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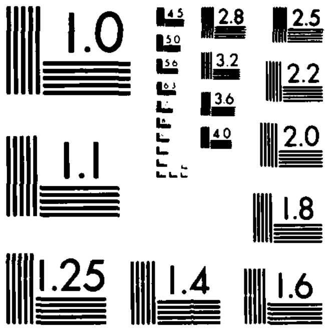

MICROCOPY RESOLUTION TEST CHART

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INTERIM TECHNICAL REPORT

ON

SINGLE MODE FIBER BENDING LOSS

AND

ITS ENVIRONMENTAL DEPENDENCE

CONTRACT NO. DAAL03-86-C-0012
CLIN: 0002AD

SPONSORED BY

U.S. ARMY LABORATORY COMMAND ARMY RESEARCH OFFICE

AND

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INTRODUCTION

The objective of this study contract is to develop a practical single mode bending loss model for special fibers critical to several future Army Weapon Systems. The model will facilitate the selection of fiber and aid the design of high speed missile payout canisters used in major Army fiber optics systems such as FOG-M and AAWS-M. The initial effort will be directed to study various bending induced loss mechanisms in fiber. A theoretical bending loss model, expressed in appropriate computer algorithms is being formulated. Practical fiber characterization schemes will be devised to yield relevant input data to the loss model. The model will then be modified to improve its adequacy for bending loss analysis. Environmental effects on fiber bending loss will be investigated. Reduction of temperature induced fiber loss of missile payout bobbins and field deployable fiber cables is the ultimate goal.

PROGRESS

In the first phase of the Basic Program we have laid the foundation for the real thrust of the project. The schedule is shown in Figure 1. An oral progress report was given to ARO and CECOM personnel at CECOM on September 26, 1986 and represents the detailed portion of this interim report. Progress is summarized and documented in this report. A copy of the oral presentation is shown as Appendix A. A literature survey was conducted and completed on the subject of single mode fiber theory and

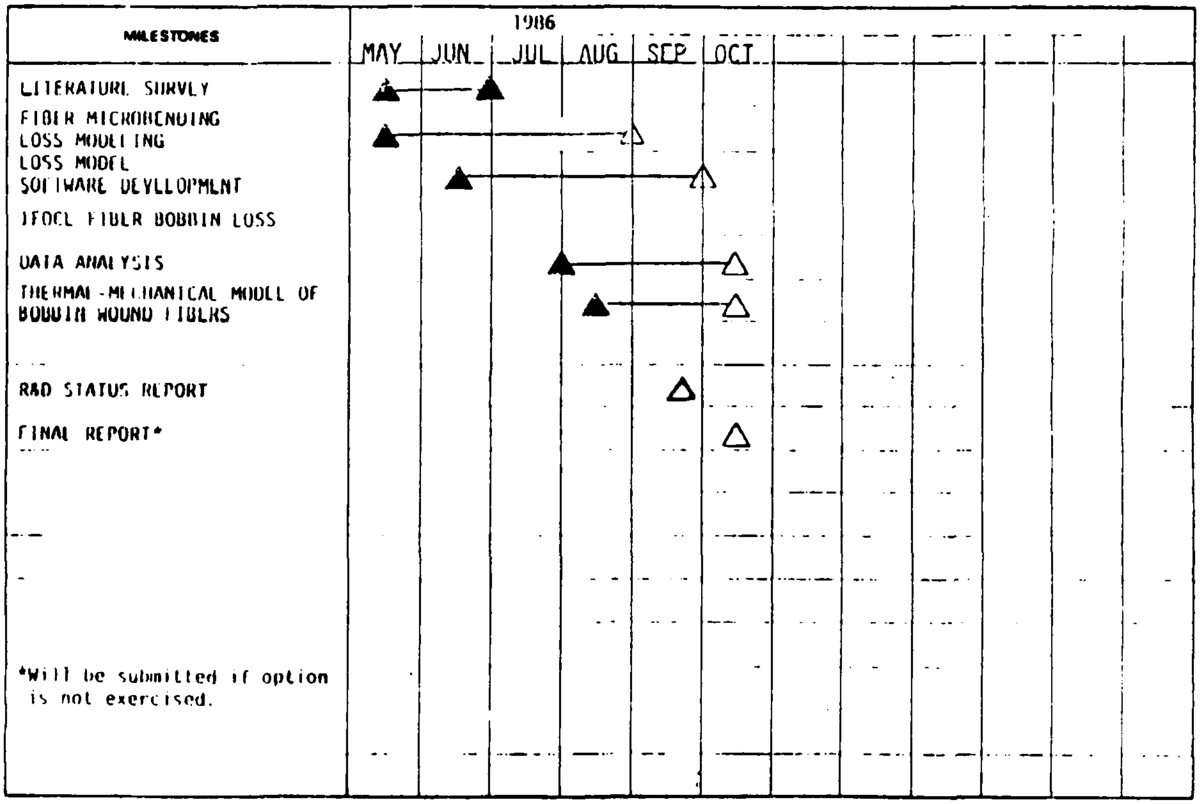


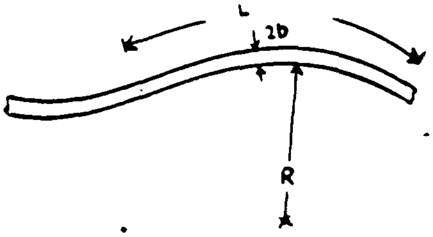
Figure 1. Basic Program Schedule

bending loss phenomena. The survey has produced a reference list of 76 titles, presented as Appendix B. It shows extensive work to date in pursuit of fundamental understanding of fiber bending loss. Numerous articles have been published on both the macrobend and microbend fiber loss study. However, many experimental data still can not be fully explained by the existing theory. There is no unified theoretical equation or a single model that adequately predict actual fiber bending loss. In addition, there are problems generated by the different analytical approaches employed during fiber bending loss research. Our immediate effort is to review these existing theories and to formulate a comprehensive single mode fiber loss model that combines the output of past research efforts with new work. The fiber parameters and measured bend loss are the inputs and the output of the mode.

The basic mechanism of bend induced loss on a single mode fiber is a mode coupling process taking place between a guided mode (HE_{11}) and the radiation modes of fiber. Specifically, the radiation modes include both cladding modes and air modes. A mode coupling into the cladding modes, in which the optical power is still trapped in the fiber cladding, often creates a slow power leakage along the fiber length. A mode coupling to the air modes will cause a radiation loss as the optical power actually radiates out the fiber. The fiber bend loss mechanism can be roughly divided into two categories, depending on its physical dimensions and the abruptness of the bend. The categories are shown in Figure 2.

- MACROBENDS: LARGE BENDING CURVATURE ($R >> b$) THAT COVERS A FIBER LENGTH OF $L >> b$

SPOOL; MANDREL
CORRUGATION PLATE
PAYOUT PEEL POINT



- MICROBENDS: SHARP BENDING CURVATURE ($R \sim b$) THAT INVOLVES LOCAL AXIAL DISPLACEMENTS OF A FEW MICRONMETERS AND SPATIAL WAVELENGTHS OF A FEW MILLIMETERS.

FIBER DIMENSION NONUNIFORMITY (CORE SIZE)
CABLING STRESS, PACKING PRESSURE
BOBBIN WINDING STRESS
ENVIRONMENTAL STRESS ON BOBBIN WOUND FIBER (LOW TEMPERATURE)

Figure 2. Types of Fiber Bending Loss

Macrobend generally refers to a bend curvature several orders larger than the optical wavelength. It can introduce radiation loss as the result of field deformation on the fiber guided mode. The radiation loss depends strongly on the bend curvature and the fiber index profile. In contrast, microbend refers to the microscopic random deviation of fiber axis from its natural straight condition defined by the original drawing of the fiber. It can be introduced on fiber by cabling, winding, and ambient environment change. Microbends often generate gentle mode coupling between the guided mode and the cladding modes of fiber and generally lead to a small optical power loss in fiber over a long length. The microbending loss is known to depend on fiber structure, jacketing material, cabling design, winding condition, and ambient conditions. In theory, microbending loss is a complex process that often requires statistical methodology to characterize the loss behavior. Nevertheless, the formula for both macrobending and microbending fiber loss employs many identical mathematics.

We started our computer model effort by working on the mathematical programming of the constant curvature bending loss of step-index single mode fiber. Marcuse has shown that the bending loss, α , can be expressed in terms of the fiber index profile and the bend radius R as: (Ref.47 in Appendix B)

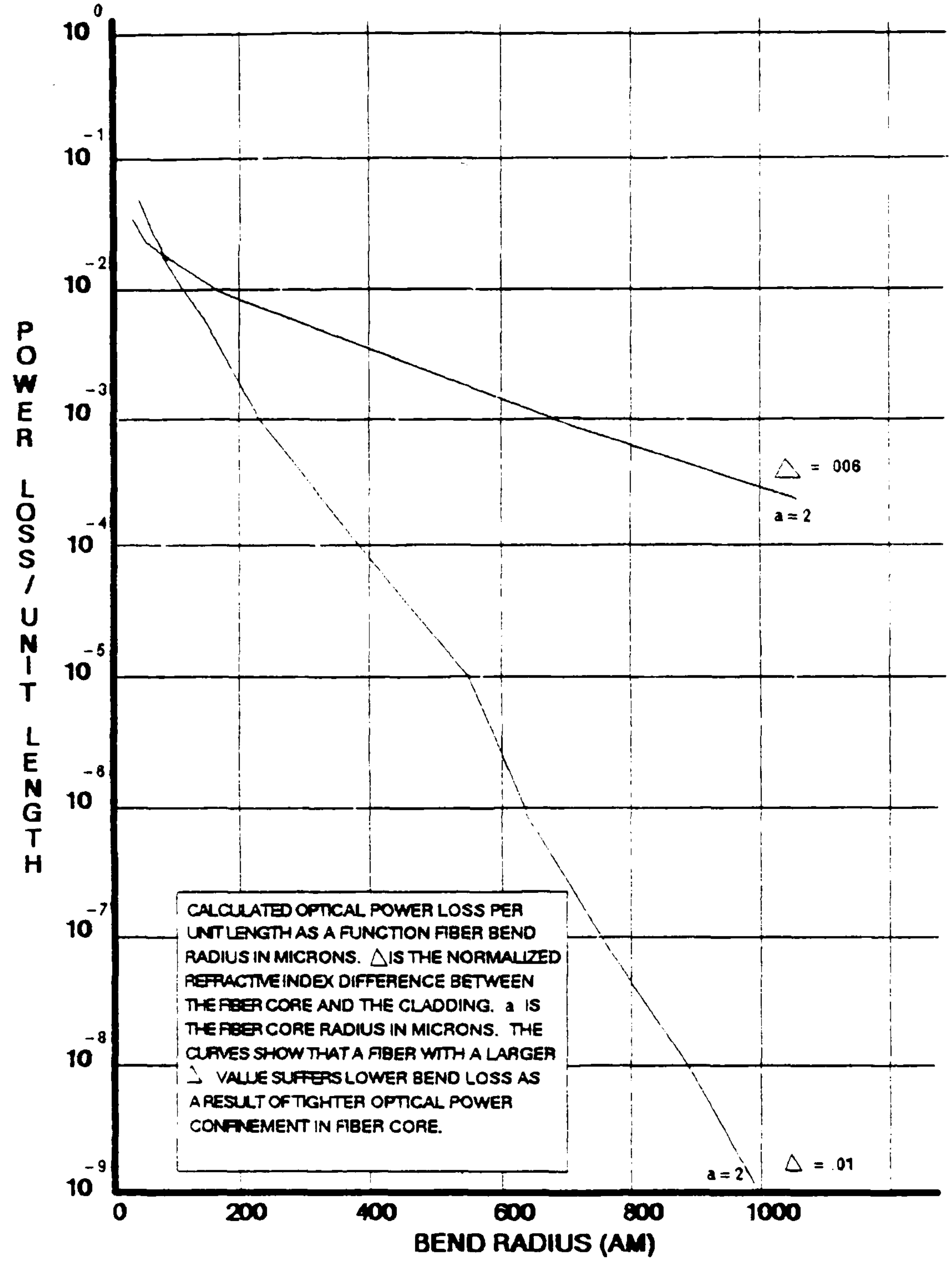
$$a = \frac{\sqrt{\pi}\kappa^2 \exp\left[- \frac{2}{3} \left(\frac{\gamma^3}{\beta 2}\right) R\right]}{2\gamma^{3/2} V^2 \sqrt{R}\, (K_{-1}(\gamma^a)\ K_{;}(\gamma^3))]}$$

where $\kappa = (n_c{}^2 \kappa^2 - \beta^2)^{1/2}$, $\gamma = (\beta^2 - n_{c\ell}{}^2 \kappa^2)^{1/2}$

$V = \kappa^a (n_c{}^2 - n_{c\ell}{}^2)^{1/2}$, $K = \frac{2\pi}{\lambda} a.$

a is the fiber cord radius. n_c and $n_{c\ell}$ are the refractive index of fiber core and cladding respectively. λ is optical wavelength.

A computer program has been written using Professional FORTRAN as its source language. This program has been tested on an IBM PC AT with math processor and should run on IBM PC, XT, or compatibles with a math coprocessor. The preliminary program listing is included as Appendix C. The program calculates the bending loss curves as a function of fiber parameters and bending radius as shown in Figure 3. It shows that the bend induced loss depends critically on the fiber core-cladding refractive index difference and the bend radius R. The next step will compare the calculated loss values with measured constant curvature bend loss data generated from fiber samples designed for use in high speed missile payout dispensers. Expected discrepancies between the two sets of loss data will be analyzed for improving the bending loss model as well as for bend loss measurement.



CALCULATED OPTICAL POWER LOSS PER UNIT LENGTH AS A FUNCTION FIBER BEND RADIUS IN MICRONS. △IS THE NORMALIZED REFRACTIVE INDEX DIFFERENCE BETWEEN THE FIBER CORE AND THE CLADDING. a IS THE FIBER CORE RADIUS IN MICRONS. THE CURVES SHOW THAT A FIBER WITH A LARGER △ VALUE SUFFERS LOWER BEND LOSS AS A RESULT OF TIGHTER OPTICAL POWER CONFINEMENT IN FIBER CORE.

Figure 3. Sample Results for
Calculated Constant Curvature Bending loss

Although this bending loss program is written specifically for a step-index single mode fiber, it is believed that it will be applicable to other single mode fibers with different refractive index profiles. This extension will be required to establish an "equivalent step-index fiber mode field size" for other fibers by matching their evanescent field tails in cladding region against an ideal step-index fiber. Theoretical analysis and fiber output spot size measurements for sample fibers will be conducted to validate this bending loss analysis concept.

One potential application for the constant curvature bending loss study is to evaluate the fiber excess loss while the fiber is subjected to a constant speed payout. The payout peel point curvature is suspected to be a major loss contributor in the fiber payout process. A mechanical model analysis on the peel point curvature in terms of fiber parameters and payout conditions is being improved on a separate project. The calculated peel point curvature will then be used in the bending loss computer program to predict the fiber loss during the payout.

Another analysis effort currently under way involves the collection of optical loss data on bobbin wound fibers and experimental data on different loss measurement techniques. Existing fiber loss data indicate that winding loss and low temperature excess loss of bobbin wound fibers are both bending loss in nature. Winding geometry and the material thermal-mechanical properties of fiber buffer layer have been identified as the prime factors in the loss analysis. Additional modeling is needed to formulate thermal mechanical effects in a bobbin wound fiber pack. The stress profile of the fiber pack will then be translated into microbending parameters and used for a fiber loss prediction.

FUTURE PLAN

The immediate plan is to expand the bending loss computer program to include periodic bend loss analysis. The periodic bend loss program will then further be expanded to cover the microbending loss analysis that will integrate the loss contributions from an ensemble of microbend perturbations in different spatial frequencies. This effort, along with bend loss measurement on sample fibers, will be the primary task in the remainder of the Basic Program. Unless the option of the proposed Optional Program is exercised, a final report for the Basic Program will be prepared to cover the finding of this study. If the option is exercised, the study effort will be continued as shown in Figure 4 in the Optional Program. The results will be presented in the form of a progress report as specified by the contract.

The critical task of the Optional Program is to devise practical fiber loss chacterization schemes that will yield relevant input data useful for the bending loss model. Preliminary loss measurements, including Optical Time Domain Reflectometer (OTDR)

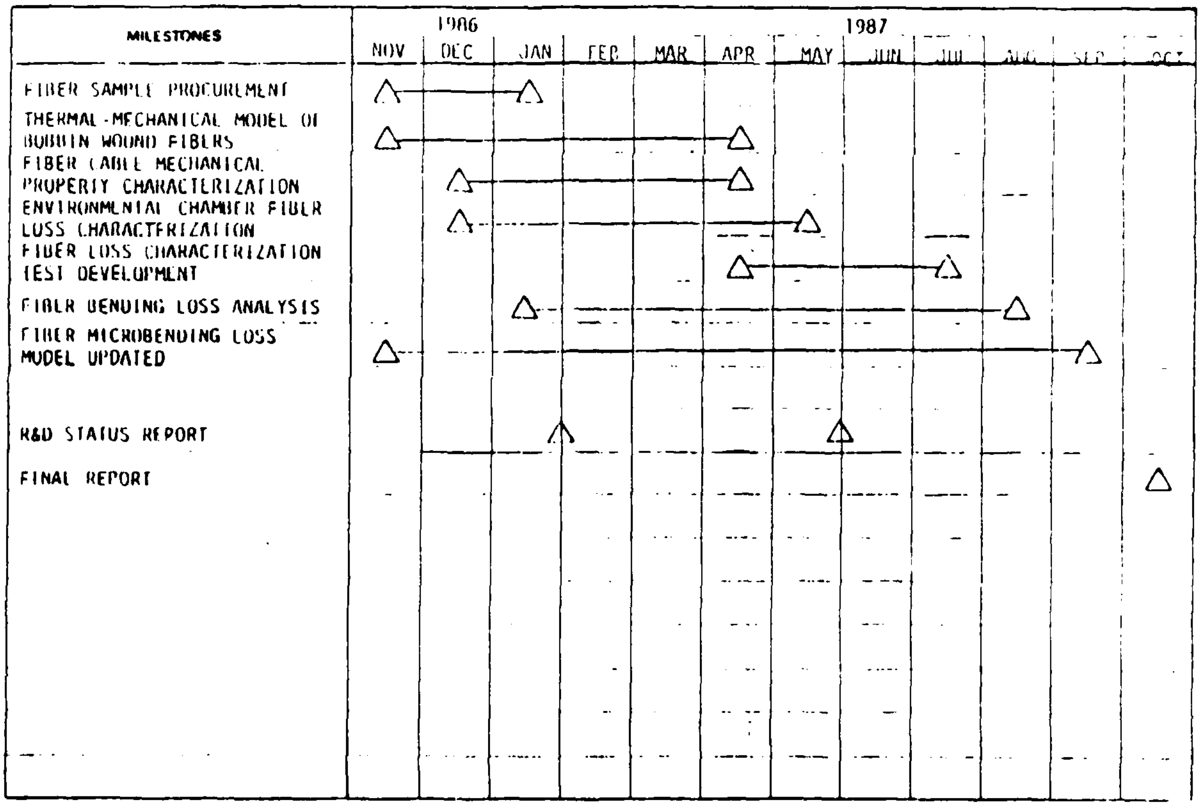


Figure 4. Option Program Schedule

and spectral loss tests, with fibers subjected to various bend profiles and perturbations, will be conducted with Government supplied fibers. Bending loss data generated by mandrel wrapping and bobbin winding will be compared with the calculated results from the bending loss model. The discrepancy analysis will be analyzed to provide new leads for the improvement of the bending loss model. Similar procedure will then be expanded to deal with both the periodic bend case and the microbending induced bobbin wound fiber loss case.

APPENDIX A

PROGRAM STATUS REVIEW
ON
SINGLE MODE FIBER BENDING LOSS
AND
ITS ENVIRONMENTAL DEPENDENCE

HUGHES

PROGRAM STATUS REVIEW
ON
SINGLE MODE FIBER BENDING LOSS
AND
ITS ENVIRONMENTAL DEPENDENCE

CONTRACT # DAAL 03-86-C0012

SPONSORED BY

U.S. ARMY LABORATORY COMMAND
ARMY RESEARCH OFFICE

PREPARED BY

H. P. HSU
SCIENTIST

HUGHES AIRCRAFT COMPANY
MISSILE SYSTEMS GROUP
SEPTEMBER 26, 1986

TECHNICAL OBJECTIVES

- STUDY THE BENDING INDUCED LOSS OF SINGLE-MODE OPTICAL FIBERS
- DEVELOP FIBER BENDING LOSS MODEL AND ANALYSIS ALGORITHMS
- ANALYZE WINDING LOSS AND LOW TEMPERATURE EXCESS LOSS OF BOBBIN WOUND FIBERS
- DEVELOP PRACTICAL TESTS THAT REVEAL FIBER BENDING LOSS SUSCEPTIBILITY

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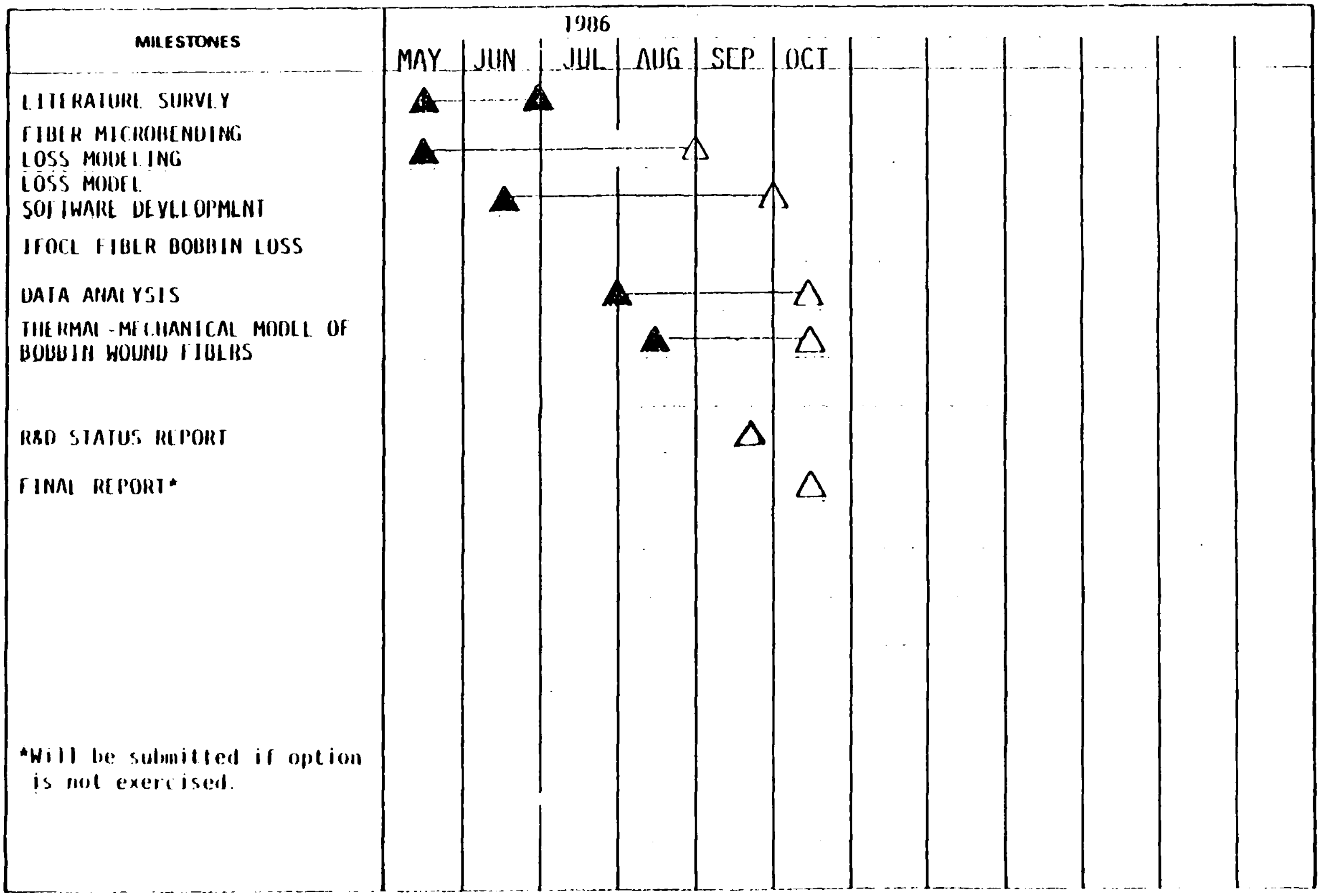
BASIC PROGRAM SCHEDULE



*Will be submitted if option is not exercised.

HPH/ 9/16/86

PROGRESS ON THE BASIC PROGRAM

- LITERATURE SURVEY COMPLETED
- COMPUTER PROGRAM FOR STEP-INDEX, SINGLE-MODE FIBER CONSTANT CURVATURE BENDING LOSS COMPLETED.
- CONSTANT CURVATURE BENDING LOSS STUDY FOR ARBITRARY INDEX PROFILE SINGLE MODE FIBER IN PROGRESS
- STUDY ON MICROBENDING LOSS MECHANISMS FOR BOBBIN WOUND FIBERS IN PROGRESS
- START THE STUDY ON THE THERMAL-MECHANICAL MODEL OF BOBBIN WOUND FIBERS

SCOPE

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- BASIC PROGRAM (MAY'86 - OCT '86)

  THEORETICAL STUDY ON FIBER BENDING LOSS

- OPTIONAL PROGRAM (NOV'86 - OCT'87)

  EXPERIMENTAL STUDY ON FIBER BENDING LOSS

HPH 9/16/86

BASIC PROGRAM

- SIX MONTHS - MAY'86 TO OCT'86
- BUDGET: $60K
- CONDUCT THEORETICAL STUDY ON OPTICAL FIBER BENDING LOSS
- STATEMENT OF WORK
  - TASK 1: FIBER BENDING LOSS THEORY AND COMPUTER MODEL
    - LITERATURE SURVEY
    - IDENTIFY BENDING LOSS MECHANISMS
    - GENERATE FIBER BENDING LOSS FORMULA
    - DEVELOP A TRANSPORTABLE BENDING LOSS COMPUTER PROGRAM
  - TASK 2: BOBBIN WOUND FIBER LOSS DATA ANALYSIS
    - REVIEW IFOCL FIBER LOSS DATA
    - GENERATE THERMAL - MECHANICAL MODEL OF BOBBIN WOUND FIBER

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LITERATURE SEARCH

- SEARCH PERIOD: 1974 - 1986

- TOTAL TITLES: 76

- SUBJECTS:

- SINGLE MODE FIBER THEORY	(22)
- SINGLE MODE FIBER MICROBENDING LOSS	(21)
- SINGLE MODE FIBER MACROBENDING LOSS	(14)
- EFFECT OF FIBER JACKET AND TEMPERATURE ON FIBER LOSS	(14)
- BOBBIN WOUND FIBER LOSS	(5)

COMPUTER PROGRAM FOR FIBER BENDING LOSS

- IBM - PC/XT/AT WITH MATH CO-PROCESSOR
- SOURCE LANGUAGE IS PROFESSIONAL FORTRAN

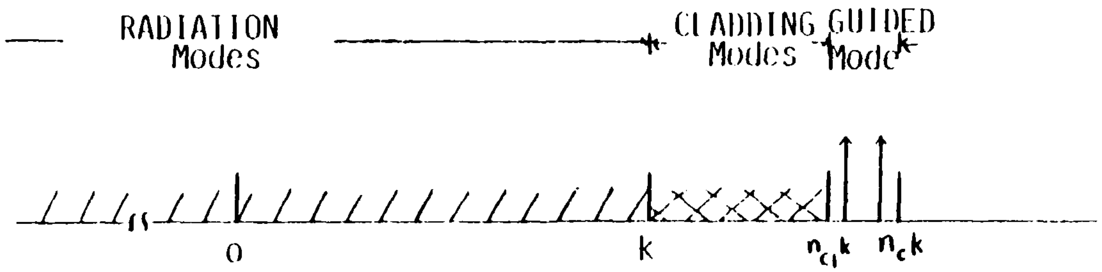
} COMPATIBLE WITH CECOM EFOCL COMPUTER

- STEP-INDEX, SINGLE-MODE FIBER
  - EIGENVALUE SEARCH FOR THE FIBER HE_{11} MODE PROPAGATION CONSTANT (β)
  - CONSTANT CURVATURE BENDING LOSS CALCULATION

FIBER BENDING LOSS MECHANISMS

MODE CONVERSION LOSS - COUPLING FROM A GUIDED MODE TO

- OTHER GUIDED MODES (MULTIMODE FIBER ONLY)
- QUASI-GUIDED MODES OR CLADDING MODES
- RADIATION MODES



n_{ca} = core refractive index

n_{cl} = cladding refractive index

k = wave number in free space $(=2\pi/\lambda)$

CONSTANT CURVATURE FIBER BENDING LOSS

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STEP INDEX FIBER (D. Marcuse)

$$\alpha = \frac{\sqrt{\pi}\,\kappa^2 \exp\left[-\frac{2}{3}\left(\frac{\gamma^3}{\beta^2}\right)R\right]}{2\,\gamma^{3/2}\,V^2\sqrt{R}\,\left[K_{-1}(\gamma a)\,K_{+1}(\gamma a)\right]}$$

where

$$\kappa^2 = n_c^2 k^2 - \beta^2$$

$$\gamma^2 = \beta^2 - n_{cl}^2 k^2$$

$$V = \frac{2\pi}{\lambda}\, a\, (n_c^2 - n_{cl}^2)^{1/2}$$

FOR SINGLE-INDEX SINGLE-MODE FIBER = $V \leq 2.405$

WHY CALCULATE CONSTANT CURVATURE FIBER BENDING LOSS

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- GENERATE AND TEST THE EIGENVALUE SEARCH PROGRAM FOR STEP-INDEX FIBER
- ARBITRARY INDEX - PROFILE FIBER CAN BE STUDIED BY DEFINING A EQUIVALENCE A STEP-INDEX PROFILE FIBER
  - MATCH THE EVANESCENT FIELD IN THE CLADDING REGION FOR BENDING LOSS STUDY
  - MATCH THE PROPOGATION CONSTANT FOR TRANSMISSION CHARACTERISTICS
  - MATCH THE FIBER SPOT-SIZE FOR FIELD CONFINEMENT ANALYSIS

SAMPLE RESULTS FOR CALCULATED CONSTANT CURVATURE BENDING LOSS

```
lambda = 1.30    delta   0.010    A = 2.50    N = 1.45

NORMALIZE FREQUENCY V = 2.477763707538318700
Beta = 6.97654975014008283

Fixed Upper Bound = 17.52042105564704140
Fixed Lower Bound = 17.34521684509057240
            BetaA = 17.43137437535020750

KapaA = 1.662412062647931070      GamaA = 1.828932119546558650

R (Spool Radius):    Alpha (Energy Loss):
           8.00      0.179352098967267E+00
          16.00      0.969923462275028E-01
          24.00      0.605672471058301E-01
          32.00      0.401157083741957E-01
          40.00      0.274413490609392E-01
          80.00      0.507715539881278E-02
         160.00      0.245790702162945E-03
         240.00      0.137397810646567E-04
         320.00      0.814648826172231E-06
         400.00      0.498856254227490E-07
         800.00      0.530591615247906E-13
        1600.00      0.848878048524951E-25
        2400.00      0.156819329220963E-36
        3200.00      0.307277062087650E-48
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        8000.00      0.260711273086556-118
       16000.00      0.648102701812552-235
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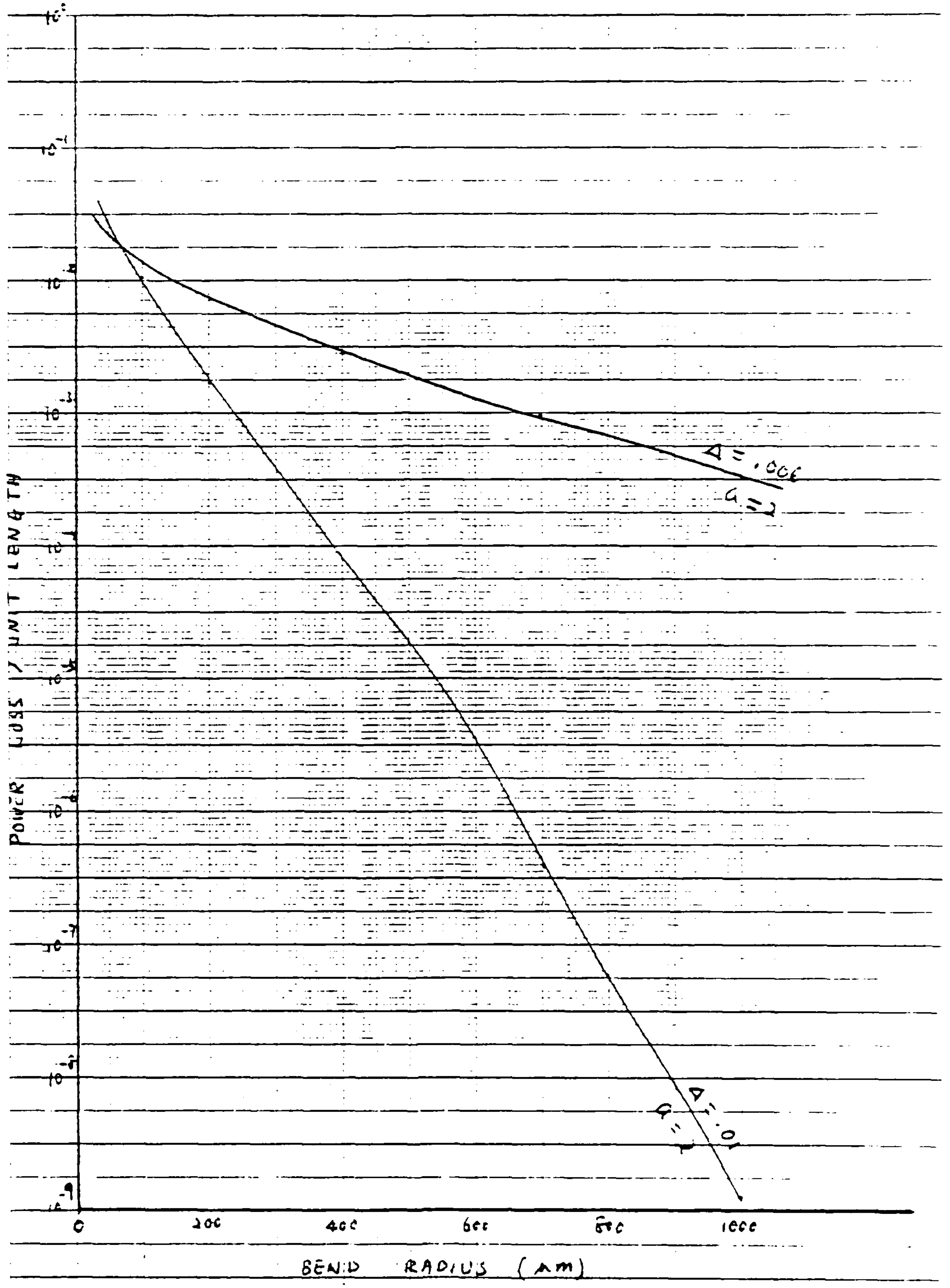
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
POWER LOSS / UNIT LENGTH
Δ = .006
a = 2
Δ = .01
a = 1
0
200
400
600
800
1000
BEND RADIUS (μm)

NEAR TERM MAJOR EVENTS

- STATUS REVIEW AT CECOM ON SEPTEMBER 26
- EVALUATE FURUKAWA VAD FIBER SUPPLIED BY CECOM
- TEST THE BENDING LOSS COMPUTER PROGRAM
- DISCUSS THE FUNDING FOR OPTION PHASE PROGRAM

BENDING LOSS COMPUTER PROGRAM DEVELOPMENT

- PERIODIC PERTURBATION LOSS PROGRAM
- MICROBENDING LOSS PROGRAM
- NON STEP INDEX PROFILE FIBER BENDING LOSS PROGRAM

TEST PLAN FOR FURUKAWA FIBER SAMPLE

- OTDR
- SPECTRAL LOSS W/WO CORRUGATED PLATE PAIR
- 90° BEND AND MANDREL WINDING
- SPOOL WINDING WO ADHESIVE
- SPOOL WINDING WITH ADHESIVE

HPH5a 9/24/86

OPTICAL LOSS OF BOBBIN WOUND FIBERS

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- ANALYZE THE OPTICAL LOSS DATA GENERATED FROM IFOCL AND EFOCL PROGRAM BOBBINS

- IDENTIFY BENDING RELATED LOSSES IN TERMS OF:
  - FIBER PARAMETERS
  - WINDING CONDITION

    WINDING SCHEME, WINDING TENSION, ADHESIVE
  - ENVIRONMENTAL DEPENDENCE

    TEMPERATURE PRESSURE

- ANALYZE BOBBIN WOUND FIBER BY THE BENDING LOSS MODEL

HPH 9/18/86

SPOOL-WOUND FIBER

SPOOL DIAMETER	LOOP LENGTH (CM)	# OF LOOPS/KM	SINGLE LOOP LOSS FOR 1 dB/KM INCREMENT (10-4 dB)
3"	23.94	4178	2.39
4"	31.92	3133	3.19
5"	39.90	2507	3.99
6"	47.88	2089	4.79
7"	56.86	1791	5.69
8"	63.84	1567	6.38
9"	71.82	1392	7.18
10"	79.80	1254	7.98

FIBER DIAMETER (MM)	# OF LOOPS/IN OF SPOOL
200	127
220	115
350	101
300	84
400	63

FIBER BENDING LOSS CHARACTERIZATION

- TRANSMISSION LOSS MEASUREMENTS WITH FIBER SUBJECTED TO
  - CORRUGATED PLATE PAIR
  - MANDREL WINDING
  - SAND PAPER SANDWICH
- OTDR
  - LONG LENGTH BOBBIN
- SPECTRAL LOSS MEASUREMENTS

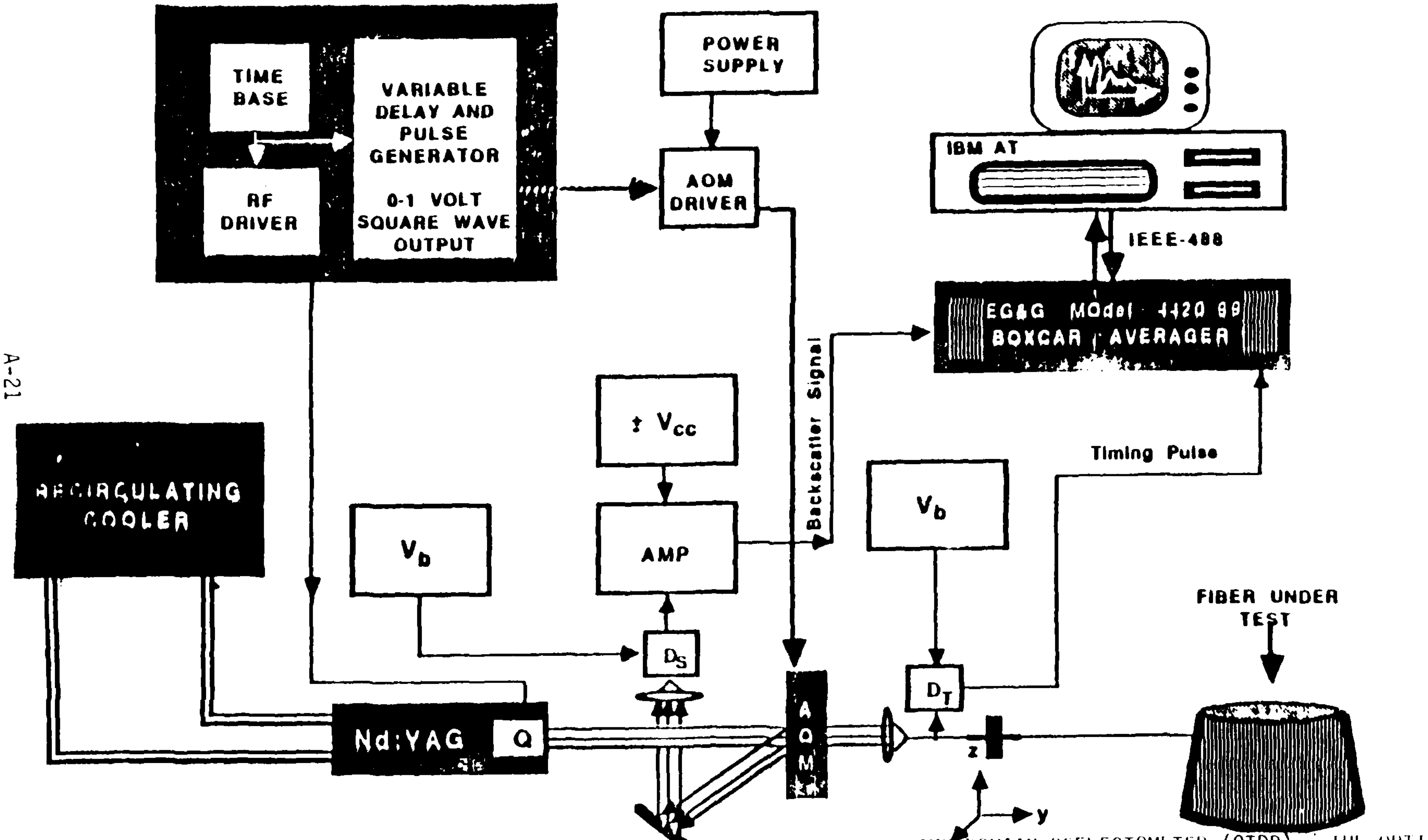
OTDR LOSS MEASUREMENT

LASER PULSE WIDTH	DISTANCE RESOLUTION
10 ns	2 m
100 ns	20 m
1 us	200 m

LOSS RESOLUTION	POWER RATIO (PIn/Pout)
1 dB	.794
0.1 dB	.977
0.01 dB	.9977
0.001 dB	.99977
0.0001 dB	.999977

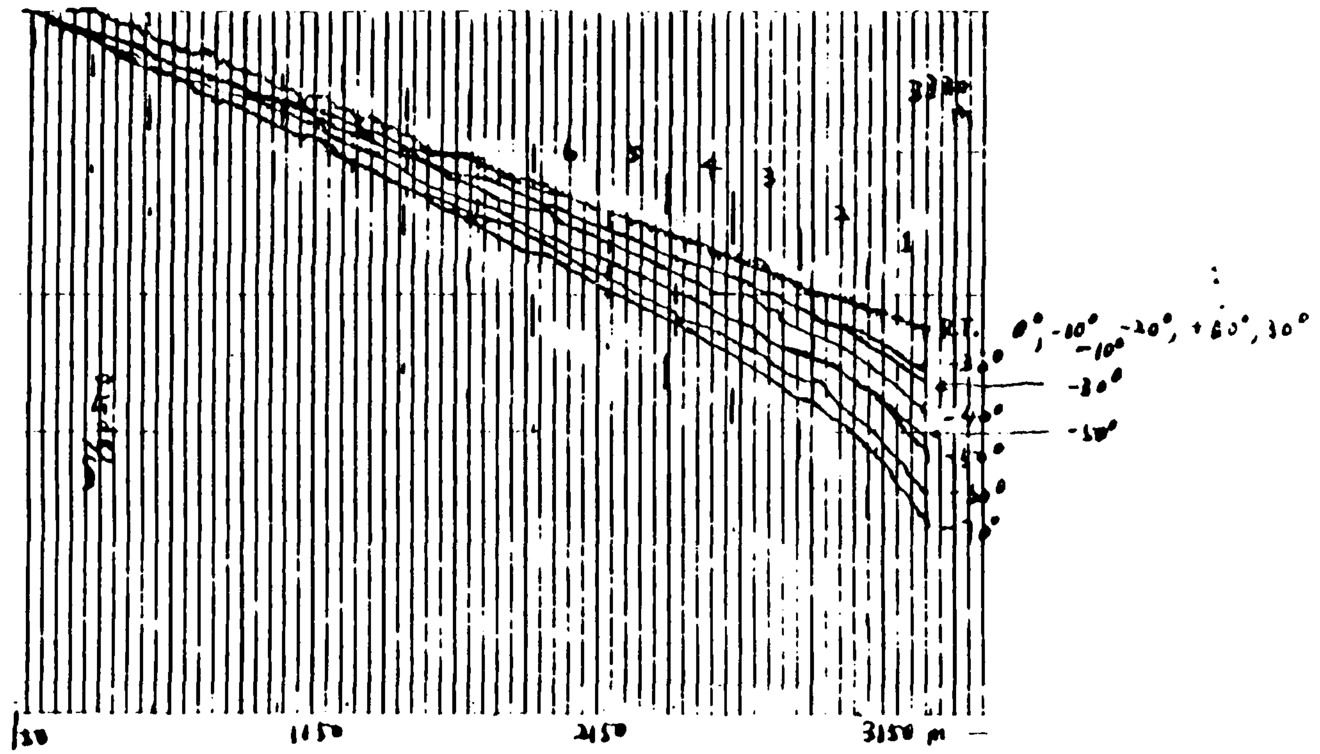
Optical Time Domain Reflectometer

HUGHES



THE FIGURE DEPICTS THE FUNCTIONAL BLOCK DIAGRAM OF LONG RANGE OPTICAL TIME DOMAIN REFLECTOMETER (OTDR). THE OPTICAL SOURCE IS AN $1.32\ \mu m$ Nd:YAG LASER. IN GaAs PHOTODIODE DETECTS, THE RETURN SIGNAL, EG&G BOXCAR AVERAGER PROVIDES THE SIGNAL NOISE RATIO ENHANCEMENT.

COMPOSED OTDR LOSS TRACES OF 3.3 Km SMF BOBBIN AT DIFFERENT TEMPERATURES

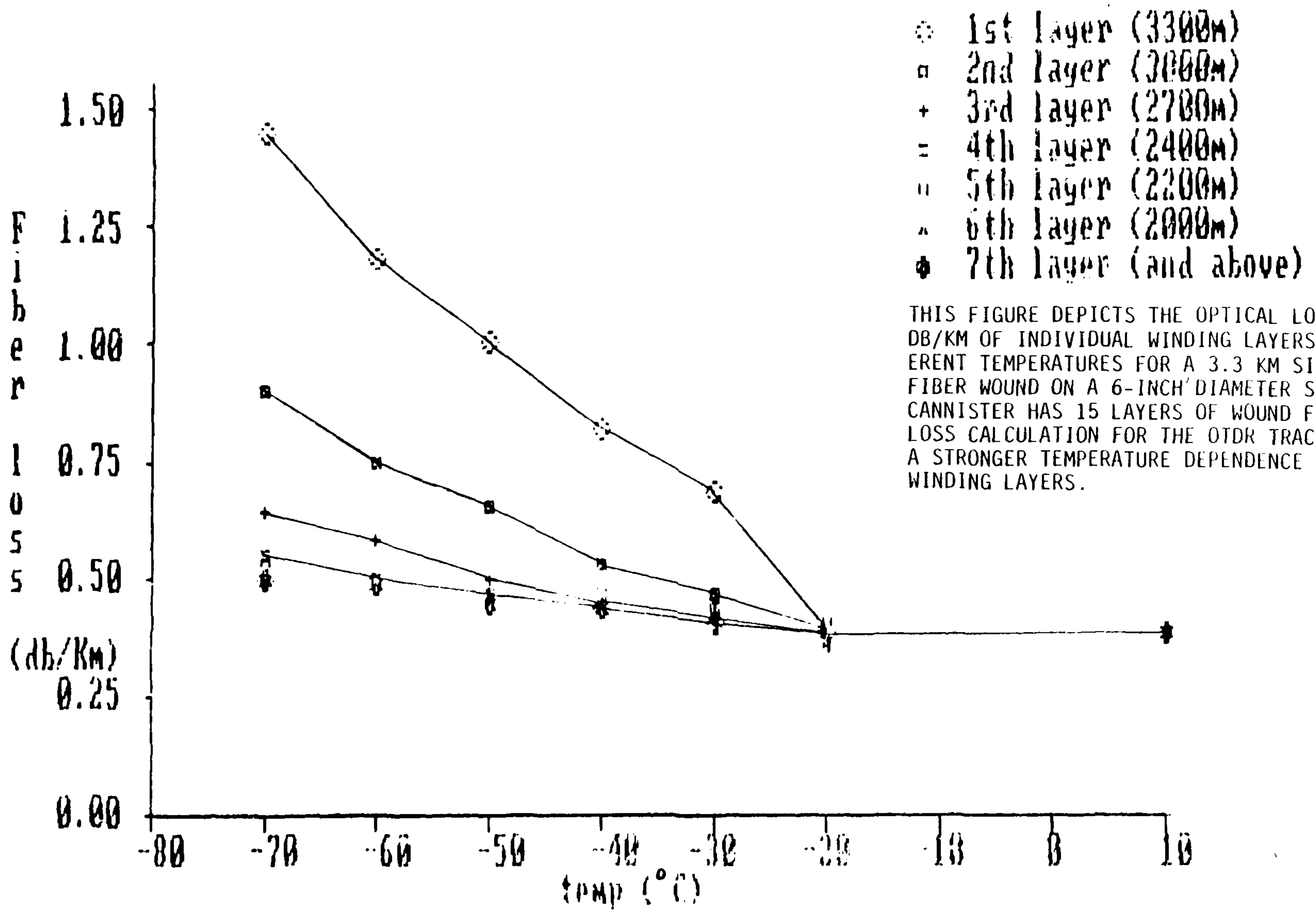


FIBER LOSS TRACES OF A 3.3 KM SINGLE MODE FIBER CANNISTER MEASURED BY AN OPTICAL TIME DOMAIN REFLECTOMETER (OTDR). THE FIBER IS PRECISION WOUND ON A 6-INCH DIAMETER PAYOUT SPOOL. THE LOSS TRACES ARE MEASURED AT DIFFERENT AMBIENT °C TEMPERATURES. THE SLOPE CHANGE OF EACH TRACE INDICATES THE FIBER LOSS DEPENDS ON THE LOCATION OR LAYER IN THE CANNISTER.

VERTICAL SCALE IS 0.25 dB/cm

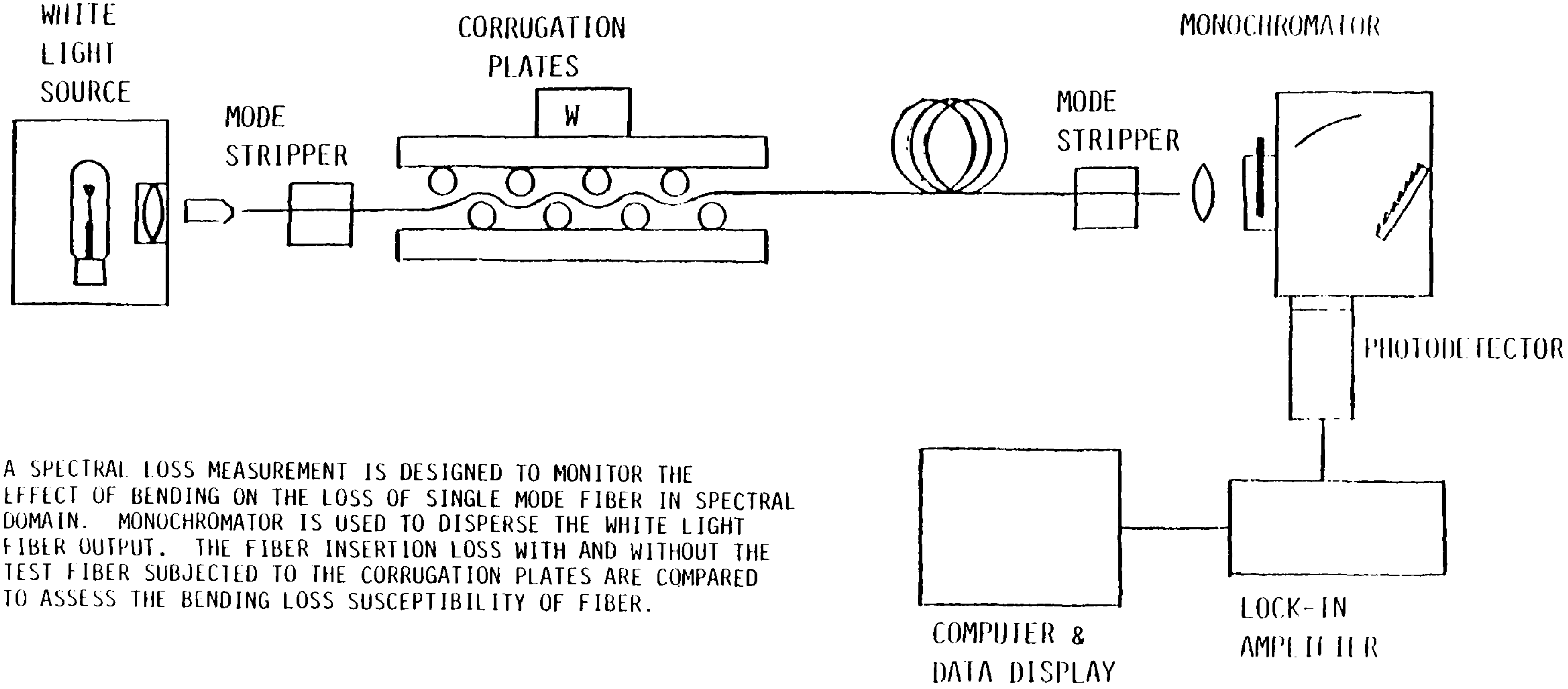
HORIZONTAL SCALE IS 100 m/cm

FIBER LOSS TEMPERATURE DEPENDENCE OF
3.3 Km SMF WOUND ON A 5" BOBBIN (TOTAL 15 LAYERS)



THIS FIGURE DEPICTS THE OPTICAL LOSS, IN DB/KM OF INDIVIDUAL WINDING LAYERS AT DIFFERENT TEMPERATURES FOR A 3.3 KM SINGLE MODE FIBER WOUND ON A 6-INCH DIAMETER SPOOL. THE CANNISTER HAS 15 LAYERS OF WOUND FIBER. THE LOSS CALCULATION FOR THE OTDR TRACES EXHIBIT A STRONGER TEMPERATURE DEPENDENCE FOR INNER WINDING LAYERS.

SPECTRAL LOSS MEASUREMENT

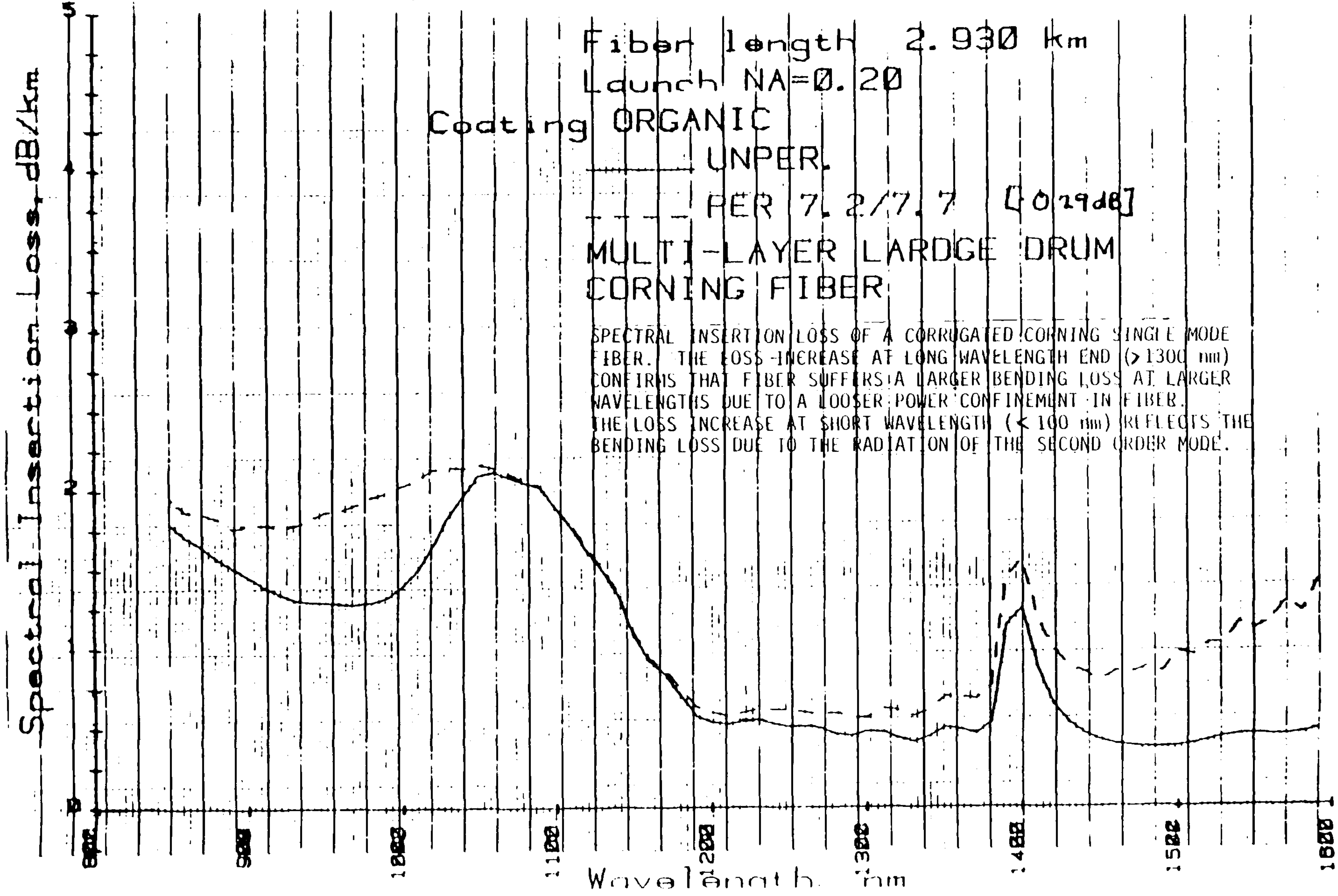


A SPECTRAL LOSS MEASUREMENT IS DESIGNED TO MONITOR THE EFFECT OF BENDING ON THE LOSS OF SINGLE MODE FIBER IN SPECTRAL DOMAIN. MONOCHROMATOR IS USED TO DISPERSE THE WHITE LIGHT FIBER OUTPUT. THE FIBER INSERTION LOSS WITH AND WITHOUT THE TEST FIBER SUBJECTED TO THE CORRUGATION PLATES ARE COMPARED TO ASSESS THE BENDING LOSS SUSCEPTIBILITY OF FIBER.

9-23-86

SPECTRAL INSERTION LOSS DATA OF A CORRUGATED CORNING SINGLE-MODE FIBER

HUGHES



SPECTRAL INSERTION LOSS OF A CORRUGATED CORNING SINGLE MODE FIBER. THE LOSS-INCREASE AT LONG WAVELENGTH END (>1300 nm) CONFIRMS THAT FIBER SUFFERS A LARGER BENDING LOSS AT LARGER WAVELENGTHS DUE TO A LOOSER POWER CONFINEMENT IN FIBER. THE LOSS INCREASE AT SHORT WAVELENGTH (< 100 nm) REFLECTS THE BENDING LOSS DUE TO THE RADIATION OF THE SECOND ORDER MODE.

BOBBIN WOUND FIBER LOSS ANALYSIS PARAMETERS

FIBER	COATING	WINDING	ENVIRONMENT
n(r)	MATERIAL		TEMPERATURE
Δn	SINGLE/DUAL	ADHESIVE	PRESSURE
CORE SIZE	THICKNESS	SPOOL GEOMETRY TENSION	
FIBER OD	THERMAL EXP. COEFF.	CROSS-OVER/ TRANSITION	
DIM. UNIFORMITY	GLASS TRANS. TEMP	CONDITIONING	
	YOUNG's MODULE		

HPHsu 9/24/86

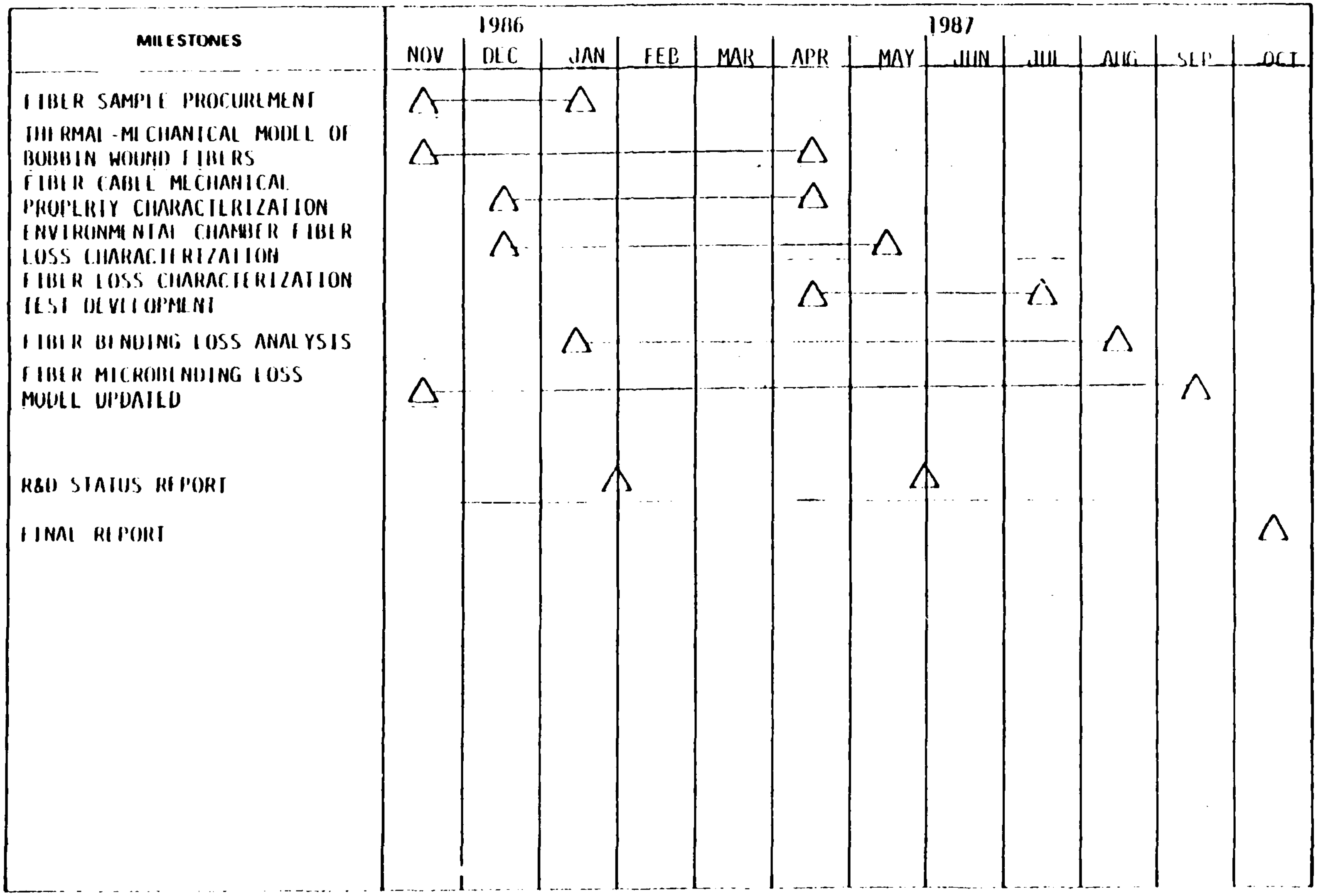
FIBER BENDING LOSS DUE TO THERMAL-MECHANICAL EFFECTS

HUGHES
HUGHES AIRCRAFT COMPANY

- LOSS MECHANISMS CAUSED BY DIFFERENTIAL THERMAL EXPANSIONS
  - BETWEEN FIBER WINDING AND METAL (ALUMINUM) SPOOL
  - BETWEEN FUSED Si AND PLASTIC BUFFER JACKET.
- LOSS MECHANISMS CAUSED BY WINDING GEOMETRY:
  - CROSS-OVER BENDING.
  - POST-CURE ADHESIVE CHARACTERISTICS.
  - BENDING DUE TO SPOOL RADIUS OF CURVATURE.
- LOSS MECHANISM CAUSED BY REDUCTION OF FIBER TENSION (IN SPOOL);
  - VISCO-ELASTIC RELAXATION AND CREEP OF BUFFER JACKET MATERIAL
  - FIBER TENSION LOSS DUE TO THERMAL CYCLING (CONDITIONING).
- LOSS MECHANISM ASSOCIATED WITH FIBER PAY-OUT:
  - PEEL-POINT CURVATURE.

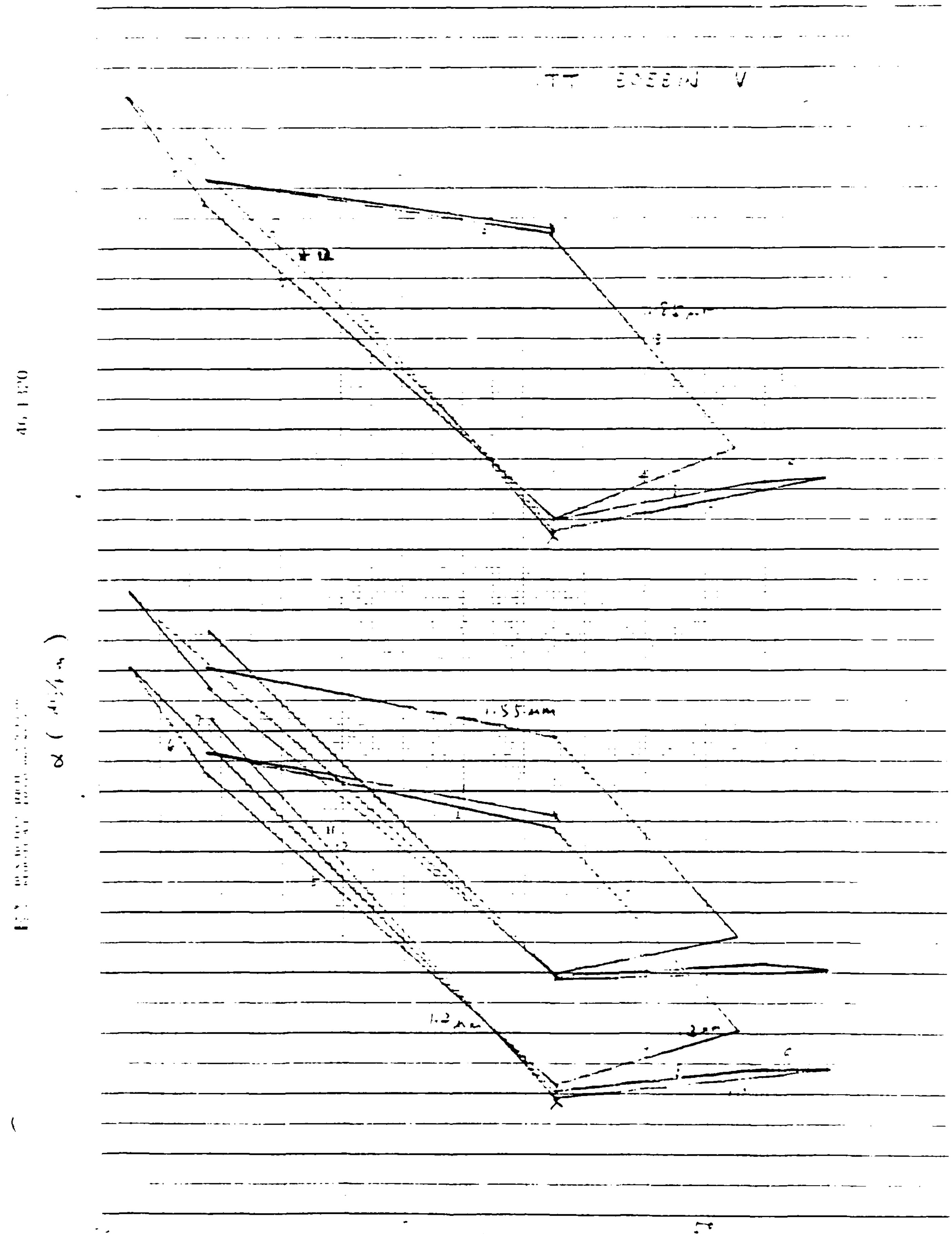
W. C. Hu
81 9. 23

OPTION PROGRAM SCHEDULE

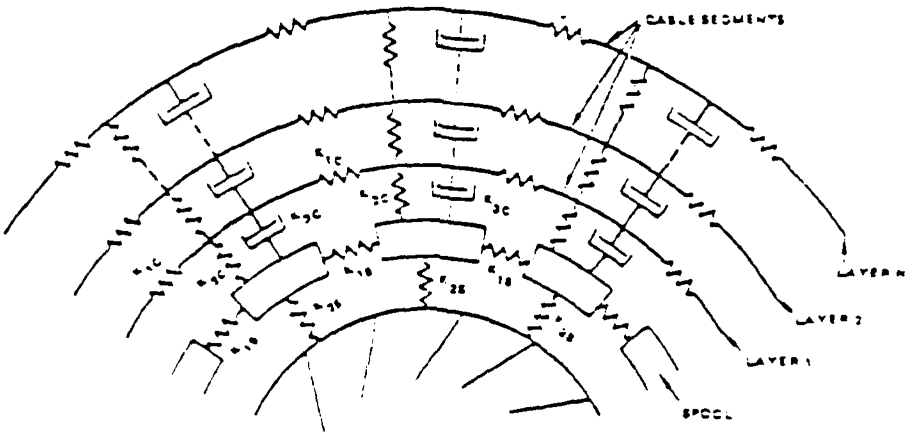


SUMMARY

- LITERATURE SURVEY INDICATES THAT MOST OF RESEARCH HAS BEEN DIRECTED TO STUDY THE FIBER BENDING LOSS UNDER CABLING CONDITION.
- LOSS ANALYSIS OF BOBBIN WOUND FIBER REQUIRES A THERMAL MECHANICAL MODEL FOR THE FIBER PACK AND MICROBENDING LOSS MODEL
- BOBBIN WOUND FIBER LOSS DATA EXHIBITED STRONG LOW TEMPERATURE EXCESS LOSS COATING AND ADHESIVE MATERIAL PHYSICAL PARAMETERS ARE CRITICAL
- LOSS MEASUREMENT SHOULD CONCENTRATE ON LONG-LENGTH SAMPLE EVALUATION TO OBTAIN RELEVANT DATA FOR THE BENDING LOSS MODEL


α (dB/km)
1.55 μm
1.2 μm
TEMP (°C)

CABLE PACK SCHEMATIC



APPENDIX B

REFERENCES

REFERENCES ON SINGLE MODE FIBER BENDING LOSS STUDY

This reference list complies the published literatures on the following subjects:

- References 1 to 22 : Single mode fiber theory
- References 23 to 43 : Single mode fiber microbending loss
- References 44 to 57 : Single mode fiber macrobending loss
- References 58 to 71 : Effect of fiber jacket and temperature on fiber loss
- References 72 to 76 : Bobbin wound fiber loss

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APPENDIX C

PROGRAM LISTING

```
c    This program calculates the normalize frequency, V, with
c  given values: lambda(wavelength), delta, A, and N(index
c  of refraction).  Futhermore, it calculates BetaA, KapaA,
c  and GamaA of the transcendental problem of the Bessel and
c  the Modified-Bessel functions.  All the values are calculated
c  in double-precision.


c  All the variables are implicitly declared real except I and M.
c  Fifty(50) elements are reserved for the arraies.
      IMPLICIT REAL*8(A-H,J-L,N-Z)
      DIMENSION RoA(50), R(50), Alpha(50), Argumt(50), Expnt(50),
     c        DeAlphaA(50), NuAlphaA(50), AlphaA(50), AlphaL(50)

c    There is an input file called "IP" that this program reads its data
c  directly from.
      OPEN (UNIT=8,FILE='IP',STATUS='OLD')
      READ(8,10) lambda, delta, A, N, aInc
      DO 5 I=1,19
      READ(8,20) RoA(I)
    5 CONTINUE
      CLOSE (UNIT=8)
   10 FORMAT(9X,F4.2,12X,F5.3,8X,F4.2,8X,F4.2,10X,F6.1)
   20 FORMAT(6X,F10.1)
      Pi=3.14159265359

c  This is where the NORMALIZE FREQUENCY, V, is calculated.
      V=((2.*Pi)/lambda)*A*N*(DSQRT(2.*delta))
      PRINT*, 'V =',V
      PRINT*, 'Inc =',aInc
      M=0

c    This where the calculation of upper and lower bound of the
c  transcendental problem is calculated.
      UpBd=N*((2*Pi)/lambda)*A
      LwBd=N*(1-delta)*((2*Pi)/lambda)*A
      BetaA=UpBd
      FxUpBd=UpBd
      FxLwBd=LwBd
   50 KapaA=DSQRT((FxUpBd**2)-(BetaA**2))
      GamaA=DSQRT((BetaA**2)-(FxLwBd**2))
      CALL Jo (KapaA, FJo)
      CALL J1 (KapaA, FJ1)
      CALL Ko (GamaA, FKo)
      CALL K1 (GamaA, FK1)
      FJ=KapaA*(FJ1/FJo)
9      FK=GamaA*(FK1/FKo)
      Error=FJ-FK
```

```
c  This accuracy can be alter to approximately 1.x10E-12
      IF (ABS(Error) .LT. 0.00000001) GOTO 200
      IF (Error .GT. 0.0) mSign=1
      IF (Error .LT. 0.0) mSign=2
      M=M+1
      IF (M .EQ. 1) GOTO 100
      IF (mFlag .NE. mSign) THEN
        UpBd=NuUpBd
        LwBd=NuLwBd
        BetaA=UpBd
      ENDIF
  100 mFlag=mSign
      CALL Bound (UpBd, LwBd, BetaA, NuUpBd, NuLwBd, delta, aInc)
      IF (M .GT. 50000) GOTO 1
      GOTO 50
  200 DO 250 I=1,19
c  Fixed Lower Bound < BetaA < Fixed Upper Bound
      Argumt(I)=(2./3.)*((((GamaA)**3)/((BetaA)**2))*(RoA(I)))
      EGamaA=(DSQRT(GamaA))**3
      Expnt(I)=DEXP(-Argumt(I))
      NuAlphaA(I)=(DSQRT(Pi)*((KapaA)**2)*Expnt(I))
      DeAlphaA(I)=4*EGamaA*(V**2)*(DSQRT(RoA(I)))*(FK1**2)
      AlphaA(I)=NuAlphaA(I)/DeAlphaA(I)
      R(I)=RoA(I)/A
      Alpha(I)=AlphaA(I)/A
      L=(Pi*R(I))/2

c  Calculation of AlphaL
      AlphaL(I)=(1-EXP(-Alpha(I)*L))/Alpha(I)
  250 CONTINUE
  260 FORMAT(' lambda =',F5.2,4X,'delta = ',F5.3,4X,'A =',
     c               F5.2,4X,'N =',F5.2,4X,'Inc = ',F5.1)
  265 FORMAT('     ')

c    Final result is printed out in the output file called "FORT9".
c  and by viewing "FORT9" will display all final result(s).
      WRITE(9,260) lambda, delta, A, N, aInc
      WRITE(9,265)
      Beta=BetaA/A
  270 FORMAT(' NORMALIZE FREQUENCY V = ',F20.18)
  272 FORMAT(' Fixed Upper Bound = ',F20.17)
  274 FORMAT(' Fixed Lower Bound = ',F20.17)
  276 FORMAT(13X,'BetaA = ',F20.17)
  278 FORMAT(' KapaA = ',F20.18,5X,'GamaA = ',F20.18)
  279 FORMAT(' Beta =',F20.17)
      WRITE(9,270) V
      WRITE(9,279) Beta
      WRITE(9,265)
      WRITE(9,272) FxUpBd
      WRITE(9,274) FxLwBd
      WRITE(9,276) BetaA
      WRITE(9,265)
      WRITE(9,278) KapaA, GamaA
      WRITE(9,265)
  280 FORMAT( 'R (Spool Radius):', '   Alpha (Energy Loss):',
     c        '    AlphaL(EnergyLoss/Length): ')
      WRITE(9,280)
  290 FORMAT(3X,F11.2,6X,E21.15,5X,E21.15)
      DO 300 I=1,19
      WRITE(9,290) R(I), Alpha(I), AlphaL(I)
  300 CONTINUE
    1 PRINT*, 'Number of loop is',M
      STOP
      END
```

```
c  -------------------------------------------------------
c  THIS SUBROUTINE CALCULATES BetaA WITH GIVEN BOUNDARY
c  -------------------------------------------------------
c
       SUBROUTINE Bound (UpBd, LwBd, BetaA, NuUpBd, NuLwBd, delta, aInc)
       IMPLICIT REAL*8(A-Z)
       X1=UpBd
       X2=LwBd
       Del=(X1-X2)/aInc
       BetaA=BetaA-Del
       NuLwBd=BetaA
       NuUpBd=BetaA+Del
       RETURN
       END

c  Following subroutines are the Bessel functions of zeroth
c  and of first order.
c  -------------------------------------------------------
c  THIS SUBROUTINE CALCULATES THE Jo(X) BESSEL FUNCTIONS
c  -------------------------------------------------------
c
       SUBROUTINE Jo (KapaA, FJo)
       IMPLICIT REAL*8(A-Z)
       X=KapaA
       IF (X .LE. 3.0) GOTO 100
       T=3.0/X
       F=0.79788456+T*(-0.00000077+T*(-0.00552740+T*(-0.00009512+T*
      c              (0.00137237+T*(-0.00072805+T*0.00014476)))))
       THETA=X-0.78539816+T*(-0.04166397+T*(-0.00003954+T*(0.00262573+T*
      c             (-0.00054125+T*(-0.00029333+T*0.00013558)))))
       Q=1.0/DSQRT(X)
       FJo=Q*F*DCOS(THETA)
       RETURN
  100  T=X*X/9.0
       FJo=1.0-T*(2.2499997-T*(1.2656208-T*(0.3163866-T*(0.0444479-T*
      c             (0.0039444-T*0.0002100)))))
       RETURN
       END

c  -------------------------------------------------------
c  THIS SUBROUTINE CALCULATES THE J1(X) BESSEL FUNCTIONS
c  -------------------------------------------------------
c
       SUBROUTINE J1 (KapaA, FJ1)
       IMPLICIT REAL*8(A-Z)
       X=KapaA
       IF (X .LE. 3.0) GOTO 100
       T=3.0/X
       F1=0.79788456+T*(0.00000156+T*(0.01659667+T*(0.00017105+T*
      c            (-0.00249511+T*(0.00113653-T*0.00020033)))))
       THETA=X-2.35619449+T*(0.12499612+T*(0.00005650+T*(-0.00637879+T*
      c            (0.00074348+T*(0.00079824-T*0.00029166)))))
       Q=1.0/DSQRT(X)
       FJ1=Q*F1*DCOS(THETA)
       RETURN
   100 T=X*X/9.0
       FJ1=X*(0.5-T*(0.56249985-T*(0.21093573-T*(0.03954289-T*
      c      (0.00443319-T*(0.00031761-T*0.00001109))))))
       RETURN
       END
```

```
c  Following subroutines are the modified-Bessel function of zeroth
c  and of first order.
c  -----------------------------------------------------------
c  THIS SUBROUTINE CALCULATES THE Ko(X) MODIFIED-BESSEL FUNCTION
c  -----------------------------------------------------------
c
      SUBROUTINE Ko (GamaA, FKo)
      IMPLICIT REAL*8(A-Z)
      X=GamaA
      IF (X .GT. 3.75) GOTO 100
      T=X*X/(3.75*3.75)
      FIo=1.0+T*(3.5156229+T*(3.0899424+T*(1.2067492+T*(0.2659732+T*
     c                     (0.0360768+T*0.0045813)))))
      GOTO 200
  100 T=3.75/X
      IF (X .GT. 85.0) X=85.0
      FIo=DEXP(X)/DSQRT(X)*(0.39894228+T*(0.01328592+T*(0.00225319+T*
     c  (-0.00157565+T*(0.00916281+T*(-0.02057706+T*(0.02635537+T*
     c  (-0.01647633+T*0.00392377))))))))
      GOTO 200
  200 IF (X .LT. 2.0) GOTO 300
      T=2.0/X
      FKo=DEXP(-X)/DSQRT(X)*(1.25331414+T*(-0.07832358+T*(0.02189568+T*
     c     (-0.01062446+T*(0.00587872+T*(-0.00251540+T*0.00053208))))))
      RETURN
  300 T=0.25*X*X
      IF (X .LT. 1.E-30) X=1.E-30
      FKo=-DLOG(0.5*X)*FIo-0.57721566+T*(0.42278420+T*(0.23069756+T*
     c       (0.03488590+T*(0.00262698+T*(0.00010750+T*0.00000740)))))
      RETURN
      END

c  ------------------------------------------------------------
c  THIS SUBROUTINE CALCULATES THE K1(X) MODIFIED-BESSEL FUNCTIONS
c  ------------------------------------------------------------
c
      SUBROUTINE K1 (GamaA, FK1)
      IMPLICIT REAL*8(A-Z)
      X=GamaA
      IF (X .GT. 3.75) GOTO 100
      T=X*X/(3.73*3.75)
      FI1=X*(0.5+T*(0.87890594+T*(0.51498869+T*(0.15084934+T*
     c          (0.02658733+T*(0.00301532+T*0.00032411))))))
      GOTO 200
  100 T=3.75/X
      IF (X .GT. 85.0) X=85.0
      FI1=DEXP(X)/DSQRT(X)*(0.39894228+T*(-0.03988024+T*(-0.00362018+T*
     c  (0.00163801+T*(-0.01031555+T*(0.02282967+T*(-0.02895312+T*
     c  (0.01787654-T*0.00420059))))))))
  200 IF (X .LT. 2.0) GOTO 300
      T=2.0/X
      FK1=DEXP(-X)/DSQRT(X)*(1.25331414+T*(0.23498619+T*(-0.03655620+T*
     c   (0.01504268+T*(-0.00780353+T*(0.00325614-T*0.00068245))))))
      RETURN
  300 T=0.25*X*X
      IF (X .LT. 1.E-30) X=1.E-30
      FK1=DLOG(0.5*X)*FI1+(1.0+T*(0.15443144+T*(-0.67278579+T*
     c  (-0.18156897+T*(-0.01919402+T*(-0.00110404-T*0.00004686))))))/X
      RETURN
      END
```

END

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