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**ESTIMATION FOR THE RASCH MODEL
WHEN BOTH ABILITY AND
DIFFICULTY PARAMETERS ARE RANDOM**

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**Estimation for the Rasch Model
when both Ability and Difficulty
Parameters are Random**

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A rectangular stamp with several lines of text, mostly illegible due to fading. The text appears to include "QUALITY INSPECTED" and "1". There is a handwritten "A-1" in the bottom left corner of the stamp area.

Estimation for the Rasch Model when both Ability and Difficulty Parameters are Random

Abstract

Estimation of the parameters of the Rasch model, a one parameter item response model, is considered when both the item parameters and the ability parameters are considered random quantities. It is assumed that the item parameters are

drawn from a $N(\overset{\text{gamma, tau to the 2nd power}}{\mu}, \overset{\text{omeron, sigma to the 2nd power}}{\sigma^2})$ distribution, and the abilities are drawn from a $N(\overset{\text{gamma, tau to the 2nd power}}{\mu}, \overset{\text{omeron, sigma to the 2nd power}}{\sigma^2})$ distribution. A variation of the EM algorithm is

used to find approximate maximum likelihood estimates of μ , σ^2 and ϕ . A second approach assumes that the difficulty parameters are drawn from a uniform distribution over part of the real line. Real and simulated data sets are discussed for illustration. *Keywords: Statistical data; Tables (data)*

Key Words: EM algorithm, Item response curve, Rasch model.

Introduction

Suppose that the responses of n examinees to k test items are assembled in an $n \times k$ matrix \tilde{Y} of binary variables, with $Y_{ij} = 1$ if the i th examinee's answer to item j is correct, and $Y_{ij} = 0$ otherwise. It will be assumed that the model for the responses is the Rasch model, i.e.

$$\begin{aligned} p_{ij} &= P(Y_{ij} = 1 \mid \theta_i, \beta_j) \\ &= \exp(\theta_i - \beta_j) / [1 + \exp(\theta_i - \beta_j)] \end{aligned} \quad (1.1)$$

where θ_i is the ability of the i th examinee and β_j is the difficulty parameter for item j . Here θ and β may take on any values on the entire real line. Given $\tilde{\theta} = (\theta_1, \dots, \theta_n)$ and $\tilde{\beta} = (\beta_1, \dots, \beta_k)$, conditional independence among the responses will be assumed, i.e.

$$p(\tilde{y} \mid \tilde{\theta}, \tilde{\beta}) = \prod_{i=1}^n \prod_{j=1}^k p_{ij}^{y_{ij}} (1 - p_{ij})^{1 - y_{ij}}. \quad (1.2)$$

The Rasch model is the simplest and probably the most widely used model in item response theory. As Thissen (1982) points out, there are situations where the Rasch model does fit test data well. However, it is overly simplistic in some situations, and so two and three parameter models (2PL and 3PL) have been proposed and studied. Estimation schemes for the 2PL and 3PL are usually much more involved than for the Rasch model. In addition, the 2PL and 3PL models require a large n in order to accurately estimate the second and third parameters of some items (Lord, 1983a). Thus when n is small, under about 200, the 2PL and 3PL models are not practical, and the Rasch model should be used. The results of this paper should be useful in these situations where the Rasch model is appropriate.

When θ and β are both considered fixed but unknown quantities, the standard maximum likelihood (ML) procedure of Birnbaum (1968) is applicable and has been studied extensively. There have been several recent proposals related to the EM algorithm (Dempster, Laird, Rubin, 1977) for estimating β or θ when θ is treated as a random sample from a normal distribution. For example, Sanathanan and Blumenthal (1978) give ML solution for parameters of this normal distribution when β is given, Bock and Aitkin (1981) and Thissen (1982) discuss methods for obtaining marginal ML estimates of β , and Rigdon and Tsutakawa (1983) discuss ML estimation of both β and the parameter of the normal distribution. In each of these cases individual ability parameters can be subsequently estimated by computing the posterior mean of θ after replacing the unknown parameters (i.e., β or the parameter of the normal distribution) by their ML estimate.

It is well known that the maximum likelihood estimate of ability is not finite for examinees that have a response pattern of all correct or all incorrect answers. The procedures mentioned in the previous paragraph possess the advantage of yielding a finite estimate of ability even in such situations. If the number of examinees is relatively small, it is likely that the response patterns for some items will consist of all zeros or all ones. In such a situation the method of maximum likelihood and the methods mentioned in the previous paragraph do not yield a finite estimate of the difficulty parameter. One of the methods proposed in this article does have the advantage of yielding finite estimates of difficulty and ability in these situations.

Lord (1983b) showed that for 3PL the maximum likelihood estimate of ability is positively biased for examinees with high ability and negatively biased for

examinees with low ability. By placing a prior distribution on the ability parameters, as is done for the methods mentioned previously and for the methods proposed here, the ability estimates are "pulled" toward the origin. Lord (1986) also indicates, again for 3PL, that Bayesian modal estimates of ability may be biased inwards, but their mean square error is smaller than that for ML.

Since the Rasch model is symmetric, in the sense that the probability of correct response depends only on the difference between the ability and difficulty parameters, the same problem of bias exists for the difficulty parameters. In this paper we deal with the case where the difficulty parameters are also treated as a random sample from some prior distribution. The use of a prior distribution for the difficulty parameters again "pulls" the estimates toward the origin.

If the parameters of the prior distribution for (θ, β) are known, then inference on θ_i or β_j can be based on the posterior distribution, given the data matrix \underline{y} . In the absence of known prior parameters, we consider replacing them by estimates obtained from the data, and thus adopt a parametric empirical Bayes (PEB) approach (Morris, 1983). One general procedure for estimating such prior parameters is by maximum likelihood, using the marginal likelihood function of the parameters. Unfortunately this approach presents insurmountable numerical problems. We propose instead an approximation suggested by the CMLF procedure of Rigdon and Tsutakawa (1983). For situations in which prior knowledge of β is diffuse and β cannot be treated as a random sample, we propose a limit of the above method by taking the prior of β to be locally uniform. Comparisons of these procedures to each other and to the MLF estimator of Rigdon and Tsutakawa (1983) are made by using simulated data sets.

Methodology

Suppose now that $\theta_1, \theta_2, \dots, \theta_n$ are selected from a normal distribution with mean zero and variance σ^2 , and that $\beta_1, \beta_2, \dots, \beta_k$ are selected from a normal distribution with mean γ and variance τ^2 . The value of (σ, γ, τ) which maximizes the marginal likelihood of the observed data y , i.e.

$$p(\underline{y} | \sigma, \gamma, \tau) = \int \int p(\underline{y}, \underline{\theta}, \underline{\beta} | \sigma, \gamma, \tau) d\underline{\theta} d\underline{\beta} \quad (2.1)$$

is called the marginal maximum likelihood estimator (MMLE). However, under the above assumptions, maximization of this quantity presents insurmountable numerical problems since multidimensional integrals must be evaluated, even if the EM algorithm of Dempster, Laird and Rubin (1977) is applied. Instead we propose a variation of the EM algorithm, which is similar to the CMLF method of Rigdon and Tsutakawa (1983). Note that the posterior density of θ_i given $(\underline{\beta}, \sigma)$ can be written

$$\begin{aligned} p(\theta_i | \underline{y}, \underline{\beta}, \sigma) &\propto p(\theta_i | \sigma) \prod_{j=1}^k p(y_{ij} | \theta_i, \beta_j) \\ &\propto \exp\left[r_i \theta_i - \theta_i^2 / 2\sigma^2 \right] / \prod_{j=1}^k [1 + \exp(\theta_i - \beta_j)] \end{aligned} \quad (2.2)$$

where

$$r_i = \sum_{j=1}^k y_{ij}$$

is the raw score of examinee i . Similarly, the posterior density of β_j given $(\underline{\theta}, \gamma, \tau)$ can be written

$$\begin{aligned} p(\beta_j | \underline{y}, \underline{\theta}, \gamma, \tau) &\propto p(\beta_j | \gamma, \tau) \prod_{i=1}^n p(y_{ij} | \theta_i, \beta_j) \\ &\propto \exp\left[-q_j \beta_j - (\beta_j - \gamma)^2 / 2\tau^2 \right] / \prod_{i=1}^n [1 + \exp(\theta_i - \beta_j)] \end{aligned} \quad (2.3)$$

where

$$q_j = \sum_{i=1}^n y_{ij}$$

Now these densities are numerically tractable, since, except for normalizing constants, they are just products of other densities which are easy to evaluate. Since neither $\underline{\theta}$ nor $\underline{\beta}$ are available, we exploit this tractability by applying the following algorithm. Start with some initial value $(\underline{\beta}_{(0)}, \sigma_{(0)}, \gamma_{(0)}, \tau_{(0)})$, set m equal to zero and repeat the following steps:

E1 Step: Compute the posterior expectations

$$\underline{\theta}^{(1)} = E(\underline{\theta} \mid \underline{y}, \underline{\beta}_{(m)}, \sigma_{(m)}) \quad (2.4)$$

and

$$\underline{\theta}^{(2)} = E(\underline{\theta}^2 \mid \underline{y}, \underline{\beta}_{(m)}, \sigma_{(m)}). \quad (2.5)$$

where $\underline{\theta}^2 = (\theta_1^2, \dots, \theta_n^2)$. These expectations are evaluated by normalizing and integrating (2.2) times θ_i and θ_i^2 for $i=1$ to n .

E2 Step: Compute the posterior expectations

$$\underline{\beta}^{(1)} = E(\underline{\beta} \mid \underline{y}, \underline{\theta}^{(1)}, \gamma_{(m)}, \tau_{(m)}) \quad (2.6)$$

and

$$\underline{\beta}^{(2)} = E(\underline{\beta}^2 \mid \underline{y}, \underline{\theta}^{(1)}, \gamma_{(m)}, \tau_{(m)}), \quad (2.7)$$

where $\underline{\beta}^2 = (\beta_1^2, \dots, \beta_k^2)$, and set $\underline{\beta}_{(m+1)} = \underline{\beta}^{(1)}$. These expectations are evaluated by normalizing and integrating (2.3) times β_j and β_j^2 for $j=1$ to k .

M Step: Set

$$\sigma_{(m+1)} = \left(\sum_{i=1}^n \theta_i^{(2)} / n \right)^{1/2} \quad (2.8)$$

$$\gamma_{(m+1)} = \sum_{j=1}^k \beta_j^{(1)} / k \quad (2.9)$$

$$\tau_{(m+1)} = \left(\sum_{j=1}^k \beta_j^{(2)} / k - [\gamma_{(m+1)}]^2 \right)^{1/2}, \quad (2.10)$$

where $\theta_i^{(\nu)}$ ($\beta_j^{(\nu)}$) is the i^{th} (j^{th}) element of the vector $\underline{\theta}^{(\nu)}$ ($\underline{\beta}^{(\nu)}$), $\nu = 1, 2$. Increment m and test for convergence. If convergence is attained to a prescribed level then stop, otherwise go to the E1 Step.

Upon Convergence, the final value $(\hat{\sigma}, \hat{\gamma}, \hat{\theta})$ of $(\sigma_{(m)}, \gamma_{(m)}, \theta_{(m)})$ maximizes the two conditional likelihood functions given by

$$U(\sigma | \underline{\beta}) = \int p(\underline{y} | \underline{\theta}, \underline{\beta}) p(\underline{\theta} | \sigma) d\underline{\theta}$$

and

$$U(\gamma, \tau | \underline{\theta}) = \int p(\underline{y} | \underline{\theta}, \underline{\beta}) p(\underline{\beta} | \gamma, \tau) d\underline{\beta}$$

where $(\underline{\bar{\theta}}, \underline{\bar{\beta}})$ satisfies the equations

$$\underline{\bar{\theta}} = E(\underline{\theta} | \underline{y}, \underline{\bar{\beta}}, \hat{\sigma})$$

and

$$\underline{\bar{\beta}} = E(\underline{\beta} | \underline{y}, \underline{\bar{\theta}}, \hat{\gamma}, \hat{\tau}).$$

In Rigdon and Tsutakawa (1983), CMLF stood for Conditional Maximum Likelihood Fixed (the difficulty parameters were fixed, i.e. not random). Keeping the same naming strategy, we call the method described here CMLR, for Conditional Maximum Likelihood Random (the difficulty parameters are considered random). It should be noted that all expectations required for this method are single integrals and must be evaluated by using numerical techniques. Gauss-Hermite quadrature formulas are appropriate (see Stroud and Secrest, 1966).

Once estimates for σ , γ , and τ are obtained, we estimate the θ 's and β 's by evaluating the means of the posterior distributions as in equations (2.4) and (2.6). Approximate interval estimates for the θ 's and β 's can be obtained by approximating the posterior distribution by a normal distribution using the posterior mean and standard deviation. That is, the interval estimate for θ_i is

$$\theta_i^{(1)} \pm z_{1-\alpha/2} \left(\theta_i^{(2)} - [\theta_i^{(1)}]^2 \right)^{1/2}$$

and for β_j the interval estimate is

$$\beta_j^{(1)} \pm z_{1-\alpha/2} \left(\beta_j^{(2)} - [\beta_j^{(1)}]^2 \right)^{1/2}$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)$ point of the standard normal distribution function.

In some cases, there is vague prior information regarding β and the assumption that β is a random sample from a common distribution may not be reasonable. One Bayesian solution to this problem is to adopt an independent uniform prior distribution on each β_j . In this case (γ, τ) does not exist, equation (2.9) is not necessary and the M Step reduces to computing $\sigma_{(m+1)}$ only. The posterior density of β_j is now replaced by

$$p(\beta_j | \underline{y}, \underline{\theta}) \propto \exp(-\beta_j q_j) / \prod_{i=1}^n [1 + \exp(\theta_i - \beta_j)]. \quad (2.11)$$

This method will be called CMLU, for Conditional Maximum Likelihood Uniform, since the prior is uniform. This method does not have the advantage of yielding finite estimates of the difficulty parameter when the response pattern for that item consists of all zeros or all ones.

An Example

We will illustrate our methods using results from a test of general knowledge regarding arthritis, which was administered to hospital patients. This data set was previously used in Tsutakawa (1984) and consists of responses to $k=47$ items by $n=162$ patients. We will compare the methods proposed here, i.e. CMLR and CMLU with the MLF method of Rigdon and Tsutakawa (1983), since these methods are similar in the way that they apply the EM algorithm. The CMLF estimates are nearly identical to the MLF estimates. The estimates of the prior parameters, or the appropriate sample statistics, are shown in Table I for the MLF method and the CMLR and CMLU methods of this article; the average of the estimated abilities are also shown. Tables II and III display the estimates of the ability and difficulty parameters, respectively. Both the ability and difficulty estimates obtained by MLF and CMLU are quite close. The estimates obtained by CMLR are somewhat less disperse than the estimates obtained by other methods.

Insert Tables I, II and III about here.

Simulations

The computer-generated data sets of Rigdon and Tsutakawa (1983) are used here to compare the performance of the various estimation procedures. The ability parameters were randomly generated from the standard normal distribution. The difficulty parameters were chosen deterministically as the 1, 3, . . . , 99 percent points of the following distributions:

- i) the standard normal,
- ii) the uniform over the interval $(-3^{1/2}, 3^{1/2})$, and
- iii) the parabolic U-shaped with density $h(x) = (5/27)x^2 + (7/36)$ for $-1.5 < x < 1.5$.

These represent sets of item parameters with difficulties (i) concentrated near the average ability, (ii) spread out uniformly, and (iii) sparse near the average. The six response matrices \underline{Y} was then randomly generated using the probabilities of correct response which depend on $\underline{\theta}$ and $\underline{\beta}$ through the relation in (1.1).

The estimates of the parameters of the prior distribution are shown in Table IV for CMLR and CMLU and for the MLF method of Rigdon and Tsutakawa (1983). The averages of the estimated abilities are also shown in this table. For MLF and CMLU (where γ and τ are not part of the model) the sample means and standard deviations are shown for comparison. The averages of the sets of estimates tend to be quite close. The major difference between the sets of estimates seems to be in the dispersion. The difficulty estimates obtained by CMLR tend to be less disperse than those for the other methods. Ability estimates from CMLR are also less disperse, but this is not as pronounced.

Insert Table IV about here

Comparisons can be made between the actual values and the estimated values since the data were simulated. A measure of the accuracy of these procedures is the root mean squared deviations (RMSD's),

$$\left\{ \sum_i (\theta_i - \hat{\theta})^2 / n \right\}^{1/2}$$

and

$$\left\{ \sum_j (\beta_j - \hat{\beta})^2 / k \right\}^{1/2}$$

where $\hat{\theta}_i$ and $\hat{\beta}_j$ are estimates of θ_i and β_j . The RMSD's for the MLF, CMLR and CMLU methods are given in Table V. In most cases the performances of the procedures are quite close. In some cases the RMSD of the CMLR estimates of difficulty are considerably less than the RMSD's for the other methods. Two of these cases occur when the distribution of the β 's was chosen to be 'U'-shaped, indicating that the CMLR method is robust with respect to the assumption that the β 's come from a normal distribution.

Insert Table V about here

The frequencies of actual values within two posterior standard deviations of the estimates are also shown in Table V. As can be seen from this table, close to 95 per cent of the estimates are within these limits, a result we would expect if the posteriors were normally distributed. This indicates that the posterior distribution may be useful in assessing the uncertainty in an estimate of $\underline{\theta}$ or $\underline{\beta}$.

Discussion

A Bayesian who has not seen the items may be inclined to assume that the prior distribution of $\underline{\beta}$ is exchangeable. This person may then find it convenient to represent the exchangeable prior through a normal distribution having a hyperparameter with a subjective prior distribution. A frequentist, on the other hand, may view $\underline{\beta}$ as a random sample from a larger population associated with a large or hypothetically large item pool. This person might then find it convenient to view this population as one having a normal distribution which can be estimated. Our first estimate, CMLR, conforms more to the latter point of view, whereas our second estimate, CMLU, is more compatible with the former when the prior for $\underline{\beta}$ is diffuse.

We feel that our method can be recommended in situations where there are relatively few examinees and there is limited information about the item response curves. When the data satisfies the assumptions for the Rasch model and n is larger, our estimates should be in close agreement with the conventional ML estimates. For small n , not only does one have problems with the nonexistence of ML estimates, but the asymptotic properties for measuring the precision of these estimates will be of limited value. Our method seems particularly suitable for handling such cases.

The extension of our approach to 2PL and 3PL is clearly possible. Such extensions would require introducing additional distributions for the additional item parameters and developing efficient techniques for numerically evaluating two and three dimensional integrals, corresponding to (2.6) and (2.7).

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Table I

Estimates of Parameters of Prior
Distribution for Arthritis Test

Method	$\Sigma \hat{\theta}_i / 162$	σ	γ	τ
MLF	0.00	0.76	-0.80*	1.17*
CMLR	0.02	0.76	-0.85	1.12
CMLU	0.01	0.78	-0.89*	1.16*

* Sample statistics used in these entries.

Table II

Estimates of Abilities for Arthritis Test

Raw Score	MLF	CMR	CMU
47	2.20	2.16	2.20
44	1.61	1.60	1.61
42	1.29	1.28	1.29
41	1.14	1.13	1.14
40	1.00	1.00	1.00
39	0.87	0.87	0.87
38	0.74	0.75	0.74
37	0.62	0.63	0.62
36	0.50	0.51	0.50
35	0.38	0.40	0.38
34	0.27	0.29	0.27
33	0.16	0.19	0.16
32	0.06	0.08	0.06
31	-0.05	-0.02	-0.05
30	-0.15	-0.12	-0.15
29	-0.25	-0.22	-0.25
28	-0.35	-0.31	-0.35
27	-0.44	-0.41	-0.44
26	-0.54	-0.50	-0.54
25	-0.64	-0.60	-0.64
24	-0.73	-0.69	-0.73
23	-0.83	-0.78	-0.83
22	-0.92	-0.87	-0.92
20	-1.11	-1.06	-1.11
19	-1.20	-1.15	-1.20
18	-1.30	-1.24	-1.30
17	-1.39	-1.34	-1.39
16	-1.49	-1.43	-1.49
13	-1.78	-1.71	-1.78
8	-2.31	-2.23	-2.31

Table III

Estimates of Difficulty Parameters for Arthritis Test

Item Score	MLF	CMLR	CMLU
149	-2.86	-2.53	-2.68
148	-2.60	-2.45	-2.59
146	-2.44	-2.30	-2.43
145	-2.37	-2.23	-2.36
143	-2.24	-2.13	-2.22
141	-2.11	-2.03	-2.10
139	-2.00	-1.91	-1.98
136	-1.84	-1.73	-1.83
135	-1.79	-1.68	-1.78
133	-1.70	-1.62	-1.68
132	-1.65	-1.60	-1.64
130	-1.57	-1.54	-1.55
123	-1.29	-1.17	-1.27
122	-1.25	-1.14	-1.23
121	-1.21	-1.13	-1.20
118	-1.11	-1.09	-1.09
117	-1.07	-1.08	-1.06
115	-1.00	-1.03	-0.99
110	-0.84	-0.75	-0.83
107	-0.75	-0.64	-0.74
103	-0.62	-0.60	-0.61
98	-0.48	-0.53	-0.47
94	-0.36	-0.33	-0.35
93	-0.33	-0.27	-0.33
88	-0.19	-0.11	-0.19
87	-0.16	-0.11	-0.16
82	-0.02	-0.08	-0.02
81	0.00	-0.07	0.01
80	0.03	-0.05	0.03
72	0.26	0.34	0.25
71	0.28	0.37	0.28
62	0.54	0.45	0.54
61	0.57	0.47	0.56
57	0.69	0.67	0.68
50	0.91	0.91	0.90
43	1.14	1.04	1.13
37	1.36	1.36	1.35
27	1.79	1.70	1.77

Table IV
Estimates of Parameters of Prior
Distribution for Simulated Data Sets

β 's	n	Method	$\Sigma \hat{\theta}_i/n$	σ	γ	τ
N(0,1)	50	MLF	-0.01	0.97	0.11*	1.09*
		CMLR	-0.00	0.93	0.11	1.00
		CMLU	-0.00	0.98	0.11*	1.10*
N(0,1)	200	MLF	-0.01	1.03	0.01*	1.03*
		CMLR	-0.00	1.02	0.01	0.99
		CMLU	-0.00	1.03	0.01*	1.02*
Uniform	50	MLF	-0.00	1.14	-0.26*	1.03*
		CMLR	-0.00	1.09	-0.25	0.94
		CMLU	0.00	1.14	-0.26	1.03*
Uniform	200	MLF	-0.00	0.97	0.09*	0.99*
		CMLR	0.01	0.95	0.11	0.96
		CMLU	-0.01	0.96	0.09*	0.98*
"U"-shaped	50	MLF	0.00	0.99	-0.04*	1.04*
		CMLR	-0.00	0.95	-0.04	0.96
		CMLU	0.00	1.00	-0.04*	1.05*
"U"-shaped	200	MLF	0.00	1.06	-0.01*	1.06*
		CMLR	0.01	1.04	0.01	1.03
		CMLU	0.00	1.05	-0.01*	1.05*

* Sample statistics used in these entries.

Table V
Comparison of Actual and Estimated Values:
Simulated Data Sets

β 's	n	Method	RMSD θ	RMSD β	Freq. of θ within 2 s.d.	Freq. of β within 2 s.d.
N(0,1)	50	MLF	0.394	0.311	48	-
		CMLR	0.355	0.319	48	48
		CMLU	0.425	0.311	48	47
N(0,1)	200	MLF	0.173	0.322	191	-
		CMLR	0.173	0.324	189	47
		CMLU	0.167	0.324	190	48
Uniform	50	MLF	0.263	0.326	48	-
		CMLR	0.266	0.324	48	46
		CMLU	0.265	0.326	48	49
Uniform	200	MLF	0.155	0.322	193	-
		CMLR	0.158	0.324	192	48
		CMLU	0.155	0.319	193	49
"U"-shaped	50	MLF	0.336	0.298	48	-
		CMLR	0.318	0.300	49	50
		CMLU	0.341	0.298	48	50
"U"-shaped	200	MLF	0.161	0.322	187	-
		CMLR	0.145	0.322	188	47
		CMLU	0.158	0.322	188	48

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