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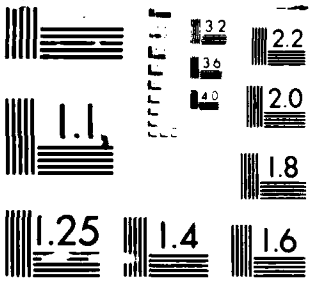

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Absorption at Optical Wavelengths in a Plasma Aerosol

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The absorption of radiation at optical wavelengths (3 cm to 0.3μ) in a plasma aerosol is examined. In particular, the mean free path is plotted as a function of wavelength for temparatures appropriate to laser-induced gas breakdown. Various formulae, one assumption used in their derivations, and their regions of validity are discussed.

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CONTENTS

INTRODUCTION

This report is concerned with the absorption of radiation at optical wavelengths in a plasma aerosol ([1]-[3]). We shall take this to include wavelengths from 3 cm to 0.3 μ.[†] The term aerosol refers to a gas-particle mixture such as that found in the atmosphere. Furthermore, the plasma is assumed to be of the type achieved by laser-induced air breakdown described in [1] and [3]. A number of conclusions may be reached just by an examination of the absorption mean free path l_ω (the path length necessary for the intensity of the impinging radiation, e.g., a laser beam, to fall to 1/e of its value).[††] More detailed information, such as the potential consequences of the dissipation of absorbed energy, depends on specifying explicitly the geometry (for both the beam and the plasma), characteristic times, and the mechanisms of plasma containment/maintenance.

ABSORPTION BEHAVIOR

For both historical and practical reasons there exists a plethora of formulae for the absorption coefficient μ_ω. The various derivations may be roughly classified into three categories: those using the Vlasov transport equation (classical), those using classical bremsstrahlung coupled with the principle of detailed balancing (semiclassical), and quantum mechanical. In all except the least rigorous derivations, these techniques result in expressions involving unpleasant integrals. Consequently, approximations are made which result in a large set of formulae, each valid under particular conditions appropriate to its derivation. The appendix of this report contains an *extensive summary* of those which are pertinent to the present study.

It can be seen from those equations that the temperature dependence of μ_ω is quite complex. Nevertheless, in considering an aerosol plasma, we shall find ourselves able to pick a nominal temperature on which to base our conclusions. We first note that because of the exponential dependence of the electron partial pressure on temperature in equations (A-22) and (A-30), there is a steep rise in absorption just prior to complete single ionization.[†††] For atmospheric pressure this occurs at about T 1.5 eV or 17,500 °K. At that point the electron pressure is about one-half the total pressure, and μ_ω is given by (A-19) with Z 1. It can be seen from that equation that μ_ω decreases with a further increase of temperature. This holds true until about 25,000 °K, when double ionization begins to dominate, and μ_ω begins to rise again ([1], p 243). Further computations require the entire set of equations (A-21), (A-22), (A-27), and (A-28) until the double ionization is close to complete, at which point (A-19) is again a good approximation but with Z 2. As the temperature rises, this procedure repeats itself, and μ_ω passes through a series of maxima and minima. The ratios of adjacent maxima to minima are only on the order of two (cf., [1], p 243) so that it is not unreasonable to examine only the maximum values. Also, as noted above, temperatures below ionization may be dismissed as having insignificant absorption. Finally, at or near breakdown for aerosol plasmas the temperature is in the range of 1 to 2 eV ([3], p 966), so that as a

---

[†] That is 10 GHz to 10^6 GHz or 4.0×10^{-5} eV to 4.0 eV.

[††] l_ω is $1/\mu_\omega$ where μ_ω is the absorption coefficient.

[†††] See equation (A-26). For larger pressures it increases, for example, at $p_{atm} = 100$, total ionization occurs at about T 28,000 °K.

general guide to absorption we may assume that there is complete single ionization; i.e., we take $T = 17{,}500$ °K and $Z = 1$.

Let us now compute ω_p and $\hat{\nu}_m$ in order to determine which approximations are valid. The ion density is given by

$$n_- = \frac{p}{2kT} = \frac{1.013 \times 10^6 \, p_{atm}}{2\,(1.38 \times 10^{-16})\,(1.75 \times 10^4)} = 2.10 \times 10^{17} \, p_{atm}. \tag{1}$$

Then, at 1 atm

$$\omega_p = (4\pi e^2 \, 2.10 \times 10^{17}/m)^{1/2} = 2.58 \times 10^{13} \text{ sec}^{-1}. \tag{2}$$

This is equivalent to a wavelength of

$$\lambda_p = \frac{6.28\,(3.0 \times 10^{10})}{2.58 \times 10^{13}} = 7.3 \times 10^{-3} \text{ cm}. \tag{3}$$

Also, from (A-9) we have

$$\hat{\nu}_m = \frac{4\pi}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{e^4\,(2.10 \times 10^{17})}{(k\;1.75 \times 10^4)^{3/2}\sqrt{m}} \; g \;\; \text{sec}^{-1}$$

$$= (6.0 \times 10^{11})\, g \;\; \text{sec}^{-1}. \tag{4}$$

Thus, $\omega >> \hat{\nu}_m$ corresponds to $\lambda << 2\pi c/\hat{\nu}_m = 0.3$ cm or freq $>> 100$ GHz.

Note that for wavelengths greater than 0.01 cm (i.e., frequencies below 3000 GHz) there is total reflection of impinging radiation. *Furthermore, since* $\hat{\nu}_m << \omega_p$, we see that for frequencies much greater than 3000 GHz conditions (ii) and (iii) of the Appendix are both satisfied. Finally, down to wavelengths of about 3.0×10^{-4} cm, condition (i′) is also valid. (More precisely, $\hbar\omega << kT$ for $\lambda >> 8.2 \times 10^{-5}$ cm $= 0.82\,\mu$, and $\hbar\omega << I_1$ for $\lambda >> 8.5 \times 10^{-6}$ cm $= 0.085\,\mu$.) In the region from 3×10^{-5} cm to 3×10^{-4} cm we use the Kramers-Unsold formula (A-30b), which is justified since $\hbar\omega << I_1$. At longer wavelengths (i) is valid, and we can use the simpler equation (A-19e), which becomes

$$\mu_\omega = 3.8 \times 10^5 \, g \, p_{atm}^2 \, \lambda^2 \tag{5}$$

To evaluate the Gaunt factor g, we first determine Λ_c. From (A-3)

$$\Lambda_c = 3.8 \times 10^3 \, \lambda \,. \tag{6}$$

The low-frequency region occurs for $\lambda >> 2.6 \times 10^{-4}$cm, which we arbitrarily set at $\lambda \geq 3 \times 10^{-3}$ cm; the remainder of our spectrum, 3×10^{-3}cm to 3×10^{-7} cm, falls in a region for which we have no simple formula for g (or $\xi_0(\omega)$).

Figure 1 contains a plot of l_ω versus wavelength. In the figure, we have marked the region where the Kramers-Unsold formula was used, and the factor $\xi_0(\omega)$ was simply set equal to two. This matches the value obtained by (5) at $\lambda = 10^{-3}$ cm, a choice of ξ_0 that differs slightly from [1], page 242.† Observe the leveling of the curve at λ equal to 2×10^{-5} cm. At even smaller wavelengths (which happen to be outside our region of concern) it has a sawtooth shape, alternately increasing and decreasing ([4], p 274). The high- and low-frequency regions used to compute g in equation (5) are also noted in the figure. We have used the table in [4], page 255, to compute g in the high-frequency region. It is only tabulated down to $\hbar\omega/kT = 0.1$; i.e., up to $\lambda = 8 \times 10^{-4}$ cm.†† From this wavelength up to the low-frequency region, g was set equal to 2.0, the bottom entry in the table. This procedure seems to have produced a good interpolation between the two regions, at least on the log-log plot. Even though the table and (A-3) include the frequency dependence of g, the slope of the curve in the figure is essentially a constant within the range of equation (5). The accuracy is, of course, not very good close to the plasma wavelength $\lambda_p = 7 \times 10^{-3}$ cm.

Finally, if we approximate (5) by setting g = 2.0 (a reasonable value for the optical region), we obtain the particularly simple formula

$$l_\omega = \frac{8 \times 10^{-5}}{p_{atm}^2 \lambda^2} \text{ [cm]}, \tag{7}$$

which is essentially the same as that in [3]. Note that if the impinging beam is not parallel (e.g., a focused laser), the beam divergence should be taken into account in computing absorption distance.

CONCLUSION

We have seen that at atmospheric pressure, there is significant absorption only at wavelengths above 3×10^{-4} cm = 3 μ. (Also, the medium is essentially a total reflector for $\lambda > 10^{-2}$ cm = 100 μ.) One obvious way to extend this range is to increase the plasma pressure. According to [3], for short periods of time during breakdown the pressure at a breakdown site may be as high as 100 atm. Whether it is feasible to maintain a pressure of this magnitude over longer periods and throughout the plasma depends on the details of the plasma maintenance and containment. (Most treatments of the maintenance of a laser-induced air plasma assume close to atmospheric pressure; c.f., [2], [5].)

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† Raizer sets g = 2.5 for a CO_2 laser ($\lambda \approx 10^{-3}$ cm), and he sets $\xi = 0.7$ for a neodymium laser ($\lambda \approx 10^{-4}$ cm). He also includes an "edge" factor of 1.2 to 1.5.

†† For λ smaller than the low-frequency region, that is, for λ from 3×10^{-5} cm to 3×10^{-3} cm, $\hbar\omega/kT$ ranges from 2.7 to 0.027.

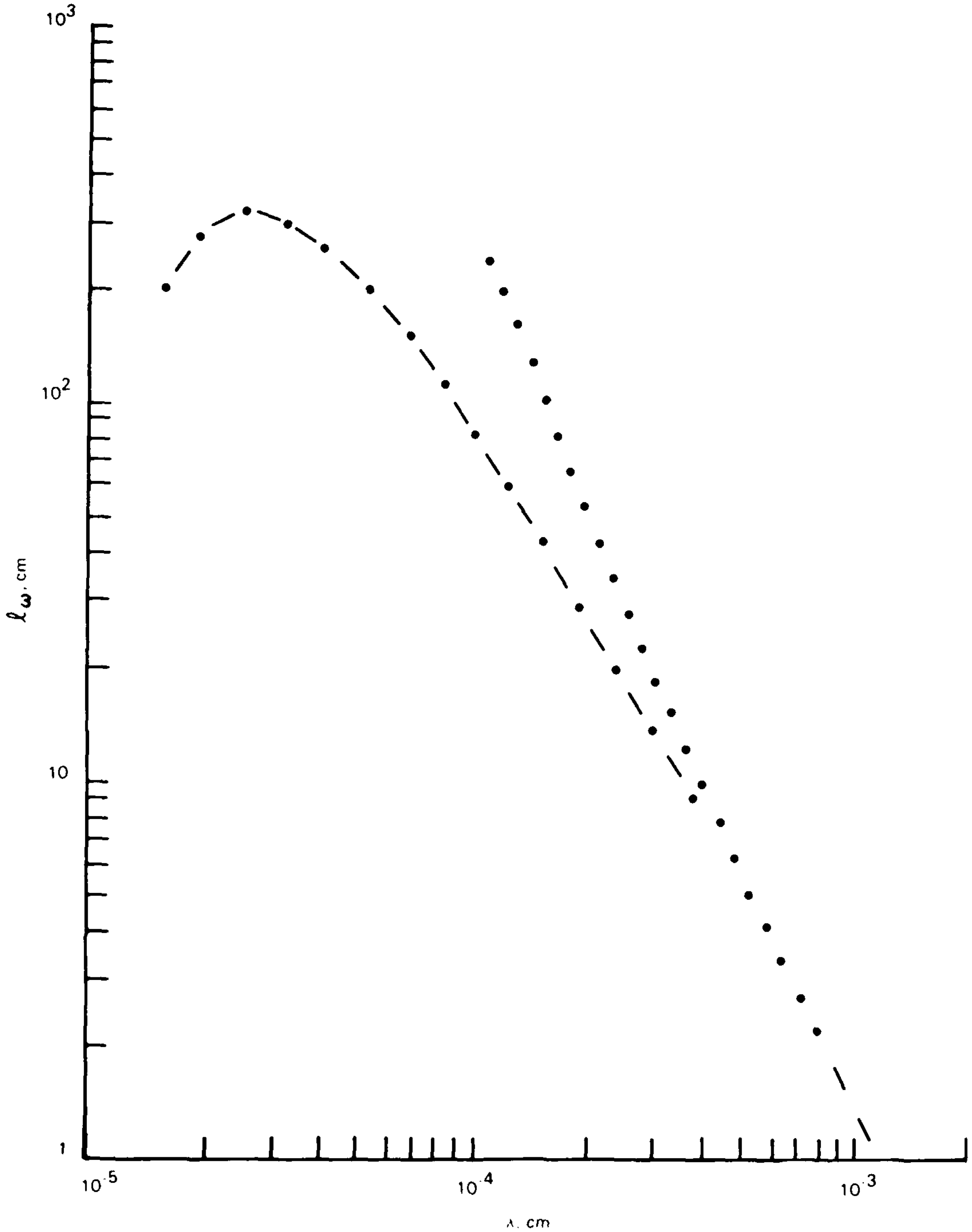

Figure 1a. Plot of absorption mean free path ℓ_ω versus the wavelength λ. The Kramers-Unsold approximation is given by • — • , equation (5), with the high-frequency approximation to g is indicated by • • • , and the mid-frequency range with $g = 2.0$ by — — .

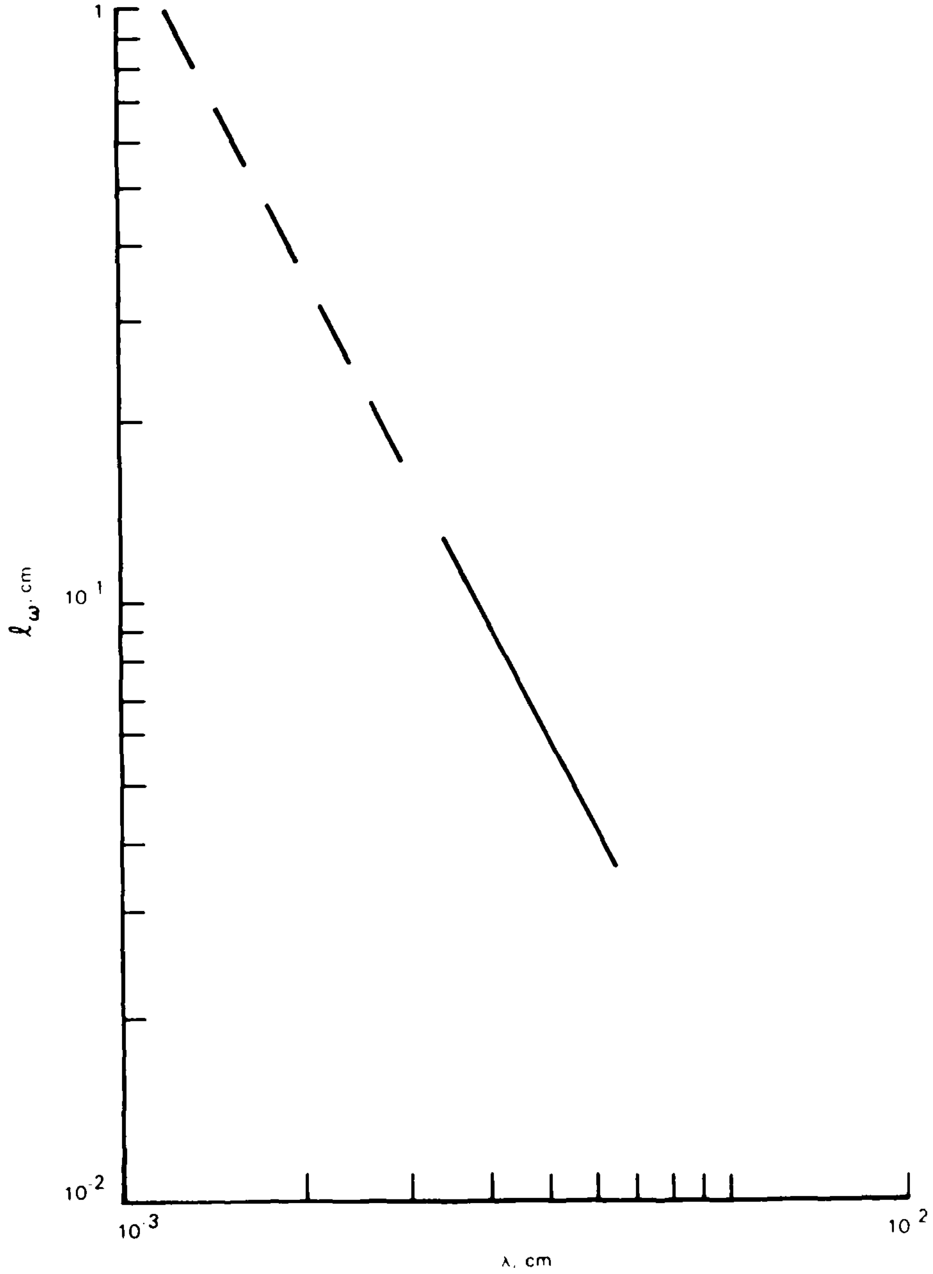

Figure 1b. Plot of absorption mean free path ℓ_ω versus the wavelength λ. The curve was computed from equation (5) with the mid-frequency g (set to 2.0) indicated by the dashes, and the low-frequency approximation (A.3a) for g shown by the solid line. Total reflection occurs for λ below 7×10^{-3} cm.

NOTATION

$c = 2.998\ 10^{10}$ cm/sec (speed of light)
$e = 4.803\ 10^{-10}$ esu (charge of electron)
g, g_A, g_B, etc. = Gaunt factor (cf. equation (A-3))
$g^{(m)}$ = statistical weight of ground state of ion of charge m
$\hbar = 6.626\ 10^{-27}/2\pi$ erg-sec (Planck's constant/2π)
$= 1.055\ 10^{-27}$ erg-sec
I = ionization potential of plasma atoms
$= Z^2 I_H$, where I_H is that of a hydrogen atom
$k = 1.380\ 10^{-16}$ joule/ °K (Boltzmann's constant)
l_ω = absorption mean free path ($= 1/\mu_\omega$)
$m = 9.109\ 10^{-28}$ grams (electron rest mass)
n_- = number density of electrons
n_m = number density of ions of charge m
Note that $Z = m + 1$
n = total number density of ions
n_+ = number density of ions when ionization is total
(i.e., assuming all ions have the same charge)
p_{atm} = pressure in atmospheres
p = pressure [dynes/cm^2]
$= 1.013\ 10^6 \times p_{atm}$
T = temperature, °K
$u^{(m)}$ = partition function for mth ion, normalized by the ground state energy
$x_- = p_-/p$, where p_- is electron partial pressure
Z = charge of nucleus for hydrogen-like atom
$\gamma = 1.781$ ($\ln(\gamma)$ = Euler's constant)
λ = wavelength in cm
μ_ω = total absorption coefficient for monochromatic radiation
of frequency ω
ν_m = effective collision frequency (for momentum change)
$\bar{\nu}_m$ = velocity-independent approximation of ν_m
ω = frequency, radians/sec = 2π frequency
Note that $\omega\ [\mathrm{sec}^{-1}] = 1.518\ 10^{15} \times (\hbar\omega)_{ev}$
ω_p = plasma frequency in radians/sec
$= (4\pi e^2 n_-/m)^{1/2}$

APPENDIX: ABSORPTION COEFFICIENT FORMULAE

FREE-FREE TRANSITIONS

One of the more useful expressions, from a viewpoint of generality and computability, for the absorption coefficient of a plasma is ([4], p 259-260)

$$\mu_\omega = \kappa_\omega \left(1 - e^{-\hbar\omega/kT}\right) \ [\mathrm{cm}^{-1}] , \tag{A-1}$$

where

$$\kappa_\omega = \frac{16\pi^2}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{1}{(mkT)^{1/2}} \, \frac{Z^2 e^6 n_+ n}{mc\,\hbar\,\omega\,\omega^2}\, g \tag{A-2}$$

and g is a so-called Gaunt factor given by ([4] p 255, [6] p 488-491, [7] equation (5-59))

$$g = \begin{cases} \dfrac{\sqrt{3}}{\pi} \ln \Lambda_C & \text{for } T < (8.9 \times 10^5)\, Z^2 \text{ and } \Lambda_C \gg 1 \quad \text{(A-3a)} \\ O(1) & \text{for } T < (8.9 \times 10^5)\, Z^2 \text{ and } \Lambda_C \approx 1 \quad \text{(A-3b)} \\ \dfrac{\sqrt{3}}{\pi} \ln \Lambda_Q & \text{for } T > (8.9 \times 10^5)\, Z^2 \quad \text{(A-3c)} \end{cases}$$

where (cf. equations (A-11) and (A-12))

$$\Lambda_C = \frac{2\,(2kT)^{3/2}}{\gamma^{5/2} Z e^2 \sqrt{m}\,\omega} = 3.11 \times 10^8 \,\frac{T^{3/2}}{Z\,\omega}$$

$$\Lambda_Q = \frac{4kT}{\gamma \hbar \omega} = 2.94 \times 10^{11} \,\frac{T}{\omega}$$

The notation O(1) is to be interpreted as "of the order 1."

Equations (A-1) and (A-2) are derived in [4] semiclassically; i.e., bremsstrahlung radiation for an electron in a coulomb potential is computed classically,† the coefficient κ_ω deduced from an application of the principle of detailed balancing plus Planck's formula for radiative equilibrium, and the factor in (A-1) added to correct for stimulated emission. The assumptions used in the derivation are

(i) Free-free transitions are the dominant absorption mechanism
(ii) $\omega \gg \omega_p$ where ω_p is the plasma frequency.
(iii) $\omega \gg \nu_m$ where ν_m is the transport collision frequency.

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†The Gaunt factor, which is not derived in [4] or [1], is discussed below.

(iv) $\hbar\omega << kT$.

Note, there is not a simple expression for g which is accurate in the crossover region $\Lambda_C \approx 1$.

Condition (i) is tantamount to ignoring energy absorbed in photoionization of the atoms or ions (bound-free transitions). It is intuitively clear that a necessary condition for this approximation is that the photon energy $\hbar\omega$ be small in comparison with I, the ionization potential of the atoms. Furthermore, it can be shown ([4] p 271-272) that, since under these conditions photons are only absorbed by highly excited atoms, the ratio of the contribution from bound-free transitions to free-free is $e^{\hbar\omega/kT}-1$ ([4] p 271-272). Thus, a *sufficient* condition for (i) is

$$\text{(i}'\text{a)} \qquad \hbar\omega << I \tag{A-4}$$

$$\text{(i}'\text{b)} \qquad \hbar\omega << kT . \tag{A-5}$$

Under these circumstances (A-1) reduces to

$$\mu_\omega = \frac{16\pi^2}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{e^6 Z^2 n_+ n}{c(mkT)^{3/2}\omega^2}\, g, \tag{A-6}$$

which is equation (6.1) of [1] (often referred to as Kramers formula). On the other hand, if $\hbar\omega \geq kT$, the semiclassical treatment of free-free transitions and even the (quantum) approximation (A-3c) lose their validity; moreover, in that case bound-free absorption dominates ([4] p 272).

Condition (ii) plays two roles in the derivation of (A-2). The first, which is relatively transparent, is that it insures that the index of refraction is close to one (cf. [1] p. 28, [6]). The application of this assumption in the derivation is fairly clear. It enters into the energy balancing computation via the relationship between the energy flux and energy density ([1] p 28), which assumes that the speed of energy propagation is equal to the speed of light in a vacuum. (Compare [6], equations (12.17) - (12.18), where c is replaced by the group velocity and n is not one.) Classically, for $\omega < \omega_p$ there is total reflection. Formulae for the absorption coefficient when ω is in the neighborhood of ω_p may be found in [8], [9], [1] p 308, and [6] p 66-68. The less obvious role for condition (ii) relates to Debye shielding. Formulae (5.8) and (5.9) of [4] are obtained by integrating over the impact parameter b for electron scattering with an upper limit of infinity. This is equivalent to ignoring the shielding, and it is valid provided $\omega >> \omega_p$ (see [10]).[†] We note that [8] includes shielding effects in the transport equation, but only gets simple expressions for the cases $\omega >> \omega_p$ and $\omega << \omega_p$.

[†] Heuristically, we note that contributions at large b only affect frequencies $\omega < v_0/b$ (where v_0 = initial electron velocity). If we take the cutoff to be the Debye length λ_D, then the error produced by extending our integration to values of b larger than λ_D is negligible at those frequencies satisfying $\omega >> v_0/\lambda_D \approx (kT/m)^{1/2}/\lambda_D = \omega_p$

Inequality (iii) expresses conditions under which the correlation of successive collisions of an electron with ions and its attendant interference of bremsstrahlung radiation may be ignored ([4] p 258). The quantity ν_m is the collision frequency for the momentum change; i.e., the mean[†] change in momentum per unit time due to collisions is $m\nu_m v$ ([1],[6]). In general ν_m is a function of speed v (cf., [6] p 51-52); however, for the purposes of (iii) we may use the expression (A-9). When ω is of the order of ν_m, an appropriate formula is (A-7), which includes the effects of correlated collisions; this reduces to the simpler expression (A-8) provided ν_m is essentially a constant over the range of velocities which are significant in the integral.

(iv) The derivation of (A-3a) and (A-3b) involves two integrations. The expression for bremsstrahlung is obtained by an integration over the electron's impact parameter [11], balancing leads to an absorption coefficient which is a function of electron initial velocity v_0, and this expression is integrated over the Maxwell distribution to obtain κ_ω. The bremsstrahlung formula is simplified via two approximations, one at low frequencies ([4] equation (5.9)) and one at high frequencies ([4] equation (5.8)). Upon balancing and integration over v_0 (cf. [6]), these two approximations lead to (A-3a) and to $g = 1$, respectively. However, the conditions of approximation are velocity dependent, so that one must show that the contribution to the integration over v_0 outside the region of approximation is insignificant. Oster [10] does this for low frequencies and shows that the condition for validity is $\Lambda_C >> 1$. Ginsberg ([6] p 489) seems to feel that the procedure is justified in both cases. (Actually, [4] only treats the high-frequency case, which leads to the Kramers formula (A-2) with $g = 1$.) We note that even if we accept the value $g \approx 1$ in the high-frequency region where $\Lambda_C << 1$ (but still requiring $\hbar\omega << 1$, of course), there is still a gap for frequencies where $\Lambda_C \approx 1$. A rough table of Gaunt factors appears in [4], p 255. The two other approximations, involving T, separate the classical region from that in which a quantum-mechanical derivation is necessary. Quantum-mechanical effects become important where the (correspondence principle) quantum minimum impact parameter $\hbar/mv_0$ approaches the classical minimum impact parameter Ze^2/mv_0^2 (c.f. [9]). In the quantum-mechanical treatment, the impact parameter does not appear explicitly; however, the results for a single electron interaction do have to be integrated over the velocity v_0. The result may be approximated by the simple expression (A-3c) under the condition $\Lambda_Q >> \sqrt{\gamma}\,\Lambda_C$; i.e., $T >> 5 \times 10^5 Z^2$ [10]. Oster [10] also shows that at low frequencies (i.e., $\Lambda_C >> 1$) the general quantum-mechanical expression reduces to the classical formula (A-3a) under the reverse inequality $T << 5 \times 10^5 Z^2$. Both these approximations also involve the condition $\hbar\omega << kT$. The region in which a classical approach is permissible may also be obtained heuristically by substituting[††] an average velocity $\langle v_0 \rangle = (kT/m)^{1/2}$ in the relation $Ze^2/mv_0^2 >> \hbar/mv_0$. Observe that in (A-3) we have interpolated over the crossover region; i.e., set the switching point at $\Lambda_C = \Lambda_Q$, which gives $T = 8.9 \times 10^5 Z^2$. Oster seems to feel that this interpolation is justified.

[†] By which is meant an average over the impact parameter with a fixed initial velocity of the electron

[††] An examination of the derivation of (A-3c) indicates that a natural choice for $\langle v_0 \rangle$ is $(2kT/\gamma m)^{1/2}$ [10]. This leads to the inequality $T << 2.8 \times 10^5$

EXPRESSIONS FOR ν_m AND g

A classical treatment of the problem utilizing the transport equations yields ([1] p 42, [6] p 51-52)

$$\mu_\omega = -\frac{4\pi e^2}{3mc\omega^2}\int_0^\infty \nu_m \frac{\omega^2}{\omega^2+\nu_m^2}\, v^3 \frac{\partial}{\partial v}(\phi v^{-2})dv^\dagger\ , \tag{A-7}$$

which is valid under the condition (i′). The factor $\omega^2/(\omega^2+\nu_m^2)$ reflects the effects of correlated collisions ([1] p 29-34, [4] p 257); i.e., condition (iii) is no longer necessary. If ν_m may be treated as a constant $\dot{\nu}_m$, then (A-7) can be integrated by parts to yield [1]

$$\mu_\omega = \frac{4\pi e^2 n_-}{mc}\frac{\dot{\nu}_m}{\omega^2+\dot{\nu}_m^2}\ . \tag{A-8}$$

Alternatively, we may define ν_m as that value for which (A-8) holds in the limit $\omega >> \nu_m$. Applying this definition to (A-7) with ϕ the Maxwell distribution yields ([6] p 52-53†† and p 490)

$$\dot{\nu}_m = \frac{4\pi}{3}\left(\frac{2\pi}{3}\right)^{1/2}\frac{Z^2e^4n_+}{(kT)^{3/2}\sqrt{m}}\, g. \tag{A-9}$$

Dropping $\hat{\nu}_m^2$ in the denominator of (A-8) and substituting (A-9) in that equation, we arrive at (A-6).

The precise expression for g bears some discussion. In elementary treatments (cf. [1] and [4]) it is simply unity. A very coarse analysis, employing only average quantities, may be performed as follows: Let $\bar{v}=(kT/m)^{1/2}$ be the average electron speed. Let r_c, the coulomb radius, be the minimum radius to which an electron of average energy can approach and still escape capture; i.e., $Ze^2/r_c = m\bar{v}^2/2 = kT/2$. Finally, let the coulomb cross section be $\sigma_c = \pi r_c^2$, and $\dot{\nu}_m = n_+\bar{v}\sigma_c$. It then follows that

$$\dot{\nu}_m = \frac{4\pi n_+ Z^2 e^4}{(kT)^{3/2}\sqrt{m}},$$

i.e.,

$$g_A = 3(3/2\pi)^{1/2} = 2.1\ . \tag{A-10}$$

A more precise treatment, employing transport equations including the Boltzmann collision term, but nevertheless averaging ν_m over the velocity distribution, yields the expression (A-9) for $\dot{\nu}_m$, with g given by ([12] p 251 and 348, [13] p 258)

---

† $\phi(v)dv$ is the electron density for speed v

†† Note that, in equation (6 13) of [6], $\bar{v} = (8kT/m)^{1/2}$ (p 52) and $\ln(0.37kT\ \ )\ \ (2/3)\ln(\ \ )^{3/2}$ $= (2/3)(\pi/\sqrt{3})\,g$, which results in (A-9)

$$g_B = \frac{\sqrt{3}}{\pi} \ln \Lambda$$

$$= \frac{\sqrt{3}}{\pi} \ln\left(\frac{3}{\sqrt{4\pi}} \frac{(kT)^{3/2}}{Ze^3 n_-^{1/2}} \right) . \qquad \text{(A-11)}$$

Note that this Λ differs from that of [4] on page 418 by a factor of one-half.

Alternatively, deducing $\hat{\nu}_m$ from a comparison of (A-7) and (A-8) in the limit $\omega >> \nu_m$ yields a Gaunt factor with a frequency dependence ([6] p 490, [9]). In particular, we have (cf. [6] p 490)

$$g_C = \frac{\sqrt{3}}{\pi} \ln \frac{2(2kT)^{3/2}}{\gamma^{5/2} Ze^2 \sqrt{m}\, \omega} , \qquad \text{(A-12)}$$

where[†] $\gamma = e^{0.577} = 1.781$. The same expression for g is found in [7] (p 148) and [10]. We note that these formulae are essentially classical and apply only to temperatures in the range ([6] p 488, [9], [10])

$$T << \frac{\gamma^2 m Z^2 e^4}{2 \hbar^2 k} \approx 5 \times 10^5 \text{ for } Z = 1. \qquad \text{(A-13)}$$

In the opposite extreme, at high temperatures, we have ([6] p *491*, [9][††], *[10]*, *[14]* p *246*)

$$g_D = \frac{\sqrt{3}}{\pi} \ln \frac{4kT}{\gamma \hbar \omega} , \qquad \text{(A-14)}$$

which must be derived from quantum-mechanical considerations. All of these expressions ((A-11) through (A-14)) are only valid for frequencies in the region in which the argument of the logarithm is much greater than one.

The variations in the equations for g tend to be the result of the somewhat arbitrary cutoffs involved in the classical approximations; nevertheless, their role can provide physical insight. For example, [9] provides the less precise expression

$$g_E = \frac{\sqrt{3}}{\pi} \ln \frac{v_T kT}{Ze^2 \omega} , \qquad \text{(A-15)}$$

which reduces to an expression very close to (A-12) if the thermal velocity v_T is taken as $(kT/m)^{1/2}$. Another approximation is to avoid the frequency dependence by replacing ω with ω_p in (A-12). This leads to

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[†] This can be written $\gamma = e^{\gamma^*}$, where γ^* is Euler's constant Care should be taken inasmuch as some references (cf , [7]) use the notation γ for γ^*

[††] [9] finds $\Lambda_Q = v_T (mkT)^{1/2}/\hbar\omega$, which requires a value for the electron thermal velocity v_T This differs from (A-16) by 4/1 781 = 2 25 in the argument of the logarithm if $v_T = (kT/m)^{1/2}$

$$g_F = \frac{\sqrt{3}}{\pi} \ln \frac{2(2kT)^{3/2}}{\gamma^{5/2} Ze^2 \sqrt{m}} \frac{\sqrt{m}}{(4\pi e^2 n_-)^{1/2}}$$

$$= \frac{\sqrt{3}}{\pi} \ln .38 \frac{(kT)^{3/2}}{Ze^3 n^{1/2}} \; . \qquad \text{(A-16)}$$

If γ is set equal to one, then this matches (A-11) up to replacement of the factor $3/\sqrt{4\pi} = 0.85$ by 0.80 . We have adopted (A-12) in conjunction with (A-1) as the most rigorously derived under conditions (i) - (iv). Note that (A-16) does not reduce to the Gaunt factor given by Raizer ([1] equation (6.1)) even though the footnote to that equation might seem to indicate that should be the case. In fact, if we multiply by 2/3 and introduce the exponent 3/2 inside the logarithm, Raizer's expression becomes

$$g_G = \frac{\sqrt{3}}{\pi} \frac{2}{3} \ln 8 \frac{(kT)^{3/2}}{Z^{3/2} e^3 n_-^{1/2}} \; , \qquad \text{(A-17)}$$

which differs from the various expressions in the exponent of Z as well as the numerical coefficients. We do not know whether this expression of Raizer is erroneous; however, it does yield reasonable results in the region of application. (If $T = 15{,}000$ °K and $p =$ atmospheric pressure, then the electron partial pressure p_e is about $10^6/2$ dynes/cm^2, $n_- = p_e/kT = 2.42 \times 10^{17}$, and we have $g = 2.23$ for (A-17) while $g = 1.67$ from equation (A-16).) Most likely (A-17) represents an approximation to the Gaunt factor for optical frequencies inasmuch as, under those circumstances, $\Lambda_C \approx 1$, and (A-12) is unacceptable.

IMPLICATIONS OF THE SAHA EQUATION

To actually compute μ_ω, we need, in addition to values for the temperature and frequency, the ion and electron densities n_+ and n_-. Both of these may be computed from the pressure under the appropriate thermodynamic assumptions. For a perfect gas in equilibrium (i.e., $T_+ = T_-$), we have

$$\begin{aligned} p_+ &= n_+ kT \\ p_- &= n_- kT \\ p &= p_+ + p_- + p_n \; , \end{aligned} \qquad \text{(A-18)}$$

where p_+, p_-, and p_n are the partial pressures of the ions, electrons and neutrals. We still have too many unknowns; however, if we assume total ionization, then $p_n = 0$, $n_- = Zn_+$, and (A-6) becomes

$$\mu_\omega = \frac{16\pi^2}{3} \left(\frac{2\pi}{3} \right)^{1/2} \frac{e^6 g}{m^{3/2} c\, (kT)^{7/2}} \frac{Zp_-^2}{\omega^2} \qquad \text{(A-19a)}$$

$$= \frac{16\pi^2}{3} \left(\frac{2\pi}{3} \right)^{1/2} \frac{e^6 g}{m^{3/2} c (kT)^{7/2}} \frac{Z^3}{(1+Z)^2} \frac{p^2}{\omega^2} \qquad \text{(A-19b)}$$

$$= 3.77 \times 10^{29} \frac{Z^3}{(1+Z)^2} \frac{g}{(T/10^4)^{7/2}} \frac{p_{atm}^2}{\omega^2} \tag{A-19c}$$

$$= 0.16 \frac{Z^3}{(1+Z)^2} \frac{g}{(T/10^4)^{7/2}} \frac{p_{atm}^2}{(\hbar\omega)_{ev}^2} \tag{A-19d}$$

$$= 1.07 \times 10^{7} \frac{Z^3}{(1+Z)^2} \frac{g}{(T/10^4)^{7/2}} p_{atm}^2 \lambda^2 \ . \tag{A-19e}$$

More generally, a gas will exist in various stages of ionization. The particle densities for a monatomic gas may be described by the Saha equation ([4] p 192-207)

$$\frac{n_{m+1} n_-}{n_m} = 2 \frac{u^{(m+1)}}{u^{(m)}} \left(\frac{mkT}{2\pi\hbar^2} \right)^{3/2} e^{-I_{m+1}/kT} , \tag{A-20}$$

where n_m is the number density of ions of charge m, n_- is the number density of electrons, u_m is the partition function for the m^{th} ion normalized by the ground state energy (see [4] equation (3.43)), and I_m is the ionization energy to remove the m^{th} electron after the first m-1 have been removed. The particle densities may be related to the partial pressures and thence to the total pressure; however, solving the resulting set of equations even for the case of a gas consisting of a single element can be quite involved. More specifically, in addition to (A-20), we have

$$n = \sum_0^{n_{max}} n_m + n_- \tag{A-21a}$$

$$n_- = \sum_0^{n_{max}} m\, n_m \ , \tag{A-21b}$$

where n is the total number density of particles. Given n, this provides $n_{max} + 2$ nonlinear equations in the same number of unknowns.

Things simplify considerably when only one level of ionization is present; i.e., when $n_m = 0$ except for one set of adjacent values, $m = m_0$ and $m = m_0 + 1$. (Note that $Z = m_0 + 1$.) The simplest, and for our purposes most useful, case is when $Z = 1$.† Then $m_0 = 0$, so that n_+, which by definition is n_{m_0+1}, equals n_-, and $n_0 = n - 2n_-$. In terms of partial pressures the left-hand side of (A-20) may be written

$$\frac{p_+ p_-}{kT\, p_0} = \frac{p_-^2}{kT\,(p-2p_-)} \ .$$

Furthermore, if we let $x_- = p_-/p$ and approximate the partition functions by the ground

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† A brief treatment of multiple ionization may be found in [4].

state statistical weights $g^{(m)}$, we get equation (6.47) of [1]

$$\frac{x_-^2}{1-2x_-} = 2\,\frac{g^{(1)}}{g^{(0)}}\left(\frac{m}{2\pi\hbar^2}\right)^{3/2}\frac{(kT)^{5/2}}{p}\,e^{-I_1/kT}$$

$$= 6.7 \times 10^3\,\frac{g^{(1)}}{g^{(0)}}\,\frac{(T/10^4)^{5/2}\,e^{-I_1/kT}}{p_{atm}}\quad . \tag{A-22}$$

These expressions may be combined with (A-6) to give a formula for the absorption coefficient which (assuming conditions (i′), (ii), (iii), (iv)) is also valid for weakly ionized gases:

$$\mu_\omega = 1.07 \times 10^7\,\frac{g}{(T/10^4)^{7/2}}\,x_-^2\,p_{atm}^2\,\lambda^2\quad . \tag{A-23}$$

Because of the exponential in (A-22), at some temperature depending on I_1, the gas will experience a sudden rise in the electron partial pressure to its asymptotic value, where $x_- = 1/2$. More precisely, solving the quadratic $x_-^2 = a(1-2x_-)$, where a is given by the right-hand side of (A-22), we find that for $a >> 1$ we have $x_- \approx 1/2$. In particular, for $a = 3$, $x_- = 0.45$. Expressing T in electron volts, letting $\frac{g^{(1)}}{g^{(0)}} = 1.9$ ([1] p 242), $p_{atm} = 1$, and taking the logarithm of the right-hand side of (A-22), this translates to

$$\ln(6.7 \times 10^3) + \ln(1.9) + \frac{5}{2}\ln(1.161\,T_{ev}) - \left(\frac{I_1}{T}\right)_{ev} > \ln(3) \tag{A-24}$$

Then, letting $\beta T_{ev} = I_{ev}$, we find

$$\beta + \frac{5}{2}\ln\beta < 8.72 + \frac{5}{2}\ln I_{ev}\quad . \tag{A-25}$$

A sufficient condition is $\beta < 5$, but taking the ionization potential for air to be $I_1 = 14.4$ eV ([1] p 242), we find $\beta < 9.7$. Thus, a sufficient condition for the complete single ionization of air is

$$T_{ev} > (I_1)_{ev}/9.7 > 1.5\ \text{eV} \tag{A-26}$$

Note, however, if the temperature rises sufficiently (2 to 3 eV), double ionization will commence. Thus, within the temperature range 1.5 eV to 2.5 eV, and for frequencies satisfying $(\hbar\omega)_{ev} << 1.5$ eV (otherwise we must consider bound-free transitions), we may use the formulae (A-19) with $Z = 1$.

INCLUSION OF BOUND-FREE TRANSITIONS

Formulae for the absorption coefficient which include the effects of both free-free and bound-free transitions are derived semiclassically in [4]. As described there ([4] p 266), the results are reasonably accurate in a more general setting. We note that in their derivation of (5.34) and also (5.41), a factor of $2\frac{g^{(m+1)}}{g^{(m)}}$ † has been replaced by 1 (a reasonably good approximation). We reintroduce this factor and an additional factor, designated $\xi(\omega)$, which is simply designed to indicate a quantum-mechanical correction (When free-free transitions dominate, $\xi(\omega)$ reduces to the Gaunt factor g. See [1], p 241-242.) Under these circumstances, we may write ([4] p 278)

$$\kappa_\omega = \sum_m \kappa_{\omega m} \qquad \text{(A-27)}$$

where

$$\kappa_{\omega m} = a \frac{2\, g^{(m+1)}}{g^{(m)}} (m+1)^2 \frac{n_m}{T^2} e^{-y_m} F_m(x)\, \xi_m(\omega) \qquad \text{(A-28a)}$$

and

$$a = \frac{16\pi^2}{3\sqrt{3}} \frac{e^6}{hck^2} = 0.96 \times 10^{-7} \ \mathrm{cm^2\, deg^2} \qquad \text{(A-28b)}$$

$$y_m = I_{m+1}/kT; \qquad x = \hbar\omega/kT \qquad \text{(A-28c)}$$

$$F_m(x) = \begin{cases} \dfrac{e^x}{x^3} & \text{for } x < y_m \\ 2y_m \dfrac{e^{y_m}}{x^3} & \text{for } x > y_m. \end{cases} \qquad \text{(A-28d)}$$

Note that y_m is x_{1m} of [4]. The ion densities are determined from (A-20) and (A-21). The absorption coefficient μ_ω is given by substituting (A-27) in (A-1), which includes the effects of stimulated emission.

Several observations are in order: First, we note that the inequalities in (A-28d) should actually read "$<<$" and "$>>$" since the formulae are approximations derived under these assumptions [4]. The point at which the two expressions are equal is actually $x = \ln(2\, y_m) + y_m$, but this occurs quite close to $x = y_m$. Second, an assumption was made in the derivation of (A-28) that the ions may be modeled as hydrogen-like atoms with charges of Z; this may not always be accurate ([4] p 275). Also, we remark that, in the derivation of (A-28), [4] sums the contributions from the free-free transitions

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† This really should be the ratio of the ion partition functions $u^{(m)}$, but it has been approximated by the ratio of the statistical weights of the ground states

and the bound-free transitions. Although the expression for the free-free transitions is only valid for $\hbar\omega << kT$, its contribution for $\hbar\omega \geq kT$ is negligible. Thus, (A-28) remains a valid approximation under quite general conditions; i.e., so long as $\omega >> \omega_p$ and $\omega >> \hat{\nu}_m$.

If we consider the case of single-level ionization with $m = 0$ (i.e., $kT << I_1$) and frequencies for which $\hbar\omega << I_1$, then (A-28) reduces to the Kramers-Unsold formula,

$$\mu_\omega = \frac{16\pi^2}{3\sqrt{3}} \frac{e^6 k}{hc} \xi_0(\omega) \, 2 \frac{g^{(1)}}{g^{(0)}} \frac{n_0 T}{(\hbar\omega)^3} e^{-(I_1 - \hbar\omega)/kT} \left(1 - e^{-\hbar\omega/kT}\right) \qquad \text{(A-29a)}$$

$$= 6.28 \times 10^{-20} \xi_0(\omega) \, 2 \frac{g^{(1)}}{g^{(0)}} \frac{n_0 T}{(\hbar\omega)^3_{ev}} e^{-(I - \hbar\omega)/kT} \left(1 - e^{-\hbar\omega/kT}\right) . \qquad \text{(A-29b)}$$

Observe that the conditions for the validity of (A-29) may be written

(a) $kT << I_1$
(b) $\hbar\omega << I_1$,

which is less stringent that (i′). Since $Z = m + 1 = 1$ and $n_1 n_- = n_-^2$, we have

$$n_0 = \frac{p_-^2}{(kT)^2} \frac{n_0}{n_1 n_-}$$

$$= \frac{x_-^2 p^2}{(kT)^2} \frac{n_0}{n_1 n_-} .$$

Using this expression and substituting (A-20) into (A-29), we find

$$\mu_\omega = \frac{16\pi^2 e^6}{3\sqrt{3}\, hck} \left(\frac{2\pi}{km} \right)^{3/2} \xi_0(\omega) \frac{x_-^2 p^2}{T^{5/2}} \frac{1}{\omega^3} \left(e^{\hbar\omega/kT} - 1\right) \qquad \text{(A-30a)}$$

$$= 0.14 \, \xi_0(\omega) \frac{x_-^2 p_{atm}^2}{(T/10^4)^{5/2}} \frac{1}{(\hbar\omega)^3_{ev}} \left(e^{\hbar\omega/kT} - 1\right) . \qquad \text{(A-30b)}$$

with x_- computed via (A-22).

Note that, up to the edge correction $e^{\hbar\Delta\omega/kT}$, (A-30b) matches equation (6.46) of [1]. Also, for $\hbar\omega << kT$ and $x_- = 1/2$, (A-30b) reduces to (A-19d) with $Z = 1$. Finally, we remark that in comparison with (A-29b), (6.15) of [1] seems to be lacking a factor of two. This occurs even if we set the factor $\xi_0(\omega)$ (which for our purposes is actually close to two) equal to one.

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