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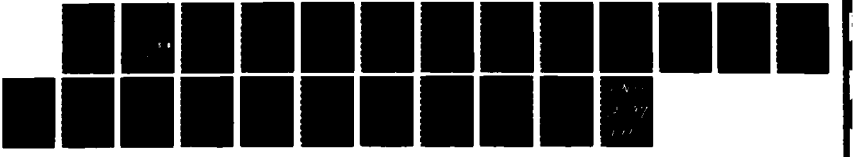

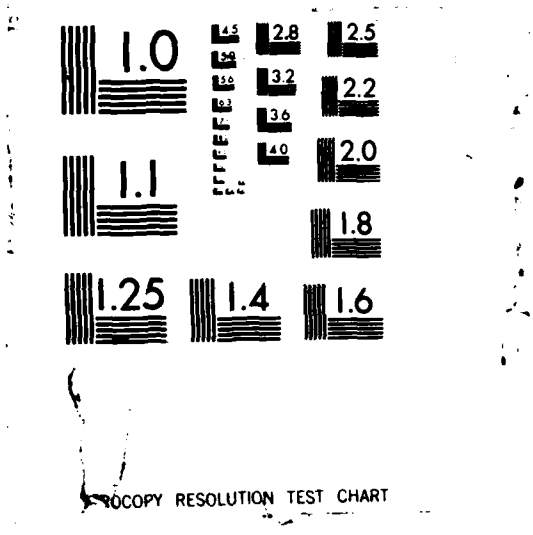
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UNIVERSITY OF MIAMI

ROSENSTIEL SCHOOL OF MARINE AND ATMOSPHERIC SCIENCE

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AN APPROACH TO THE ANALYSIS OF THE MODIFICATION OF SURFACE WAVES BY DEPTH-VARYING CURRENT FIELDS

by

Richard A. Skop

November 1986

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An approach is developed for ~~the analysis of~~ the modifications of surface waves by depth-varying current fields. The approach is based on an approximate dispersion relation for wave-current interactions derived from the governing equations of the problem (the inviscid Orr-Sommerfeld equations) coupled with the wave kinematic/wave action formulation of surface wave propagation. The approach is particularly useful in that it focuses directly on the wave parameters of interest (the amplitude, frequency, direction, and wavelength of the wave) and eliminates the requirement to solve the inviscid Orr-Sommerfeld equations to derive these parameters. The validity of the approach is demonstrated by comparisons with exact Orr-Sommerfeld solutions.

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INTRODUCTION

The modifications of surface waves by a depth-varying current field are described mathematically by the inviscid Orr-Sommerfeld equations of hydrodynamic stability (Peregrine (3)). The solution of these equations is required to derive the wave parameters of interest (the amplitude, frequency, direction, and wavelength of the wave). This solution can be difficult to obtain, even numerically.

An alternative approach to the analysis of the modifications of surface waves by depth-varying current fields is developed here. The approach is based on an approximate dispersion relation for wave-current interactions derived from the governing Orr-Sommerfeld equations (Skop (5)) coupled with the wave kinematic/wave action formulation of surface wave propagation (Bretherton and Garrett (1), Crapper (2)). The approach is particularly useful in that it focuses directly on the wave parameters of interest and eliminates the requirement to solve the inviscid Orr-Sommerfeld equations to derive these parameters.

The validity of the approach is demonstrated by comparisons with exact Orr-Sommerfeld solutions.

THE GOVERNING EQUATIONS

Let t deonote time and let x_1, x_2, and z define a Cartesian coordinate system with $z = 0$ as the undisturbed free surface of the water column and decreasing with depth. Within some region of this space, a current field is imposed. The horizontal components of the current field are denoted by U_1 and U_2 and the vertical component by W. The current field is allowed to vary in space and time.

A surface wave is taken to propagate from some region of space where the current field vanishes into the region of space where the

current field is imposed. We wish to ascertain how the surface wave is modified by the current field.

We assume only that the imposed current field varies slowly in the horizontal coordinates and time relative to the wavelength and period of the surface wave. Then, as viewed by the surface wave, the imposed current field appears to vary only in the z coordinate over at least several wavelengths of propagation or periods of oscillation of the surface wave. Further, under the slowly varying assumption, the continuity equation for the imposed current field appears locally as $\partial W/\partial z = 0$ to the surface wave. Hence, as viewed by the surface wave, W can be put equal to zero without loss of generality.

We denote the velocitites and pressures associated with the surface wave as u_1^*, u_2^*, w^*, and p^* and write the total fluid velocities and pressure as the sum of those quantities associated with the surface wave and those quantities associated with the imposed current field. Substituting the total fluid velocities and pressure into the Euler equations, linearizing with respect to the surface wave quantities, and applying the slowly varying assumption to the imposed current field, we find the equations that govern the local variations of the surface wave as

$$\frac{\partial u_1^*}{\partial x_1} + \frac{\partial u_2^*}{\partial x_2} + \frac{\partial w^*}{\partial z} = 0 \tag{1a}$$

$$\frac{\partial u_1^*}{\partial t} + U_1 \frac{\partial u_1^*}{\partial x_1} + U_2 \frac{\partial u_1^*}{\partial x_2} + w^* \frac{\partial U_1}{\partial z} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x_1} \tag{1b}$$

$$\frac{\partial u_2^*}{\partial t} + U_1 \frac{\partial u_2^*}{\partial x_1} + U_2 \frac{\partial u_2^*}{\partial x_2} + w^* \frac{\partial U_2}{\partial z} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x_2} \tag{1c}$$

$$\frac{\partial w^*}{\partial t} + U_1 \frac{\partial w^*}{\partial x_1} + U_2 \frac{\partial w^*}{\partial x_2} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z} \tag{1d}$$

where ρ specifies the density of the water. At the mean free surface $z = 0$, the local kinematic and dynamic boundary conditions on the surface wave are obtained as

$$\frac{\partial \eta^*}{\partial t} + U_1 \frac{\partial \eta^*}{\partial x_1} + U_2 \frac{\partial \eta^*}{\partial x_2} - w^* = 0 \tag{2a}$$

$$p^* - \rho g \eta^* + \gamma \left(\frac{\partial^2 \eta^*}{\partial x_1^2} + \frac{\partial^2 \eta^*}{\partial x_2^2}\right) = 0 \tag{2b}$$

Here, η^* is the elevation of the surface wave, g is the acceleration due to gravity, and γ is the coefficient of surface tension.

We seek wavelike solutions to equations (1) and (2) as

$$u_1^* = u_1(z) \exp[i(k_1 x_1 + k_2 x_2 - \omega t)] \tag{3a}$$

$$u_2^* = u_2(z) \exp[i(k_1 x_1 + k_2 x_2 - \omega t)] \tag{3b}$$

$$w^* = w(z) \exp[i(k_1 x_1 + k_2 x_2 - \omega t)] \tag{3c}$$

$$p^* = p(z) \exp[i(k_1 x_1 + k_2 x_2 - \omega t)] \tag{3d}$$

$$\eta^* = \eta \exp[i(k_1 x_k + k_2 x_2 - \omega t)] \tag{3e}$$

where k_1 and k_2 are the components of the wave number vector of the surface wave and where ω is its radial frequency. In accord with the slowly varying nature of the imposed current field relative to the surface wave, k_1, k_2, ω, and the amplitude functions of equations (3) can vary with x_1, x_2, and t but, again, only slowly over at least several wavelengths of propagation or periods of oscillation of the surface wave. Substituting equations

approach to the analysis of the modification of surface waves by depth-varying current fields. The approach is particularly useful in that it focuses directly on the wave parameters of interest and eliminates the requirement to solve the inviscid Orr-Sommerfeld equations to derive these parameters.

AN APPROACH TO THE ANALYSIS OF THE MODIFICATION OF SURFACE WAVES BY DEPTH-VARYING CURRENT FIELDS

The Wave Kinematic Equations

The surface wave phase function ϕ is defined by

$$\phi = k_1 x_1 + k_2 x_2 - \omega t \tag{6}$$

Recalling that k_1, k_2, and ω are slowly varying funcitons of x_1,x_2, and t, we have

$$k_1 = \frac{\partial \phi}{\partial x_1}, \quad k_2 = \frac{\partial \phi}{\partial x_2}, \quad \omega = -\frac{\partial \phi}{\partial t} \tag{7}$$

Taking the mixed second derivatives of ϕ, we find the wave kinematic equations as

$$\frac{\partial k_1}{\partial x_2} - \frac{\partial k_2}{\partial x_1} = 0 \tag{8a}$$

$$\frac{\partial k_1}{\partial t} + \frac{\partial \omega}{\partial x_1} = 0 \tag{8b}$$

$$\frac{\partial k_2}{\partial t} + \frac{\partial \omega}{\partial x_2} = 0 \tag{8c}$$

Equations (8) are not linearly independent and serve to determine only two of the three phase measures. The dispersion relation for the surface wave is required to complete the wave kinematic equations.

An Approximate Dispersion Relation for Wave-Current Interactions

Following some algebra, equations (4) can be reduced to a single equation for w as

$$\frac{\partial^2 w}{\partial z^2} - \left(k^2 + \frac{k_1 \frac{\partial^2 U_1}{\partial z^2} + k_2 \frac{\partial^2 U_2}{\partial z^2}}{k_1 U_1 + k_2 U_2 - \omega}\right) w = 0 \tag{9}$$

where we have denoted the absolute value of the wave number vector by $k = (k_1^2 + k_2^2)^{1/2}$. Similarly, equations (5) can be reduced to a single condition on w at the mean free surface z = 0 as

$$(k_1 U_1 + k_2 U_2 - \omega)^2 \frac{\partial w}{\partial z} =$$

$$\left[k^2\left(g + \frac{\gamma k^2}{\rho}\right) + (k_1 U_1 + k_2 U_2 - \omega)\left(k_1 \frac{\partial U_1}{\partial z} + k_2 \frac{\partial U_2}{\partial z}\right)\right] w \tag{10}$$

Equations (9) and (10) can be further simplified by introducing the effective current field U defined by

$$U = \frac{k_1}{k} U_1 + \frac{k_2}{k} U_2 \tag{11}$$

Denoting the surface wave celerity by $c = \omega/k$, we then find

$$\frac{\partial^2 w}{\partial z^2} - \left(k^2 + \frac{\frac{\partial^2 U}{\partial z^2}}{U - c}\right) w = 0 \tag{12a}$$

while, at the free surface,

$$(U - c)^2 \frac{\partial w}{\partial z} = [(g + \frac{\gamma k^2}{\rho}) + (U - c) \frac{\partial U}{\partial z}] w \qquad (12b)$$

Let us assume, for the moment, that the magnitude of the effective current field U is smaller than some characteristic celerity c_0 of the surface wave. We can then write

$$U = \varepsilon c_0 \overline{U} \qquad (13a)$$

where $\overline{U}$ is a dimensionless velocity of order unity and where ε is a dimensionless smallness parameter. Also, we can seek perturbation solutions for c and w as

$$c = c_0 + \varepsilon c_1 + \ldots \qquad (13b)$$

$$w = w_0 + \varepsilon w_1 + \ldots \qquad (13c)$$

Substituting equations (13) into equations (12) and equating like powers of ε, we find the boundary value problem for w_0 as

$$\frac{\partial^2 w_0}{\partial z^2} - k^2 w_0 = 0 \qquad (14a)$$

$$c_0^2 \frac{\partial w_0}{\partial z} - (g + \frac{\gamma k^2}{\rho}) w_0 = 0 \qquad \text{at } z = 0 \qquad (14b)$$

and that for w_1 as

$$\frac{\partial^2 w_1}{\partial z^2} - k^2 w_1 = - \frac{\partial^2 \overline{U}}{\partial z^2} w_0 \qquad (15a)$$

$$c_0^2 \frac{\partial w_1}{\partial z} - (g + \frac{\gamma k^2}{\rho}) w_1 =$$

$$2c_0(c_0\overline{U} - c_1) \frac{\partial w_0}{\partial z} - c_0^2 \frac{\partial \overline{U}}{\partial z} w_0 \qquad \text{at } z = 0 \qquad (15b)$$

The solution to equation (14a) that vanishes as $z \to -\infty$ is

$$w_0 = e^{kz} \tag{16a}$$

and, from equation (14b), we find

$$c_0 = \sigma(k)/k \tag{16b}$$

where

$$\sigma(k) = \sqrt{gk}\ (1 + \frac{\gamma k^2}{\rho g})^{1/2} \tag{17}$$

We recognize $\sigma(k)$ as the dispersion relation for the surface wave in a region of space where the current field vanishes and c_0 as the surface wave celerity in this same region of space. Further, we see that our assumption that the effective current field U is smaller than c_0 is satisfied for very long deepwater waves ($k \to 0$) and for very short capillary waves ($k \to \infty$). If surface tension is neglected, the assumption is still satisfied for very long deepwater waves.

Substituting for w_0 in equation (15a) and using variation of parameters, we obtain the particular solution for w_1 as

$$w_1 = - e^{-kz} \int_{-\infty}^{z} \frac{\partial \overline{U}}{\partial \xi} e^{2k\xi}\, d\xi \tag{18a}$$

and, from equation (15b), we find, after some algebraic manipulation and integration by parts,

$$c_1 = 2k\ c_0 \int_{-\infty}^{0} \overline{U}\, e^{2kz} dz \tag{18b}$$

Substituting equations (16b) and (18b) into equation (13b) and using equation (13a), the perturbation solution for the surface wave celerity is determined as

$$c = \frac{\sigma(k)}{k} + 2k \int_{-\infty}^{0} U\, e^{2kz}\, dz \tag{19}$$

and replacing the effective current field U by its definition from equation (11), we find the approximate dispersion relation

$$\omega = \sigma(k) + 2k \left[k_1 \int_{-\infty}^{0} U_1\, e^{2kz}\, dz + k_2 \int_{-\infty}^{0} U_2\, e^{2kz}\, dz\right] \tag{20}$$

Equation (20) was first derived by Stewart and Joy (6) and later modified to account for water of finite depth by Skop (5). Skop also noticed that equation (20) was not limited to very long deepwater waves or very short capillary waves. For, on integrating equation (20) repeatedly by parts, one obtains

$$\omega = \sigma(k) + (k_1 U_{1S} + k_2 U_{2S}) - \left(\frac{k_1}{2k} \frac{\partial U_{1S}}{\partial z} + \frac{k_2}{2k} \frac{\partial U_{2S}}{\partial z}\right) + O\left(\frac{1}{k}\right) \tag{21}$$

where the subscript "S" denotes the current and its derivatives at the free surface. Equation (21) is, through terms in the surface shear, identical to the asymptotic dispersion relation obtained by Peregrine and Smith (4) for short gravity or gravity-capillary waves ($k \gg 1$) riding on a depth-varying current field. Hence, equation (20) provides a doubly asymptotic approximation to the exact dispersion relation for surface waves on a depth-varying current field.

Given a specified current field, equations (8) and (20) serve to determine the three phase measures -- k_1, k_2 and ω -- for the surface wave throughout x_1, x_2 and t. To complete the analysis of the modifications

of the surface wave by a depth-varying current field, an expression allowing the calculation of the surface wave amplitude η is required.

The Wave Action Equation

The surface wave action function A is defined by

$$A = E/\sigma \tag{22a}$$

where E, the energy density of the wave, is given by

$$E = \frac{1}{2}\rho g \eta^2 \left(1 + \frac{\gamma k^2}{\rho g}\right) \tag{23a}$$

The wave action obeys the conservation law

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x_1}(c_{g1}A) + \frac{\partial}{\partial x_2}(c_{g2}A) = 0 \tag{24}$$

Here, c_{g1} and c_{g2} are the components of the group velocity of the surface wave and are found from the dispersion relation as

$$c_{g1} = \frac{\partial \omega}{\partial k_1} \quad , \quad c_{g2} = \frac{\partial \omega}{\partial k_2} \tag{25}$$

Equation (24) was originally derived by Bretherton and Garrett (1) on the basis of Hamiltonian dynamics. A more physically motivated derivation of this equation can be found in Crapper (2).

Equation (24) provides the required expression for determining the surface wave amplitude η throughout x_1, x_2 and t.

Remarks

Equations (8) are purely kinematical in nature whereas equation (24) is purely dynamical. The accuracy of the approach that has been developed

here for the analysis of the modifications of surface waves by depth-varying current fields thus depends entirely on the accuracy of the approximate dispersion relation defined by equation (20). This dispersion relation appears in two contexts in the approach: first, to complete the wave kinematic equations and second, to determine the components of the group velocity that arise in the wave action equation.

As has already been noted, equation (20) is asymptotic to the exact dispersion relation for waves on a depth-varying current field for long deepwater waves ($k \to 0$) and for short gravity or gravity-capillary waves ($k \gg 1$). Hence, the overall accuracy of the approximate dispersion relation is determined by its ability to mimic the exact dispersion relation at intermediate wave numbers. Skop (5) has demonstrated, by example, that this ability is excellent for two depth-varying current fields for which Taylor (7) obtained exact dispersion relations in his study of hydraulic breakwaters. The first of these was a uniform current extending from the surface to some depth d, while the second was a uniformly sheared current extending from the surface and vanishing at some depth d.

We wish to examine here a significantly more complex depth-varying current field than either of the two current fields previously considered by Skop.

THE SUBMERGED JET

We consider the depth-varying current field defined by

$$U_1 = \begin{cases} 0 & 0 > z > -\alpha d \\ \dfrac{U_0}{1-\alpha}\left[\alpha + \dfrac{z}{d}\right] & -\alpha d > z > -d \\ -\dfrac{U_0}{1-\alpha}\left[2 - \alpha + \dfrac{z}{d}\right] & -d > z > -(2-\alpha)d \\ 0 & -(2-\alpha)\,d > z \end{cases} \tag{26a}$$

$$U_2 = 0 \tag{26b}$$

As shown in Figure 1, this current field represents a symmetric submerged jet. The maximum velocity in the jet is $-U_0$ and occurs at $z = -d$. The width of the jet is $2d(1 - \alpha)$ and the jet velocity goes to zero at $z = -\alpha d$ and $z = -2(2 - \alpha)d$. If one desires, the jet can be thought of as a model of an internal wave trapped at a thermocline.

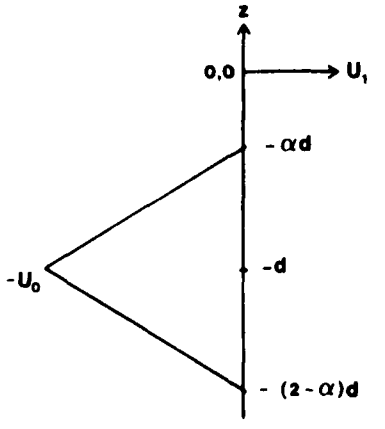

Figure 1. Schematic of the current field characterizing a symmetric submerged jet.

We take a surface wave propagating from $x = -\infty$ into the jet. Then, in the surface wave phase function, we can put $k_1 = k$ and $k_2 = 0$ without loss of generality. We also assume, for this example, that surface tension is unimportant and set the coefficient of surface tension $\gamma = 0$. Substituting equations (17) and (26) into equation (20), we obtain the approximate dispersion relation as

$$\Omega = \sqrt{\kappa} - \frac{F}{2(1-\alpha)} e^{-2\alpha\kappa}[1 - 2e^{-2(1-\alpha)\kappa} + e^{-4(1-\alpha)\kappa}] \tag{27}$$

where the dimensionless frequency Ω, dimensionless wave number κ, and the Froude number F are defined by

$$\Omega = \sqrt{d/g}\ \omega,\ \kappa = kd,\quad F = U_0/\sqrt{gd} \tag{28}$$

The exact dispersion relation for the submerged jet is derived in the Appendix. We find it satisfies the fifth order polynomial in Ω

$$p_5\Omega^5 + p_4\Omega^4 + p_3\Omega^3 + p_2\Omega^2 + p_1\Omega + p_0 = 0 \tag{29}$$

where

$$p_5 = 4(1-\alpha)^3 \tag{30a}$$

$$p_4 = 2(1-\alpha)^2 F\ \{[e^{-2(1-\alpha)\kappa} - 2]e^{-2\kappa} + 2(1-\alpha)\kappa + e^{-2\alpha\kappa}\} \tag{30b}$$

$$\begin{aligned} p_3 = (1-\alpha)\{&2F^2[(1-\alpha)\kappa - 1 + e^{-2(1-\alpha)\kappa}] \\ &+ 2F^2[(1-\alpha)\kappa e^{-2(1-\alpha)\kappa} + e^{-2(1-\alpha)\kappa} - 1]e^{-2\kappa} \\ &- F^2(1 + e^{-2\alpha\kappa})(e^{-2(1-\alpha)\kappa} - 2)\, e^{-2(1-\alpha)\kappa} \\ &+ F^2[2(1-\alpha)\kappa - 1](1 + e^{-2\alpha\kappa}) - 4(1-\alpha)^2\kappa\} \end{aligned} \tag{30c}$$

$$p_2 = F\{2(1-\alpha)^2\kappa[e^{-2(1-\alpha)\kappa} - 2]\, e^{-2\kappa}$$
$$- 2(1-\alpha)^2\kappa[2(1-\alpha)\kappa - 1]$$
$$- F^2(1 + e^{-2\alpha\kappa})[(1-\alpha)\kappa\, e^{-2(1-\alpha)\kappa}$$
$$+ e^{-2(1-\alpha)\kappa} - 1]e^{-2(1-\alpha)\kappa}$$
$$+ F^2(1 + e^{-2\alpha\kappa})[(1-\alpha)\kappa - 1 + e^{-2(1-\alpha)\kappa}]$$
$$+ 2(1-\alpha)^2\, \kappa(e^{-2\alpha\kappa} - 1)\} \qquad (30d)$$

$$p_1 = (1-\alpha)F^2\{2\kappa[(1-\alpha)\kappa e^{-2(1-\alpha)\kappa}$$
$$+ e^{-2(1-\alpha)\kappa} - 1]e^{-2\kappa}$$
$$- \kappa(e^{-2\alpha\kappa} - 1)[e^{-2(1-\alpha)\kappa} - 2]e^{-2(1-\alpha)\kappa}$$
$$- 2\kappa[(1-\alpha)\kappa - 1 + e^{-2(1-\alpha)\kappa}]e^{-2\alpha\kappa}$$
$$+ \kappa[2(1-\alpha)\kappa - 1](e^{-2\alpha\kappa} - 1)\} \qquad (30e)$$

$$p_0 = F^3\kappa(e^{-2\alpha\kappa} - 1)[(1-\alpha)\kappa e^{-4(1-\alpha)\kappa}$$
$$+ e^{-4(1-\alpha)\kappa} + (1-\alpha)\kappa - 1] \qquad (30f)$$

As mentioned previously, the roots of equation (29) as a function of κ for fixed values of α and F provide information on both the dispersion relation for the surface wave and the stability of the imposed current field. The root that is relevant as being the dispersion relation for the incoming surface wave is identified by its asymptotic behavior from equation (21) that $\Omega \to \sqrt{\kappa}$ as $\kappa \to \infty$.

Figures 2, 3, 4, and 5 show comparisons of the approximate and exact dispersion relations for the symmetric submerged jet. In Figure 2, the comparisons are made for $\alpha = 0$ and values of $F = 0.25$, 0.50, and 1.00 which represent a progressively stronger jet. In Figures 3, 4, and 5, the value of F is fixed (at 0.25, 0.50, and 1.00, respectively) and α takes the values $\alpha = 0.00$, 0.05, and 0.20 which represent a progressively

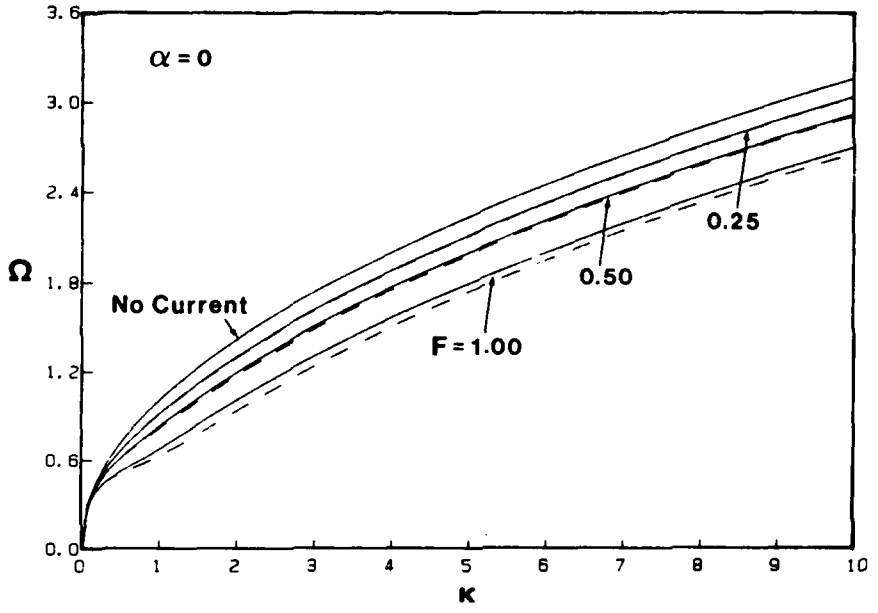

Figure 2. The dispersion relation for water waves on a symmetric submerged jet as a function of the Froude number F with the submergence parameter α = 0. Increasing values of F correspond to progressively stronger jets. Exact solution (——); approximate solution (---).

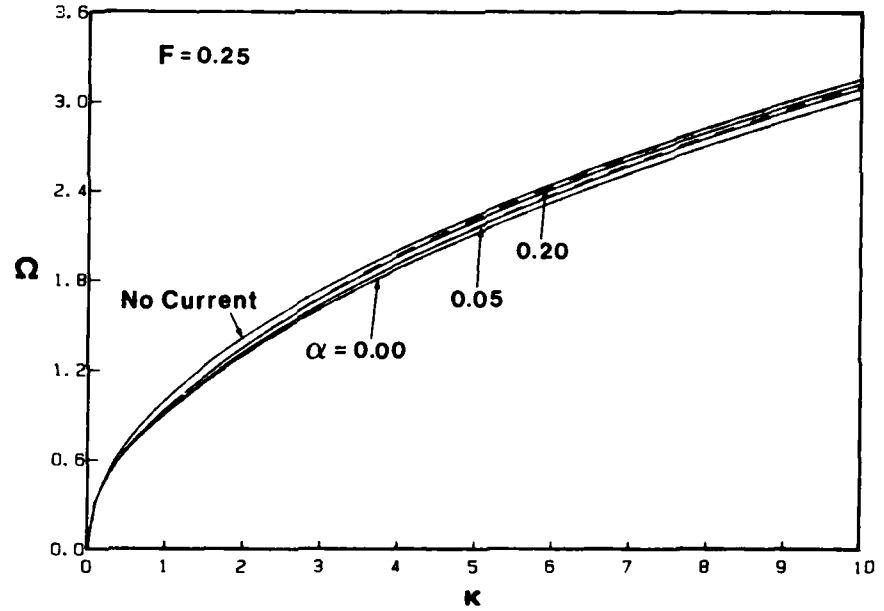

Figure 3. The dispersion relation for water waves on a symmetric submerged jet as a function of the submergence parameter α with the Froude number F = 0.25. Increasing values of α correspond to progressively narrower jets. Exact solution (——); approximate solution (---).

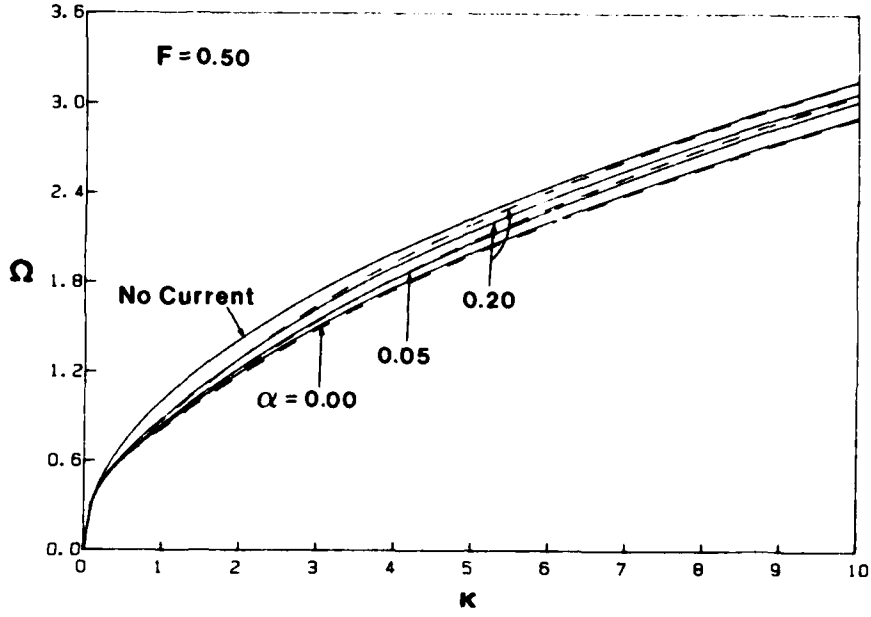

Figure 4. The dispersion relation for water waves on a symmetric submerged jet as a function of the submergence parameter α with the Froude number F = 0.50. Increasing values of α correspond to progressively narrower jets. Exact solution (——); approximate solution (---).

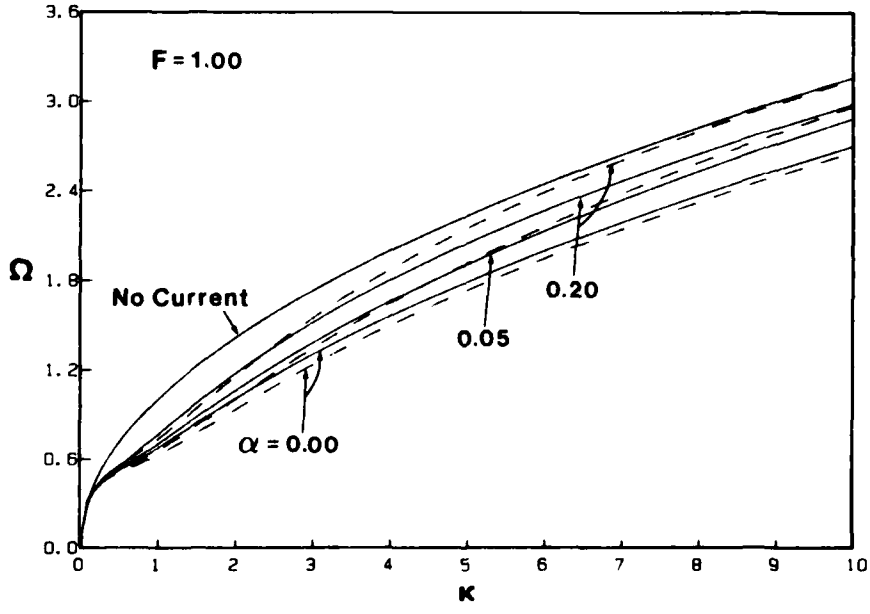

Figure 5. The dispersion relation for water waves on a symmetric submerged jet as a function of the submergence parameter α with the Froude number F = 1.00. Increasing values of α correspond to progressively narrower jets. Exact solution (——); approximate solution (---).

narrower jet. In each figure, the dispersion relation $\Omega = \sqrt{\kappa}$, applicable to a region where the current field vanishes, is shown for reference.

The general observation to be made from Figures 2 through 5 is that the approximate dispersion relation provides a highly satisfactory representation to the exact dispersion relation for the submerged jet throughout wave number space. The representation is best for the weakest and widest jets and degrades with increasing jet strength and decreasing jet width.

A second observation to be made from these figures is that a totally submerged current field can appreciably modify the dispersion relation for a surface wave vis-a-vis its dispersion relation in a region where the current field vanishes. This fact could have important implications for the understanding and interpretation of oceanic features sensed by radar.

CONCLUSIONS

An approach has been developed for the analysis of the modifications of surface waves by depth-varying current fields. The approach is based on an approximate dispersion relation for wave-current interactions derived from the governing equations of the problem (the inviscid Orr-Sommerfeld equations) coupled with the wave kinematic/wave action formulation of surface wave propagation. The wave kinematic/wave action formulation arises purely from kinematic and dynamic considerations; hence, the accuracy of the approach depends entirely on the accuracy of the approximate dispersion relation with respect to the exact dispersion relation. We have demonstrated here and elsewhere (Skop (5)) that the approximate dispersion relation provides a highly satisfactory repre-

sentation to the exact dispersion relation for a variety of depth-varying current fields ranging from simple to complex in structure.

The approach is particularly useful in that it focuses directly on the wave parameters of interest (the amplitude, frequency, direction, and wavelength of the wave) and eliminates the requirement to solve the inviscid Orr-Sommerfeld equations to derive these parameters.

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APPENDIX — DISPERSION RELATION FOR THE SUBMERGED JET

With $U_2 = 0$ and $k_1 = k$, the effective velocity U defined by equation (11) becomes $U = U_1$. Noting from equation (26a) that $\partial^2 U_1/\partial z^2 = 0$, equation (12a) for the vertical velocity component w becomes

$$\frac{\partial^2 w}{\partial z^2} - k^2 w = 0 \tag{A1}$$

Since $U_1 = 0$ and $\partial U_1/\partial z = 0$ at the free surface, the boundary condition there is, from equation (12b) with $\gamma = 0$,

$$c^2 \frac{\partial w}{\partial z} = gw \qquad \text{at } z = 0 \tag{A2}$$

Discontinuities in the velocity gradient of the submerged jet occur at $z = -\alpha d$, $-d$, and $-(2 - \alpha)d$. Across these discontinuities, the vertical velocity and the pressure must be continuous. Hence, we have

$$w \quad \text{and} \quad (U_1 - c)\frac{\partial w}{\partial z} - \frac{\partial U_1}{\partial z} w \tag{A3}$$

must be continuous at $z = -\alpha d$, $-d$, and $-(2 - \alpha)d$.

We seek solutions for equation (A1) as

$$w = \begin{cases} a_1 e^{kz} + b_1 e^{-kz} & 0 > z > -\alpha d \\ a_2 e^{k(z+\alpha d)} + b_2 e^{-k(z+\alpha d)} & -\alpha d > z > -d \\ a_3 e^{k(z+d)} + b_3 e^{-k(z+d)} & -d > z > -(2 - \alpha)d \\ a_4 e^{k[z+(2-\alpha)d]} & -(2 - \alpha)d > z \end{cases} \tag{A4}$$

where a_1 through a_4 and b_1 through b_3 are constants of integration. Apply-

ing the boundary and continuity conditions specified by equations (A2) and (A3), recalling that $c = \omega/k$, and introducing the dimensionless parameters defined by equation (28), we find

$$\Omega^2(a_1 - b_1) = \kappa(a_1 + b_1) \tag{A5a}$$

$$a_1 e^{-\alpha\kappa} + b_1 e^{\alpha\kappa} = a_2 + b_2 \tag{A5b}$$

$$\Omega(a_1 e^{-\alpha\kappa} - b_1 e^{\alpha\kappa}) = \Omega(a_2 - b_2) + \frac{F}{(1-\alpha)}(a_2 + b_2) \tag{A5c}$$

$$a_2 e^{-(1-\alpha)\kappa} + b_2 e^{(1-\alpha)\kappa} = a_3 + b_3 \tag{A5d}$$

$$(F\kappa + \Omega)[a_2 e^{-(1-\alpha)\kappa} - b_2 e^{(1-\alpha)\kappa}] + \frac{F}{(1-\alpha)}[a_2 e^{-(1-\alpha)\kappa} + b_2 e^{(1-\alpha)\kappa}] = (F\kappa + \Omega)(a_3 - b_3) - \frac{F}{(1-\alpha)}(a_3 + b_3) \tag{A5e}$$

$$a_3 e^{-(1-\alpha)\kappa} + b_3 e^{(1-\alpha)\kappa} = a_4 \tag{A5f}$$

$$\Omega[a_3 e^{-(1-\alpha)\kappa} - b_3 e^{(1-\alpha)\kappa}] - \frac{F}{(1-\alpha)}[a_3 e^{-(1-\alpha)\kappa} + b_3 e^{(1-\alpha)\kappa}] = \Omega a_4 \tag{A5g}$$

Elimination of the constants of integration from equations (A5) leads to the exact dispersion relation given by equation (29).

END

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