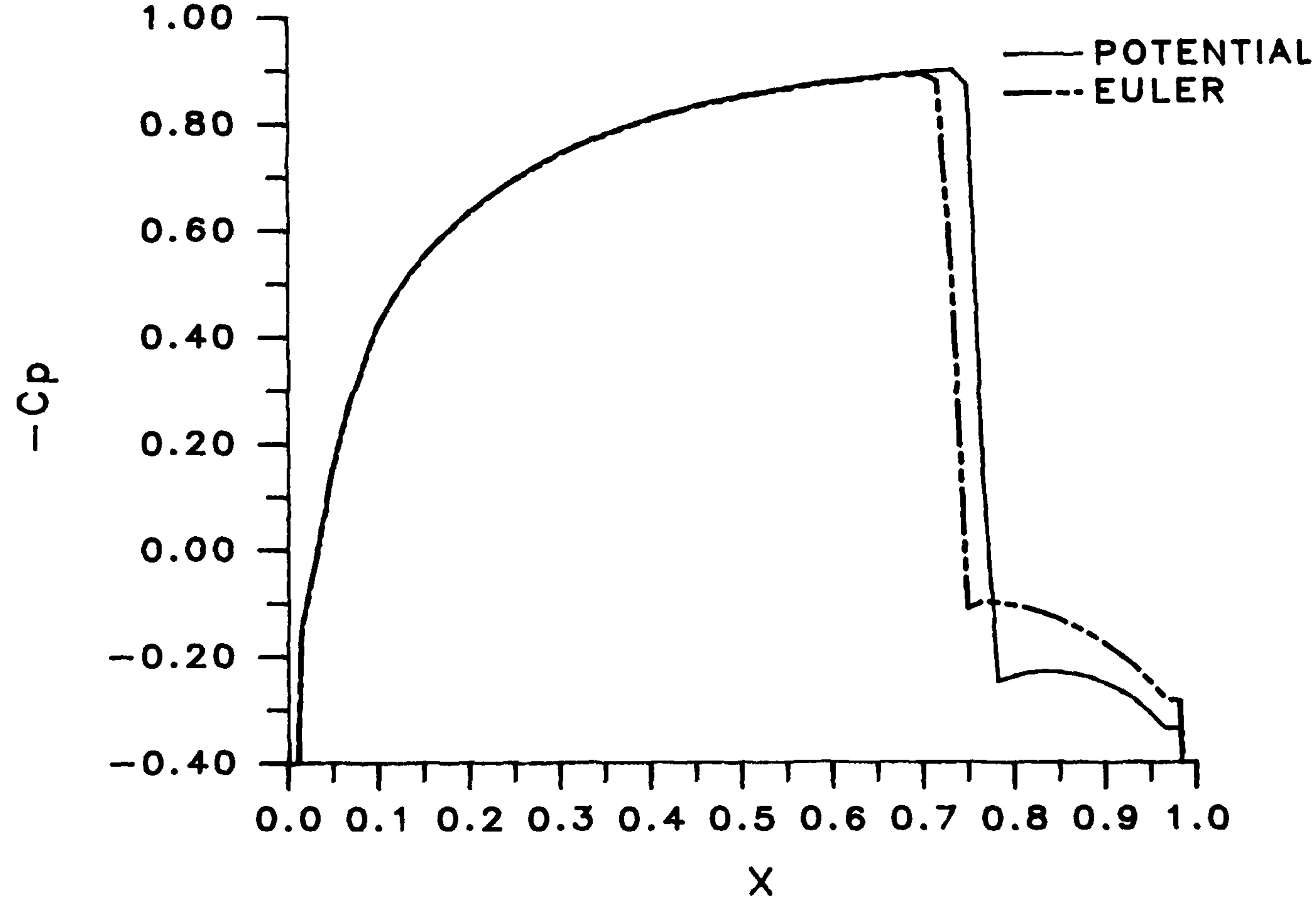

Fig. B2.1 Pressure Coefficient for Transonic Flow over a NACA0012 Airfoil at $M_\infty = 0.85$, Composite Velocity Formulation

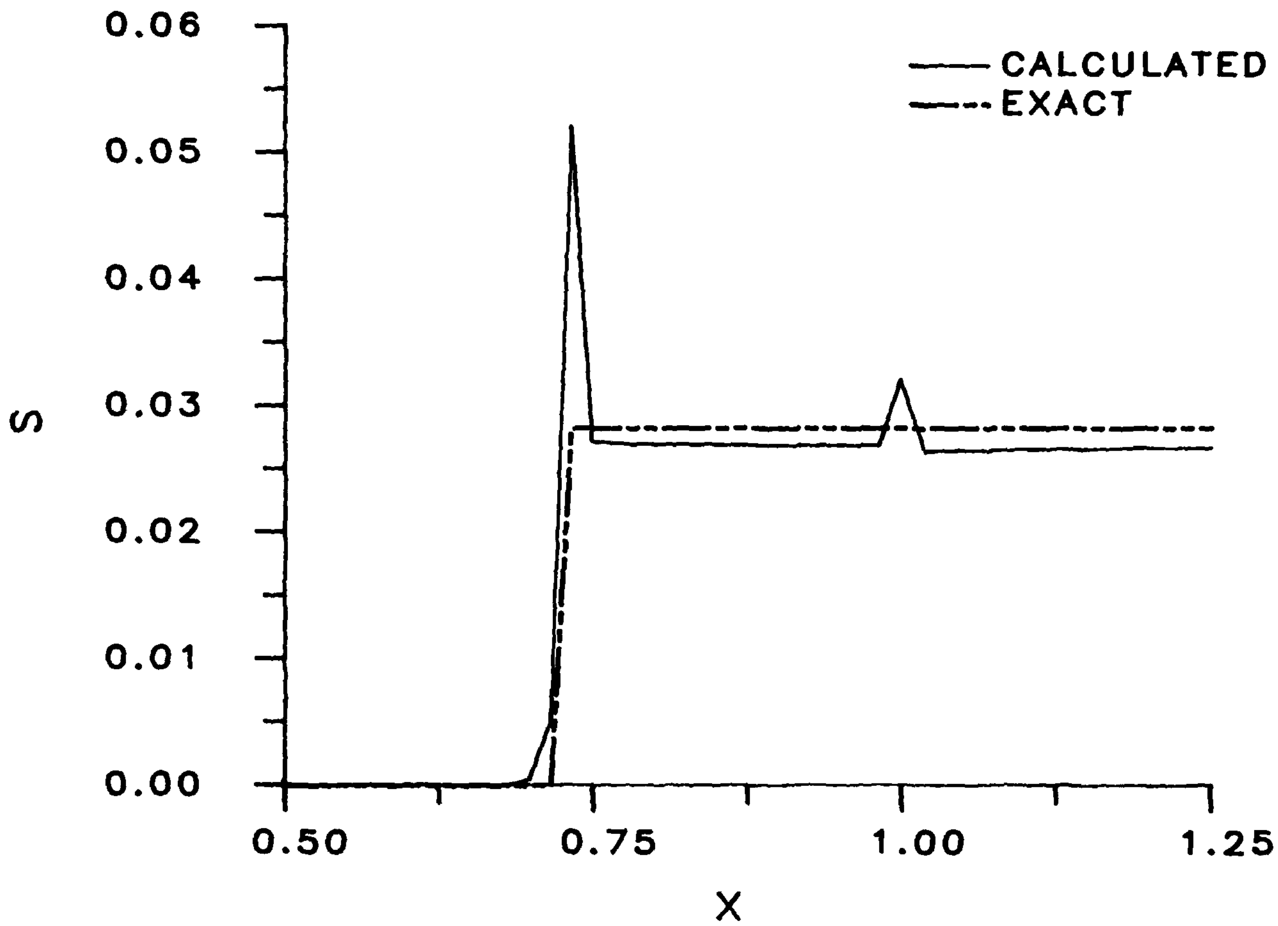

Fig. B2.2 Entropy Distrubition for Composite Velocity Euler Solution

B.2 CONSISTENT STRONGLY IMPLICIT ITERATIVE PROCEDURES FOR TWO-DIMENSIONAL UNSTEADY AND THREE-DIMENSIONAL SPACE-MARCHING FLOW CALCULATIONS

P.K. Khosla and S.G. Rubin

(To be submitted for publication)

Introduction

The success of a numerical formulation for the solution of two-dimensional time accurate or three-dimensional spatial accurate flow, e.g., RNS marching or global relaxation, depends largely on the solution algorithm. A non-iterative, unconditionally stable, consistent procedure should provide maximum efficiency. For large time increments (Δt) i.e., steady state calculations, consistency is not critical; rather, the technique should have strong convergence properties. For moderate Δt and transient flows, or moderate Δx for spatial marching, consistency of the numerical scheme plays an important role.

The most generally applied implicit and consistent formulation is the ADI factorization due to Douglas and Gunn(1). The Briley/McDonald(2) and Beam/Warming(3) schemes are based on this ADI factorization. There are, however, a number of problems with the ADI technique that have been encountered by various investigators. These are (i) an "instability" associated with the boundary condition, which may be attributable to the choice of intermediate boundary conditions and can only be controlled with smaller values of Δt, (ii) poor rates of convergence for $\Delta t \geq 1$, i.e. steady state problems (see Nietubicz(4) and Ghia et al(5)) and (iii) $O(\Delta t^2)$ accuracy of the factorization, which may not be desirable with certain types of discontinuous initial conditions. For the ADI method and also Crank-Nicholson scheme, it can be shown that odd and even time steps decouple for larger Δt. Thus the solution is oscillatory. Additional time smoothing is required in such cases to make the technique stable and convergent. For $\Delta t >> 1$, inconsistent and $O(\Delta t)$ techniques, e.g. CSIP, typically possess better convergence properties. Ghia et al.[5] have presented comparisons of the CSIP and the CADI method for both steady and unsteady internal flows. The advantages of the CSIP for steady calculations has been reaffirmed. Although the CSIP has also been used by these investigators for unsteady flow computations, it may be shown that the consistency and hence the time accuracy of the CSIP will deteriorate as the spatial mesh width h is refined, i.e., for a fixed time step, $\frac{\nu\Delta t}{h^2} >> 1$. A similar limitation exists for other relaxation procedures, e.g. LSOR, SSOR. This deficiency of relaxation techniques has not been discussed in any of the earlier time accurate references cited here. Therefore, there is currently no single implicit solution algorithm that has both the desirable properties of consistency (small Δt), rapid convergence to the steady state ($\Delta t >> 1$) and can also render an $O(\Delta t)$ scheme consistent so as to be applicable when first order accuracy is desirable.

In the present investigation a number of simple remedies have been investigated to render any of the well known relaxation procedures consistent. These are (i) a modified predictor-corrector SIP or CSIP procedure (ii) a multi-grid predictor-corrector scheme and (iii) a new algorithm based on the Sherman-Morrison[6] formula. The first two remedies require considerable programming steps and results in two step procedures similar to the ADI technique. The third formulation is very simple to implement, requires little modification of existing codes and results in less than 5% additional computational effort. Furthermore, it is a single step procedure that can be applied to to achieve either $O(\Delta t)$ or $O(\Delta t^2)$ accuracy. Intermediate boundary conditions are not required for either of the procedures. All of the algorithms have been tested successfully with the SIP on a model problem. These algorithms have been investigated for application to flow over an expanding airfoil.

MODIFIED SIP OR CSIP

The new procedures are described here for the model problem:

$$\phi_t = \nu\nabla^2\phi \tag{1}$$

A second order accurate Crank-Nicholson scheme ($\theta = 1/2$) leads to the following discrete form of the equations:

$$\frac{\phi^{n+1}_{ij} - \phi^{n}_{ij}}{\Delta t} = \nu A(\theta\phi^{n+1} + (1-\theta)\phi^{n})ij, \tag{2}$$

where the various diagonals in the matrix operator A are typically of $O(1/h^2)$; h is the mesh spacing and n the temporal index. This equation can be rewritten as:

$$(I - \nu\Delta t\theta\ A)\phi^{n+1} = (I + \Delta t\nu(1-\theta)A)\phi^{n}\ , \tag{3}$$

An exact solution for ϕ^{n+1} is obtained when the coefficient matrix $(I-\nu\Delta t\theta A)$ can be inverted. For matrices arising from general two dimensional (Navier-Stokes, Euler or RNS) operators, this requires the use of Gaussian elimination or a variant thereof. For large systems, where many length scales must be resolved, such a solution algorithm is extremely inefficient and prohibitively expensive. An approximate factorization is usually preferable. This factorization should be efficient and ideally generate an approximate solution which is within the truncation error of the scheme.

The CSIP is such a procedure. Application of the SIP for the inversion of $(I - \nu\Delta t\theta A)$ introduces the diagonal elements or "corner points" $\phi^n_{i-1,j+1}$, $\phi^n_{i+1,j-1}$, etc. These are treated explicitly and iterated upon in order to achieve the converged solution. A closer examination of the coefficients of these terms in the inversion algorithm reveals, that as noted previously these diagonals are of $O(\frac{\nu\Delta t}{h^2})$. Thus an accurate and consistent prediction of ϕ^{n+1}_{ij} requires that $\frac{\nu\Delta t}{h^2} << 1$. This restricts the choice of Δt to unacceptable small values for $h << 1$. In the following section, a number of simple remedies have been investigated for improving the consistency of the SIP procedure. These remedies are quite general and can be used with other relaxation procedures too.

Predictor-Corrector

In order to apply a single solution algorithm for both steady and unsteady or 3-D marching problems, i.e., a single formulation retaining the most desirable features for all values of Δt (or Δx marching), the present authors have investigated a number of two-step and one-step techniques. Intermediate boundary conditions are not required.

(i) The matrix A resulting from the spatial or "non-marching" discretization is rewritten as:

$$A\phi = \nu(\phi^{n+1}_{yy} + \varepsilon\phi^{n+1}_{zz} + (1-\varepsilon)\phi^n_{zz}) = M\phi^{n+1} + (1-\varepsilon)N\phi^n. \tag{4}$$

Equation (2) can then be written as:

$$(I-\nu\Delta t\theta M)\phi^{n+1} = (I + \nu\Delta t(1-\theta)A)\phi^n + \theta(I-\varepsilon)N\phi^n \tag{5}$$

In the predictor step, the coefficient matrix $(I - \nu\Delta t\theta M)$ with $\epsilon = \epsilon_o$ and $\theta = 1$ is inverted by the SIP. The error arising from the diagonals $\phi^n_{i-1,j+1}$ and $\phi^n_{i+1,j-1}$ can be made independent of the mesh size by an appropriate choice of the parameter ϵ. This provides a reasonable predictor for ϕ^{n+1}_{ij}. The error due to the inconsistency is within the truncation error of the numerical approximation of the scheme. A second-order accurate corrector step repeats the procedure with $\epsilon_o = 1$ and $\theta = \frac{1}{2}$. The choice of the appropriate value of ϵ_o is crucial to the success of the method. The following expression was selected in the test problem:

$$\epsilon_o = \frac{\Delta t/h^2}{1 + \Delta t/h^2} \qquad (6)$$

Other expressions or values of ϵ_o can also be chosen and still provide the required consistency. It should be noted that for $(\Delta t/h^2) << 1$, the order of inconsistency is bounded and independent of the mesh spacing h. For $\Delta t/h^2 >> 1$, $\epsilon \simeq 1$ and the usual SIP algorithm is recovered. A model problem with Dirichlet boundary conditions was chosen to test the predictor-corrector SIP technique. Calculations for $\nu = \frac{1}{10}$ were considered for (17x17) and (51x51) grids and $\Delta t = 1$. For a single time step, the maximum error in the numerical solution is less than 2% and 7%, respectively. Additional corrector iterations with $\epsilon = 1$ will further reduce this error. For comparison, the standard SIP on a (51x51) grid incurs almost a 100% error. The method requires the splitting of various derivatives in a manner similar to the ADI method. The choice of ε is quite arbitrary and may even depend upon the nature of the differential equation and the flow parameters.

SIP-Multigrid

A complementary approach to improve the consistency of the CSIP involves the application of a multigrid procedure. Since consistency error is amplified for fixed Δt and fine meshes, a course mesh predictor minimizes this error. The predictor step is then repeated on a succession of finer grids, the fine grid corrector step will not require additional iterations. Solutions obtained in this fashion are also $O(\Delta t^2, \Delta y^2, \Delta z^2)$; however, local convergence is enhanced by the multigrid smoothing process. Results similar to the one obtained in the previous method are also obtained by this technique. However, the programming complexity increases. This technique can always be employed with the procedures described in this paper. A new iterative procedure that is based on this idea and the application of sparse matrix direct solver is being currently investigated (reference C4) for the solution of the reduced Navier-Stokes equations. The preliminary results are quite encouraging.

C. Stone's-SIP

The consistency of the SIP procedure can be improved considerably by using a second-order factorization similar to the one originally proposed by Stone. In the present section a number of techniques similar in character to Stone's procedure will be discussed. Although most of these techniques are unstable for steady state calculations (some more than the others), they provide simple extensions of SIP for time consistent computations, fairly large $\Delta t \sim 1$ and highly stretched grids. Some additional inexpensive remedies are suggested which not only improve the stability for larger Δt, but also improve the consistency.

Stone[6] had proposed, the following second-order factorization. This cancels the corner values of ϕ.

$$\phi^{n+1}_{i-1,j+1} = \phi^{n}_{i-1,j+1} - \alpha(\phi^{n}_{i-1,j} + \phi^{n}_{i,j+1} - \phi^{n}_{i,j})$$
$$+ \alpha(\phi^{n+1}_{i-1,j} + \phi^{n+1}_{i,j+1} - \phi^{n+1}_{i,j})$$

$$\phi^{n+1}_{i+1,j-1} = \phi^{n}_{i+1,j-1} - \alpha(\phi^{n}_{i+1,j} + \phi^{n}_{i,j-1} - \phi^{n}_{i,j})$$
$$+ \alpha(\phi^{n+1}_{i+1,j} + \phi^{n+1}_{i,j+1} - \phi^{n+1}_{i,j})$$

Clearly the terms evaluated at the previous time level have a spatial truncation error of $O(\Delta x \Delta y)$ when α is chosen to be unity. This happens to be the simplest method for making the method consistent. Originally, Stone's technique was abandoned because of the complexity of the factorizing procedure for more than one unknown.. However, the following simple implementation has been found to be equivalent.

Given the algebraic system

$$A_1\phi_{i,j-1} + D_1\phi_{i-1,j} + B_1\phi_{i,j} + C_1\phi_{i,j+1} + E_1\phi_{i+1,j} \qquad (7)$$

the solution algorithm can be described as:

$$\phi_{i,j} = GM_{i,j} + E_{i,j}\phi_{i,j+1} + F_{i,j}\phi_{i+1,j} .$$

The elimination of the lower triangular terms $\phi_{i-1,j}$ and $\phi_{i,j-1}$ along with the cancellation of the corner points $\phi_{i+1,j-1}$ and $\phi_{i-1,j+1}$ can be carried out in a single step as:

$$\phi_{i,j-1} = (1 - \alpha F_{i,j-1})^{-1} [GM_{i,j-1} + (E_{i,j-1} - \alpha F_{i,j-1})\phi_{i,j}$$

$$+ \alpha F_{i,j-1} \phi_{i+1,j} + F_{i,j-1} \{\phi^n_{i+1,j-1} - \alpha(\phi^n_{i+1,j}+\phi^n_{i,j-1}-\phi^n_{i,j})\}] \tag{8}$$

and a similar expression for $\phi_{i-1,j}$. Substituting these in the governing equations, we get, the following recurrence relation for $F_{i,j}$ (say),

$$F_{i,j} = \frac{-\{C_1 + \alpha \frac{D_1 E_{i-1,j}}{1-\alpha E_{i-1,j}}\}}{B_1 + A_1 \frac{E_{i,j-1}-\alpha F_{i,j-1}}{1-\alpha F_{i,j-1}} + D_1 \frac{F_{i-1,j}-\alpha E_{i-1,j}}{1-\alpha E_{i-1,j}}}$$

It may be noticed that the values of $F_{i,j}$ do not depend upon the grid spacing and $F_{i,j}/(1-F_{i,j-1})$ etc. are $O(1)$. The group of terms being computed explicitly in equation (8) are thus of $O(\Delta x \Delta y)$. As such, the error does not grow in a space maraching or time dependent problems due to the initial guess of the solution. A general block version of this algorithm for any number of equations and unknowns has been coded and as a test is being applied to the three dimensional boundary region equations for supersonic flow past a cone. A second technique allows for the inclusion of an additional diagonal (corresponding to $\phi_{i+1,j+1}$) in the sparse LU factorization of the preconditioning matrix M. In this case, the source terms computed at the nth time level can further be cancelled by addition and subtraction of a grouping of the form $[\phi_{i+1,j+1} - \alpha(\phi_{i+1,j}+\phi_{i,j+1}-\phi_{i,j})]$. The storage required for the factorization step in this case increases from 3N to 4N. Other groupings can be devised to achieve similar results. For $\alpha = 0$, all the methods reduce to the simple SIP which has been used by the present authors for coupled system and for a large number of steady state computations.

The cancellation of the corner points $\phi_{i-1,j+1}$ and $\phi_{i+1,j-1}$ can also be performed by changing the truncation error of the governing equation. For example, the conservation form of the following equation

$$\frac{\partial \phi}{\partial t} + A_x + B_y = 0$$

where A and B are functions of ϕ, ϕ_x and ϕ_y, etc.

Then,

$$(Ax)_{j-1} + (By)_{i+1} - (Ax)_{i,j} - (By)_{i,j}$$

$$(Ax)_{j+1} + (By)_{i-1} - (Ax)_{i,j} - (By)_{i,j}$$

can be used to cancel the corner values of $\phi_{i-1,j+1}$ and $\phi_{i+1,j-1}$. This procedure worked quite well for steady subsonic potential flows. Additional iterations with $\alpha = 1$ did not diverge when used to compute flow past a biconvex airfoil. However, this may be fortuitous. Unfortunately, this technique is quite complicated and becomes prohibitively cumbersome for more than one unknown.

In order to eliminate the sensitivity of Stone's procedure to the value of α, a rank one improvement of the iterative procedure was found to be quite useful. This is achieved by the application of a Sherman-Morrison formula. It must be emphasized that all iterations, required at a given time step for nonlinear convergence, can be performed with smaller values of α. A value of $\alpha = 0.9$ has been utilized for this purpose.

Sherman-Morrison Formula[7]

This formula inverts a matrix of the following form:

$$A = B + UV^T$$

The matrix B is such that B^{-1} can easily be computed and U and V are two vectors. Then

$$A^{-1} = B^{-1} - \frac{B^{-1}UV^TB^{-1}}{1 + V^TB^{-1}U} \tag{9}$$

Wilf[8] used this formula to invert any non-singular square matrix. The computational cost is $O(N^3)$ which is comparable to Gaussian elimination. Memory requirements are $O(N^2)$ for N large. As a direct solver the Sherman-Morrison Formula is not competitive with iterative techniques. However, it can be usefully employed for improving the consistency of iterative techniques. Most iterative methods are based on some type of splitting of the coefficient matrix, e.g.,

$$A\phi = b \ . \tag{10}$$

can be written as

$$(M+N)\phi = b \tag{11}$$

For most of the matrices arising in flow problems, M^{-1} can be computed with reasonable computational effort; however, the error matrix N cannot be decomposed in the required form, i.e.,

$$N \neq UV^T \tag{12}$$

Therefore equation (9) cannot be usefully exploited. However, the linear equation (11) can be written as:

$$(M + N\,\hat{\phi}\hat{\phi}^T)\phi = b \ , \tag{13}$$

where $\hat{\phi} = \phi/\langle\phi^T,\phi\rangle$ is the unit vector. Equation (13) is non-linear and can be solved iteratively for ϕ as:

$$(M + U\hat{\phi}^{T^n})\phi^{n+1} = b \tag{14}$$

where

$$U = (N\hat{\phi})$$

The solution of (14) can be written as:

$$\phi_{n+1} = M^{-1}b - \frac{(M^{-1}U)\langle\hat{\phi}_n^T M^{-1}b\rangle}{1 + \langle\hat{\phi}_n^T M^{-1}U\rangle} \tag{15}$$

This solution further improves the symmetry property, as well as, the time-consistency of the solution. From equation (15), it can be seen that the additional effort required in the present case, as compared to the relaxation procedure based on the preconditioning by M^{-1}, is associated with the evaluation of two scalar products. $M^{-1}b$ and $M^{-1}U$ can be computed in the same loop with very small increase in the computation. The overall increase in time is between 2% to 5%. The maximum error is less than 2% for a (17x17) grid with $\Delta t = 1$. The problem under consideration has smooth initial conditions, with the exact solution having a ϕ_t of O(1). For a problem with sudden heating of the boundary, the initial conditions are discontinuous. In such a case, time consistency of the solution requires the use of a smaller Δt, at least in the initial stages of the calculation. The error matrices for SIP with and without the Sherman-Morrison update are $N\phi$ and $(N\phi)\dfrac{\phi^T}{\phi^T\phi}$ respectively. Since

$$||(N\phi)\frac{\phi^T||}{\phi^T\phi} \le \frac{||N\phi||\ ||\phi^T||}{\phi^T\phi} \le ||N\phi|| ,$$

the influence of the error matrix is reduced with the application of the Sherman-Morrison formula.

Error Analysis

The first order implicit formulation of the time dependent equation can be written as:

$$[I + \Delta t(M+N)]\phi_{n+1} = \phi_n + \Delta t\ b$$

where the index corresponds to a time step and (M+N) is the coefficient matrix arising out of the spatial terms. The time consistency of the solution procedure with Sherman-Morrison update, can be investigated by rewriting the lefthand side of the above equation as:

$$[I + \Delta t\ M + \Delta t\ N\ \hat{\phi}_n\ \hat{\phi}_n^T]\phi_{n+1} + \Delta t\ \{N\ \hat{\phi}_{n+1}\ \hat{\phi}_{n+1}^T - N\ \hat{\phi}_n\ \hat{\phi}_n^T\}\ \phi_{n+1}$$

where $\hat{\phi}$ represents the normalized unit vector defined earlier.

Using Taylor series expansion, it can be shown that the error term is given as

$$N\ \hat{\phi}_n\ \hat{\phi}_n^T - N\ \hat{\phi}_{n+1}\ \hat{\phi}_{n+1}^T = \Delta t\ [2\ \frac{\langle\phi_{n+1},\ \phi_t\rangle}{\langle\phi_{n+1},\ \phi_{n+1}\rangle}\ N\ \hat{\phi}_{n+1}\ \hat{\phi}_{n+1}^T$$

$$- \{N\ \phi_t\ \hat{\phi}_{n+1}^T + N\ \hat{\phi}_{n+1}\ \phi_t^T\}] + O(\Delta t^2)$$

Clearly, the error term is first order in time provided ϕ_t is O(1) and N is such that the spatial error does not dominate the error term as the grid is refined. In such a case, the limits $\Delta t \to 0$ will render the system consistent. The strongly implicit procedure with $\alpha = 0$, which has been extensively used in previous calculations, does not lead to a consistent splitting of the coefficient matrix. The error matrix N, amplifies the error as $h \to 0$, leading to a limitation that $(\Delta t/h^2) << 1$ for consistency. However, for $\alpha = 1$, the spatial error is not amplified as $h \to 0$. Thus consistency is achieved if Δt and $h \to 0$ independently. The procedure is also applicable to second order time accurate technique.

Applications

The technique has been tested on a variety of simple model problems. Both linear diffusion and nonlinear Burger equations have been investigated to test the validity of the procedure. In addition, the unsteady flow past a biconvex airfoil was also considered. The results for simple diffusion are described below.

The heat conduction equation is given as:

$$\phi_t = \frac{1}{Re} \nabla^2 \phi$$

The exact solution to this problem is

$$\phi(t,x,y) = e^{\frac{-2\beta^2 t}{Re}} \sin\beta x \sin\beta y + e^{\frac{-t}{Re}} e^{-x(1-\beta^2)^{1/2}} \sin\beta y$$

Exact initial and boundary conditions have been imposed on the numerical solution. This problem was chosen, because the boundary conditions and the solution are time dependent. The maximum residue on 17x17 and 51x51 uniform grids are depicted in Figures B2.1 and B2.2 for both the consistent SIP and the inconsistent SIP procedures. No additional iterations have been performed for either case. Both techniques include the Sherman-Morrison procedure. This improves the accuracy of the standard or inconsistent SIP method as well as the new procedure. On coarser grids, the two results are similar; however, on the fine grid there is significant difference. This reflects the severe inconsistency of the standard SIP procedure. The new formulation, however, retains the error of the order of truncation error. This is even true for finer grids and large values of Δt.

As another example of the applicability of the new algorithm, the composite velocity solution past a biconvex heaving airfoil at $M_\infty = 0.65$ has also been computed. Computations have been carried out for number of time steps on a 75×33 non-uniform grid. In these calculations, the non-linearity has been treated via picard iteration. At least one additional iteration is performed after time stepping and before the density is updated. In all the calculations, only three density updates have been performed. For comparison purposes, solutions using a direct solver have also been obtained. Three iterations on density are also performed for these exact computations. The results of these calculations are depicted in Figs. B2. Figures (B2.3) shows the coefficient of pressure at different normalized time levels. These calculations have been performed with $\Delta t = 0.1$ and 0.5. The results are very similar at similar time levels. For comparison the pressure time history at the mid chord has been depicted in Fig. B2.4 along with a direct solver solution, see B.4. The two solutions are in excellent agreement.

As another test problem the laminar boundary region equations for flow over a 10° cone at $M_\infty = 2$ has also been considered. The boundary region equations contain all cross flow diffusion terms and are solved, using the coupled algorithm. Solution for this case is well known. The present consistent CSIP requires only two iteration as compared to 5 to 6 iteration with the old CSIP to achieve the same order of accuracy for each marching step.

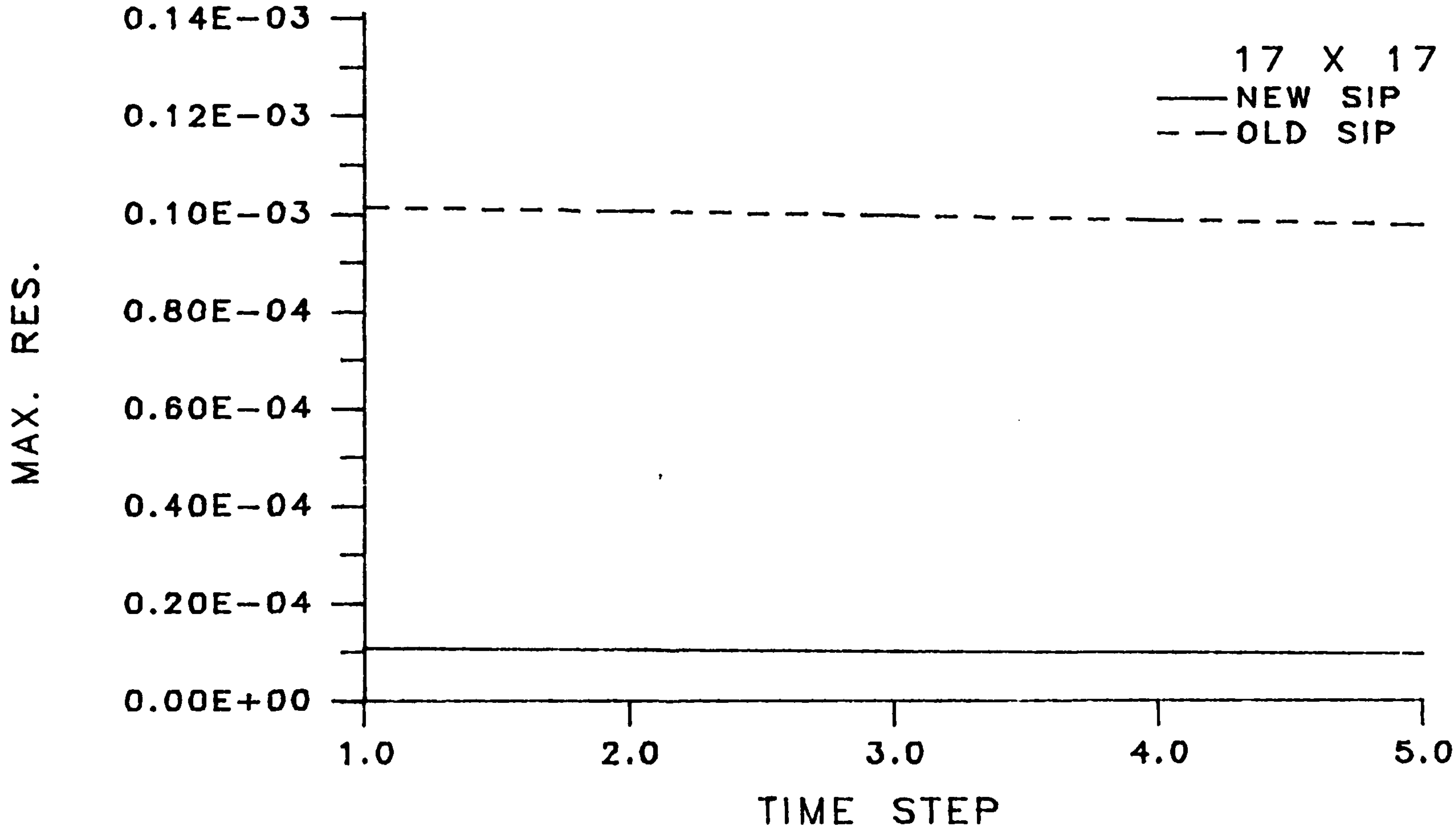
0.14E-03
0.12E-03
0.10E-03
0.80E-04
0.60E-04
0.40E-04
0.20E-04
0.00E+00
MAX. RES.
1.0
2.0
3.0
4.0
5.0
TIME STEP
17 X 17
NEW SIP
OLD SIP

FIG. B2.1

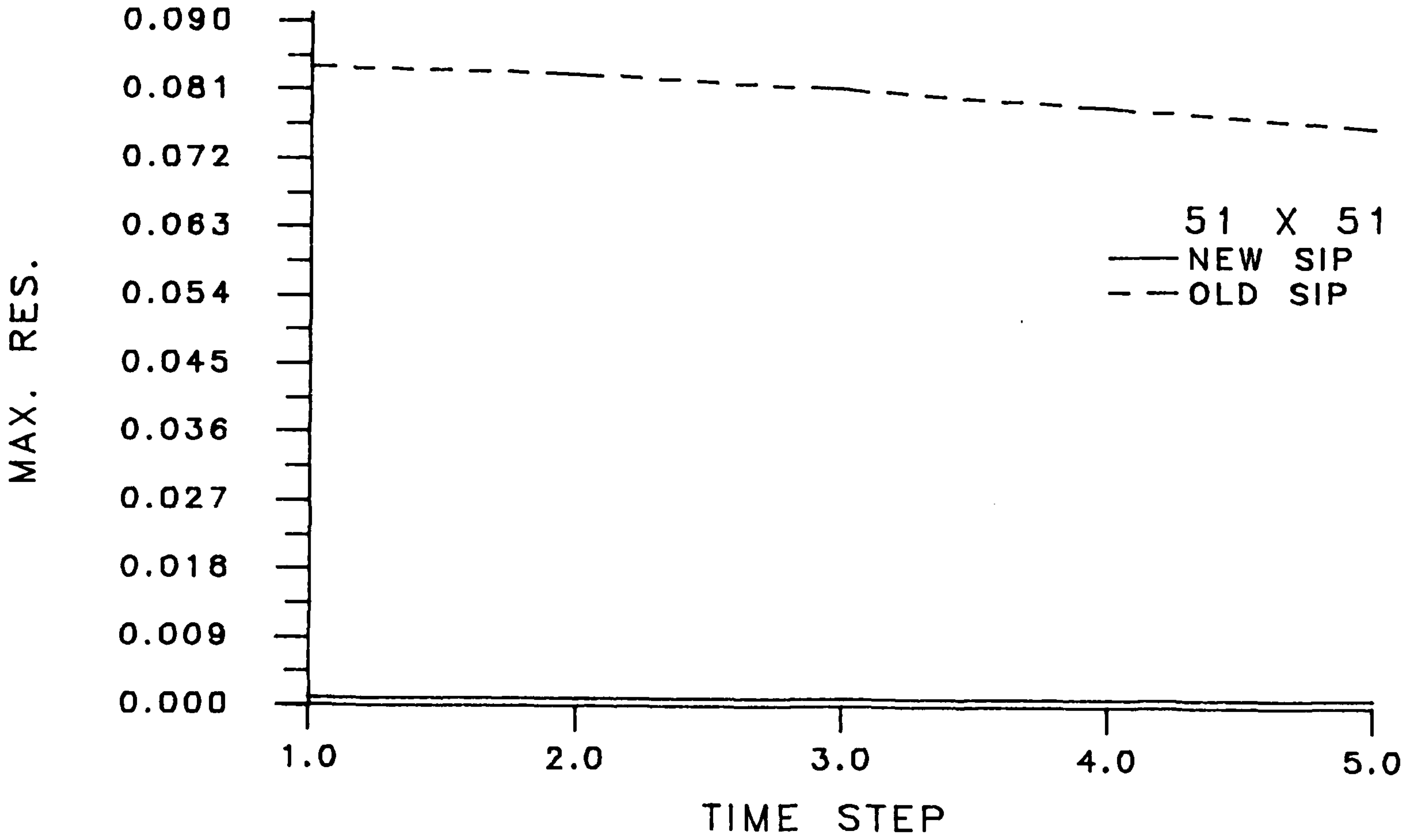
0.090
0.081
0.072
0.063
0.054
0.045
0.036
0.027
0.018
0.009
0.000
MAX. RES.
1.0
2.0
3.0
4.0
5.0
TIME STEP
51 X 51
NEW SIP
OLD SIP

FIG. B2.2

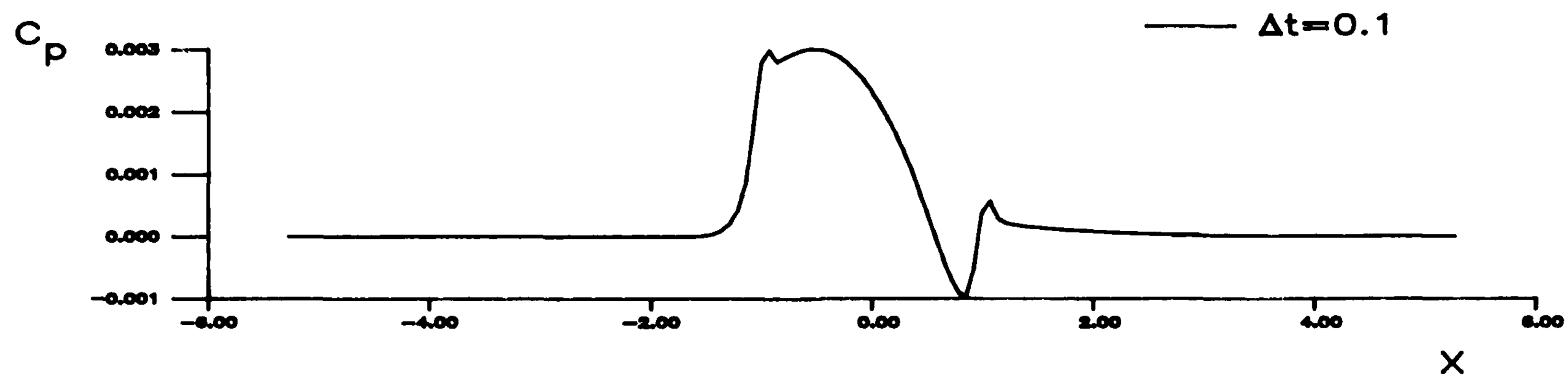

Fig. B2.3a Pressure Coefficient at Normalized Time $=10, M_\infty=0.65$

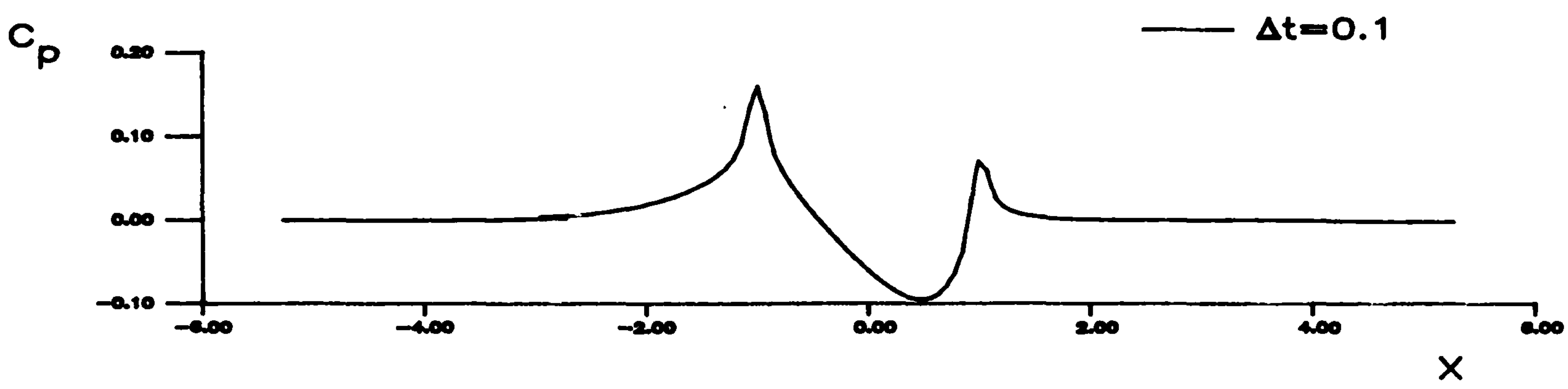

Fig. B2.3b Pressure Coefficient at Normalized Time =50,M_∞=0.65

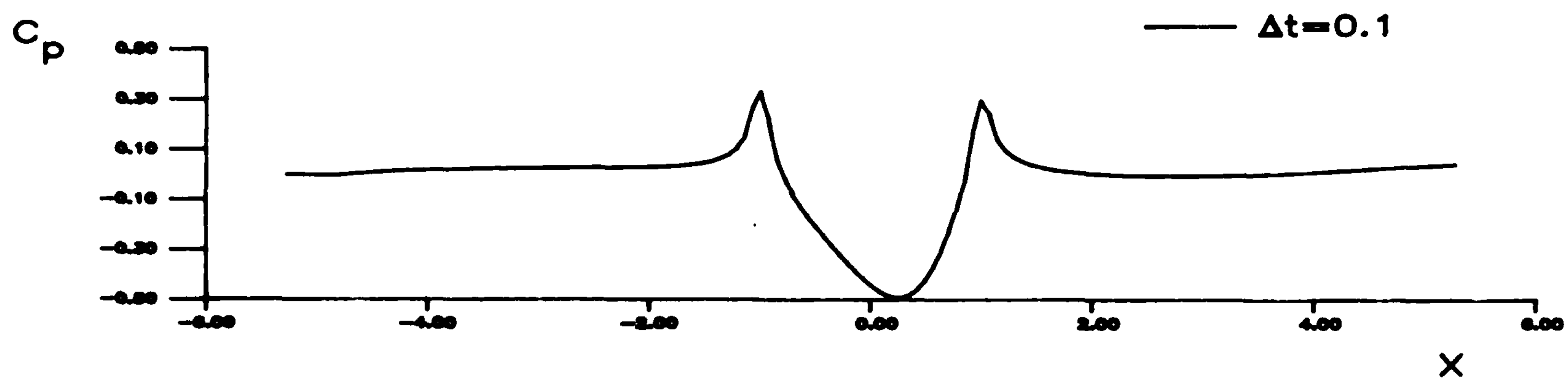

Fig. B2.3c Pressure Coefficient at Normalized Time =100, M_∞=0.65

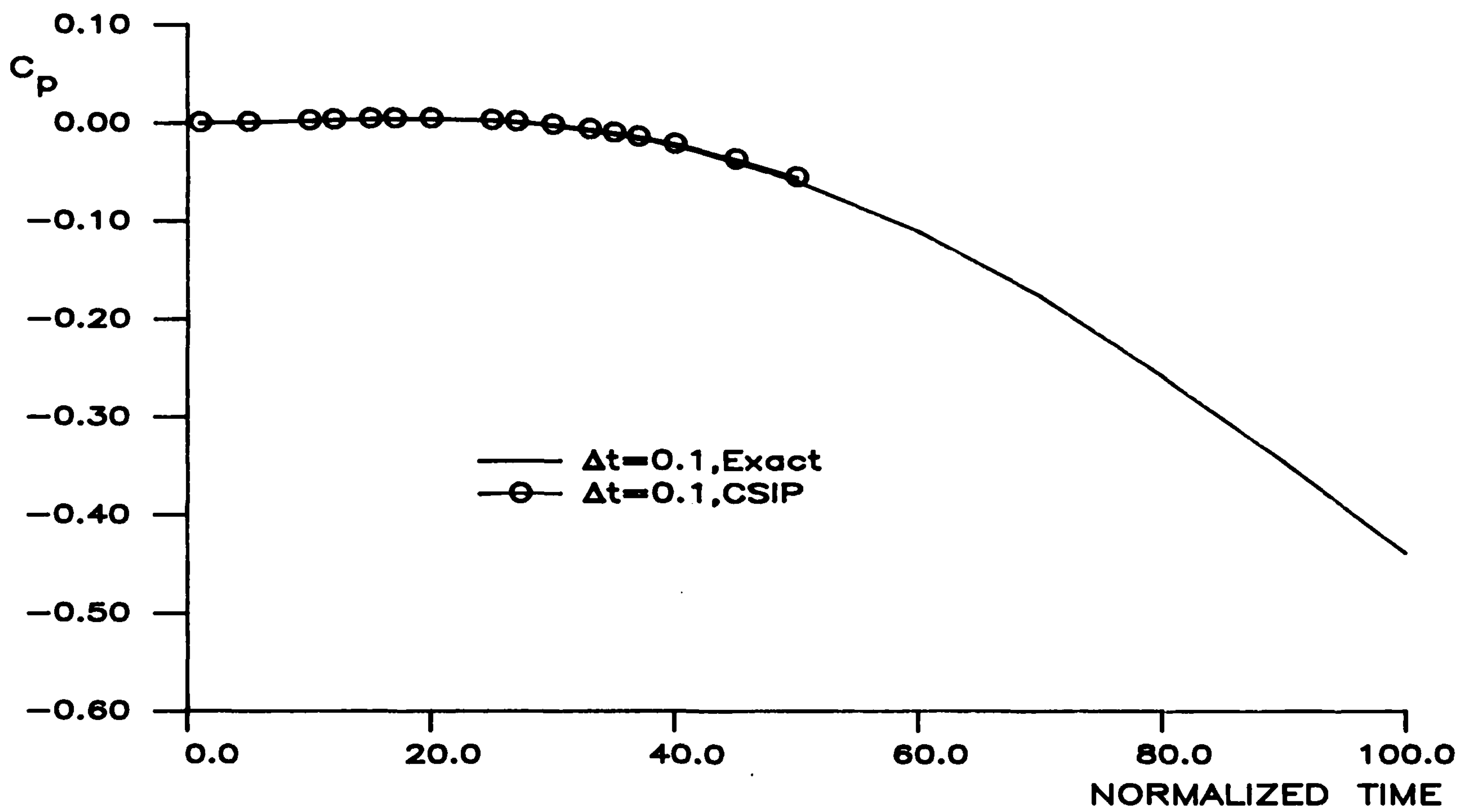

Fig.B2.4 Pressure Time History at Mid−Chord of Biconvex Airfoil,M_∞=0.65

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8. Wilf, H.S. (1959), "Matrix Inversion by Rank Annihilation of a Rank," Mathematical Methods for Digital Computers, John Wiley & Sons Inc., New York, pp. 73-77. Also see J. of Soc. Ins. & Appl. Math., Vol. 7, pp. 149-151.

B.3 A COUPLED STRONGLY IMPLICIT PROCEDURE FOR REDUCED NAVIER-STOKES EQUATIONS

Some solutions for the unsteady, compressible Reduced Navier-Stokes equations obtained using the Sherman and Morrison technique were presented at the AIAA 24TH AEROSPACE SCIENCES MEETING , held in Reno, Nevada. This algorithm is suitable only for a H-type grid. Since, there are some inherent advantages in a C-type grid for many flow problems, an algorithm that can be used for both the H-type and the C-type grid would be more desirable. Such an algorithm has been developed. This algorithm is based on the coupled strongly implicit (CSIP) procedure of Stones.

The quasi-linearized form of the governing finite difference equations together with the boundary conditions can be written as

$$A_{i,j} V^n_{i-1,j-1} + B_{i,j} V^n_{i-1,j} + C_{i,j} V^n_{i-1,j+1} +$$

$$D_{i,j} V^n_{i,j-1} + E_{i,j} V^n_{i,j} + F_{i,j} V^n_{i,j+1} +$$

$$G_{i,j} V^n_{i+1,j-1} + H_{i,j} V^n_{i+1,j} + I_{i,j} V^n_{i+1,j+1} = J_{i,j} \quad (1)$$

where $V_{i,j}$ is a vector of unknown $\rho_{i,j}$, $u_{i,j}$ and $v_{i,j}$ at the grid points (x_i , y_j) , $A_{i,j}$, $B_{i,j}$, ... , $I_{i,j}$ are known 3X3 coefficient matrices , $J_{i,j}$ is a known 3X1 matrix and 'n' refers to the present time level.

The CSIP is developed from the following approximate LU decomposition.

$$V^n_{i,j} = P_{i,j} + Q_{i,j} V^n_{i,j-1} + R_{i,j} V^n_{i-1,j} \quad (2)$$

Taylor series expansion is used to approximate $V^n_{i-1,j+1}$, $V^n_{i-1,j-1}$ and $V^n_{i+1,j-1}$ that appear in (1). We write

$$V^n_{i-1,j+1} \approx V^{n-1}_{i-1,j+1} + \varepsilon_1 (V^n_{i,j+1} - V^{n-1}_{i,j+1})$$

$$+ \varepsilon_2 \{ (V^n_{i-1,j} - V^{n-1}_{i-1,j}) - (V^n_{i,j} - V^{n-1}_{i,j}) \} \quad (3a)$$

$$V^n_{i-1,j-1} \approx V^{n-1}_{i-1,j-1} + \varepsilon_1 (V^n_{i,j-1} - V^{n-1}_{i,j-1})$$

$$+ \varepsilon_2 \{ (V^n_{i-1,j} - V^{n-1}_{i-1,j}) - (V^n_{i,j} - V^{n-1}_{i,j}) \} \quad (3b)$$

$$V^n_{i+1,j-1} \approx V^{n-1}_{i+1,j-1} + \varepsilon_1 (V^n_{i,j-1} - V^{n-1}_{i,j-1})$$

$$+ \varepsilon_2 \{ (V^n_{i+1,j} - V^{n-1}_{i+1,j}) - (V^n_{i,j} - V^{n-1}_{i,j}) \} \quad (3c)$$

where $0 \le \varepsilon_1 , \varepsilon_2 \le 1$ are some constants. From (1) , (2) and (3) we obtain the recursion relation

$$P_{i,j} = \text{Function}\,(P_{i,j+1} , Q_{i,j+1} , R_{i,j+1} ,$$
$$P_{i+1,k} , Q_{i+1,k} , R_{i+1,k} \;\; k=j \text{ or } j+1)$$

Similar relations are obtained for $Q_{i,j}$ and $R_{i,j}$.

We start the solution procedure by solving for $P_{M,j}$, $Q_{M,j}$ and $R_{M,j}$. The outflow boundary condition $(p_x)_{i=M} = 0$ enables us to solve for $P_{M,j}$, $Q_{M,j}$ and $R_{M,j}$ from the governing equations. The boundary conditions for j=N ($\rho_{i,n} = 1$, $u_{i,N} = u_\infty$) together with the continuity equation for j=N-1/2 yield $P_{i,N}$, $Q_{i,N}$ and $R_{i,N}$, $2 \le i \le M$. Since, $P_{M,j}$, $Q_{M,j}$ and $R_{M,j}$ and $P_{i,N}$, $Q_{i,N}$ and $R_{i,N}$ $2 \le i \le M$ are all known the recursion relation can be used to obtain $P_{i,j}$, $Q_{i,j}$ and $R_{i,j}$ for i=M-1,M-2,....,2 in that order. The continuity equation at j=3/2 and the boundary conditions for j=1 ($\rho_{i,1} = 1$, $u_{i,1} = u_\infty$) are then used to solve for $v^n_{i,1}$. From $\rho^n_{1,j}$, $u^n_{1,j}$, $v^n_{1,j}$, $1 \le j \le N$ and $\rho_{i,1}$, $u_{i,1}$, $v_{i,1}$, $2 \le i \le M$ and $P_{i,j}$, $Q_{i,j}$ $R_{i,j}$, $2 \le j \le N$, $2 \le i \le M$ one can solve for $V_{i,j}$ for all values of i and j using equation (2).

The unsteady breakdown of the laminar calculations for the flow past a sine-wave geometry was studied using this algorithm to explore the possibilities of obtaining unsteady, unsymmetric solutions for large Reynolds numbers. So far such a solution has not been obtained. Instead it was found that even without any imposed conditions of symmetry in the wake the solution for R=400,000 neither converges nor exhibits any tendency towards unsymmetric shedding. The behaviour in time of the wall shear stress for R=400,000 is shown in Fig.B3.1. The size of the separated region

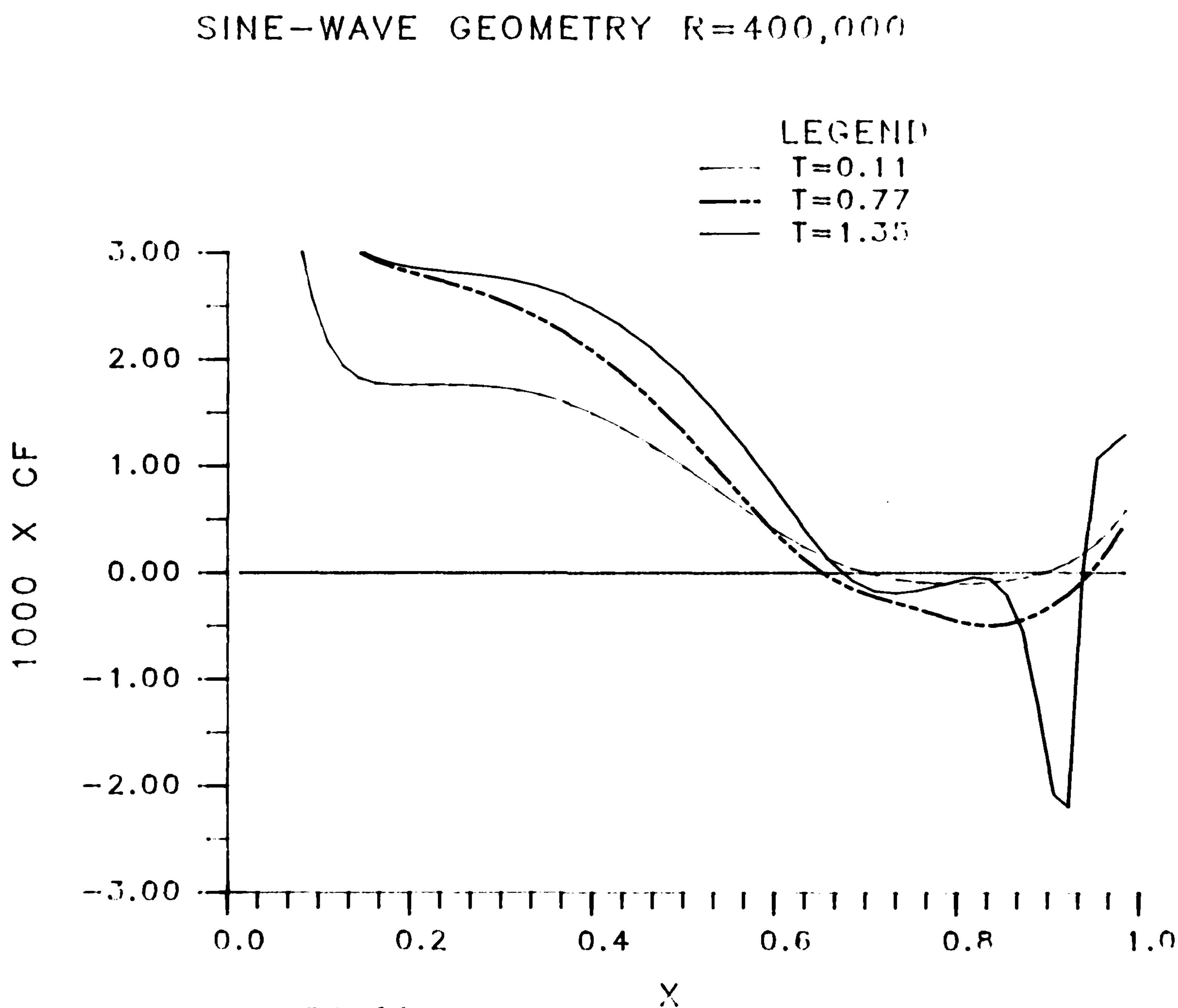
SINE-WAVE GEOMETRY R=400,000
LEGEND
T=0.11
T=0.77
T=1.35
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
1000 X CF
0.0
0.2
0.4
0.6
0.8
1.0
X

FIG. B3.1

initially increases with time and for large values of t the separated bubble breaks up into two and also the negative peak in the wall shear distribution increases unboundedly with time. Further analysis is required to understand this phenomenon better.

B.4 BLOCK ITERATIVE PROCEDURE FOR THE SOLUTION OF FULL AND REDUCED NAVIER-STOKES EQUATIONS

Most relaxation methods slow down when the number of grid points increase. Usually, such procedures are acceleralted by using either conjugate gradient or multi-grid techniques. For algebraic equations arising from large Reynolds number flows on stretched grids, both methods require special considerations and become quite problem dependent. There is no single procedure which the CFD community can reliably utilize, without many changes in the codes, that will work for a large class of problems. Usually the separated flow regions require special attention in applying multi-grid or other acceleration techniques. In view of these problems, a new technique is being investigated. This is based on a direct solver and is iterative in character. The relaxation process is carried out by simultaneously solving the equations on large blocks of the grid and iterating between these blocks. Typically, direct solvers require very large amounts of memory and become slow as the numbe of grids increases. However, they can be used on smaller blocks of the grid in a fairly efficient fashion. In this manner longer wave lengths of the solution error can be treated more effectively; the length is determined by block size.

The block iteration procedure consists of directly solving for values on subgrids of the main grid. It has been found experimentally that selection of the subgrids and the order in which the iteration takes place has a significant effect on convergence rate. The method has been tested on the Laplace equation in the unit square. The fastest grid sequence found so far is described in Fig. B4.1, where the subgrid numbers show the sequence in which they are iterated. The first two subgrids split the main grid in half and a third grid that overlaps the other two is solved last. This subgrid sequence converges in 6 iterations (the error criterion was the difference of the computed and exact solutions and was less than 10^{-4}) for a 21 by 21 grid, and 7 iterations for a 101 by 101 grid. Without the third grid the iteration count for the 21 by 21 case was 25.

The main grid was further divided into smaller grids as shown in Fig. B4.2. The first four grids were solved in an upward sweep followed by a downward sweep for the three overlapping grids. This was done to ensure a uniform propagation of information over the solution domain. As expected this grid strategy needed more iterations to converge. For a 21 by 21 grid, 15 iterations were required, and 25 iterations were required for a 101 by 101 grid. The convergence histories for these grid strategy in a model problem are shown in Figures B4.3 and B4.4 respectively. Apparently the method's sensitivity to main grid size increases with the number of subgrids.

It would seem initially that this strategy would be inferior to directly solving for the entire grid (given enough computer memory). For nonlinear problems however, iteration is required anyway to converge the nonlinearities. The amount of time required by sparse matrix solvers increases superlinearily with the number of unknowns ($O(n^{1.5})$ is optimum). Therefore one iteration of this subgrid strategy can be faster than an iteration of a fully direct solution. If the number of iterations using the

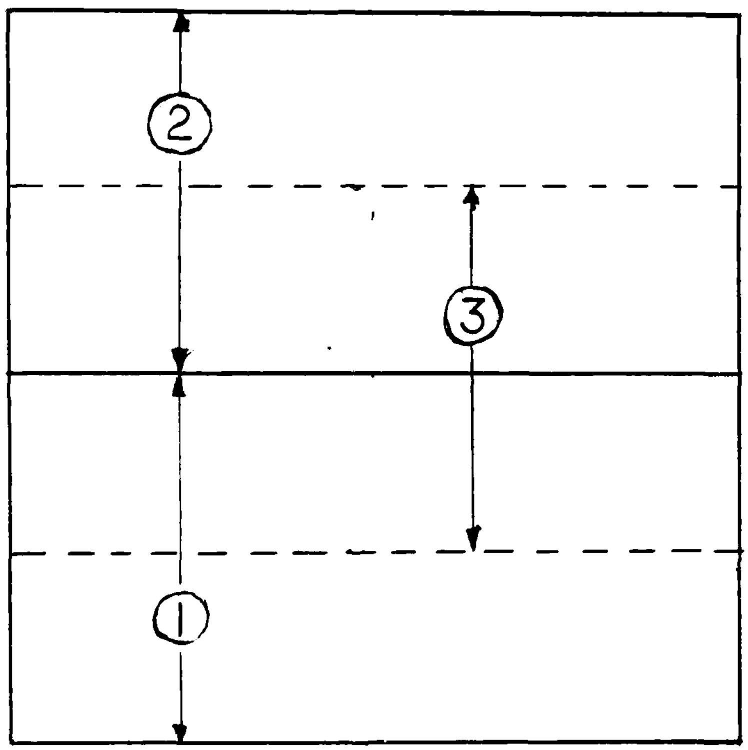

FIG. B4.1. 3 SUBGRIDS STRATEGY (NUMBER DETERMINES THE ORDER IN WHICH THE ITERATIONS PROCEED).

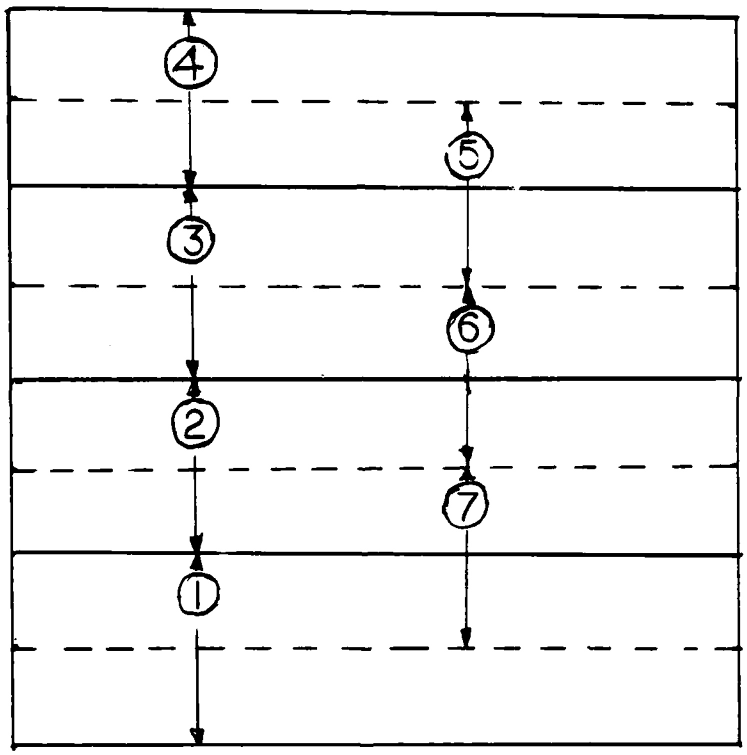

FIG. B4.2. 7 SUBGRIDS STRATEGY.

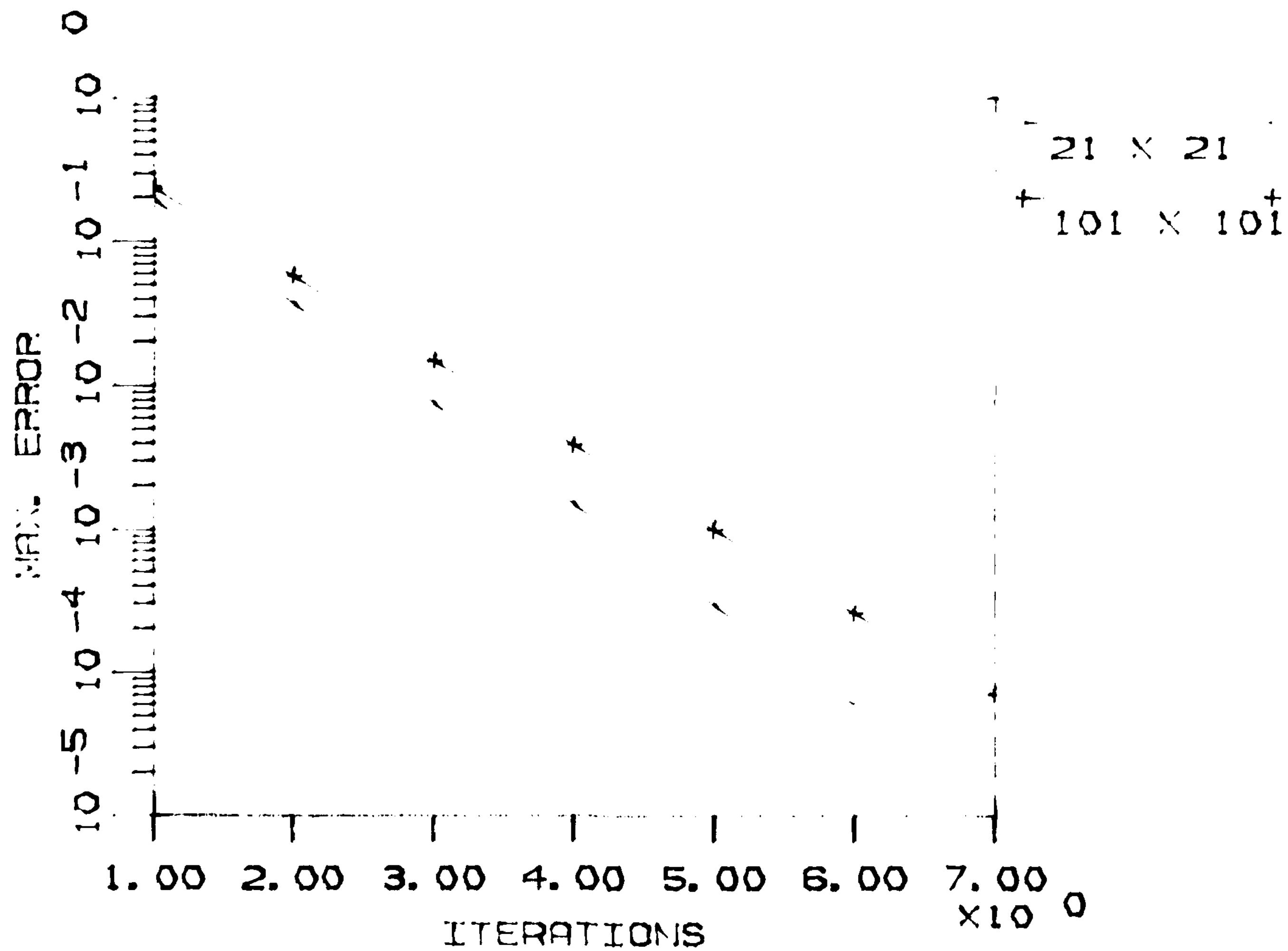

FIG. B4.3 CONVERGENCE HISTORY - 3 GRIDS

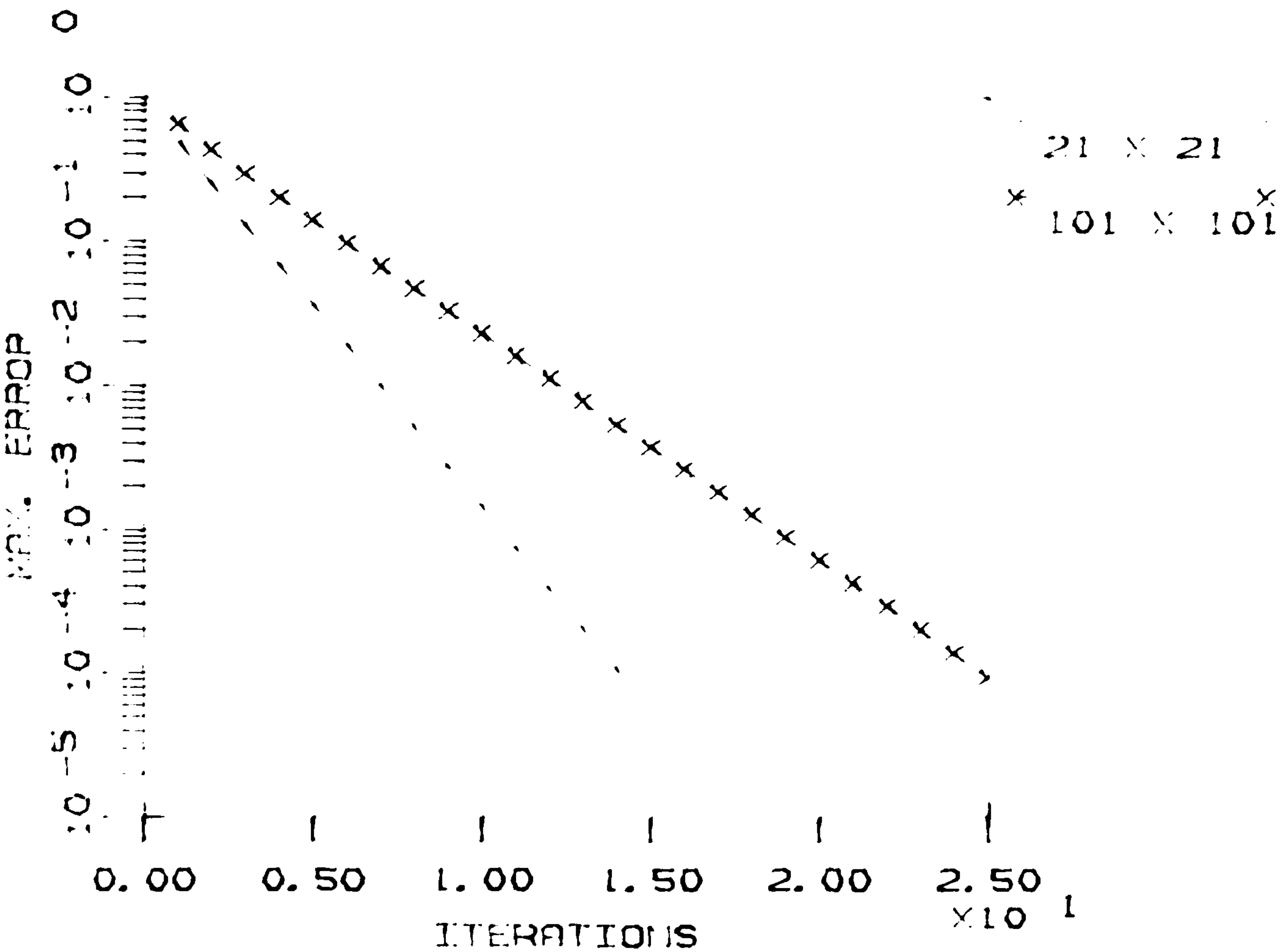

FIG. B.4.4. CONVERGENCE HISTORY - 7 GRIDS subgrid strategy is about the same as the number required to converge the nonlinearities, the subgrid strategy can be faster. The procedure has been applied to the solution of flow in a driven cavity and the steady laminar flow past a NACA 0012 airfoil at Re = 2000 and M_∞ = .72. Similar rates of convergence have been obtained for these problems.

C. TECHNICAL PUBLICATIONS 2/1/85 - 2/1/86

(1) Ramakrishnan, S.V. and Rubin, S.G., "Numerical Solution of Unsteady Compressible Reduced Navier-Stokes Equations" (to be submitted to AIAA Journal or Computers & Fluids).

(2) Khosla, P.K. and Rubin, S.G., "Consistent Time and Space Marching Coupled Strongly Implicit Algorithms " (to be submitted to Computers and Fluids or J. Computational Physics).

(3) Rubin, S.G. and Gordnier, R., "Composite Velocity, Potential Euler and RNS Solutions" (in preparation).

(4) Khosla, P.K. and Bender, E., "A 'Direct Solver' Iterative Formulation and Application to Viscous Interacting Flows" (in preparation).

D. PROFESSIONAL PERSONNEL 2/1/85 - 2/1/86

S.G. Rubin, Professor of Aerospace Engineering

P.K. Khosla, Professor of Aerospace Engineering

S.V. Ramakrishnan, Research Assistant (PhD student)

R. Gordnier, Research Assistant (PhD student)

M.S. "Transonic Viscous and Euler Solution Using a Composite Velocity Procedure", March 1985.

E. Bender, Research Assistant (PhD student)

E. OTHER ACTIVITIES AND INTERACTIONS 2/1/85 - 2/1/86

(i) Papers, Presentations, Seminars, Short Courses

(1) Ramakrishnan, S.V. and Rubin, S.G., "Numerical Solution of Unsteady Compressible Reduced Navier-Stokes Equations", Paper No. AIAA 86-0205, AIAA 24th Aerospace Sciences Meeting, Reno, NE, January 1986.

(2) Gordnier, R., "Transonic Viscous and Inviscid Solutions Using a Composite-Velocity Procedure", Paper No. AIAA 86-0074, AIAA 24th Aerospace Sciences Meeting, Reno, NE, January 1986.

(3) Reddy, D.R., Delaney, R. and Rubin, S.G., "Reduced Navier-Stokes Relaxation Procedure for Three-Dimensional Internal Flows with Interaction", SAE Aerospace Technology Conference, Long Beach, CA, October 1985.

(4) Khosla, P.K. and Rubin, S.G., "Consistent Strongly Implicit Iterative Procedures", (submitted to 10th International Conference on Numerical Methods in Fluid Dynamics, Beijing, July 1986).

(5) Rubin, S.G., "Global Relaxation Procedures for a Reduced Form of the Navier-Stokes Equations"

a. Pennsylvania State University, June 1985

b. Institute for Computer Applications in Science & Engineering, (ICASE), NASA Langley Research Center, Hampton, VA, May 1985

c. Allison Gas Turbine Division, General Motors Corporation, Indianapolis, IN, February 1985

d. University of Tennessee, Knoxville, TN, July 1985. Short Course on Finite Element Analysis in Fluid Mechanics and Heat Transfer.

(6) Khosla, P.K., "Subsonic and Transonic Reduced Navier-Stokes Techniques and Applications", International Conference on Fluid Dynamics, Tokyo, Japan, September, 1985.

(ii) <u>Consulting and Advisory Functions 2/1/85 - 2/1/86</u>

S.G. Rubin: Principal Investigator

(a) Joint NASA/DOD Panel on Hypersonic Flow Research and Graduate Training, NASA Headquarters, May 1985.

(b) NASA Lewis Computational Mechanics Advisory Committee. To advise the Lewis Research Center on Computational Mechanics research and development of a Computational Mechanics Institute under the auspices of the University Space Research Association. Meetings were held during 1984 and 1985.

(c) NASA Aerospace and Research Technology Subcommittee (ARTS) of the Office of Aeronautics and Space Technology (OAST). Appointed December, 1985.

(d) Discussions with Dr. J. Shang of Wright-Patterson Air Force Base for interaction on CFD and for sabbatical/consulting arrangement starting later in 1986. Agreement in principle has been reached.

(e) Consultant to AVCO Corporation, Everett, Mass on CFD related problems, November 1984 - March 1985.

(f) Consultant to Allison Gas Turbine Division, General Motors Corporation on use of Reduced Navier-Stokes Methodology for problems in internal flows, October 1984 - Present.

(g) Advisor to Aerospace Corporation on problems associated with viscous interacting flows and CFD, September 1979 - Present.

F. OTHER INFORMATION

The Reduced Navier Stokes (RNS), Composite Velocity (CV) and Coupled Strongly Implicit Procedure (CSIP) have been and continue to be applied for other agencies and by several other investigators. Recent publications based on these ideas include: R.H. Pletcher (Iowa State University) for J. Heat Transfer (to appear 1986), M. Israeli and M. Rosenfeld at the AIAA 6th CFD meeting in Cincinnati, June 1985, M. Barnett and R.T. Davis in Computers and Fluids (to appear 1986), H. Raven and M. Hoekstra at several hydrodynamics symposia (most recently in Washington, D.C., June 1985) and by B. Laksminarayana and co-workers at Pennsylvania State University at the AIAA 24th Aerospace Sciences Meeting in Reno, January 1986. These procedures are currently being investigated for hydrodynamics problems at MARIN, the National Maritime Institute of the Netherlands by H. Raven and M. Hoekstra, for transonic viscous boattail configurations at the NASA Langley Research Center with R. Wilmoth, for primitive variable formulations with subsonic viscous/inviscid interaction at ONR with T.C. Tai as technical monitor, at the Allison Gas Turbine Division of General Motors by D. Reddy for internal flow problems, for the NASA Lewis Research Center under J. Adamcyzk and B. Anderson for internal and hypersonic flows, respectively. The AFOSR sponsored work has also been referenced in numerous papers during the past year and very similar ideas appear in the work of P. Bradshaw, Imperial College and R. Consteix at CERT, France.

It has been approximately five years since the formulations considered here were first proposed by the present investigators. These methods have been shown to be accurate and efficient procedures for two-dimensional steady flows when combined with the coupled strongly implicit, conjugate gradient and global relaxation algorithms. The initial applications to two-dimensional unsteady and three-dimensional steady problems, that have been reported here, have further established the utility of such techniques. Until such time as a fully coupled, time-dependent, compressible Navier-Stokes solver becomes cost and computer efficient for solving general viscous flow problems, the procedures discussed herein will continue to be highly competitive for a significant class of aerodynamic configurations and viscous/inviscid interactions.

END

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