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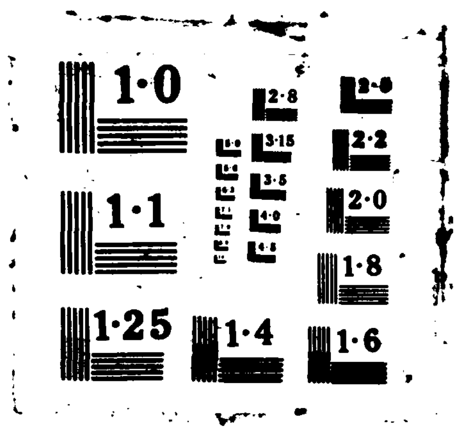
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ON THE INTENSITY OF CROSSINGS BY A SHOT NOISE PROCESS

by

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ON THE INTENSITY OF CROSSINGS BY A SHOT NOISE PROCESS

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Summary. The crossing intensity of a level by a shot noise process with a monotone impulse response is studied. It is shown that the intensity can be naturally expressed in terms of a marginal probability. Also some examples are given to illustrate how the marginal probability can be obtained.

AMS 1980 Subject Classification: 60K99.

Key Words and Phrases: Level crossing, shot noise.

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1. Introduction.

Consider the shot noise process

$$X(t) = \sum_{\tau \leq t} h(t - \tau), \quad t \in R,$$

where the τ's are the points of a stationary Poisson process on R with mean rate $\lambda > 0$, and h, the impulse response, is a non-negative function on $[0,\infty)$ such that

i) h is non-increasing,

ii) h is finite except possibly at zero, and

iii) $\int_u^\infty h(x)dx < \infty$ for some large u.

By Daley (1971), Theorem 1, the conditions (ii) and (iii) ensure that $X(t) < \infty$ a.s. for each t.

Observe that the sample function of X increases only at the points of η. Thus it is unambiguous to define that X upcrosses the level u at t, where $u \geq 0$, if $X(t-) \leq u$ and $X(t) > u$. For $u \geq 0$, write N_u for the point process (cf. Kallenberg (1976)) that consists of the points at which upcrossings of level u by X occur. Thus for each Borel set B, $N_u(B)$ denotes the number of upcrossings of u by X in B. N_u is a stationary point process, which may be viewed as a thinned process of η. The purpose of this paper is to derive the following result.

Theorem 1. For each $u \geq 1$, $EN_u[0,1] = \lambda P[u - h(0) < X(0) \leq u]$.

Note that the "downcrossing" intensity of a level by X is also given by Theorem 1.

It is worth mentioning that similar problems were treated by Rice (1944), and Bar-David and Nemirovsky (1972) in other settings. A result in the latter paper can be reduced to one which is similar to Theorem 1. However, our assumptions on h are considerably simpler.

We prove Theorem 1 in Section 2 using an approach which appears to be most natural for the present purpose. In Section 3, we illustrate the manner in which Theorem 1 can be made useful for a number of situations.

2. Derivation.

It is convenient to enumerate the points of η in $(-\infty, 0)$ by letting ρ_i be the ith largest point of η to the left of zero for $i = 1,2,\ldots$. The ρ_i are well-defined with probability one (w.p.1), and $-\rho_1$, $\rho_1 - \rho_2$, $\rho_2 - \rho_3, \ldots$ are independent and identically distributed (i.i.d.) exponential random variables. The following result is useful.

Lemma 2. For each $i = 1,2,\ldots$, $P[X(\rho_i -) = \sum_{j \geq i+1} h(\rho_i - \rho_j)] = 1$ where $X(\rho_i -)$ denotes the left-hand limit of X at ρ_i. From this, it follows immediately that $X(\rho_i -)$ is independent of ρ_i, and $X(\rho_i -)$ has the same distribution as $X(0)$.

Proof. Let $i \geq 1$ be fixed. Since h is monotone, it is almost everywhere continuous. Using the continuity of $\rho_i - \rho_j$, $j \geq i + 1$, we obtain

$$\lim_{\varepsilon \downarrow 0} h(\rho_i - \rho_j - \varepsilon) = h(\rho_i - \rho_j) \text{ w.p.1} \quad \text{for } j \geq i + 1.$$

Also by the monotonicity of h, $h(\rho_i - \rho_j - \varepsilon) \leq h(\rho_{i+1} - \rho_j)$ for

$0 < \varepsilon < \rho_i - \rho_{i+1}$, $j \geq i + 2$, where $\sum_{j \geq i+2} h(\rho_{i+1} - \rho_j)$ is equal in distribution to $X(0)$ which is finite w.p.1. Thus it follows from dominated convergence that w.p.1,

$$\lim_{\varepsilon \downarrow 0} X(\rho_i - \varepsilon) = \lim_{\varepsilon \downarrow 0} \sum_{j \geq i+1} h(\rho_i - \rho_j - \varepsilon) = \sum_{j \geq i+1} h(\rho_i - \rho_j) . \quad \square$$

Proof of Theorem 1. By stationarity, it apparently suffices to show that $EN_u(B)$ equals $\lambda m(B)P[u - h(0) < X(0) \leq u]$ for each Borel set B in $(-\infty, 0)$, where $m(B)$ denotes the Lebesgue measure of B. Since $X(\rho_i) = h(0) + \sum_{j \geq i+1} h(\rho_i - \rho_j)$, it follows from Lemma 2 that w.p.1,

$$N_u(B) = \sum_{i \geq 1} 1(u - h(0) < X(\rho_i -) \leq u, \ \rho_i \in B),$$

where $1(\cdot)$ is the indicator function. Applying the facts that $X(\rho_i -)$ is independent of ρ_i and $X(\rho_i -)$ is equal in distribution to $X(0)$, we get

$$EN_u(B) = \sum_{i \geq 1} E1(u - h(0) < X(\rho_i -) \leq u) E1(\rho_i \in B)$$

$$= P[u - h(0) < X(0) \leq u]\lambda m(B). \quad \square$$

3. Marginal Distribution.

The usefulness of Theorem 1 obviously depends on the availability of the marginal probability $P[u - h(0) < X(0) \leq u]$. The Laplace transform of $X(0)$ is (cf. Gilbert and Pollak (1960))

(3.1) $$L(s) = Ee^{-sX(0)} = exp\{-\lambda \int_0^\infty (1 - e^{-sh(x)})dx\}, \quad s \geq 0.$$

For some impulse responses h, the distribution of $X(0)$ can be expressed analytically, while for a class of others, a recursive method due to Gilbert and Pollak (1960) is applicable. If it is of interest to study the asymptotic crossing intensity for increasingly high levels, certain Tauberian theorems (cf. Embrechts et. al. (1985)) are useful. We consider three examples.

(a) Suppose $h(x) = \begin{cases} \infty, & x = 0 \\ -\log x, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$. Then

$$L(s) = exp\{-\lambda \int_0^\infty (1 - e^{-sx})e^{-x}dx\}, \quad s \geq 0,$$

which is the Laplace transform of the Bessel density (cf. Feller (1971)):

$$f(x) = e^{-(x+\lambda)} \sqrt{\frac{\lambda}{x}}\; I_1(2\sqrt{\lambda x}), \quad x > 0.$$

(b) For $h(x) = e^{-x}$, $x \geq 0$, Gilbert and Pollak (1960) showed that the density f of $X(0)$ can be obtained recursively as follows:

$$f(x) = \begin{cases} \dfrac{e^{-\lambda\gamma}}{\Gamma(\lambda)} x^{\lambda-1}, & 0 < x < 1, \\ \\ x^{\lambda-1}\left[\dfrac{e^{-\lambda\gamma}}{\Gamma(\lambda)} - \lambda \int_1^x f(y-1)y^{-\lambda}dy\right], & x \geq 1, \end{cases}$$

where γ is Euler's constant.

(c) Assume that h is boundedly supported, say, on $[0,1]$. Then by a change-of-variable, (3.1) becomes

$$L(s) = exp\{-\lambda + \lambda \int_{[0,\infty)} e^{-sy} \mu(dy)\}$$

where μ is a probability measure on $[0,\infty)$ such that

$$\mu(B) = \text{Lebesgue measure of } \{0 \le x \le 1 : h(x) \in B\}$$

for each Borel set B in $[0,\infty)$. Thus $X(0)$ has a compound Poisson distribution. For h satisfying certain regularity conditions, Embrecht et. al. (1985) showed that

$$P[X(0) > x] \sim \frac{exp\{-\lambda[1 - \psi(t)] - e^{-\lambda} - t(x - 1)\}}{t\sqrt{2\pi\lambda\psi''(t)}} \text{ as } x \to \infty$$

where $\psi(s) = \int e^{-su} \mu(du)$, and t satisfies $\lambda\psi'(t) = x$.

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