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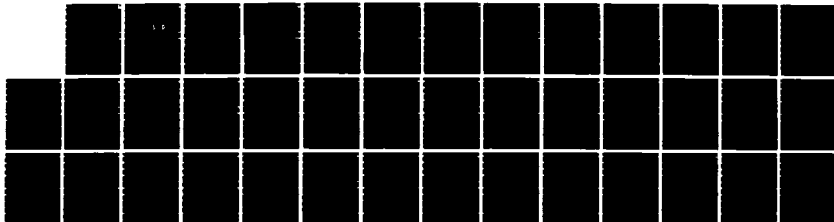
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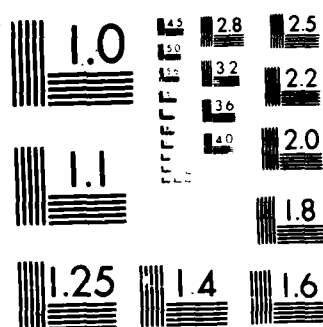
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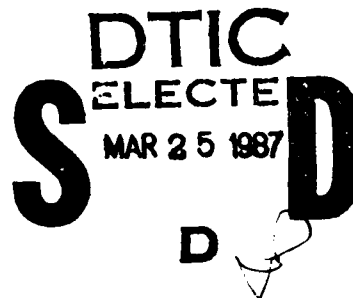
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THEORETICAL BACKGROUND FOR MODELING ON NON-STATIONARY CHANNELS

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Technical Report



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19 ABSTRACT (Continue on reverse if necessary and identify by block number) Models of randomly time-variant, frequency-selective, communication channels commonly employ statistical representations of channel input-output system functions, such as the time-variant impulse response or the time-variant transfer function, with Gaussian first order statistics. The additional assumption that the channel is WSSUS, that is, the second order statistics of the channel system functions are wide-sense stationary in time (or equivalently uncorrelated in Doppler) and uncorrelated in delay (or equivalently wide-sense-stationary in carrier frequency), leads to simple and elegant doubly-stationary channel model formulations. The assumption of double-stationarity holds for a very diverse range of narrow-band channels, and the Gaussian-WSSUS model has received much attention and use. The applicability of such a model to some advanced modem concepts of current interest, such as meteor burst links and wideband HF communication techniques, however, cannot always be justified.					
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19. ABSTRACT (Continued)

Such channels are known to be rapidly non-stationary in carrier frequency, time or both. This paper presents an approach to modeling such doubly non-stationary channels. First the use of mutual coherence functions for the statistical representation of time-variant channel system functions is briefly reviewed. This is followed by the presentation of new time- and frequency-separable forms of the mutual coherence functions that admit doubly non-stationary channel models. The WSSUS model is then shown to be a degenerate case of this more general model.

SUMMARY

The standard approach to the modeling of linear time-variant, frequency-selective communication channels has been reviewed and extended to allow for statistical variation in both time and frequency. One particular model that allows the channel statistics to vary in a piece-wise linear fashion over time and frequency has been addressed in detail. The theoretical aspects of the model have been emphasized. A discussion of the use of the DNS model for practical communication system analysis can be found in Reference 2.

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SECTION I INTRODUCTION

The modeling of frequency-selective fading linear channels has been a topic of research for several decades, much of it based upon the work of Bello (Reference 1). These linear channels can be represented by baseband equivalent input-output system functions such as a time-variant impulse response or a time-variant transfer function. The non-specular component of the channel system functions is usually viewed as the output of a random process, and the random variations of the channel can be described quantitatively by the statistical parameters of the associated random process.

When analyzing the effects of fading on most communication systems, one can assume that the statistical description of the channel is invariant in time and frequency. This assumption leads to Bello's so-called WSSUS channel model, to be discussed below. However, some communication systems operate over propagation media with fading statistics that change rapidly in time (e.g. meteor burst modem) or vary in frequency over the bandwidth of the transmitted signal (e.g. a direct sequence spread-spectrum HF modem). The WSSUS model is inappropriate for the analysis of such systems. This paper presents a doubly non-stationary (DNS) channel model to accommodate systems such as these. In the following, we first review Bello's general approach to statistical channel modeling, and then derive time and frequency separable forms of the channel statistics that allow the channel model to propagate no energy outside an arbitrary region in time-frequency space. We then show that a weighted combination of an arbitrarily large number of such models forms the desired DNS model. Lastly, we discuss one particular weighting that causes the channel statistics to vary in a piece-wise linear fashion in time-frequency space.

SECTION 2 GENERAL BACKGROUND MATERIAL ON CHANNEL MODELING

2.1 LOWPASS EQUIVALENT CHANNEL REPRESENTATION.

The standard complex baseband signal representations will be used to express transmitted and received waveforms. The transmitted signal is denoted $z(t)$ in the time domain, and $Z(f)$ in the frequency domain. $w(t)$ and $W(f)$ denote the time and frequency domain representations of the received signal. Both transmitted and received signals are referenced to some fixed (angular) frequency ω_0 (in units of radians/second), though $Z(f)$ and $W(f)$ need not be centered about that frequency. The signal actually sent by the transmitting terminal can be expressed in the time domain by either side of the following equation.

$$\text{Real}\{ z(t) e^{j\omega_0 t} \} = I_z(t) \cos \omega_0 t - j Q_z(t) \sin \omega_0 t \quad (1)$$

The complex waveform $z(t)$ equals $I_z(t) + jQ_z(t)$, where $I_z(t)$ and $Q_z(t)$ are by definition real waveforms and j denotes the square root of -1 . The signal actually seen at the receiving terminal can similarly be expressed in the time domain as

$$\text{Real}\{ w(t) e^{j\omega_0 t} \} = I_w(t) \cos \omega_0 t - j Q_w(t) \sin \omega_0 t \quad (2)$$

where $w(t)$ equals $I_w(t) + jQ_w(t)$, and again $I_w(t)$ and $Q_w(t)$ are real waveforms. The channel model generates the received signal by passing the complex transmitted signal through a time-varying, linear system, as shown below.

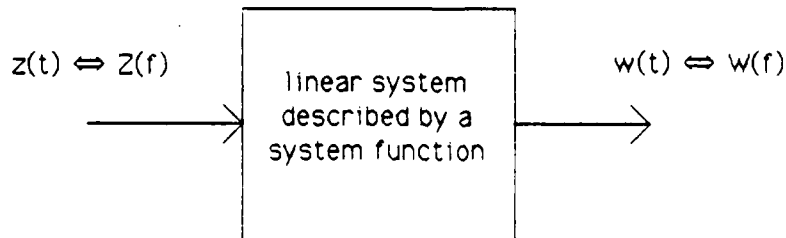


Figure 1. Block diagram of complex baseband channel model.

Bello in Reference 1 defines twelve system equations that express the output, either $w(t)$ or $w(f)$, in terms of the input, either $z(t)$ or $Z(f)$. Four of these are used in the sequel, and are reproduced in the following equations:

$$w(t) = \int z(t-\xi) g(t,\xi) d\xi \quad (3)$$

$$w(t) = \int Z(f) T(f,t) \exp(j2\pi ft) df \quad (4)$$

$$w(f) = \int Z(f-\nu) G(f-\nu,\nu) d\nu \quad (5)$$

$$w(t) = \iiint z(t-\xi) \exp(j2\pi\nu t) U(\xi,\nu) d\nu d\xi \quad (6)$$

The variable t is time in units of seconds, f is frequency offset from the nominal center frequency ($\omega_0/2\pi$) in units of hertz, ξ is delay in units of seconds, and ν is doppler frequency shift in units of hertz.

Bello named these four system functions and gave them physical interpretations. The Input Delay-Spread Function, $g(t,\xi)$, is the channel output at time t resulting from the excitation of the channel to a impulse ξ seconds before t . The Output Doppler-Spread Function, $G(f,\nu)$, is the spectral response ν hertz above f , resulting from the excitation of the channel to a frequency impulse (i.e., time domain cissoid) at f . The Time-Variant Transfer Function, $T(f,t)$, is the transfer function in the frequency variable f at time t . Finally, suppose the transmitted signal is decomposed into infinitesimal elements that are first delayed, and then doppler shifted before being linearly combined to form the channel output. Then the Delay-Doppler Spread Function, $U(\xi,\nu)$, is the complex weighting given to the component delayed by ξ seconds and shifted by ν hertz.

Reference 1 shows that the four system functions are related by Fourier transforms in one or both arguments. The various relationships are mathematically expressed by Equations 7 through 18 and illustrated by Figure 2 (where arrows point in the direction of the Fourier transforms).

$$g(t,\xi) = \int T(f,t) e^{j2\pi\xi f} df \quad (7)$$

$$g(t,\xi) = \int U(\xi,\nu) e^{j2\pi\nu t} d\nu \quad (8)$$

$$g(t, \xi) = \int \int G(f, \nu) e^{j2\pi(\nu t + \xi f)} d\nu df \quad (9)$$

$$T(f, t) = \int g(t, \xi) e^{-j2\pi \xi f} d\xi \quad (10)$$

$$T(f, t) = \int G(f, \nu) e^{j2\pi \nu t} d\nu \quad (11)$$

$$T(f, t) = \int \int U(\xi, \nu) e^{j2\pi(\nu t - \xi f)} d\nu d\xi \quad (12)$$

$$U(\xi, \nu) = \int g(t, \xi) e^{-j2\pi \nu t} dt \quad (13)$$

$$U(\xi, \nu) = \int G(f, \nu) e^{j2\pi \xi f} df \quad (14)$$

$$U(\xi, \nu) = \int \int T(f, t) e^{j2\pi(\xi f - \nu t)} df dt \quad (15)$$

$$G(f, \nu) = \int T(f, t) e^{-j2\pi \nu t} dt \quad (16)$$

$$G(f, \nu) = \int U(\xi, \nu) e^{-j2\pi \xi f} d\xi \quad (17)$$

$$G(f, \nu) = \int \int g(t, \xi) e^{-j2\pi(\nu t + \xi f)} dt d\xi \quad (18)$$

2.2 STATISTICAL FORMULATION OF RANDOM CHANNELS.

Random channels can be characterized by statistical descriptions of any one of these four system functions. It is common to assume the real and imaginary components of the system functions are independent and have Gaussian, zero-mean first order statistics. The stochastic properties of the random system functions are then completely characterized with the addition of the second order statistics. The second order statistics are expressed by the correlation properties of the system functions in the form of what we term "mutual coherence functions". The mutual coherence functions of the four system functions are defined as follows.

$$R_g(t, s; \xi, \eta) = \overline{g(t, \xi)^* g(s, \eta)} \quad (19)$$

$$R_T(f, l; t, s) = \overline{T(f, t)^* T(l, s)} \quad (20)$$

$$R_G(f, l; \nu, \mu) = \overline{G(f, \nu)^* G(l, \mu)} \quad (21)$$

$$R_U(\xi, \eta; \nu, \mu) = \overline{U(\xi, \nu)^* U(\eta, \mu)} \quad (22)$$

where * denotes the complex conjugate operation and the overbar indicates the ensemble averaging operation. As was the case with the system functions, from any one of the mutual coherence functions one can derive the other three. They are interrelated by double or quadruple Fourier transforms as discussed in References 1, expressed mathematically by Equations 23 through 34 and shown pictorially in Figure 3.

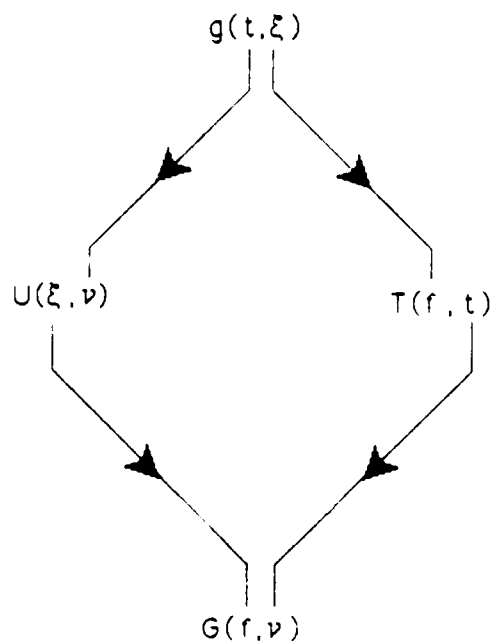


Figure 2. Fourier transform relationships between the system functions g , T , G , and U .

$$R_g(t,s;\xi,\eta) = \iint R_T(f,l;t,s) e^{j2\pi(\eta l - \xi f)} df dl \quad (23)$$

$$R_g(t,s;\xi,\eta) = \iint R_U(\xi,\eta;\nu,\mu) e^{j2\pi(\mu s - \nu t)} d\nu d\mu \quad (24)$$

$$R_g(t,s;\xi,\eta) = \iiint R_G(f,l;\nu,\mu) e^{j2\pi(\mu s - \nu t + \eta l - \xi f)} d\nu d\mu df dl \quad (25)$$

$$R_T(f,l;t,s) = \iint R_g(t,s;\xi,\eta) e^{j2\pi(\xi f - \eta l)} d\xi d\eta \quad (26)$$

$$R_T(f,l;t,s) = \iint R_G(f,l;\nu,\mu) e^{j2\pi(\mu s - \nu t)} d\nu d\mu \quad (27)$$

$$R_T(f,l;t,s) = \iiint R_U(\xi,\eta;\nu,\mu) e^{j2\pi(\mu s - \nu t + \xi f - \eta l)} d\nu d\mu d\xi d\eta \quad (28)$$

$$R_U(\xi,\eta;\nu,\mu) = \iint R_g(t,s;\xi,\eta) e^{j2\pi(\nu t - \mu s)} dt ds \quad (29)$$

$$R_U(\xi,\eta;\nu,\mu) = \iint R_G(f,l;\nu,\mu) e^{j2\pi(\eta l - \xi f)} df dl \quad (30)$$

$$R_U(\xi,\eta;\nu,\mu) = \iiint R_T(f,l;t,s) e^{j2\pi(\eta l - \xi f + \nu t - \mu s)} df dl dt ds \quad (31)$$

$$R_G(f,l;\nu,\mu) = \iint R_T(f,l;t,s) e^{j2\pi(\nu t - \mu s)} dt ds \quad (32)$$

$$R_G(f,l;\nu,\mu) = \iint R_U(\xi,\eta;\nu,\mu) e^{j2\pi(\xi f - \eta l)} d\xi d\eta \quad (33)$$

$$R_G(f,l;\nu,\mu) = \iiint R_g(t,s;\xi,\eta) e^{j2\pi(\nu t - \mu s + \xi f - \eta l)} dt ds d\xi d\eta \quad (34)$$

2.3 WSSUS CHANNEL MODEL.

The WSSUS channel model is obtained from the above general statistical form by assuming the first and second order statistics are constant over (or independent of) time and carrier frequency. The time independence is called the wide-sense-stationary (WSS) assumption and the carrier frequency independence is called the uncorrelated scatterer (US) assumption. The effect of each of the WSS and US assumptions upon the form of mutual coherence functions is first treated

separately in the discussion to follow. Then taken together the WSSUS channel follows immediately.

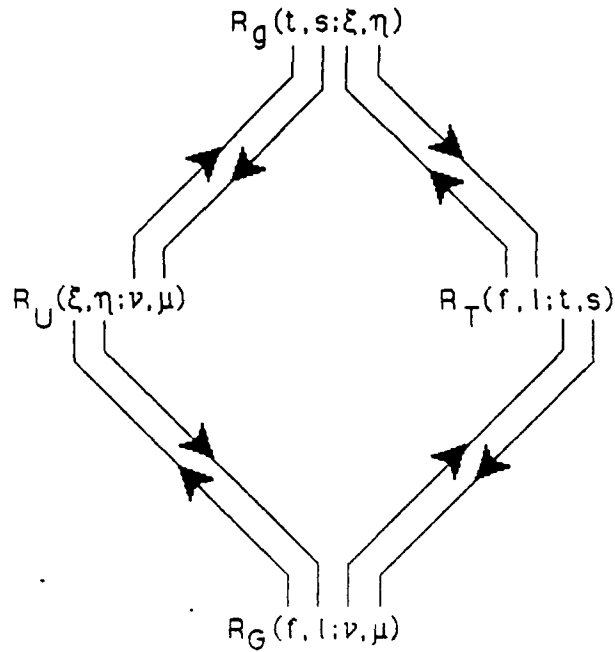


Figure 3. Fourier transform relationships between the mutual coherence functions of the system functions g , T , G and U .

The WSS assumption causes a simplification of the functional form of all four mutual coherence functions discussed above. The WSS forms of the mutual coherence functions, denoted with a tilde throughout the sequel, are as follows.

$$R_g(t, s; \xi, \eta) = \tilde{R}_g(s - t; \xi, \eta) \quad (35)$$

$$R_T(f, l; t, s) = \tilde{R}_T(f, l; s - t) \quad (36)$$

$$R_G(f, l; \nu, \mu) = \tilde{R}_G(f, l; \nu) \delta(\mu - \nu) \quad (37)$$

$$R_U(\xi, \eta; \nu, \mu) = \tilde{R}_U(\xi, \eta; \nu) \delta(\mu - \nu) \quad (38)$$

Substituting $s+\tau$ for t allows the WSS forms of Equations 23 through 34 to be rewritten as follows.

$$\tilde{R}_g(\tau; \xi, \eta) = \iint \tilde{R}_T(f, l; \tau) e^{j2\pi(\eta l - \xi f)} df dl \quad (39)$$

$$\tilde{R}_g(\tau; \xi, \eta) = \int \tilde{P}_U(\xi, \eta; \nu) e^{j2\pi\nu\tau} d\nu \quad (40)$$

$$\tilde{R}_g(\tau; \xi, \eta) = \iiint \tilde{P}_G(f, l; \nu) e^{j2\pi(\nu\tau + \eta l - \xi f)} d\nu df dl \quad (41)$$

$$\tilde{R}_T(f, l; \tau) = \iint \tilde{R}_g(\tau; \xi, \eta) e^{j2\pi(\xi f - \eta l)} d\xi d\eta \quad (42)$$

$$\tilde{R}_T(f, l; \tau) = \int \tilde{P}_G(f, l; \nu) e^{j2\pi\nu\tau} d\nu \quad (43)$$

$$\tilde{R}_T(f, l; \tau) = \iiint \tilde{P}_U(\xi, \eta; \nu) e^{j2\pi(\nu\tau + \xi f - \eta l)} d\nu d\xi d\eta \quad (44)$$

$$\tilde{P}_U(\xi, \eta; \nu) = \int \tilde{R}_g(\tau; \xi, \eta) e^{-j2\pi\nu\tau} d\tau \quad (45)$$

$$\tilde{P}_U(\xi, \eta; \nu) = \iint \tilde{P}_G(f, l; \nu) e^{j2\pi(\eta l - \xi f)} df dl \quad (46)$$

$$\tilde{P}_U(\xi, \eta; \nu) = \iiint \tilde{R}_T(f, l; \tau) e^{j2\pi(\eta l - \xi f - \nu\tau)} df dl d\tau \quad (47)$$

$$\tilde{P}_G(f, l; \nu) = \int \tilde{R}_T(f, l; \tau) e^{-j2\pi\nu\tau} d\tau \quad (48)$$

$$\tilde{P}_G(f, l; \nu) = \iint \tilde{P}_U(\xi, \eta; \nu) e^{j2\pi(\xi f - \eta l)} d\xi d\eta \quad (49)$$

$$\tilde{P}_G(f, l; \nu) = \iiint \tilde{R}_g(\tau; \xi, \eta) e^{j2\pi(-\nu\tau + \xi f - \eta l)} d\tau d\xi d\eta \quad (50)$$

The above equations show that the mathematical relationships between the non-singular factors of the WSS forms of the mutual coherence functions, like the general forms themselves, are related by Fourier transforms. Figure 4 shows these Fourier transform relationships. The non-singular factors in the WSS forms of the mutual coherence functions have the following physical interpretations.

$\tilde{R}_g(\tau; \xi, \eta)$ Cross correlation of pair $[g(t, \xi), g(t, \eta)]$ for fixed ξ and η .

$\tilde{R}_T(f, l; \tau)$ Cross correlation of pair $[T(f, t), T(l, t)]$ for fixed f and l .

$\tilde{P}_G(f, l; \nu)$ Cross spectrum of pair $[T(f, t), T(l, t)]$ for fixed f and l .
 $\tilde{P}_G(f, f; \nu)$ as a function of ν is the power spectral density of the fluctuations of $T(f, t)$ in the variable t .

$\tilde{P}_U(\xi, \eta; \nu)$ Cross spectrum of pair $[g(t, \xi), g(t, \eta)]$ for fixed ξ and η .
 $\tilde{P}_U(\xi, \xi; \nu)$ as a function of ν is the power spectral density of the fluctuations of $g(t, \xi)$ in the variable t .

The WSS assumption affects the nature of the system functions. $G(f, \nu)$ and $U(\xi, \nu)$ have the character of non-stationary white noise in the variable ν . $g(t, \xi)$ and $T(f, t)$ have the character of stationary colored noise in the variable t .

The US assumption causes a different simplification in the forms of the mutual coherence functions. These US forms, denoted with a circumflex throughout the sequel, are as follows.

$$R_g(t, s; \xi, \eta) = \hat{P}_g(t, s; \xi) \delta(\xi - \eta) \quad (51)$$

$$R_T(f, l; t, s) = \hat{R}_T(l - f; t, s) \quad (52)$$

$$R_G(f, l; \nu, \mu) = \hat{R}_G(l - f; \nu, \mu) \quad (53)$$

$$R_U(\xi, \eta; \nu, \mu) = \hat{P}_U(\xi; \nu, \mu) \delta(\xi - \eta) \quad (54)$$

Substituting Ω for $l - f$ allows the US of Equations 23 through 34 to be rewritten as follows.

$$\hat{P}_g(t, s; \xi) = \int \hat{R}_T(\Omega; t, s) e^{j2\pi\xi\Omega} d\Omega \quad (55)$$

$$\hat{P}_g(t, s; \xi) = \int \int \hat{P}_U(\xi; \nu, \mu) e^{j2\pi(\mu s - \nu t)} d\nu d\mu \quad (56)$$

$$\hat{P}_g(t, s; \xi) = \int \int \int \hat{R}_G(\Omega; \nu, \mu) e^{j2\pi(\mu s - \nu t + \xi\Omega)} d\nu d\mu d\Omega \quad (57)$$

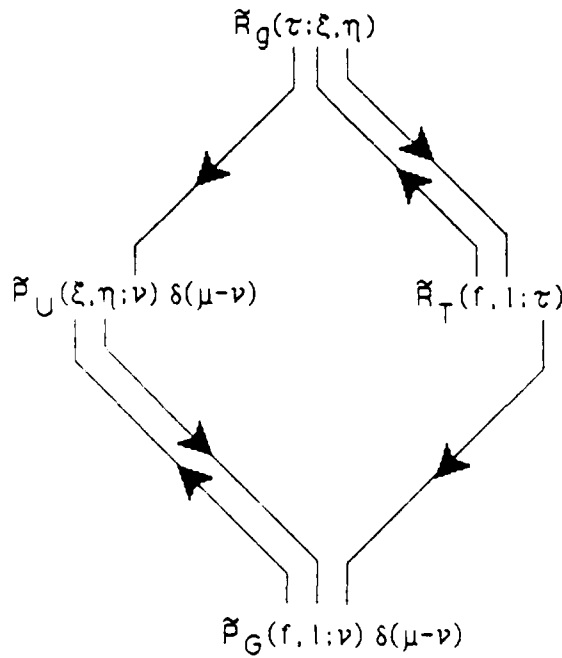


Figure 4. Fourier transform relationships between the WSS forms of the mutual coherence functions.

$$\hat{P}_g(t, s; \xi) = \int \hat{R}_T(\Omega; t, s) e^{j2\pi\xi\Omega} d\Omega \quad (55)$$

$$\hat{P}_g(t, s; \xi) = \iint \hat{P}_U(\xi; \nu, \mu) e^{j2\pi(\mu s - \nu t)} d\nu d\mu \quad (56)$$

$$\hat{P}_g(t, s; \xi) = \iiint \hat{R}_G(\Omega; \nu, \mu) e^{j2\pi(\mu s - \nu t + \xi\Omega)} d\nu d\mu d\Omega \quad (57)$$

$$\hat{R}_T(\Omega; t, s) = \int \hat{P}_g(t, s; \xi) e^{-j2\pi\xi\Omega} d\xi \quad (58)$$

$$\hat{R}_T(\Omega; t, s) = \iint \hat{R}_G(\Omega; \nu, \mu) e^{j2\pi(\mu s - \nu t)} d\nu d\mu \quad (59)$$

$$\hat{R}_T(\Omega; t, s) = \iiint \hat{P}_U(\xi; \nu, \mu) e^{j2\pi(\mu s - \nu t - \xi\Omega)} d\nu d\mu d\xi \quad (60)$$

$$\hat{P}_U(\xi; \nu, \mu) = \int \int \hat{P}_g(t, s; \xi) e^{j2\pi(\nu t - \mu s)} dt ds \quad (61)$$

$$\hat{P}_U(\xi; \nu, \mu) = \int \hat{R}_G(\Omega; \nu, \mu) e^{j2\pi\xi\Omega} d\Omega \quad (62)$$

$$\hat{P}_U(\xi; \nu, \mu) = \int \int \int \hat{R}_T(\Omega; t, s) e^{j2\pi(\xi\Omega + \nu t - \mu s)} d\Omega dt ds \quad (63)$$

$$\hat{R}_G(\Omega; \nu, \mu) = \int \int \hat{R}_T(\Omega; t, s) e^{j2\pi(\nu t - \mu s)} dt ds \quad (64)$$

$$\hat{R}_G(\Omega; \nu, \mu) = \int \hat{P}_U(\xi; \nu, \mu) e^{j2\pi\xi\Omega} d\xi \quad (65)$$

$$\hat{R}_G(\Omega; \nu, \mu) = \int \int \int \hat{P}_g(t, s; \xi) e^{j2\pi(\nu t - \mu s - \xi\Omega)} dt ds d\xi \quad (66)$$

The Fourier transform relationships between the US forms of the various mutual coherence functions are shown by Figure 5. The non-singular factors in the US forms of the mutual coherence functions have the following physical interpretations.

$\hat{P}_g(t, s; \xi)$ Cross spectrum of pair $[T(f, t), T(f, s)]$ for fixed t and s .
 $\hat{P}_g(t, t; \xi)$ as a function of ξ is the power spectral density of the fluctuations of $T(f, t)$ in the variable f .

$\hat{R}_T(\Omega; t, s)$ Cross correlation of pair $[T(f, t), T(f, s)]$ for fixed t and s .

$\hat{R}_G(\Omega; \nu, \mu)$ Cross correlation of pair $[G(f, \nu), G(f, \mu)]$ for fixed ν and μ .

$\hat{P}_U(\xi; \nu, \mu)$ Cross spectrum of pair $[G(f, \nu), G(f, \mu)]$ for fixed ν and μ .
 $\hat{P}_U(\xi; \nu, \nu)$ as a function of ξ is the power spectral density of the fluctuations of $G(f, \nu)$ in the variable f .

The US assumption affects the nature of the system functions. $g(t, \xi)$ and $U(\xi, \nu)$ have the character of non-stationary white noise in the variable ξ . $G(f, \nu)$ and $T(f, t)$ have the character of stationary colored noise in the variable f .

The WSS and US assumptions taken together is called the WSSUS assumption. The WSSUS form of the mutual coherence functions, denoted with an over-bar through-out the sequel, are given below.

$$R_g(t, s; \xi, \eta) = \bar{P}_g(s-t; \xi) \delta(\xi - \eta) \quad (67)$$

$$R_T(f, l; t, s) = \bar{R}_T(l-f; s-t) \quad (68)$$

$$R_G(f, l; \nu, \mu) = \bar{P}_G(l-f; \nu) \delta(\mu - \nu) \quad (69)$$

$$R_U(\xi, \eta; \nu, \mu) = \bar{P}_U(\xi; \nu) \delta(\mu - \nu) \delta(\xi - \eta) \quad (70)$$

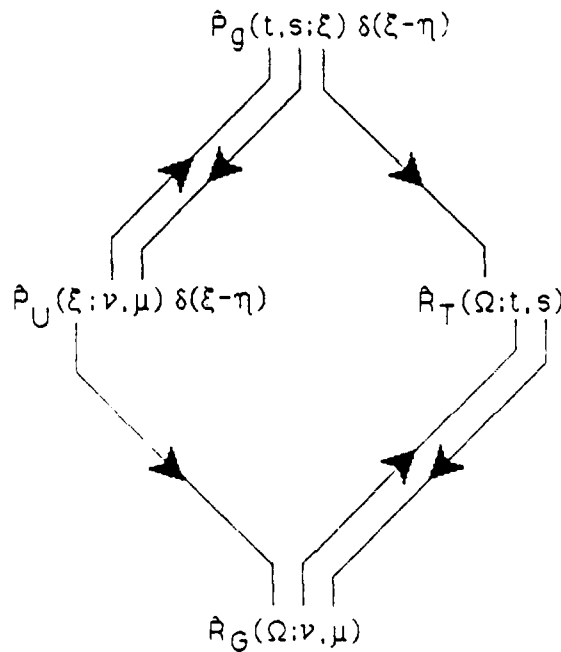


Figure 5. Fourier transform relationships between the US forms of the mutual coherence functions.

Substituting $l+\Omega$ for f and $s+\tau$ for t allows the WSSUS forms of Equations 23 through 34 to be written as follows.

$$\bar{P}_g(\tau; \xi) = \int \bar{R}_T(\Omega; \tau) e^{j2\pi\xi\Omega} d\Omega \quad (71)$$

$$\bar{P}_g(\tau; \xi) = \int \bar{P}_U(\xi; \nu) e^{j2\pi\nu\tau} d\nu \quad (72)$$

$$\bar{P}_g(\tau; \xi) = \int \int \bar{R}_G(\Omega; \nu) e^{j2\pi(\nu\tau + \xi\Omega)} d\nu d\Omega \quad (73)$$

$$\bar{R}_T(\Omega; \tau) = \int \bar{P}_g(\tau; \xi) e^{-j2\pi\xi\Omega} d\xi \quad (74)$$

$$\bar{R}_T(\Omega; \tau) = \int \bar{P}_G(\Omega; \nu) e^{j2\pi\nu\tau} d\nu \quad (75)$$

$$\bar{R}_T(\Omega; \tau) = \int \int \bar{P}_U(\xi; \nu) e^{j2\pi(\nu\tau - \xi\Omega)} d\nu d\xi \quad (76)$$

$$\bar{P}_U(\xi; \nu) = \int \hat{P}_g(\tau; \xi) e^{-j2\pi\nu\tau} d\tau \quad (77)$$

$$\bar{P}_U(\xi; \nu) = \int \hat{R}_G(\Omega; \nu) e^{j2\pi\xi\Omega} d\Omega \quad (78)$$

$$\bar{P}_U(\xi; \nu) = \int \int \hat{R}_T(\Omega; \tau) e^{j2\pi(\xi\Omega - \nu\tau)} d\Omega d\tau \quad (79)$$

$$\hat{R}_G(\Omega; \nu, \mu) = \int \int \hat{R}_T(\Omega; t, s) e^{j2\pi(\nu t - \mu s)} dt ds \quad (80)$$

$$\hat{R}_G(\Omega; \nu, \mu) = \int \hat{P}_U(\xi; \nu, \mu) e^{j2\pi\xi\Omega} d\xi \quad (81)$$

$$\hat{R}_G(\Omega; \nu, \mu) = \int \int \hat{P}_g(t, s; \xi) e^{j2\pi(\nu t - \mu s - \xi\Omega)} dt ds d\xi \quad (82)$$

The Fourier transform relationships between these are shown schematically in Figure 6. The non-singular factors in the WSSUS forms of the mutual coherence functions have the following physical interpretations.

- $\bar{P}_g(\tau; \xi)$ Delay cross-power spectral density. $\bar{P}_g(0; \xi)$ is the Delay power density spectrum.
- $\bar{R}_T(\Omega; \tau)$ Time-frequency correlation function.
- $\bar{P}_G(\Omega; \nu)$ Doppler cross-power spectral density. $\bar{P}_G(0; \nu)$ is the Doppler power density spectrum.
- $\bar{P}_U(\xi; \nu)$ Scattering function. The power spectral density in delay and Doppler of $T(f, t)$

The WSSUS assumption affects the nature of the system functions. $g(t, \xi)$ has the character of non-stationary white noise in the variable ξ and stationary colored noise in the variable t . $U(\xi, \nu)$ has the character of non-stationary white noise in both variables. $G(f, \nu)$ has the character of non-stationary white noise in the variable ν and stationary colored noise processes in the variable f . $T(f, t)$ has the character of stationary colored noise in both variables.

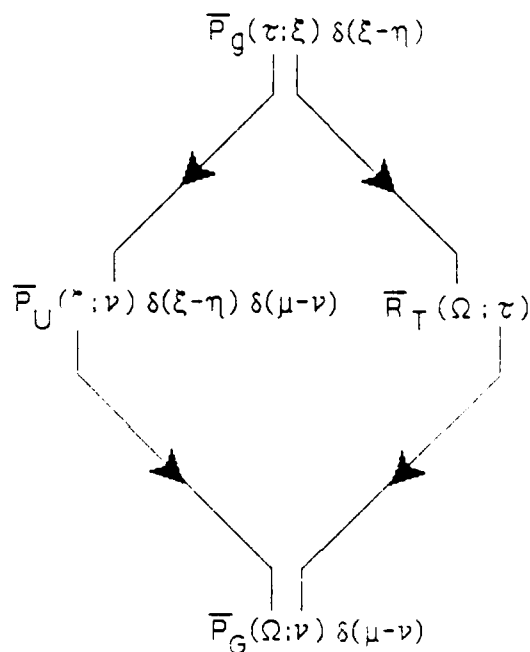


Figure 6. Fourier transform relationships between the WSSUS forms of the mutual coherence functions.

SECTION 3 SEPARABLE FORMS OF THE MUTUAL COHERENCE FUNCTIONS

3.1 TIME-SEPARABLE FORMS.

Recall from Equations 37 and 38 that the WSS assumption leads to a singularity of the form $\delta(\mu-\nu)$ in the mutual coherence functions R_G and R_U . Here we consider replacing $\delta(\mu-\nu)$ in Equations 37 and 38 with some arbitrary non-singular factor, denoted $\tilde{\Gamma}(\mu-\nu)$. Then we show the corresponding effects upon R_T and R_G . We start with a pair of functions, say $\tilde{\chi}(\cdot)$ and $\tilde{\Gamma}(\cdot)$, that are related by the following one-dimensional Fourier transformation.

$$\tilde{\Gamma}(x) = \int \tilde{\chi}(s) e^{-j2\pi xs} ds \quad (83)$$

From Equation 27 we have

$$R_T(f, l; t, s) = \int \int R_G(f, l; \nu, \mu) e^{j2\pi(\mu s - \nu t)} d\nu d\mu \quad (84)$$

Presuming $R_G(f, l; \nu, \mu)$ is in the form $\tilde{P}_G(f, l; \nu) \tilde{\Gamma}(\mu - \nu)$, we can write

$$R_T(f, l; t, s) = \int \tilde{P}_G(f, l; \nu) \left[\int \tilde{\Gamma}(\mu - \nu) e^{j2\pi(\mu s - \nu t)} d\mu \right] d\nu \quad (85)$$

Replace $\mu s - \nu t$ with $\nu(s - t) + (\mu - \nu)s$ to get the following equation.

$$R_T(f, l; t, s) = \int \tilde{P}_G(f, l; \nu) e^{j2\pi\nu(s - t)} \left[\int \tilde{\Gamma}(\mu - \nu) e^{j2\pi(\mu - \nu)s} d\mu \right] d\nu \quad (86)$$

Substituting $\nu + \beta$ for μ and $d\beta$ for $d\mu$ yields

$$R_T(f, l; t, s) = \int \tilde{P}_G(f, l; \nu) e^{j2\pi\nu(s - t)} d\nu \int \tilde{\Gamma}(\beta) e^{j2\pi\beta s} d\beta \quad (87)$$

From Equation 43 and 83 we get

$$R_T(f, l; t, s) = \tilde{R}_T(f, l; s - t) \tilde{\chi}(s) \quad (88)$$

Therefore replacing the singularity $\delta(\mu - \nu)$ in the WSS form of R_G with some

non-singular function $\tilde{\Gamma}(\mu-\nu)$ introduces the solely time-dependent factor $\tilde{\mathcal{S}}(s)$ into the WSS form of R_T . Equation 88 could be rewritten as

$$R_T(f,l;s-\tau,s) = \tilde{R}_T(f,l;\tau) \tilde{\mathcal{S}}(s), \quad (89)$$

so that all of the s , or time, dependence has been separated out of \tilde{R}_T and is in $\tilde{\mathcal{S}}(s)$. A similar treatment relates the time-separable and WSS forms of the remaining mutual coherence functions. The following equations show all of the time-separable forms and should be compared directly with Equations 35 through 38.

$$R_g(t,s;\xi,\eta) = \tilde{R}_g(s-t;\xi,\eta) \tilde{\mathcal{S}}(s) \quad (90)$$

$$R_T(f,l;t,s) = \tilde{R}_T(f,l;s-t) \tilde{\mathcal{S}}(s) \quad (91)$$

$$R_g(f,l;\nu,\mu) = \tilde{R}_g(f,l;\nu) \tilde{\Gamma}(\mu-\nu) \quad (92)$$

$$R_g(\xi,\eta;\nu,\mu) = \tilde{R}_g(\xi,\eta;\nu) \tilde{\Gamma}(\mu-\nu) \quad (93)$$

The WSS form then is just a special case of the time-separable form, in particular, when $\tilde{\mathcal{S}}(s) \equiv 1$ and $\tilde{\Gamma}(\mu-\nu) = \delta(\mu-\nu)$. Figure 7 is the more general time-separable version of Figure 4. The functions \tilde{R}_g , \tilde{R}_T , \tilde{R}_g and \tilde{R}_g are still related mathematically by Equations 39 through 50.

3.2 FREQUENCY-SEPARABLE FORMS.

Recall from Equations 51 and 54 that the US assumption leads to a singularity of the form $\delta(\xi-\eta)$ in the mutual coherence functions R_g and R_g . Here we consider replacing $\delta(\xi-\eta)$ in Equations 15 and 18 with some arbitrary non-singular factor, denoted $\tilde{\Gamma}(\xi-\eta)$. Then we show the corresponding effects upon R_T and R_g . We start with a pair of functions, say $\mathcal{S}(\cdot)$ and $\tilde{\Gamma}(\cdot)$, that are related by the following one-dimensional Fourier transformation.

$$\tilde{\Gamma}(x) = \int \mathcal{S}(l) e^{-j2\pi xl} dl \quad (94)$$

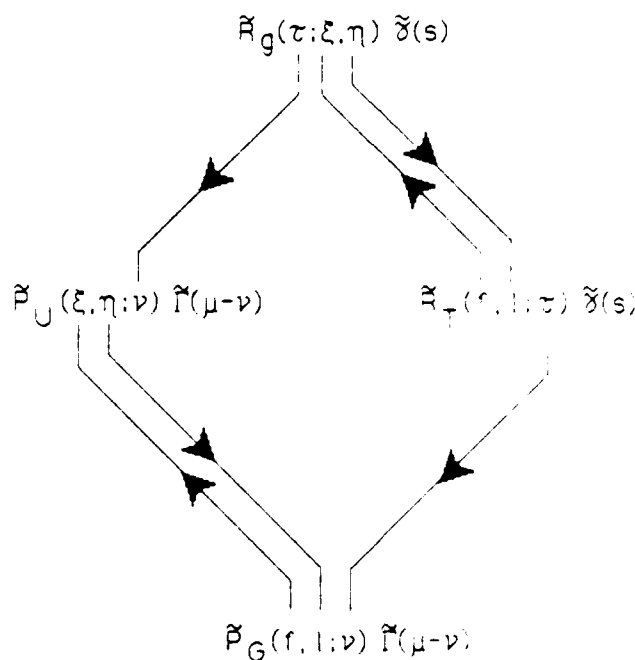


Figure 7. Fourier transform relationships between the time-separable forms of the mutual coherence functions.

From Equation 26 we have

$$R_T(f, l; t, s) = \iint R_g(t, s; \xi, \eta) e^{j2\pi(\xi f - \eta l)} d\xi d\eta \quad (95)$$

Presuming $R_g(t, s; \xi, \eta)$ is in the form $\hat{P}_g(t, s; \xi) \hat{\Gamma}(\xi - \eta)$, we can write

$$R_T(f, l; t, s) = \int \hat{P}_g(t, s; \xi) \left[\int \hat{\Gamma}(\xi - \eta) e^{j2\pi(\xi f - \eta l)} d\eta \right] d\xi \quad (96)$$

Replace $\xi f - \eta l$ with $-\xi(l-f) + (\xi - \eta)l$ to get the following equation.

$$R_T(f, l; t, s) = \int \hat{P}_g(t, s; \xi) e^{-j2\pi\xi(l-f)} \left[\int \hat{\Gamma}(\xi - \eta) e^{j2\pi(\xi - \eta)l} d\eta \right] d\xi \quad (97)$$

Substituting $\xi - \beta$ for η and $-d\beta$ for $d\eta$ yields

$$R_T(f, l; t, s) = \int \hat{P}_g(t, s; \xi) e^{-j2\pi\xi(l-f)} d\xi \int \hat{\Gamma}(\beta) e^{j2\pi\beta l} d\beta \quad (98)$$

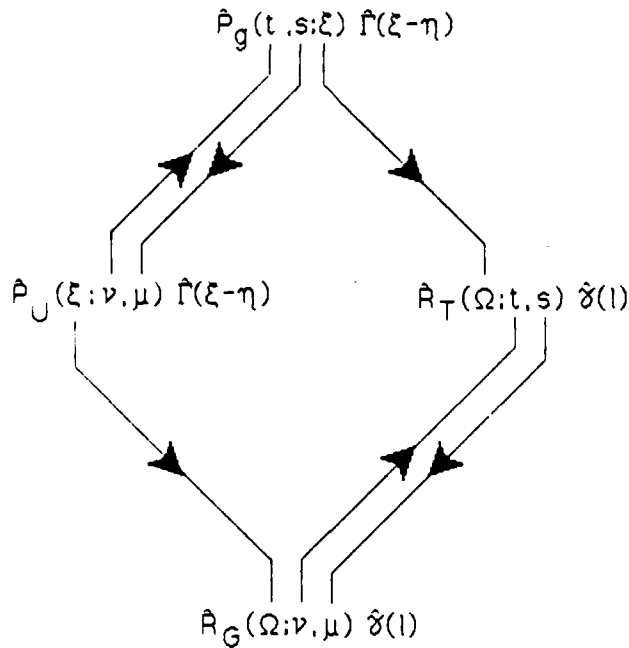


Figure 8. Fourier transform relationships between the frequency-separable forms of the mutual coherence functions.

From Equations 58 and 94 we get

$$R_T(f, l; t, s) = \hat{R}_T(l-f; t, s) \hat{\mathcal{J}}(l) \quad (99)$$

Therefore replacing the singularity $\delta(\xi - \eta)$ in the US form of \hat{P}_g with some non-singular function $\hat{\Gamma}(\xi - \eta)$ introduces the solely frequency-dependent factor $\hat{\mathcal{J}}(l)$ into the US form of R_T . Equation 99 could be rewritten as

$$R_T(l - \Omega, l; t, s) = \hat{R}_T(\Omega; t, s) \hat{\mathcal{J}}(l), \quad (100)$$

so that all of the l , or frequency, dependence has been separated out of \hat{R}_T and is in $\hat{\mathcal{J}}(l)$. A similar treatment relates the frequency-separable and US forms of the remaining mutual coherence functions. The following equations show all of the frequency-separable forms and should be compared directly with Equations 51 through 54.

$$R_g(t, s; \xi, \eta) = \hat{P}_g(t, s; \xi) \hat{\Gamma}(\xi - \eta) \quad (101)$$

$$R_T(f, l; t, s) = \hat{R}_T(l-f; t, s) \hat{\mathcal{J}}(l) \quad (102)$$

$$R_G(f, l; \nu, \mu) = R_G(l-f; \nu, \mu) \delta(l) \quad (103)$$

$$R_U(\xi, \eta; \nu, \mu) = \hat{P}_U(\xi; \nu, \mu) \hat{\Gamma}(\xi - \eta) \quad (104)$$

The US form then is just a special case of the frequency-separable form, in particular, when $\delta(l) \equiv 1$ and $\hat{\Gamma}(\xi - \eta) = \delta(\xi - \eta)$. Figure 8 is the more general frequency-separable version of Figure 5. The functions \hat{P}_G , \hat{R}_T , \hat{R}_G and \hat{P}_U are still related mathematically by Equations 25 through 66.

3.3 TIME- AND FREQUENCY-SEPARABLE FORMS.

The derivations of the two previous subsections can easily be drawn together to provide the time- and frequency-separable forms of the mutual coherence functions in terms of the WSSUS forms and the arbitrary Fourier transform functional pairs $[\tilde{\chi}(\cdot), \tilde{\Gamma}(\cdot)]$ and $[\hat{\chi}(\cdot), \hat{\Gamma}(\cdot)]$. These forms are shown in the following four equations and should be compared with Equations 67 through 70.

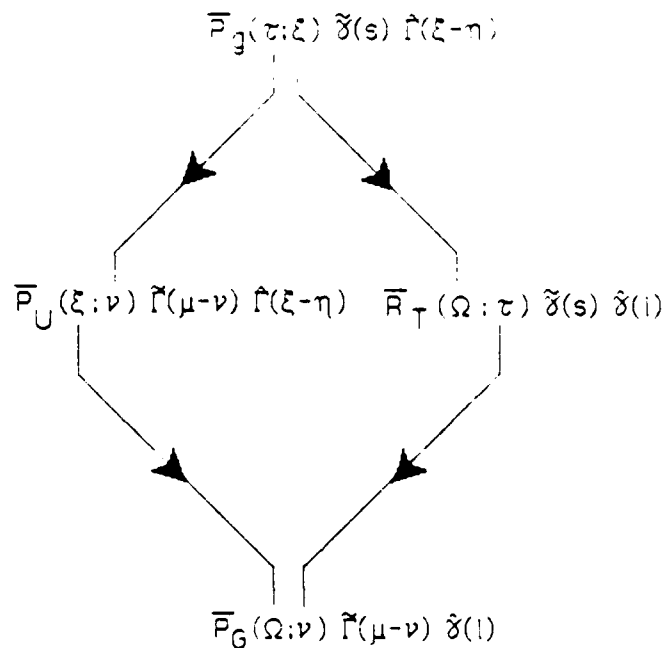


Figure 9. Fourier transform relationships between the time- and frequency-separable forms of the mutual coherence functions.

$$R_G(t,s;\xi,\eta) = P_G(s-t;\xi) \Gamma(\xi-\eta) \chi(s) \quad (105)$$

$$R_T(f,l;t,s) = \bar{R}_T(1-f;s-t) \tilde{\chi}(s) \hat{\chi}(l) \quad (106)$$

$$R_G(f,l;\nu,\mu) = \bar{P}_G(1-f;\nu) \tilde{\Gamma}(\mu-\nu) \hat{\chi}(l) \quad (107)$$

$$R_U(\xi,\eta;\nu,\mu) = \bar{P}_U(\xi;\nu) \tilde{\Gamma}(\mu-\nu) \hat{\Gamma}(\xi-\eta) \quad (108)$$

The WSSUS form is just a special case of the time- and frequency-separable form. Figure 9 is the more general version of Figure 6. The functions \bar{P}_G , \bar{R}_T , \bar{P}_G and \bar{P}_U are still related mathematically by Equations 71 through 82.

3.4 UTILITY OF THE SEPARABLE FORMS FOR THE DNS CHANNEL MODEL.

The general forms of the four mutual coherence functions, defined previously by Equations 19 through 22, provide complete four dimensional statistical characterizations of the channel. These forms are clearly non-stationary in time and frequency, but do not lend themselves to an intuitively appealing, or even manageable, parameterization. The assumption of separability has provided the more tractable, but less general, forms of the mutual coherence functions given by Equations 105 through 108. These forms are too restrictive to be used for direct characterization of arbitrary variation of the channel statistics in time and frequency. However, these do provide a formalism to turn the channel off over any region in the frequency-time space. This capability provides the basis for very general non-stationary channel models.

Consider the separable form of R_T given by Equation 106. With two variable substitutions, $\Omega = 1 - f$ and $\tau = s - t$, this equation can be rewritten as

$$R_T(1-\Omega,l;s-\tau,s) = \bar{R}_T(\Omega;\tau) \tilde{\chi}(s) \hat{\chi}(l) \quad (109)$$

$\bar{R}_T(\Omega;\tau)$ specifies a WSSUS channel model, and thereby contains the requisite information to model the channel over any range of time and carrier frequency that the channel remains approximately stationary. The $\tilde{\chi}(s) \hat{\chi}(l)$ factors provide the flexibility to drive the right side of Equation 109 to zero at arbitrary regions of time and carrier frequency, even when Ω and τ equal 0. A value of zero corresponds to no energy being propagated.

In the next section, an array of channel models will be combined to form a composite channel. Each will have distinct $\tilde{\chi}(s) \tilde{\chi}(l)$ factors to impede energy propagation outside overlapping regions of time and carrier frequency. The $\overline{R}_T(\Omega;\tau)$ factor for each component model will provide a WSSUS approximation of the channel statistics within the associated time-frequency regions of non-zero $\tilde{\chi}(s) \tilde{\chi}(l)$. This non-stationary composite model provides an intuitively appealing means to parameterize the channel -- by specifying a two dimensional WSSUS characterization, in the form of $\overline{R}_T(\Omega;\tau)$, from region to region over arbitrarily large intervals of time and carrier frequency.

SECTION 4 THE DOUBLY NON-STATIONARY (DNS) CHANNEL MODEL

Recall from Section 2.3 that a WSSUS channel model can be specified by any one of the four two-dimensional mutual coherence functions. The most intuitively appealing of these parameterizations is the scattering function, denoted \bar{P}_U in Section 2.3, a two-dimensional power spectrum in the delay-doppler domain, exhibiting no variation in time or carrier frequency. This section presents a non-stationary channel model that allows gradual variation in the scattering function with time and frequency. The "gradual" restriction on the variation of the scattering function with frequency and time allows the channel representation over any small region of time and frequency to be well approximated with the WSSUS formalism. A channel having this locally stationary characteristic is sometimes called a quasi-WSSUS or QWSSUS channel.

In this development of the non-stationary channel representation, direct use of the scattering function is inconvenient, and we will instead use its double Fourier transform, denoted previously by \bar{R}_T . In addition the frequency-time domain is allowed to be arbitrarily large, but is partitioned into a rectangular grid with the frequency granulation indexed by m and the time granulation indexed by n as shown in Figure 10.

4.1 COMBINING MULTIPLE WSSUS SYSTEM FUNCTIONS TO OBTAIN DNS SYSTEM FUNCTIONS.

We assume that an independent random WSSUS system function, say the time-variant transfer function, is available for each grid line crossing point. Each such transfer function exhibits distinct statistics completely specified by one of the mutual coherence functions, for example \bar{R}_T . For this development, we augment the notational conventions for system functions and mutual coherence functions with the grid point indices. Thus the time-variant transfer function at the (m,n) th grid point is denoted as $T_{n,m}(f,t)$, and the corresponding WSSUS mutual coherence function is denoted as $\bar{R}_T|_{n,m}(\Omega,\tau)$, which in accord with Equations 8 and 20, and the augmented notation for T , is written as

$$\overline{R}_T|_{n,m}(\Omega, \tau) = \overline{T_{n,m}(1-\Omega, s-\tau)^* T_{n,m}(1, s)} . \quad (110)$$

The time and frequency origin of $T_{n,m}(f, t)$ is taken relative to the associated grid point coordinates T_n and F_m . We desire to combine the independent time-variant transfer functions, variously offset in time and frequency to their associated grid points, in such a way that the resulting composite time-varying transfer functions satisfy the following objectives:

1. The composite system function should be QWSSUS over time-frequency regions that are small in comparison to the local grid cell size.
2. In the vicinity of a grid point, the composite system function should exhibit QWSSUS statistics that match the WSSUS mutual coherence function corresponding to that grid point.
3. The QWSSUS statistics of the composite system function should vary smoothly between the grid lines.
4. If identical WSSUS mutual coherence functions are specified for every grid point, the composite system functions should degenerate back to a WSSUS channel characterization spanning the entire grid.

The composite system functions require a few more extensions to our notational conventions. We denote the absolute frequency variable as f_a , which is the abscissa of Figure 10, and the absolute time variable as t_a , which is the ordinate of Figure 10. These are related to the the grid point relative frequency and time variables, f and t , by offsets to the associated grid point co-ordinates by $f = f_a - F_m$ and $t = t_a - T_n$. Also, system functions that are not assumed to have WSSUS statistics are expressed in bold type. This will allow, for example, the system functions $T_{n,m}(f, t)$ and $\overline{T}_{n,m}(f, t)$ to be defined differently below.

The above objectives can be well satisfied by a composite time-variant transfer function of the form

$$T(f_a, t_a) = \sum_{m=0}^M \sum_{n=0}^N w_{n,m}(f_a - F_m, t_a - T_n) T_{n,m}(f_a - F_m, t_a - T_n) \quad (111)$$

where $w_{n,m}(f_a - F_m, t_a - T_n)$ is assumed to be everywhere real and positive. We now explore the suitability of this definition of $T(f_a, t_a)$. First define the weighted, and therefore non-WSSUS, transfer function $T_{n,m}(f, t)$ as

$$T_{n,m}(f, t) = w_{n,m}(f, t) T_{n,m}(f, t) . \quad (112)$$

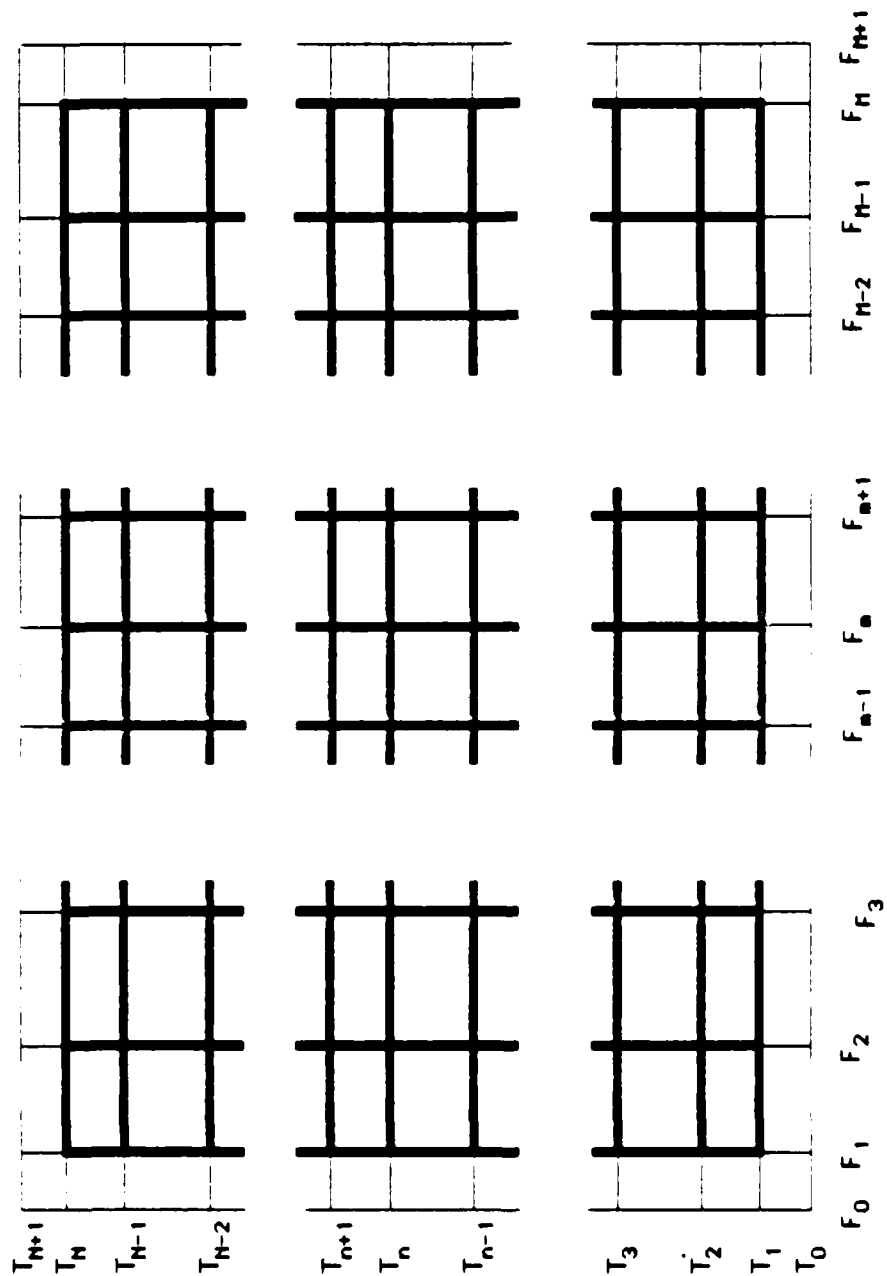


Figure 10 Diagram of grid partitioning of large region of the time-frequency space

Its mutual coherence function is then given by

$$R_T|_{n,m}(f,l;t,s) = \overline{T_{n,m}(f,t)^* T_{n,m}(l,s)} \quad , \text{ or} \quad (113)$$

$$R_T|_{n,m}(l-\Omega,l;s-\tau,s) = \overline{T_{n,m}(l-\Omega,s-\tau)^* T_{n,m}(l,s)} \quad (114)$$

$$= w_{n,m}(l-\Omega,s-\tau) w_{n,m}(l,s) \overline{R_T|_{n,m}(\Omega,\tau)} \quad (115)$$

Then Equation 111 can be rewritten in terms of $T_{n,m}$ as follows.

$$T(f_a,t_a) = \sum_{m=0}^M \sum_{n=0}^N T_{n,m}(f_a-F_m,t_a-T_n) \quad (116)$$

Its mutual coherence function is given by the following equation.

$$R_T(f_a,l_a;t_a,s_a) = \sum_{m=0}^M \sum_{n=0}^N \sum_{m=0}^M \sum_{n=0}^N \overline{T_{n,m}(f_a-F_m,t_a-T_n)^* T_{n',m'}(l_a-F_{m'},s_a-T_{n'})} \quad (117)$$

Since the time-variant transfer functions associated with distinct grid points are independent, the above equation can be re-written as

$$R_T(f_a,l_a;t_a,s_a) = \sum_{m=0}^M \sum_{n=0}^N R_T|_{n,m}(f_a-F_m,l_a-F_m;t_a-T_n,s_a-T_n) \quad (118)$$

or, using the substitution of Equation 111 and replacing f_a and t_a with $l_a-\Omega$ and $s_a-\tau$, respectively, one arrives Equation 119 below.

$$R_T(l_a-\Omega,l_a;s_a-\tau,s_a) = \sum_{m=0}^M \sum_{n=0}^N w_{n,m}(l_a-F_m-\Omega,s_a-T_n-\tau) w_{n,m}(l_a-F_m,s_a-T_n) \overline{R_T|_{n,m}(\Omega,\tau)} \quad (119)$$

Now the character of w needed to satisfy our objectives can be ascertained. For all channels of interest, it can be assumed that there exists some Ω_{\max} and τ_{\max} called respectively the maximum correlation frequency and the maximum

correlation time, for which $\bar{R}_T|_{n,m}(\Omega, \tau)$ is essentially zero when $|\Omega|$ is less than Ω_{\max} or $|\tau|$ is less than τ_{\max} . We henceforth require that the weighting function w is chosen so that

$$w_{n,m}^2(l, s) \approx w_{n,m}(l - \Omega, s - \tau) w_{n,m}(l, s) \quad (120)$$

for all n, m, l, s and all Ω and τ such that $|\Omega| < \Omega_{\max}$ or $|\tau| < \tau_{\max}$. That is, the right side of Equation 120 may not change appreciably for values of Ω and τ for which the amplitude of $\bar{R}_T|_{n,m}(\Omega, \tau)$ much exceeds zero. For this restriction on the choice of w , Equations 115 and 119 can be rewritten as follows.

$$R_T|_{n,m}(l - \Omega, l; s - \tau, s) \approx w_{n,m}^2(l, s) \bar{R}_T|_{n,m}(\Omega, \tau) \quad (121)$$

$$R_T(l_a - \Omega, l_a; s_a - \tau, s_a) \approx \sum_{m=0}^M \sum_{n=0}^N w_{n,m}^2(l_a - F_m, s_a - T_n) \bar{R}_T|_{n,m}(\Omega, \tau) \quad (122)$$

A comparison of Equation 121 with Equation 109 of Section 3.3 indicates that the non-WSSUS mutual coherence function for each weighted WSSUS system function at any grid point can be put in time- and frequency- separable form. The choice of w , within the restrictions stated above, allows complete control over the separable factors $\tilde{\gamma}(s)$ and $\tilde{\gamma}(l)$, and their associated Fourier transforms, $\tilde{\Gamma}(\mu - \nu)$ and $\tilde{\Gamma}(\xi - \eta)$, discussed in Section 3. These four factors are henceforth denoted $\tilde{\gamma}_n(s)$, $\tilde{\gamma}_m(l)$, $\tilde{\Gamma}_n(\mu - \nu)$ and $\tilde{\Gamma}_m(\xi - \eta)$ to conform to the notation of this section, and to emphasize that one is free to vary these separable factors for each point in the grid. Each $w_{n,m}(l, s)$ should be chosen equal to the square root of the desired product of $\tilde{\gamma}_n(s)$ and $\tilde{\gamma}_m(l)$ at the (n, m) th grid point.

4.2 CHOICE OF WEIGHTING FUNCTION w LEADING TO PIECE-WISE-LINEAR VARIATION OF R_T IN TIME AND FREQUENCY.

$R_T(f_a, l_a; t_a, s_a)$ will vary in a piece-wise-linear fashion in time and carrier frequency if the time and frequency dependent factors of the separable form of $R_T|_{n,m}$ at each grid point takes the following form.

$$\begin{aligned} \tilde{\gamma}_n(s) = & \quad \underline{\wedge}(s/\Delta T_n^\uparrow) \text{Rect}(s/\Delta T_n^\uparrow - 0.5) + \\ & \quad \underline{\wedge}(s/\Delta T_n^\downarrow) \text{Rect}(s/\Delta T_n^\downarrow + 0.5) \end{aligned} \quad (123)$$

$$\tilde{\gamma}_m(l) = \triangleleft\{1/\Delta F_m^\uparrow\} \text{Rect}\{1/\Delta F_m^\uparrow - 0.5\} + \triangleleft\{1/\Delta F_m^\downarrow\} \text{Rect}\{1/\Delta F_m^\downarrow + 0.5\} \quad (124)$$

where

$$\Delta T_n^\uparrow = T_{n+1} - T_n = \Delta T_{n+1}^\downarrow \quad (125)$$

$$\Delta T_n^\downarrow = T_n - T_{n-1} = \Delta T_{n-1}^\uparrow \quad (126)$$

$$\Delta F_m^\uparrow = F_{m+1} - F_m = \Delta F_{n+1}^\downarrow \quad (127)$$

$$\Delta F_m^\downarrow = F_m - F_{m-1} = \Delta F_{n-1}^\uparrow \quad (128)$$

and $\triangleleft\{\cdot\}$ and $\text{Rect}\{\cdot\}$ are defined below.

$$\triangleleft\{x\} = \begin{cases} 1 - |x| & \text{for } -1 < x \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (129)$$

$$\text{Rect}\{x\} = \begin{cases} 1 & \text{for } -1/2 < x \leq 1/2 \\ 0 & \text{otherwise} \end{cases} \quad (130)$$

The weighting function $w_{n,m}$, resulting from the above form of $\tilde{\gamma}_n(s)$ and $\tilde{\gamma}_m(l)$, is given by the following equations.

$$w_{n,m}(l, s) = [\tilde{\gamma}_n(s) \tilde{\gamma}_m(l)]^{1/2} = \quad (131)$$

$$\left[\begin{array}{c} \triangleleft\{s/\Delta T_n^\uparrow\} \text{Rect}\{s/\Delta T_n^\uparrow - 0.5\} \\ \triangleleft\{s/\Delta T_n^\downarrow\} \text{Rect}\{s/\Delta T_n^\downarrow + 0.5\} \end{array} \right]^{1/2} \times \left[\begin{array}{c} \triangleleft\{1/\Delta F_m^\uparrow\} \text{Rect}\{1/\Delta F_m^\uparrow - 0.5\} \\ \triangleleft\{1/\Delta F_m^\downarrow\} \text{Rect}\{1/\Delta F_m^\downarrow + 0.5\} \end{array} \right]^{1/2}$$

SECTION 5

LIST OF REFERENCES

1. Bello, P.A. , "Characterization of randomly time-variant linear channels," IEEE Trans. Commun. Syst., vol. CS-11, pp 360-393, December 1963.
2. Sawyer, B. E. , "A computer implementation of a doubly non-stationary channel model for simulation-aided analyses of communication systems," Mission Research Corporation technical note MRC-N-724, April 1986.

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