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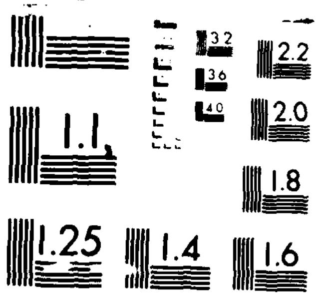

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THE PMC SYSTEM LEVEL FAULT MODEL: CARDINALITY PROPERTIES OF THE IMPLIED FAULTY SETS

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ABSTRACT

In this paper, we consider one aspect of the PMC system level fault model, the properties of the implied faulty sets. For τ-diagnosable systems that have at most τ faulty units, we give lower bounds on the cardinality of the maximal implied faulty sets, then we show that these bounds are greatest lower bounds and we indicate how these results may be used in diagnosis algorithms.

I. INTRODUCTION

The PMC system level fault model [PRE67] consists of a set of units $U = \{u_1, u_2, \ldots, u_n\}$ capable of testing one another and a set of ordered pairs $\{(u_i, u_j) \mid u_i \textit{ tests } u_j\}$ describing the organization of the tests. The model is defined by the fault-test relationship which specifies the test outcome $a_{i,j}$ in terms of the status of both the unit u_i applying the test and the unit u_j being tested. If u_i is nonfaulty, then $a_{i,j} = 0$ if u_j is nonfaulty and $a_{i,j} = 1$ if u_j is faulty, and if u_i is faulty, the test outcome $a_{i,j} = 0 \textit{ or } 1$, independent of the status of u_j. A collection of all test outcomes is called a syndrome. The model can be represented by the directed graph $G = (U,E)$, in which the vertices in U are the units and the edges in E are the tests between units. The test outcomes are the edge labels of the graph, and thus G has both 0-edges and 1-edges. The model has been studied extensively and among topics that have been addressed are conditions for τ-diagnosability ([PRE67], [HAK74], [ALL75], [CHW81], [KEN84]) and algorithms for system diagnosis ([KAM75], [MEY78], [MAD77], [MEY81], [DAH84], [DAH85]).

Given a syndrome, the diagnosis problem consists of identifying the set of faulty units F_S and the set of nonfaulty units G_S. A system is τ-diagnosable if and only if all faulty units can be identified from the syndrome whenever the system has at most τ faulty units [PRE67]. For a given syndrome, a partition (G,F) is consistent with the syndrome if every test among units in G has a 0 outcome and every test from a unit in G to a unit in F has a 1 outcome. Diagnosis of a τ-diagnosable system with at most τ faulty units requires identifying the unique consistent partition (G_S, F_S) such that $\|F_S\| \leq \tau$.

We recall that for a given syndrome, the implied nonfaulty set $G(u_i)$ for the unit u_i is the set of all units that are implied nonfaulty if u_i is assumed to be nonfaulty and the implied faulty set $L(u_i)$ is the set of all units that are implied faulty if u_i is assumed to be nonfaulty [KAM75]. Thus, if we define a *0-path* in the graph G as a path in which every edge is a 0-edge, we see that

$$G(u_i) = \{u_i\} \cup \{u_j \mid \textit{there is a 0-path from } u_i \textit{ to } u_j\},$$

and

$$L(u_i) = \{u_j \mid \textit{there exists } u_p \textit{ in } G(u_i),\ u_q \textit{ in } G(u_j) \text{ and } \textit{either } a_{p,q} = 1 \textit{ or } a_{q,p} = 1 \textit{ or both}\}.$$

It is clear that if $L(u_i) \cap G(u_i) \neq \phi$, then the unit u_i is faulty. Many diagnosis algorithms take advantage of this fact by declaring such units faulty and concentrating on the problem of diagnosing the resulting reduced system. For example, if a system is τ-diagnosable and has at most τ faulty units, the algorithm in [MEY81] identifies the set of faulty units if there exists at least one faulty unit u_i such that either $L(u_i) \cap G(u_i) \neq \phi$ or $\|L(u_i)\| \geq \tau + 1$. Only τ-diagnosable systems in which no two units test each other are known to have this property [MAD77], [MEY83]. The structural constraints associated with self-implicating systems [DAH85] are even stronger.

In this paper we do not impose structural constraints on the test organization, and we analyze the properties of the implied faulty sets only under the assumptions that the system is τ-diagnosable and that the number of faulty units is not greater than τ. The main thrust of our effort is directed at obtaining lower bounds on the cardinality of the maximal implied faulty sets associated with not only the units in F_S, but also the units in G_S. We then present a brief example of how these bounds can simplify diagnosis. All proofs for this paper are contained in [KEN86].

II. IMPLIED FAULTY SETS OF FAULTY UNITS

When $\tau \leq 2$, the cardinality of the maximal implied faulty sets associated with the faulty units can be obtained without much difficulty.

Theorem 1: If S is τ-diagnosable, if $1 \leq \|F_S\| \leq \tau$, and if $\tau \leq 2$, at least one unit u_i in F_S exists such that $\|L(u_i)\| \geq \tau + 1$.

The next result shows that for the implied faulty sets associated with faulty units this lower bound is actually the greatest lower bound.

Lemma 1: To the integers $\tau = 1$ and $\tau = 2$ correspond at least one τ-diagnosable system S that has τ faulty units and one syndrome such that:

(i) $L(u_i) \cap G(u_i) = \phi$ for every unit u_i in S,

(ii) $\|L(u_i)\| = \tau + 1$ for every faulty unit u_i, and

(iii) $\|L(u_i)\| = \tau$ for every nonfaulty unit u_i.

When $\tau > 2$, we obtain the following results.

Theorem 2: If S is τ-diagnosable, if $1 \leq \|F_S\| \leq \tau$, and if $\tau > 2$, at least one unit u_i in F_S exists such that either $L(u_i) \cap G(u_i) \neq \phi$ or $\|L(u_i)\| \geq \tau - k + 1$, where k is the least integer such that $\tau \leq 6k + 2$.

The next result shows that for $\tau > 2$ the lower bound given in Theorem 2 is actually the greatest lower bound on the cardinality of the maximal $L(u_i)$ associated with the faulty units.

Lemma 2: To every integer $\tau > 2$ corresponds at least one τ-diagnosable system S that has τ faulty units and one syndrome such that:

(i) $L(u_i) \cap G(u_i) = \phi$ for every unit u_i in S,

(ii) $\|L(u_i)\| = \tau - k + 1$ for every faulty unit u_i, where k is the least integer such that $\tau \leq 6k + 2$, and

(iii) $\|L(u_i)\| = \tau$ for at least one nonfaulty unit u_i.

Theorems 1 and 2 show that the set of values of τ may be partitioned into intervals of length 6, except for the first interval that is of length 2. For τ-diagnosable systems in which both $1 \leq ||F_S|| \leq \tau$ and $L(u_i) \cap G(u_i) = \phi$ for all u_i in S, Theorem 1 implies that if $\tau \leq 2$, at least one faulty unit u_i exists such that $||L(u_i)|| \geq \tau + 1$, and Theorem 2 implies that if $\tau \leq 8$, at least one faulty unit u_i exists such that $||L(u_i)|| \geq \tau$, if $\tau \leq 14$, at least one faulty unit u_i exists such that $||L(u_i)|| \geq \tau - 1$, and so forth.

III. IMPLIED FAULTY SETS OF ALL UNITS

It is clear that $||L(u_i)|| \leq \tau$ whenever the unit u_i in nonfaulty, and therefore when $\tau \leq 2$, the consideration of nonfaulty units does not result in an improvement of the lower bound on the cardinality of $L(u_i)$.

Theorem 3: If S is τ-diagnosable, if $1 \leq ||F_S|| \leq \tau$, and if $\tau \leq 2$, at least one unit u_i in S exists such that $||L(u_i)|| \geq \tau + 1$.

When $\tau > 2$, Lemma 6 not only shows that the lower bound given in Theorem 2 is the greatest lower bound, but also that the unit with the maximal implied faulty set may be nonfaulty. We now improve the lower bound on the cardinality of the maximal $L(u_i)$ by considering not only the implied faulty sets associated with the faulty units, but also the implied faulty sets associated with the nonfaulty units.

The following theorem extends Theorem 3 by considering the implied faulty sets of both faulty and nonfaulty units.

Theorem 4: If S is τ-diagnosable, if $1 \leq ||F_s|| \leq \tau$, and if $\tau > 2$, at least one unit in S exists such that either $L(u_i) \cap G(u_i) \neq \phi$ or $||L(u_i)|| \geq \tau - k + 1$, where k is the least integer such that $\tau \leq 7k + 2$.

Lemma 6 shows that for $3 \leq \tau \leq 8$ the lower bound on the cardinality of the maximal implied faulty set given in Theorem 2 is the greatest lower bound. The next lemma proves a similar result for $\tau > 8$.

Lemma 3: To every integer $\tau > 8$ corresponds at least one τ-diagnosable system S

that has τ faulty units and one syndrome such that:

(i) $L(u_i) \cap G(u_i) = \phi$ for every u_i in S,

(ii) $||L(u_i)|| \leq \tau - k + 1$ for every u_i in S, where k is the least integer such that $\tau \leq 7k + 2$,

(iii) $||L(u_i)|| = \tau - k + 1$ for at least one faulty unit u_i, and

(iv) $||L(u_i)|| = \tau - k + 1$ for at least one nonfaulty unit u_i.

Theorems 3 and 4 show that the set of values of τ may be partitioned into intervals of length 7, except for the first interval of length 2. Thus, for a τ-diagnosable system in which $1 \leq ||F_s|| \leq \tau$ and $L(u_i) \cap G(u_i) = \phi$ for all u_i in S, Theorem 3 implies that if $\tau \leq 2$, at least one unit u_i exists such that $||L(u_i)|| \geq \tau + 1$, and Theorem 4 implies that if $\tau \leq 9$, at least one unit u_i exists such that $||L(u_i)|| \geq \tau$, if $\tau \leq 16$, at least one unit u_i exists such that $||L(u_i)|| \geq \tau - 1$, and so forth.

IV. CONCLUSION

We have presented results concerning the properties of the implied faulty sets in the PMC system level fault model. Unlike previous work on implied faulty set properties, we made no structural properties assumptions, only that the system was τ-diagnosable and had at most τ faulty units. The results are not only interesting in themselves, but also because of their implications in the diagnosis process.

Given a τ-diagnosable system S and the implied faulty and nonfaulty sets for each unit, we can identify the set $F_0 = \{u_i \mid L(u_i) \cap G(u_i) \neq \phi\}$. If S has at most τ faulty units, then $||F_0|| \leq \tau$. In this case, removing from S the units in F_0 and all tests involving these units produces a reduced system $(S - F_0)$ that is $(\tau - ||F_0||)$-diagnosable. The results of this paper outline the properties of the maximal implied faulty sets in the reduced system $(S - F_0)$. If $(\tau - ||F_0||) \leq 2$, then the units with the maximal $||L(u_i)||$ are faulty. If $3 \leq (\tau - ||F_0||) \leq 9$, then there exists at least one unit

u_i such that $\|L(u_i)\| \geq \tau$. If $\|L(u_i)\| > \tau$, then u_i is obviously faulty. When $\|L(u_i)\| = \tau$, the implied faulty and nonfaulty sets are the basis of a consistent partition of S. If the remainder set $N(u_i) = S - (L(u_i) \cup G(u_i))$ contains no 1-edges then $\{G(u_i) \cup N(u_i), L(u_i)\}$ is a minimal consistent partition of S such that $\|L(u_i)\| = \tau$, and thus u_i must be nonfaulty and $F_S = L(u_i)$. Also note that if $\|L(u_i)\| = \tau$ and $N(u_i)$ contains at least one 1-edge, then any consistent partition $\{G,F\}$ of S in which u_i is in G is such that $\|F\| > \tau$, and thus u_i is faulty.

As an example of a system that is easily diagnosed using this approach, consider the 6-diagnosable, 13 unit system proposed by Madden [MAD77]. Figure 1 shows the test outcome/incidence matrix B of this system. The i^{th} row and j^{th} column element b_{ij} is equal to the test outcome $a_{i,j}$ (0 or 1) if u_i tests u_j and b_{ij} is equal to x if u_i does not test u_j. For each i in $\{1,2,\ldots,13\}$, Figure 2 lists the indices of the units u_j in the implied faulty set $L(u_i)$, in the implied nonfaulty set $G(u_i)$, and in the remainder set $N(u_i) = S - (L(u_i) \cup G(u_i))$. Note that $L(u_i) \cap G(u_i) = \phi$ for all units u_i, i in $\{1,2,\ldots,13\}$. The units such that $\|L(u_i)\| = 6$ are easily diagnosed. For example, the unit u_8 has $\|L(u_8)\| = 6$ and $N(u_8) = \{u_7, u_{10}, u_{11}, u_{12}, u_{13}\}$. The set $N(u_8)$ contains the 1-edge (u_7, u_{10}) among others, thus u_8 must be faulty. On the other hand, the unit u_2 has $\|L(u_2)\| = 6$ and $N(u_2) = \{u_4\}$, and thus the set of faulty units is $F_S = L(u_2) = \{u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}\}$.

Figure 1 - Test Outcome/Incidence Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	x	x	x	x	x	x	x	1	1	x	x	x	x
2	0	x	0	x	0	0	x	1	1	1	1	1	1
3	0	0	x	x	0	x	x	1	1	1	1	1	1
4	0	x	0	x	x	x	0	1	1	1	1	1	1
5	0	x	x	x	x	0	0	1	x	1	1	1	1
6	0	x	x	x	0	x	x	x	1	1	1	1	1
7	x	x	x	x	x	x	x	x	x	1	1	1	1
8	1	1	x	1	x	x	x	x	0	x	x	x	x
9	x	x	x	1	x	1	x	0	x	x	x	x	x
10	x	1	1	1	1	x	1	x	x	x	x	x	x
11	x	1	1	1	x	1	1	x	x	x	x	x	x
12	x	1	1	1	1	1	1	x	x	x	x	x	x
13	x	1	1	1	1	1	1	x	x	x	x	x	x

Figure 2 - The indices of the units in $L(u_i)$, $G(u_i)$, and $N(u_i)$ for each u_i in S

i	$\{j \mid u_j \in L(u_i)\}$	$\{j \mid u_j \in G(u_i)\}$	$\{j \mid u_j \in N(u_i)\}$
1	{8,9}	{1}	{2,3,4,5,6,7,10,11,12,13}
2	{8,9,10,11,12,13}	{1,2,3,5,6,7}	{4}
3	{8,9,10,11,12,13}	{1,2,3,5,6,7}	{4}
4	{8,9,10,11,12,13}	{1,2,3,4,5,6,7}	ϕ
5	{8,9,10,11,12,13}	{1,5,6,7}	{2,3,4}
6	{8,9,10,11,12,13}	{1,5,6,7}	{2,3,4}
7	{10,11,12,13}	{7}	{1,2,3,4,5,6,8,9}
8	{1,2,3,4,5,6}	{8,9}	{7,10,11,12,13}
9	{1,2,3,4,5,6}	{8,9}	{7,10,11,12,13}
10	{2,3,4,5,6,7}	{10}	{1,8,9,11,12,13}
11	{2,3,4,5,6,7}	{11}	{1,8,9,10,12,13}
12	{2,3,4,5,6,7}	{12}	{1,8,9,10,11,13}
13	{2,3,4,5,6,7}	{13}	{1,8,9,10,11,12}

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