**Technical Report 718** 

## Setting Battalion Recruiting Missions

Abraham Nelson, Chester E. Phillips, and Edward J. Schmitz

Manpower and Personnel Policy Research Group Manpower and Personnel Research Laboratory







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## Setting Battalion Recruiting Missions

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FOREWORD

The Manpower and Personnel Policy Research Group is concerned with understanding the effect of Army personnel policies on enlistments. In recent research, recruiting missions have been found to influence the enlistment of "high-quality" individuals. This research examines recruiting missioning models to develop an alternative to the current system, which more closely models the production capability of individual recruiting battalions.

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EDGAR M. JOHNSON Technical Director

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#### EXECUTIVE SUMMARY

#### Requirement:

To improve the procedure for missioning the Army's recruiting battalions by developing a model that uses "efficient" recruiting missions as the basis for mission setting.

#### **Procedure:**

The model currently used by the Army projects enlistment contracts from a stepwise ordinary least squares (OLS) regression procedure. This procedure has several problems, including the extensive use of dummy variables, variables with unstable effects, and the fact that high-producing battalions are considered outliers rather than units to which other battalions should be compared.

This research uses data envelopment analysis (DEA) to determine efficient production for recruiting battalions. First, current missions are evaluated to determine whether current missioning is consistent. A set of efficient battalions is then estimated using past contract production.

#### Findings:

It was shown that the current stepwise OLS model does not set missions consistently across all recruiting battalions. Battalions were also not consistent in the number of high-quality contracts produced, with some battalions making better use of recruiting resources than others. A set of "efficient" missions were formulated. It was shown that missions for male NPS I-IIIA high school graduates should have been set lower while those for high school seniors could have been higher.

#### Utilization of Findings:

The analysis has shown that at current recruiting resource levels a redistribution would allow the Army to better monitor the relative performance of missions to produce a greater number of total contracts. Further, the model develops individual recruiting battalions. An extension of this research would enable the Army to set missions that assure maximum utilize ion of recruiting resources.

#### SETTING BATTALION RECRUITING MISSIONS

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#### SETTING BATTALION RECRUITING MISSIONS

#### I. INTRODUCTION

The U.S. Army Recruiting Command (USAREC) is responsible for recruiting 130,000 individuals yearly. This is accomplished with a budget of over 600 million dollars, which includes a "sales force" of 5000 recruiters. A key activity in the recruiting process is the establishment of recruiting missions for the 56 recruiting battalions managed by USAREC. These missions serve two major purposes. First, missions set the standard for battalion commander performance by describing the recruiting objectives that must be achieved during the next quarter. Mission achievement plays the critical role in the battalion commander's performance evaluation. A commander who consistently fails to achieve his assigned mission is not evaluated favorably.

Second, the missioning process also involves resource allocation and market appraisal. Ideally, the process should take into account the size of the available market, the ease of recruiting in that market, and the resources available to capitalize on the market.

In this paper, a new approach for setting missions is examined. A brief description of recent economic analyses of recruiting productivity and the present missioning process is provided in section II. In section III, descriptions of the methodology and models used in this paper are presented. Estimation results are presented in section IV. In the final section, policy recommendations drawn from these results are discussed.

#### II. BACKGROUND

#### Recent Mission Trends

The end of the draft in 1973 marked the beginning of a new era for the U.S. Army. Recruiting would now rely on market forces and mechanisms to enlist soldiers, rather than the direct and implicit powers of conscription. Due to the uncertain nature of these factors, it became critical to accurately predict future enlistments so that realistic goals could be established.

Over the past several years, the Army has been relatively successful in obtaining "high quality" personnel. [In this paper, the term "high quality" will refer to non-prior service high school graduate males (GMA) and seniors (GSA) acoring in the upper 50th percentile of the Armed Forces Qualification Test (AFOT). These individuals are considered to be in test categories I-IIIA.]. This period of success came after several years of disappointing recruiting (prior to FY81). Several reasons can be offered for the improved recruiting picture including high civilian unemployment rates, increased enlistment bonuses, the inception of the Army College Fund (ACF), improved resource management, and a new aggressive advertising campaign ("Be All You Can Be"). Advertising expenditures and the number of recruiters were also substantially increased. Recently, however, a general decline in the unemployment rate has led to a more difficult recruiting environment which is expected to continue because demographic trends indicate that the youth population will decrease substantially through the early 1990s. Given the present budgetary environment, it will be even more difficult to acquire greater resources to meet recruiting objectives. Continued recruiting success will require a more efficient allocation of current resources.

Figure 1 shows high quality contracts on a quarterly basis since FY82. (Prior to FY83, separate missions were recorded for GMAs and SMAs but the contract totals were aggregated). Beginning in FY83, to meet its total high quality goal, (GSMA), the Army has overachieved SMA missions to make up for a shortfall in GMA contracts.

High quality contracts for the Army's five recruiting brigades during the second quarter of FY85 are shown in Table 1. During this period not one brigade was able to meet its GMA mission. On the other hand, three brigades met their SMA mission. (In this paper, data for the second quarter of FY55 are used for detailed cross-sectional analysis.)



Figure 1. Quarterly Male AFQT I-IIIA HSDG and HSSR Contracts and Missions.

#### Table 1

			UARTER, FYE			
	G	RADUATE			SENIOR	
Brigade	Miss.	Prod.	Diff.	Miss.	Prod.	Diff.
A	2,360	1,830	-530	916	1,146	+230
B	1,792	1,465	-327	759	783	+ 24
С	1,562	1,494	- 68	616	505	- 66
D	3,220	2,502	-718	1,299	1,270	- 29
E	1,796	1,568	-228	687	782	+ 95
Total	10,730	8,859	-1,871	4,277	4,531	+254

## MISSIONS VERSUS PRODUCTION

#### Mission Modeling

The first enlistment supply estimates were obtained from aggregate time series models, such as Fechter (1978), Grissmer et al. (1978), and Cooper (1977). These models have several common features. They all found high military pay and high unemployment rates as factors significantly increasing the supply of quality enlistments.

These studies were primarily interested in the aggregate impact of policy and environmental factors on recruiting. However, it is not sufficient for USAREC to project only total enlistment supply. The Recruiting Command is organized into 56 battalions; each is responsible for recruiting in a different region of the country. USAREC needs to determine how to set missions for each of these battalions.

Several models have been developed to project battalion contracts. The first such model was the Analysis for Management of Recruiting Resources and Operations (AMRRO) by Souder et al. (1976). This research examined the impact of such factors as qualified military available (QMA) population, unemployment rate, attitude toward the military, advertising, recruiters, canvassers, and geographic area on Army contracts. A log-linear single equation econometric model was estimated using cross-sectional data over the

FY74-75 period. The final specification included battalion specific unemployment rates, attitude toward the military, and the number of recruiters to determine the numbers of supply limited contracts (I-IIIA male non-prior service high school graduates). Contracts were estimated for battalions using battalion specific data along with the national parameter estimates. Table 2 illustrates the types of explanatory variables used in these models.

#### fable 2

Factor	et.al.	$\frac{\text{Charnes}}{(1982)}$		Daula- Smith (1985)	Dertouzos (1985)		
Populacion		*	*	*	*		*
H. S. Seniors		*					
Unemployment	*	*	*	*	*	*	*
Recruiters	*	*	*	*	*	*	*
Recruiter Aides		*					
Enlist. Propensity	*	*		*			*
Relative Pay			*	*	*	*	
Family Income		*	*				
Nat. Advertising			*	*		*	
Local Advertising		*	*	*		*	
Competition		*	*	*			*
High Quality Mission				*	*	*	
Low Quality Mission				*	*	*	
Mission Tradeoff						*	
Minority Population				*			
Recruiter Experience		*	*	*			*
Other Enlistments				*			
Non-Grad. Enlistments				*			
Leads	*						
Pop. Density	*						

FACTORS INCLUDED IN CROSS-SECTIONAL RECRUITING MODELS

Four recent econometric analyses have been performed which enable a better understanding of the relationship between missions and contract (competition) production. Fairchild et al. (1984) found other services contracts, unemployment rates, population, relative military/civilian pay,

family income, the number of recruiters, local advertising, and national advertising to be statistically significant factors in an instrumental variables battalion supply equation. Daula and Smith (1985) also examined the relationship between missions and contract production. They divided battalions into those achieving high quality missions (demand constrained) and those that were not (supply constrained). They found high quality missions to be a significant factor affecting contract production for demand constrained battalions. Dertouzos (1985) and Dertouzos and Polich (1985) found that low quality missions significantly reduced high quality contract production.

An operations research (OR) approach has also been recently applied to the missioning problem. Charnes et al. (1982) used data envelopment analysis (DEA) to examine battalion contract production. Their model related the contracting of different categories of personnel to the same kinds of factors used in econometric models. However, the DEA approach differed from the econometric methodology in three important ways. First, DEA determined the maximum production obtained for a given level of resources, rather than average production. Second, DEA optimized on each observation, whereas econometric models (statistical regression) optimized across all observations. Third, DEA permitted multiple outputs to be evaluated from a single model. While their approach was innovative and provided insights into recruiting inefficiencies, the data were from 1981 and problems existed with the specification used. This included the use of low quality contracts nongraduates and test category IV graduates)as desirable outputs. (DEA will be described in further detail in section III.)

#### The Enlistment Projection Model

USAREC presently uses the Enlistment Projection Model (EPM) to estimate recruiting missions for battalions and brigades. Each quarter, separate equations are estimated for each brigade for three different missions (GSMAs, GSMBs, IIIB male graduates and seniors, and I-IIIA graduate females). A multiplicative exponential formulation is used with a stepwise regression procedure. Explanatory variables typically used in the EPM include qualified male military available population, number of Army recruiters, number of DoD

recruiters, unemployment rate, recruiter experience, and enlistment propensity. In addition, other battalion specific factors (dummy variables) frequently enter the equations to adjust for battalions performing significantly above or below the average for their brigade. Cross-sectional parameter estimates are made using the previous four quarters of data.

While producing reasonable contract estimates, the EPM has a number of limitations. First, all brigades are estimated separately. This means that each brigade essentially has its own mission-resource equation. Because of the stepwise selection procedure, not only do coerificients differ across brigades, but variables are often omitted from equations. For example, during one quarter recruiter experience may contribute positively to missioning in brigade A and negatively in brigade B. This situation could be reversed in the following quarter. For another quarter, this variable may not be included at all. The presence of "dummy" factors for specific battalions further limits the comparability of resources and missions. Finally, the estimation of separate equations for each type of mission fails to take into account their interrelationships. For example, the production of GSMB contracts may be related to the number of GSMA contracts produced.

#### III. MODEL METHODOLOGY

The methodology used in the analysis is described in this section. The same data and general relationships existing in the EPM are incorporated, but DEA is used to estimate the specific relationships.

Two general approaches have been developed for setting missions. The first is to set missions on the basis of labor market factors and recruiting resources that affect production. This is the approach used in the FPM.

The second approach is to use missions to spur production. Daula and Smith (1985), for example, estimated that increasing high quality missions would increase production. Dertouzos and Polich (1985) found tradeoffs between high and low quality missions; low quality missions took resources away from high quality recruiting. Using the incentive effect of missions would be desirable if the only interest is to maximize the number of quality

contracts in the near term. However, econometric research has not yet reached agreement on how both high and low quality missions impact on production. It is also not clear how to manage the explicit effects of differential mission setting in the long run, if performance directly influences workload. In addition, there are issues concerning production efficiency that must be addressed prior to considering the impact on any resource allocation, let alone mission changes.

The factors of production that are employed in the EPM model are also used in the models presented here. Factors such as labor market conditions and recruiting resources are first related to battalion contract production. Missions are ien set directly on the basis of efficient production. USAREC can then monitor "efficient" production (missions) versus actual production and use this information external from the model to assess performance rewards and penalties.

#### Data Envelopment Analysis

A number of approaches are available to estimate production at the battalion level. In this research DEA is used to develop relationships between recruiting resources and contract production. This section provides an overview of DEA, a discussion of its desirable features for mission setting, and a description of the empirical model.

The DEA methodology provides estimates of empirical frontier production functions or best practice frontiers. It requires no a priori specification of a parametric functional form or choice of weights. It also requires only the minimum assumption of convexity and monotonicity for the production possibility set, and uses a postulate of minimum extrapolation from observed data to estimate a frontier production function (See Banker, Charnes, and Cooper; 1984). Estimates of the relative technical efficiency of production (or decision making) units are given by DEA. This measure reflects a production unit's position relative to the "best practice" frontier.

DEA provides the capability to examine different aspects of production activities. information is provided which permits examination of production characteristics such as efficiencies, local returns to scale, and marginal

rates of substitution and transformation. The capability to accommodate multiple outputs, which is essential in the examination of not-for-profit organizations, is afforded by the DEA methodology. Sources and amounts of inefficiencies are also identified.

Since the measurement of efficiency is a primary concern of this paper, it is appropriate to define it here. A decision making unit is said to be efficient if there is no way to produce more outputs with the same level of inputs or to produce the same amount with lower input levels. This definition can be extended to the notion of relative efficiency. Relative efficiency of a unit implies that when compared to other relevant units, it exhibits no inefficiencies.

Three basic notions of efficiency have been identified by economists: technical, allocative, and scale efficiency. Figure 2 illustrates technical and allocative efficiency for units producing two outputs. The line segment passing through point E and F is the isorevenue line. It represents all combinations of outputs A and B which can be sold for a fixed amount of money at a given set of prices. Unit A is both technically and allocatively efficient. Unit B is technically efficient, but suffers from an allocative inefficiency of amount BD, since it is not producing an optimally priced combination of outputs. Unit C is technically and allocatively inefficient. To become technically efficient it would need to increase production until reaching the frontier (distance BC).

Scale efficiency is illustrated in Figure 3. This is an example of production possibilities with one input used to produce one output. Units A and C are both on the production frontier, and hence are technically efficient. Note that the slope, input-output ratio, of the line segment passing through points O, A, and B is smaller than the slope of the line segment passing through points O and C. Hence, unit A is scale inefficient since it does not achieve the highest input-output ratio available to firms operating with this technology. Unit B has the same input-output ratio as unit A, but is both technically and scale inefficient.







Input

Figure 3. Technical and Scale Efficiency

This research focuses on technical efficiency in its analysis of contract production. It is not clear that the location of allocative and scale inefficiencies have any utility in this setting. The identification of allocative inefficiency would only be useful if battalion commanders could buy and sell resources at an established market rate. Having the ability to correct for scale inefficiencies would imply that USAREC could readily adjust the size of battalions to take advantage of economies of scale. In the short run (when missions are set) this is clearly not possible. It may not even be possible in the long run, since battalion scale may largely be determined by geography, not technology.

There are many measures of technical efficiency that can be determined for a decision making unit. The particular measure employed is dictated by the focus of the problem being analyzed. The focus could be either input reduction or output augmentation. These aspects are addressed by measuring either input or output efficiency. Input efficiency measures the efficiency of the use of inputs given an output level, employing the input possibility set to represent the production technology. Output efficiency determines the efficiency of the outputs produced from a given level of inputs using the output possibility set to represent the production technology. In this paper the focus is on output efficiency measures, in particular the output efficiency of recruiting battalions.

Farrell in 1957 introduced the concept of technical efficiency and provided a method for measuring it. His measure departed from the traditional "averaging" practices of economics and statistics. It was independent of costs or prices, and took into account all factors of production simultaneously. In his approach, technical and scale versus price (or allocative) inefficiencies were disentangled. Farrell's approach, which theoretically accommodates multiple outputs, was only applied to single output numerical examples. No computational procedure was developed for the multiple output problem.

Charnes, Cooper, and Rhodes (CCR) (1978,1981) extended Farrell's approach to the multiple output-multiple input situation. (The CCR DEA formulation is described in the appendix.) This formulation provided a

measure of aggregate technical and scale inefficiency. In Banker (1984) and Banker, Charnes, and Cooper (1984), the CCR formulation was refined, so that technical and scale inefficiencies could be separated and the most productive scale size determined. The solution to the Banker, Charnes, and Cooper (BCC) formulation estimated the technical efficiency of decision making units. The BCC paper provided a formal link between DEA and economic production theory.

Banker and Morey (1986) further extended the analysis capability of DEA. They made it possible to accommodate not only discretionary inputs and outputs (those resources within the control of the producer or decision making unit manager), but also nondiscretionary (exogenously fixed and beyond the control of the producer or manager) inputs and outputs. The machematical program of this DEA formulation is described below.

Suppose there are L decision making units that use the same M inputs to produce N outputs. The  $X_{ik}$ ,  $Y_{jk}>0$  are the observed levels of input i and output j, respectively, for the  $k^{th}$  decision making unit. Let D and F represent the index set for the discretionary and nondiscretionary inputs, respectively. Let o denote the index of the unit being evaluated. The Banker and Morey output efficiency formulation then has the following linear programming format:

Maximize  $\theta + \phi \left( \sum_{j=1}^{N} s_{j}^{+} + \sum_{i \in D} s_{i}^{-} \right) + \phi \sum_{i \in F} s_{i}^{-}$ Subject to  $\sum_{k=1}^{L} \lambda_{k} X_{ik} + s_{i}^{-} = X_{i0} \quad i=1, \dots, M$   $\theta Y_{j0} - \sum_{k=1}^{L} \lambda_{k} Y_{jk} + s_{j}^{+} = 0 \quad j=1, \dots, N$   $\sum_{k=1}^{L} \lambda_{k} \quad = 1$   $\lambda_{k}, \quad s_{j}^{+}, s_{i}^{-} \geq 0 \quad i=1, \dots, M$   $j=1, \dots, N$ 

where  $\phi$  and  $\phi'$  are non-Archimedian infinitesimal quantities with  $\phi'$  smaller than  $\phi$ . The decision variables in this mathematical program are  $\theta$ ,  $\lambda_k$ ,  $s_j^+$ , and  $s_i^-$ . The variables  $s_j^+$  and  $s_i^-$  are nonnegative slack variables associated with the output and input inequalities, respectively, representing unused capability of the decision making unit. The variable  $\theta$  denotes the intensity factor which indicates how much the output levels of the unit being evaluated can be proportionately increased before equalling the level of at least one of the outputs of a composite unit (a convex combination of the inputs and outputs of other units) which guarantees the highest possible efficiency rating.

This optimization problem examines the convex combination or weighted average of observed inputs and outputs to find those which compare favorably with the input and output levels of the unit being evaluated. Units compare favorably if they use no more than the level of discretionary inputs used by the unit being evaluated, while producing at least  $\theta$  times the output levels. At the same time, the composite units use no more of the nondiscretionary inputs than used by the unit being evaluated.

The solution to the above linear programming problem supplies information which is valuable in the evaluation and analysis of a decision making unit. Efficiency scores of the units being rated are provided. A "best" subset of units which are used in the evaluation, is also provided. Members of this subset, identified by the  $\lambda_k$  in the optimal solution, are efficient and together generate an efficient facet. The optimal values of the  $\lambda_k$ s indicate the extent to which the k<sup>th</sup> unit is used in an evaluation. The larger its value, the larger the role the k<sup>th</sup> unit has in the evaluation. The set of all units which employ the same "best" subset form a comparison group. Members of this group utilize similar technologies to convert inputs into outputs.

Marginal rates of substitution of inputs and transformation of outputs can be estimated from the optimal values of the dual variables associated with the constraints, which are available after a solution to the linear program is found. This is accomplished by first using the DEA solution to

compute interior points of each linear segment of the frontier production function. This is done because partial derivatives exist only for interior points of linear surfaces. Hence meaningful dual values exist only for interior points. DEA is again applied using these additional interior points. The optimal dual values obtained from this solution can then be used to estimate the desired marginal rates. The negative of the ratio of the optimal dual values associated with the output constraints is the marginal rate of transformation of outputs. The marginal rates of substitution of inputs are estimated in a similar fashion. It should be noted that these are local rates. They are the slopes of the segments of the piece-wise linear frontier production function. The rates are the same for all points on an efficient facet.

For each unit rated inefficient relative to the other observed units, the solution provides information making it possible to estimate the level of inputs and outputs necessary for that unit to be rated efficient. This level is referred to as the "value if efficient." The estimation of this level of inputs and outputs is accomplished by using the "CCR projection" (Charnes, Cooper, and Rhodes; 1981) or a variant of it. Model Formulation

As previously stated, DEA is used to determine whether recruiting battalions are operating in an "efficient" manner given their existing resource levels and the environment (or market) in which they must function. First, a model is developed to evaluate current missions. Actual missions are compared to missions estimated by the DEA model. This comparison is followed by an analysis of contracts to determine at what levels recruiting battalions should have been producing given current resources and market factors. The unit of measure in both cases is the recruiting battalion. (While the Army has 56 recruiting battalions, data limitations confine the analysis to 54 of them.

Each model incorporates two outputs. For the missioning evaluation model, outputs include GMA and GSA missions. These two categories are separated because different missions are formulated for each group (even though the EPM includes them as a single dependent variable). Hosek and

Peterson (1986) discuss how the production function is likely to be different for the two groups, since seniors are centrally located and easier to make contact with, but may have a lower enlistment propensity than graduates. The two groups also exhibit different enlistment patterns during the year. For example, SMAs are permitted to remain in the Delayed Entry Program for up to one year while GMAs may only be permitted a few months stay. SMAs are also more likely to sign contracts during certain times of the year (autumn and winter).

The production model incorporates contracts, rather than missions, as outputs. This procedure permits an analysis of actual versus "efficient" missions and contract production, as well as a comparison of the two processes.

Identical inputs are included in each model. The mission evaluation model treats all inputs as discretionary and uses the CCR formulation. Both discretionary and non-discretionary inputs are used in the production model and it is solved using the Banker and Morey DEA formulation. Three discretionary inputs are used in this analysis. Two characterize the recruiting force itself: the number of production recruiters and the average level of recruiting experience in the battalion. Production recruiters are those recruiters assigned individual recruiting "missions" or quotas. For this analysis, recruiting experience is defined as the percentage of recruiters in the battalion with nine months or more production experience. This variable has been included in several enlistment models including the EPM and Daula-Smith model. The assumption is, the greater the experience level, the higher the mission and contract production. The third discretionary input, local advertising, has also been found to be statistically significant, positively influencing enlistments in models. This variable is composed of battalion-level advertising expenditures including newspapers and other local media. All data were collected quarterly at the battalion level.

Non-discretionary variables in the production model include quarterly battalion area unemployment rates (Bureau of Labor Statistics), the number of all Department of Defense recruiters (Defense Manpower Data Center and

Recruit Market Network), enlistment propensity (Youth Attitude Tracking Survey), and the number of qualified I-IIIA non-prior service males living within the battalion (USAREC). As previously stated, these resources are considered discretionary in the mission model. All of the data concerning inputs were obtained from the EPM data base. Output data were obtained directly from USAREC. The second quarter of FY85 (January-March) was chosen as the focus for this analysis.

#### IV. MODEL ESTIMATION RESULTS

#### Missioning Model

The missioning model was used to evaluate missions to determine whether they are set in a consistent fashion. Presumably, if the same factors are considered equally when setting each battalion's mission, DEA will rate all battalions efficient. If all are not efficient, the model results will indicate which battalions are missioned lower than others with similar resources operating in a similar market.

Thirty eight of the 54 recruiting battalions were found to be technically efficient. The 16 inefficient missions were scattered throughout 4 of the 5 recruiting brigades. As shown in Table 3, for these battalions to be efficient, their missions should have been higher. Differences between actual and "efficient" missions ranged from less than one (battalion MG, seniors) to 88 (battalion ML, graduates). Production Model

The next task was to examine contract production. This included an evaluation of each battalion's relative efficiency in contracting high quality individuals and a determination of the levels at which these battalions should be producing. Because outputs were to be examined, output inefficiencies were estimated.

		GRADUATE	1		SENIOR		EFFICIENCY SCORE
Batt.	<u>Miss</u> .	DEA.	Diff.	<u>Miss</u> .	DEA.	Diff.	
MA	134	150	-16	54	60	-6	0.963
MB	147	202	-55	57	80	-23	0.963
MC	142	190	-48	56	77	-21	0.970
MD	105	177	-72	37	65	-28	0.933
ME	193	197	-4	55	66	-11	0.989
MF	179	191	-12	73	78	-5	0.949
MG	172	173	-1	70	70	0	0.998
MH	212	231	-19	79	90	-11	0.946
MI	126	137	-11	64	69	-5	0.963
MJ	138	147	-9	48	55	-7	0.989
MK	139	144	-5	52	62	-10	0.995
ML	214	302	-88	86	121	-35	0.860
MM	191	216	-25	73	82	-9	0.906
MN	237	257	-20	97	105	-8	0.930
MO	232	246	-14	87	95	-8	0.959
MP	242	286	-44	91	111	-20	0.919

#### MISSION EVALUATION MODEL ACTUAL VERSUS EFFICIENT MISSION BY BATTALION INEFFICIENT UNITS

Overall, 37 battalions were scored efficient. Findings for the 17 inefficient battalions are shown in Table 4. Most of the same battalions that were judged to be inefficiently producing contracts were also determined to be inefficiently missioned. Eleven battalions were inefficient in both models, indicating that many battalions missioned lower than others with similar resources and market conditions, also produced fewer contracts.

How do DEA "efficient" estimates compare with those provided by the EPM estimates, actual missions, and actual production? To make these comparisons, it was necessary to examine output values if efficient for all inefficient battalions. Both outputs had to be summed to allow a direct comparison with EPM estimates. Actual missions present the largest total (15,007). This represents the Army's desired output. The next highest total was calculated for DEA estimates (14,295), followed by actual contract production (13,390) and EPM estimates (13,325). Because of the estimation

#### Table 4

	G	RADUAT	ES	-	SENIORS		EFF. SCORE
Batt.	Actual	DEA.	Pct.Less	Actua	<u>DEA</u> .	Pct.Less	
PA	120	174	30.88	47	74	36.34	0.691
PB	96	168	42.76	44	81	45.55	0.572
PC	95	130	26.97	65	89	26.97	0.730
PC	148	162	8.69	69	76	8.69	0.913
PE	128	150	14.90	45	55	18.60	0.851
PF	110	121	8.95	55	67	17.61	0.911
PG	188	226	16.94	60	125	52.13	0.831
PH	127	156	18.43	61	75	18.43	0.816
PI	238	240	0.97	145	146	0.97	0.990
PJ	188	191	1.57	64	81	21.44	0.984
PK	218	245	10.97	102	125	18.40	0.890
PL	159	215	25.93	63	107	40.96	0.741
PM	182	207	12.21	94	107	12.21	0.878
PN	189	223	15.14	89	126	29.67	0.849
PO	207	255	18.95	92	134	31.56	0.810
PP	109	123	11.29	62	71	13.09	0.887
PQ	225	236	4.71	93	119	22.06	0.953

DEA PRODUCTION MODEL ACTUAL VS. EFFICIENT PRODUCTION FOR INEFFICIENT UNITS

procedure, DEA estimates will always be greater than or equal to actual output. If a unit is determined to be efficient, the value if efficient will equal actual production, but if the unit is not efficient, this value will always exceed production. In this case even if all battalions produced efficiently, the desired output (mission) would not have been obtained.

Mission shares indicate what portion of the total mission would be allocated to each recruiting brigade. As seen in Table 5 the shares differ between actual missions, contracts, EPM estimates, and DEA estimates. The shares estimated by DEA were closer to actual contract production than were missions or EPM estimates in all brigades except one. Results indicate that to become efficient, brigade D would have had to produce a greater share of the high quality contracts. While the other brigades would also be required to produce more contracts, their shares would decrease due to the increased

share in D. This is consistent with both current mission shares and EPM estimates.

Table 5

Brigade	Mission	EPM	Contracts	DEA
A	.218	.228	.222	.226
B	.170	.171	.168	.159
C	.145	.137	.153	.147
D	.301	.317	.282	. 300
E	.165	.146	.176	.169

# COMPARISON OF DISTRIBUTIONS BY BRIGADE

In Table 6, average efficient values by brigade are compared to mean quarterly battalion production for fiscal years 1983-1985. This table indicates that the DEA estimate is less than actual mission but greater than the EPM estimates. In brigades A and D, DEA estimates fall above mean FY85 production but below the FY83 mean. In the other cases, DEA estimates fall below average FY85 production (possibly due to seasonal differences in production).

Table 6

DEA VERSUS MEAN QUARTERLY PRODUCTION

Brigade	<u>FY83</u>	<u>FY84</u>	<u>FY85</u>	DEA
A	290.5	236.3	243.9	268.7
В	288.0	229.2	233.2	226.9
С	207.0	190.3	215.8	209.9
D	361.7	286.7	292.3	330.2
E	265.3	225.1	272.0	267.8

#### **Evaluation** Groups

The facets used in the evaluation of each battalion are generated by efficient battalions similar to the evaluated unit in size and input and output ratios. Table 7 shows the number of times in the production model a unit being evaluated was compared to an efficient unit in the same brigade(excluding self) versus another brigade. Most evaluation battalions were located outside their own brigade, indicating that battalions frequently are closer in size and resources to those in brigades other than their own. This varied considerably, however. Battalions in brigades A and B were far more likely to be evaluated within the brigade (16 out of 36 times and 17 of 39 times, respectively) than those in brigade D (1 out of 55) or E (5 out of 36).

Table 7

	No. Times Comparison Element			
Brigade	Inside Brigade	Outside Brigade		
٨	16	20		
В	17	22		
C	10	34		
D	1	54		
E	<u>5</u> 49	31		
Total	49	$\frac{31}{161}$		

DEA EVALUATION GROUPS

#### Marginal Rates of Substitution and Elasticities

Marginal rates of transformation (MRT) of outputs and marginal rates of substitution (MRS) of inputs were estimated from the production model. Only dual values different from zero and the infinitesimal employed to accommodate the non-Archimedian formulation, were used. The medians of these marginal rates were then calculated and reported in Table 8. A MRT(SMA, GMA) of - 2.16 suggests that if six more GMAs are recruited, 13 fewer SMAs will be acquired.

This tradeoff indicates that GMA are about twice as difficult to recruit as GSA in this instance. A MRS(Local Ads, Number of Recruiters) of -.40 suggests that if local advertising is increased by two units, the number of recruiters can be decreased by five. The median marginal rates of substitution between other pairs of discretionary inputs can be interpreted in a similar fashion. It cannot be overemphasized that these medians are calculated from local marginal rates. Hence these results and those presented below must be interpreted with care.

Table 8

#### DEA ESTIMATES FOR MARGINAL RATES OF TRANSFORMATION AND SUBSTITUTION

	Median Rates
MRT (HSDG/HSSR)	-2.16
MRS (Local Adv./No. of Recruiters	40
MRS (Local Adv./Recruiter Experience	11
MRS (No. of Recruiters/Recruiter Experience)	15

Point elasticities were calculated from the optimal dual variable values of the DEA solution. Again, only duals which were different from zero or the infinitesimal, were employed. Elasticities were calculated for the average input and output level of each facet and are presented in Table 9, together with the elasticities from both the Fairchild and Daula and Smith models. (The headings D.C. and S.C. in the table denote demand constrained and supply constrained, respectively.) Missing values for variables signify that they were not used in that model.

In terms of the DEA results, the median elasticities for GMAs and SMAs differ. Those for SMAs are larger in all cases except for local advertising. This implies that the production of SMA contracts is generally more responsive to changes in resource levels and market conditions than the production of GMA contracts.

#### Table 9

	DEA		Daula-Smith		
	HSDO	HSSR	<u>D.C.</u>	<u>s.c.</u>	Fairchild
Number of Recruiters	.0001	.0003	.83*	.96	1.11*
Recruiter Experience	.0000	.00005	.08	.08	.09
Local Advertising	.08	.05	.05	.08	.10*
Unemployment	.66	.98	.83*	.96*	• 59*
Army Percent of DOD	1.24	1.74	-	-	-
Enlist. Propensity	.41	11.33	.38*	.28	-
Qualified Military Avail.	.98	4.00	.23*	.20	.30*

#### MEDIAN DEA ESTIMATED ELASTICITIES OF AVERAGE FACET VALUES

#### \* Indicates Significance at the .05 Level

It is difficult to compare elasticities across models, however. DEA is a nonparametric methodology, whereas the other models for which elasticities are presented, are persmetric. They are estimated using multivariate log-linear statistical models. Nevertheless, in most instances where comparisons are possible, the DEA median elasticities are close to the other results.

#### Mission Reallocation

The production analysis provides an option for mission reallocation. Four categories of battalions were identified: those achieving mission and efficient, not achieving mission and efficient, achieving mission but inefficient, and achieving neither mission nor efficiency. Output levels if efficient were then used to calculate net mission changes by output and category. Results are shown in Figure 4. Each quadrant is divided into two sections. The upper portion indicates the number of battalions falling into each category. The lower portion indicates the aggregate mission change determined by the model.



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Achieved Mission



Figure 4. Candidates for Mission Reallocation.

For GMAs the greatest mission change occurs for battalions which did not achieve missions but found to be efficient (-1181). Missions spear to have been set so high that they could not be met by most recruiting battalions, even if operating efficiently. Those which were inefficient and did not make missions should also have had a lower mission. The important finding here is that inefficient production is only part of the reason for their failure to meet missions. Even had they been efficient, they still would have missed their GMA mission by almost 300 contracts. For those who achieved missions, only a small increase would have been recommended (+99). This is because only 5 battalions achieved their GMA mission during this quarter. In total, the GMA mission should have been almost 1400 persons lower than assigned.

SMA missions could have been higher (+662) than observed. This is consistent with the observation that this mission was achieved by most battalions. Battalions in three of the four quadrants experience net increases, the largest (+528) coming from those which were efficient and achieved SMA missions. In contrast to the GMA results, those battalions which were inefficient and did not achieve missions would experience a net increase in mission because if these battalions had been efficient they would have exceeded the current mission.

#### V. SUMMARY AND CONCLUSIONS

The purpose of this analysis has been to present an alternative approach to missioning recruiting battalions. The current missioning model (the EPM) is estimated for battalions at the brigade level using a stepwise OLS regression model requiring 15 equations for each quarter. The variables in each equation may vary both in their selection and signs. Dummy variables are used extensively (26 were used in the second quarter of FY85).

The DEA formulation used in this research is a different approach to missioning. This approach has several desirable features. By comparing missions and production to "best practice" frontiers, technically efficient

battalions can be identified. Multiple outputs can be accommodated in a single model. Comparison groups of "like" units can be formulated. DEA can be used to develop missions based on "best practice" production rather than "average" production as is the case using OLS.

Although limited to a single period of data, the analysis has yielded several new findings. During the period examined, missions were not set in a consistent fashion based upon resources and market factors. Approximately 30 percent of the battalions were missioned below others with similar resources operating in a similar market. The same was true for production. Again, about 30 percent were producing below others. Eleven battalions were rated inefficient in both models.

Currently, battalion commanders are rated on the basis of how well their battalions achieved their recruiting missions. This analysis has shown, however, that some of these missions were not met because they were simply set too high given available recruiting resources. The use of DEA provides an alternative approach for identifying battalions not achieving missions because of technical inefficiencies, excessively high missions, or a combination of these factors.

The next steps towards implementing a DEA missioning model would be to develop the capability to estimate outputs given a level of resources. The model could then be run in conjunction with the EPM to compare estimates. An extension of this type would require the use of additional periods of data.

It would be useful to support any missioning model with an independent assessment of the missioning process. Relevant factors not included in the EPM data base need to be identified. The mission-setting process should be examined to determine why EPM missions are adjusted. If additional data are available for battalions that were classified inefficient, it should be reviewed to determine whether inefficiencies do indeed exist, or whether factors have been omitted from the model.

Several technical issues concerning the DEA model warrant attention. A multiperiod model may be desirable. Several quarters of data could be incorporated in a single model by either deseasonalizing the data or using an extension of a procedure developed by Banker and Morey (1985) which

incorporates categorical variables. Different functional forms could be examined, such as a piecewise Cobb-Douglas form developed by Banker and Maindiratta (1986). Finally, the use of a DEA approach not requiring non-archimedian constructs could be examined. This would make it easier to estimate and interpret the duals, and hence the marginal rates of substitution and transformation. These refinements could improve the model, depending upon the specific objective.

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It has been shown that DEA has considerable promise as a missioning tool. Inconsistency in mission setting and technical inefficiency are likely to exist at the battalion level. Its magnitude cannot be estimated using the current missioning model. This technique shows promise as a more consistent missioning technique which can provide additional useful information on production possibilities.

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#### Appendix

Charnes Cooper Rhodes DEA Mathematical Model

Minimize  $h_0 = \sum_{i=1}^{M} v_i X_{i0}$  $\sum_{j=1}^{N} w_j Y_{j0}$ 

Subject to

 $\begin{array}{c} M \\ \nabla \mathbf{v}_{i} \mathbf{X}_{ik} - \sum_{j=1}^{N} \mathbf{w}_{j} \mathbf{Y}_{jk} \leq 0 \\ \mathbf{i} = 1 \\ \end{array}$  k=1,...,L

 $w_j, v_i \ge \varepsilon > 0$   $j=1,\ldots,N$  $i=1,\ldots,M$ 

where

L = the number of decision making units (DMU) M = the number of inputs N = the number of outputs X<sub>ik</sub> = the i<sup>th</sup> input for the k<sup>th</sup> DMU Y<sub>jk</sub> = the j<sup>th</sup> output for the k<sup>th</sup> DMU v<sub>i</sub> = the weight on the i<sup>th</sup> input w<sub>j</sub> = the weight on the j<sup>th</sup> output o = the subscript for the unit being evaluated.

This problem has a linear programming equivalent format. (See Charnes, Cooper, and Rhodes; 1981). Abusing notation the equivalent format is

The formulation above has the corresponding dual form

e.

0

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Maximize 
$$\theta$$
 +  $\varepsilon \begin{pmatrix} \nabla & \mathbf{s}^{-} & \mathbf{i} & \nabla & \mathbf{s}^{+} \\ \mathbf{j} & \mathbf{j} & \mathbf{j} & \mathbf{j} & \mathbf{j} \\ \theta \mathbf{Y}_{\mathbf{j}0} & - & \sum_{j=1}^{N} \lambda_{k} \mathbf{Y}_{\mathbf{j}k} + \mathbf{s}^{+} \\ \mathbf{j} & = 0 \qquad \mathbf{j} = 1, \dots, \mathbf{N} \\ \\ & \sum_{i=1}^{M} \lambda_{k} \mathbf{X}_{\mathbf{i}k} + \mathbf{s}^{-} \\ \mathbf{i} = 1 \qquad \mathbf{i} = 1, \dots, \mathbf{M} \\ & \lambda_{k}, \mathbf{s}^{+} \\ \mathbf{j}, \mathbf{s}^{-} \\ \mathbf{i} & \geq 0 \qquad \mathbf{i} = 1, \dots, \mathbf{M} \\ & \lambda_{k}, \mathbf{s}^{+} \\ \mathbf{j}, \mathbf{s}^{-} \\ \mathbf{i} & = 0 \qquad \mathbf{i} = 1, \dots, \mathbf{M} \\ \end{pmatrix}$