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ROBUSTNESS OF PREALLOCATED PREFERENTIAL DEFENSE WITH ASSUMED ATTACK SIZE AND PERFECT ATTACKING AND DEFENDING WEAPONS

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ABSTRACT

The problem is to protect a set of t targets by n perfect interceptors against an attack by m perfect weapons. If the defender solves for an optimal preallocated preferential defense and associated game value assuming m_1 attackers, and the attacker knows the assumption of the defender and utilizes m_2 attackers, he may be able to achieve significantly more damage than had the defender assumed that there would be m_2 attackers. The paper treats the robustness of preallocated preferential defense to assumptions about the size of the attack and presents results of an alternative approach.

PREFACE

This study was conducted as part of the Independent Research Program of the Institute for Defense Analyses, under which significant issues of general interest to the defense research community are investigated.

CONTENTS

I. INTRODUCTION

In [8] Strauch analyzed preallocated preferential defense of ICBMs, where the attacker does not know the number of interceptors assigned to defend each ICBM. Using the expected survival rate as the objective function, he treated the problem as a two-person zero-sum game and solved for the optimal strategies of the attacker and defender. He assumed that both the attacker and the defender are aware of the size of the attacking missile force and interceptor forces to be used in the engagement, and that both the attacking weapons and defending weapons are perfect. In this paper we seek to explore the results under Strauch's model when the defender is unable to determine the size of the attacking force and employs a strategy optimized with respect to an incorrect assumption.

The preallocated preferential defense problem with perfect weapons was also studied by Matheson in [5]. Later, he solved the more general problem with imperfect defenders [6]. Three preallocated preferential defense computer models allow for imperfect as well as perfect defenders ([3], [4] and [7]). While the present paper treats in some detail the case of perfect attacking and defending weapons, the same approach can be used to explore the preallocated preferential defense problem with imperfect weapons.

Section II contains a summary of Strauch's model, including its optimal strategies and resultant payoff.

Section III discusses theoretical and computational aspects of the determination of expected target survival rate when the defender employs the preallocated preferential defense specified by Strauch but the attacker behaves differently.

Section IV presents numerical results covering a wide spectrum of attack and defense resources and strategies.

Section V compares results given in Section IV with those for robust preallocated preferential defense as treated in [1]. A computer model implementing the theory of [1] is documented in [2].

II. STRAUCH'S GAME

In this section is summarized the basic framework and the results of the game presented by Strauch in [2]. The essential outline of this scenario is a two-player, zero-sum game where one player, using m attacking missiles, attempts to destroy a field of t targets defended by his opponent with n interceptors. All targets are of equal value. Each missile or interceptor may attack or defend one target. At a given target all the interceptors defending it have one opportunity to attack one incoming missile each. Afterwards, the surviving missiles, if any, proceed to attack the target. We assume perfect reliability, i.e., the probability that an interceptor will destroy an attacking missile and the probability that any missile which survives interception will destroy the target are one. Thus, a target is destroyed if the missiles attacking it outnumber the interceptors defending it. The attacker and the defender assign, respectively, missiles and interceptors to the targets without either party being aware of his opponent's exact allocation. It is also assumed that both players are able to ascertain correctly the size of the force their opponents have deployed.

We wish to find the optimal strategies available to the attacker and the defender. The attacker's strategy is specified as a set of x_i's, each defined as the ratio of the number of targets assigned i attacking missiles per target to the total number of targets:

$$\begin{aligned} x &= \text{attacker's strategy} \\ &= (x_0, x_2, \ldots, x_i, \ldots, x_m), \end{aligned}$$

where

$$x_i = \begin{array}{l}\text{the fraction of targets} \\ \text{assigned i attacking missiles}\end{array} \quad .$$

Each strategy is subject to the following conditions:

1) All the targets are accounted for:

$$\sum_{i=0}^{m} x_i = 1 \quad .$$

2) The number of missiles assigned is equal to the number available:

$$\sum_{i=0}^{m} (i \cdot x_i) = \mu = m/t = \text{mean number of missiles per target.}$$

The defender's strategy is defined similarly:

$$y = \text{defender's strategy}$$
$$= (y_0, y_2, \ldots, y_j, \ldots, y_n),$$

where

$$y_j = \text{the fraction of targets assigned } j \text{ interceptors,}$$

$$\sum_{j=0}^{n} y_j = 1$$

and

$$\sum_{j=0}^{n} (j \cdot y_j) = \nu = n/t = \text{mean number of interceptors per target.}$$

Given any pair of strategies, we are able to calculate the expected number of targets that will survive. Let us consider the situation at a single target. The probability that it will be attacked by i missiles and be defended by j interceptors is equal to the product of x_i and y_j. We know that a target will be saved if j is greater than or equal to i. Thus the probability of survival for any target is equal to the sum of all $x_i \cdot y_j$ where $j \geq i$:

$$v = S(x, y) = \sum_{i=0}^{m} \sum_{j=i}^{n} x_i \cdot y_j \quad . \tag{2.1}$$

Since the expected value of a sum of random variables is equal to the sum of the expected values of the random variables, the expected number of targets that will survive is $t \cdot v$. Thus, v is also equal to the expected value for the ratio of the targets that survive over all the targets. We assume that the attacker desires to minimize (and the defender to maximize) this value.

There exist two alternative cases, henceforth known as the attack dominated game and the defense dominated game. The attack and defense dominated games correspond respectively to the expected survival rate for any target being less than or greater than 1/2. If μ and ν are integers, then the game is attack dominated if $\mu > \nu$ and defense dominated otherwise. (When μ and ν are not integers this relation does not hold exactly. This situation arises because the defender wins ties and the allocations must be in integers.) For simplicity's sake, we only exhibit the solution for this integer case in the following analysis and in the examples of Section IV. (For the general solution, consult the Appendix of [2].)

If the game is attack dominated, ℓ, the maximum number of interceptors a defender would wish to place at any one target, is $2\mu-1$. The optimal strategy for the attacker is to assign his missiles in such a way that the number of missiles attacking any target appears to be a number chosen randomly between 1 and ℓ, or:

$$x_i = 1/\ell, \; 1 \leq i \leq \ell \quad . \tag{2.2}$$

The optimal strategy for the defender is to do the same thing at as many targets as possible, with the remaining targets receiving zero interceptors:

$$y_j = 2\nu/(\ell(\ell+1)), \; 1 \leq j \leq \ell \tag{2.3a}$$

$$y_0 = 1 - 2\nu/(\ell+1) \quad . \tag{2.3b}$$

The probability of survival for a given target is:

$$v = \nu/(2\mu-1) \quad . \tag{2.4}$$

If the game is defense dominated, then $\ell = 2\nu$. The optimal strategy for the defender is to assign interceptors in such a way that the number of interceptors for each target appears to have been chosen randomly between zero and ℓ:

$$y_j = 1/(\ell+1), \; 0 \leq j \leq \ell \quad . \tag{2.5}$$

The optimal strategy for the attacker is to attack as many targets as possible, as if the number of missiles chosen for each target was selected randomly between 1 and ℓ:

$$x_i = 2\mu/(\ell(\ell+1)),\ 1 \le i \le \ell \tag{2.6a}$$

$$x_0 = 1 - 2\mu/(\ell+1) \quad . \tag{2.6b}$$

Under this regime, the probability of survival is:

$$v = 1-\mu/(2\nu+1) \quad . \tag{2.7}$$

To illustrate this result we consider two numerical examples. In both cases the defender has n = 6000 interceptors to protect t = 1000 targets:

1) Let m (the number of attacking missiles) = 9000. Then

$$\mu = m/t = 9, \quad \nu = n/t = 6 \quad .$$

Both μ and ν are integers; the game is attack dominated and

$$\ell = 2\mu-1 = 17 \quad .$$

The attacker's optimal strategy is:

$$x_i = 1/\ell = 1/17, \quad 1 \le i \le 17 \quad .$$

The defender's optimal strategy is:

$$y_0 = 1-2\nu/(\ell+1) = 1/3$$

$$y_j = 2\nu/(\ell(\ell+1)) = 2/51, \quad 1 \le j \le 17 \quad .$$

The expected survival rate is:

$$v = \nu/(2\mu-1) = 6/17 = .3529 \quad .$$

2) Let $m = 3000$. Then

$$\mu = m/t = 3, \quad \nu = n/t = 6$$

The game is defense dominated and

$$\ell = 2\nu = 12 \quad .$$

The attacker's optimal strategy is:

$$x_0 = 1-2\mu/(\ell(\ell+1)) = 7/13$$

$$x_i = 2\mu/(\ell(\ell+1)) = 6/156, \quad 1 \leq i \leq 12 \quad .$$

The defender's optimal strategy is:

$$y_j = 1/(\ell+1) = 1/13, \quad 0 \leq j \leq 12 \quad .$$

The expected survival rate is:

$$v = 10/13 = .7692 \quad .$$

In summary, there exist an optimal attack strategy which we will call x^* and an optimal defense strategy which we call y^*. As discussed above, both x^* and y^* are determined by the players using the information available to them, namely, the number of attacking missiles, the number of interceptors and the number of targets, or:

$$x^* = x^*(m, n, t) \quad , \quad y^* = y^*(m, n, t) \quad .$$

For each combination of m, n, and t, the pair of optimal strategies x^* and y^* defines an expected target survival rate, which we denote by v^*:

$$v^* = S(x^*, y^*) \quad .$$

A player, whether attacker or defen`er, will not do worse than v^* if he plays his optimal strategy x^* or y^*. Thus, v^* represents an equilibrium for this game, and we call v^* the "optimal game value."

Note that a proof of the results presented in this section may be found in the Appendix of [2].

III. ALTERNATIVE STRATEGIES

The assumption that the defender is aware of the correct attack size is a strong one. Consider the situation where the defender optimizes against an attack m^A, choosing $y^A = y^*(m^A,n,t)$. If the actual attack size turns out to be m, different from m^A, then the defender may not be acting optimally and thus subject to exploitation by the attacker.

The attacker will behave in one of two ways:

I) The attacker will use x^* regardless of what the defender does.

II) The attacker will be able to discover the defender's strategy, y^A, and optimize against it.

Assumption I implies that the attacker is unable to establish with certainty the defender's planning strategy. From Section II, we are able to solve explicitly for x^* and y^A given m, n, t, and m^A. To find the survival rate of this scenario, we simply apply equation 2.1 to x^* and y^A. We denote the resulting value for the expected target survival rate as

$$v_I = S(x^*, y^A) \quad .$$

We would expect v_I to be less than or equal to v^* for a given m, n, and t, since the defender is not utilizing the correct optimal strategy and thus may suffer as a result. He will at best achieve a survival rate equal to the optimal game value, since the attacker is employing x^*.

Under Assumption II, the attacker is informed of the defender's strategy. (Actually, simply knowing m^A would be sufficient because the attacker may then reconstruct y^A with the information available to him.) Now that he is aware of how the defender is deviating from the optimal defensive strategy, the attacker can construct a strategy, which we denote by x^A, that will enable him to take full advantage of y^A. To find x^A, we set up a linear minimization program with equation 2.1 as the objective function:

$$\min \sum_{i=0}^{m} \sum_{j=i}^{n} x_i \cdot y_j^A$$

subject to:

$$\sum x_i = 1$$

$$\sum i \cdot x_i = m/t$$

$$x_i \geq 0, \; i=0, \ldots, m \quad .$$

The fact that the attacker knows the y_j^A's makes the objective function linear. The two linear constraints are identical to the conditions which define a permissible attacker's strategy. Therefore, any simplex routine should provide a solution to the program, with the optimal attacker's strategy and the resulting expected target survival rate:

$$x^A = x^A(m, t, y^A)$$

$$v_{II} = S(x^A, y^A) \quad .$$

We would expect v_{II} to be less than or equal to both v^* and v_I, because the attacker is able to use the optimal attack x^A against y^A rather than simply x^*.

IV. EXAMPLES

We illustrate the methodology and results of Section III by examining solutions for three different numbers of interceptors: n = 3000, 6000, and 9000. The number of targets for all the examples is set at 1000. For each value of n we solve for v^*, v_I and v_{II}, under various defender assumptions regarding the attack size. The attack sizes range from 1000 to 18,000 for n = 3000 and 6000, and from 1000 to 24,000 for n = 9000.

Case I: t = 1000, n = 3000

A) Let $m^A \leq 3000$.

When the anticipated number of attacking missiles is less than the number of interceptors, the defender believes he dominates the game and employs the same strategy so long as m is equal to or less than n.

The defender's strategy y^A is:

$$y_j^A = 1/7 \quad , \quad 0 \leq j \leq 7 \quad .$$

For $m \leq 3000$, Figure 4.1a shows that v^*, v_I, and v_{II} are equal. Clearly, so long as the actual attack size is less than the defense size, the defender will be deploying his interceptors optimally even if m^A is not the actual attack size. A single strategy will guarantee the defender the optimal game value for all $m \leq n$.

As Figure 4.1a demonstrates, v_I remains equal to v^* even when the real attack size is greater than the assumed attack size. Apparently, the use of x^* against y^A will not give the attacker any result less than the optimal game value. This is not surprising. Either player in a two-person zero sum game

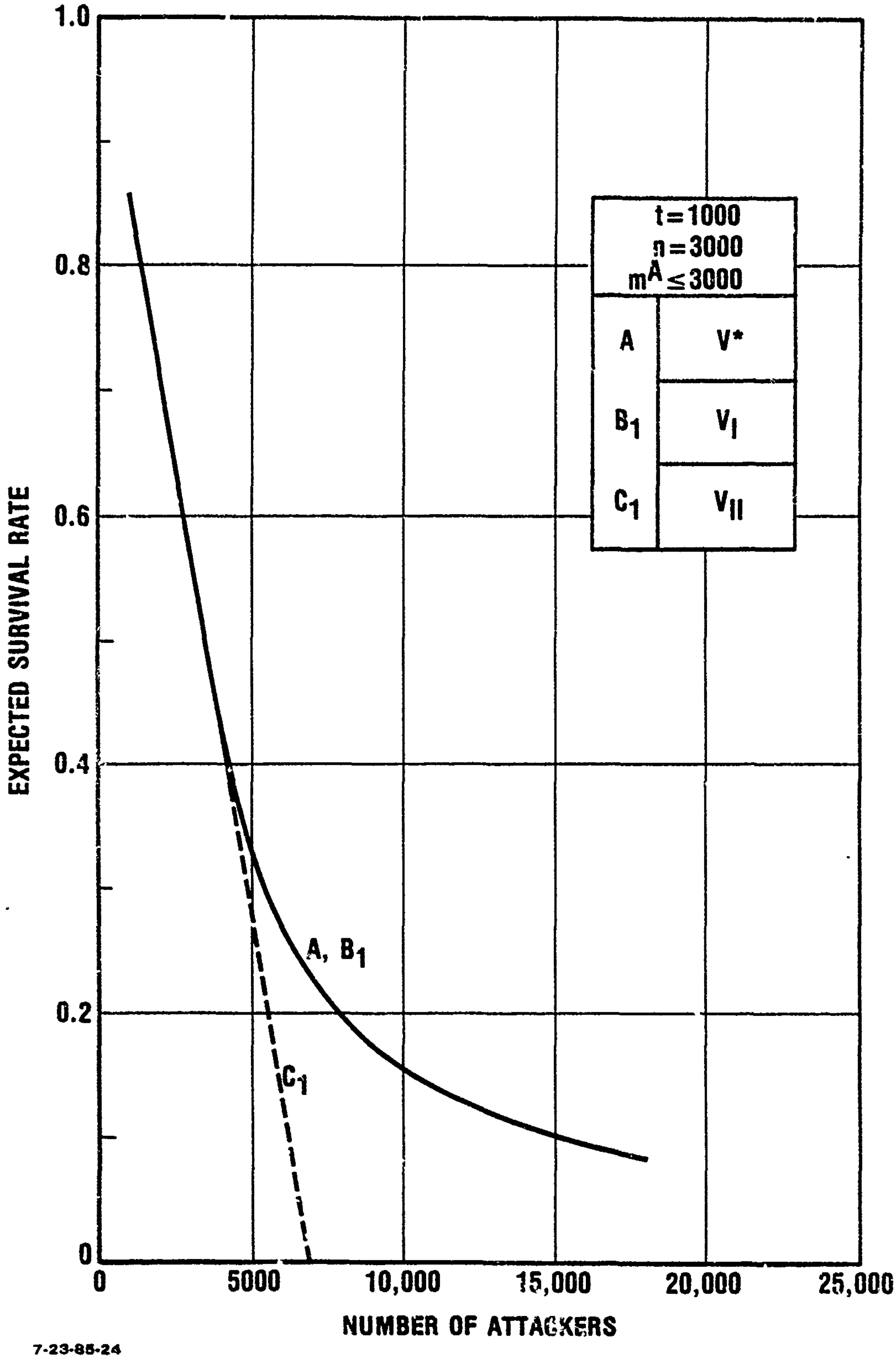

Figure 4.1a. EXPECTED SURVIVAL RATES FOR $n = 3000$, $m^A \leq 3000$

can expect the game value if he employs any combination of his "active" strategies, if his opponent uses his optimal strategy. A valid "combination of active strategies for the defender" in this game is one in which no target is assigned more than ℓ interceptors, as defined in Section II. Now let us suppose that the game is attacker dominated and the defender prepares against the correct attack size. The maximum number of missiles, ℓ, he assigns to any target must be greater than the maximum number he assigns when he believes the game to be defender dominated. Thus, whenever the defender believes he dominates and the attacker employs x*, the expected rate of survival for the targets will not differ from v* for any attack size.

Curve C_1 of the same graph, however, shows that v_{II} does deviate significantly from v*. Every attack size greater than 4000 results in an expected survival rate less than v*. At m = 7000, all the targets are destroyed. Since the maximum number of interceptors allocated to a target is 6, the attacker, by attacking each target with 7 missiles, can knock them all out. No longer restricted to using x*, the attacker is able to capitalize on y^A by using x^A.

We now look at what happens when the defender believes that the game is attacker dominated.

B) Let $m^A = 6000$

The defender's strategy y^A is:

$$y_0^A = 1/2$$

$$y_j^A = 1/22 \qquad 1 \leq j \leq 11$$

Comparing curves A and B_2 of Figure 4.1b, we note that for $m < 6000$ v_I is smaller than v*, but after the two converge at

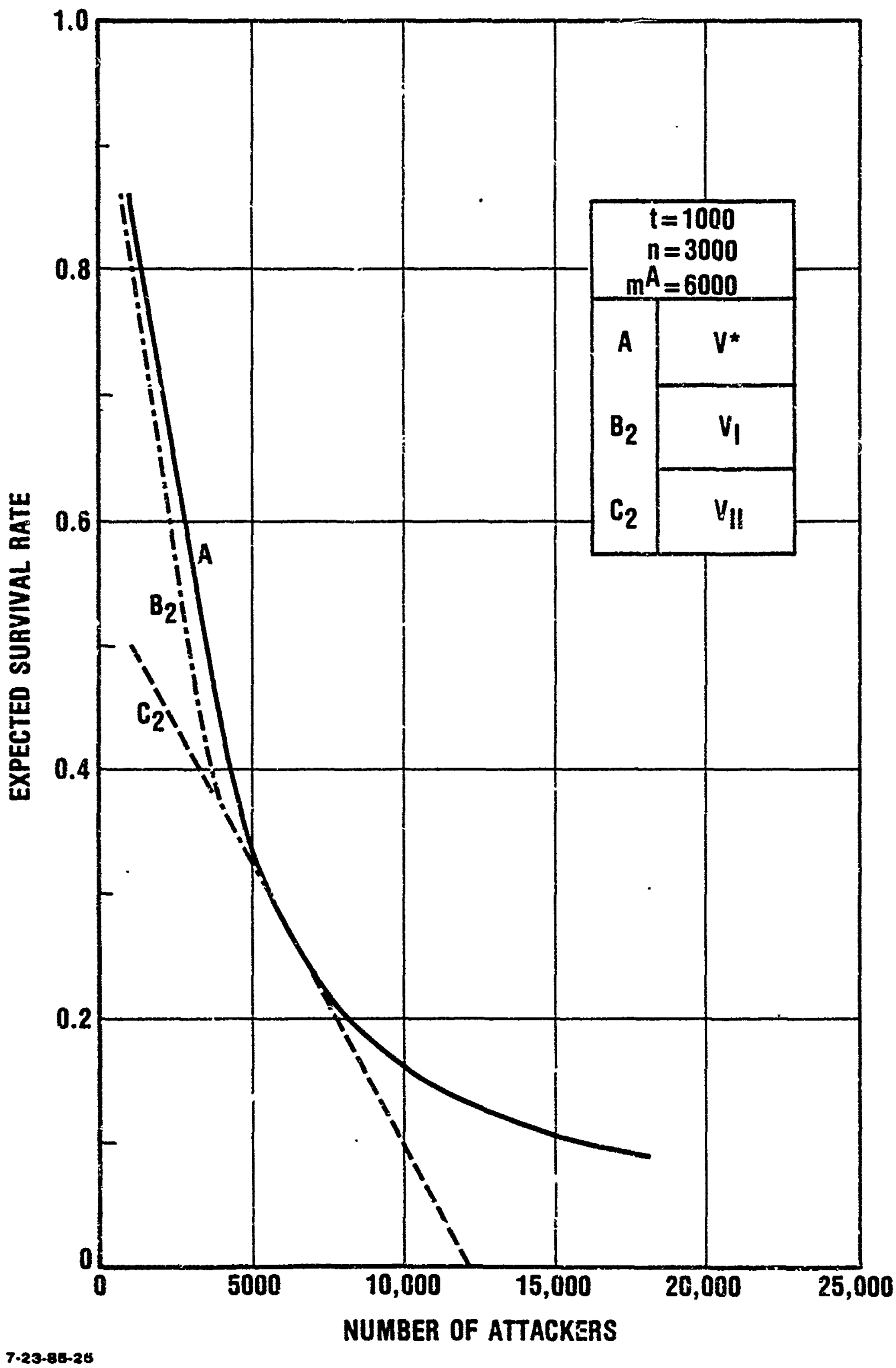

Figure 4.1b. EXPECTED SURVIVAL RATES FOR $n = 3000$, $m^A = 6000$

6000, v_I remains equal to v^*. The reason why v_I does not equal v^* for the entire range of attack sizes is that (unlike in A) the maximum number of missiles the defender puts at a target when he overestimates the attack size is larger than ℓ. For example, let the actual attack size be 3000. For $\mu = 3$ and $\nu = 3$, $\ell = 6$, whereas the maximum interceptors the defender deploys at a target under y^A is 11. Therefore, y^A does not satisfy the condition we described earlier for a valid "combination of active strategies." Intuitively, one may consider this as having "wasted" interceptors by putting more than ℓ at any target. When the defender underestimates the attack size ($m^A \leq m$), this is no longer the case. v_I and v^* are equal.

v_{II} (curve C_2 in Figure 4.1b), as expected, is significantly lower than v_I and v^*. At m = 1000 the attacker is able to take advantage of the fact that half of the targets are undefended even though the interceptors outnumber the attacking missiles. The expected survival rate is less than 60% of the optimal game value. Between 1000 and 6000, the difference shrinks until the expected attack size equals the real attack size. The defender's strategy is optimal (only) at this point. For $m > 6000$, v_{II} diverges from v^* and is reduced to zero at m = 12,000. Since the maximum number of interceptors located at any target is 11, the attacker can destroy all the targets by attacking each target with 12 missiles. m = 12,000 is the minimum attack size that makes this deployment feasible.

C) Let $m^A = 9000$.

The defender's strategy is:

$$y_0^A = 2/3$$

$$y_j^A = 1/51 \ , \quad 1 \leq j \leq 17 \ .$$

As Figure 4.1b and 4.1c show, the qualitative behavior for $m = 9000$ is very similar to that for $m = 6000$. When the defender overestimates the attack size, v_I is lower than v^*, but when he underestimates, they are equal. With v_{II}, when he overestimates attack strength ($m < m^A$), he suffers by leaving too many targets without protection. When he underestimates attack strength he leaves his defenses too widespread and is obliterated. Quantitatively v_{II} ($m^A = 9000$) is lower than v_I ($m^A = 6000$) for small attack sizes. But when the attack size turns out to be large it takes 6000 more attacking missiles to destroy all the targets.

Figure 4.1d combines Figures 4.1a, 4.1b and 4.1c to allow cross-comparison.

<u>Case II</u>: $t = 1000$, $n = 6000$

A) Let $m^A \leq 6000$.

The defender believes that he dominates. The defender's strategy is:

$$y_j^A = 1/13 \ , \quad 0 \leq j \leq 12 \quad .$$

The values for v^*, v_I, and v_{II} are displayed graphically on curves A, B_1, and C_1 of Figure 4.2. A review of Figure 4.1a verifies that the qualitative results are comparable to that for Case I. Just as in Case I, when the defender believes that he dominates, v_I is identical to v^*. v_{II} is equal to v^* as well, until the game becomes attacker dominated. When the attacker is able to use x^A, all the targets are annihilated at $m = 13{,}000$. For attack sizes less than 6000, the defender behaves optimally. But when the game turns out to be attacker dominated, y^A leaves the defender very vulnerable.

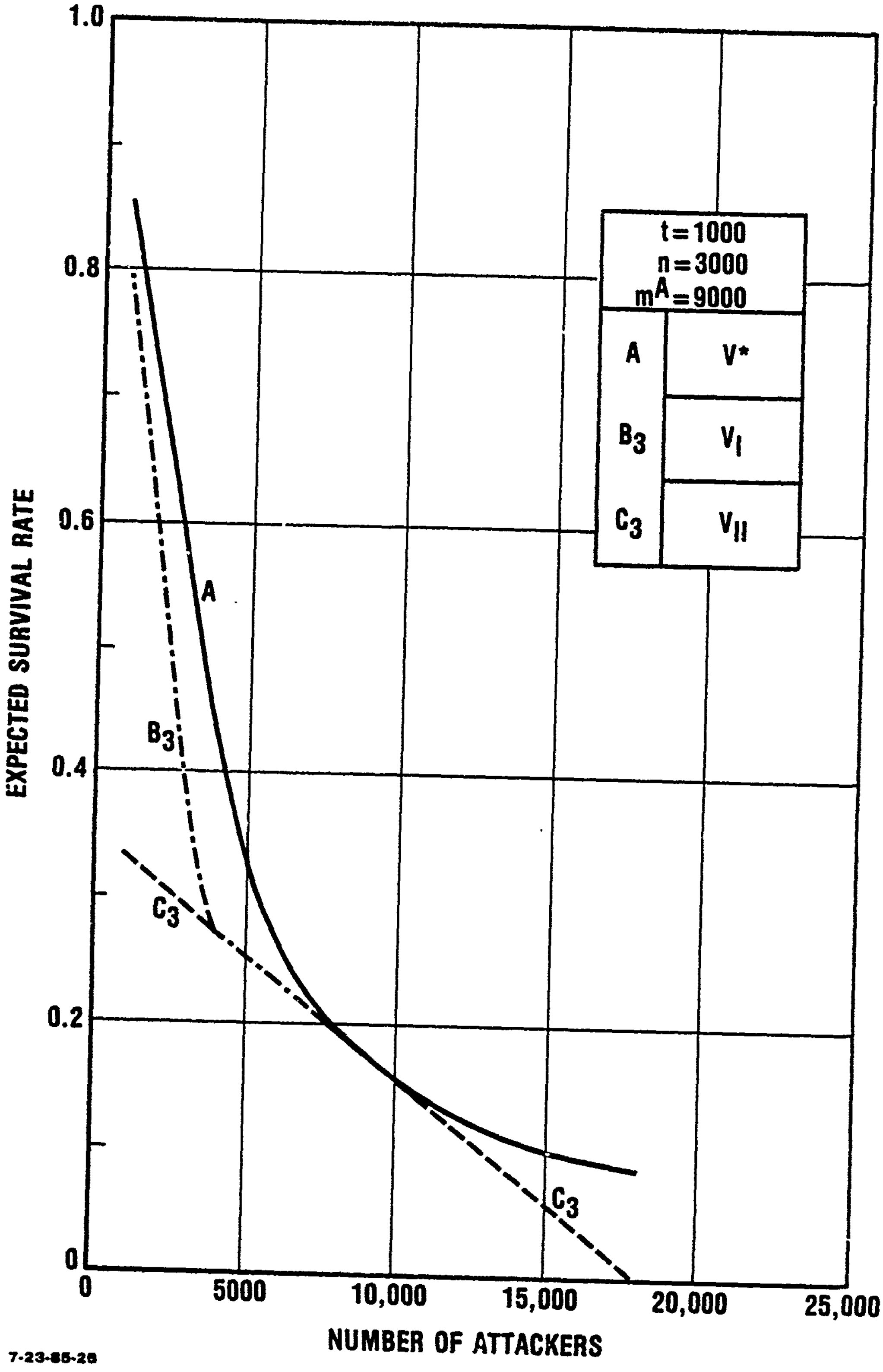

Figure 4.1c. EXPECTED SURVIVAL RATES FOR $n = 3000$, $m^A = 9000$

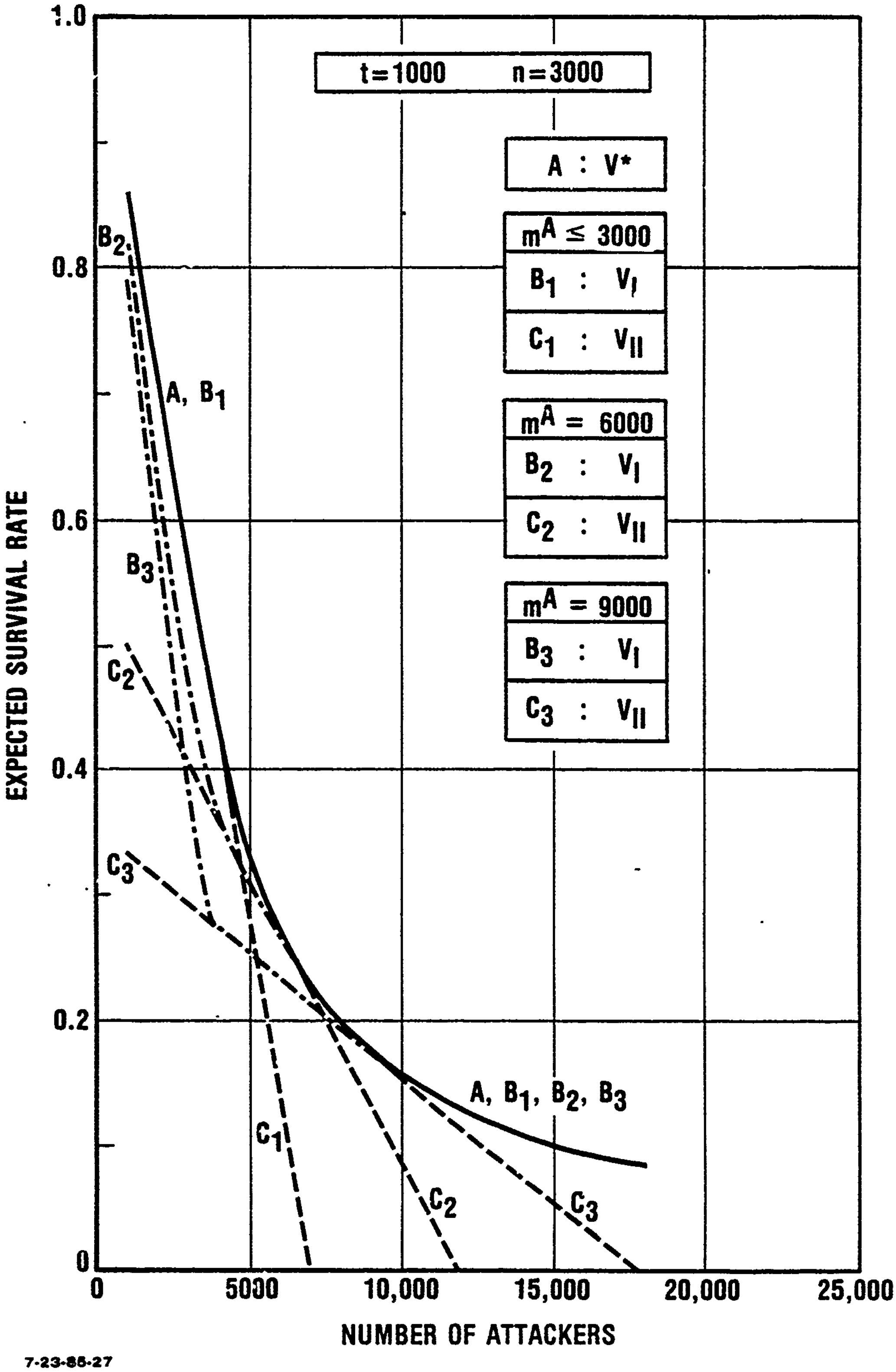

Figure 4.1d. EXPECTED SURVIVAL RATES FOR n = 3000

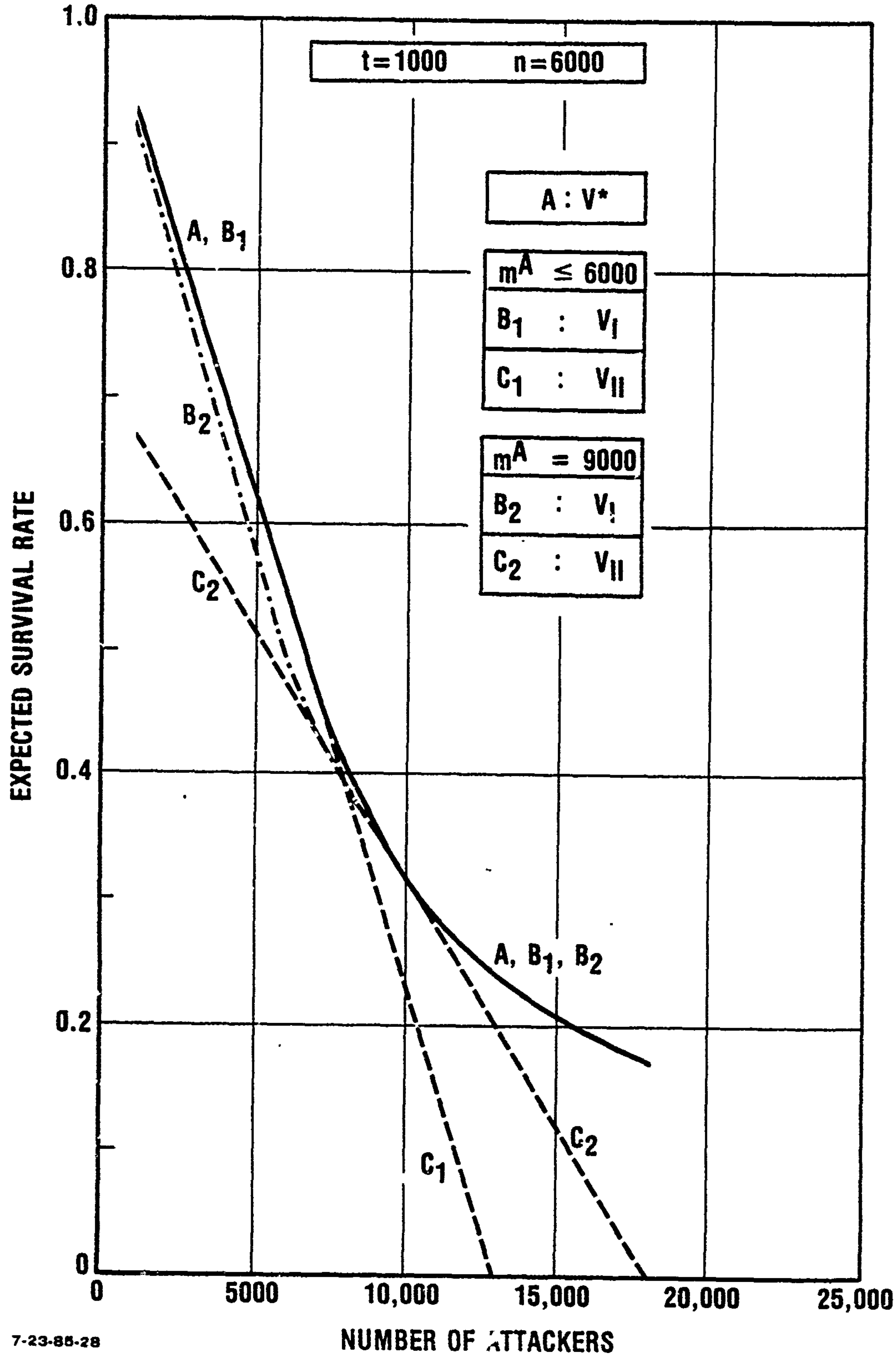

Figure 4.2. EXPECTED SURVIVAL RATES FOR n = 6000

B) Let $m^A = 9000$.

The defender believes the attacker dominates the game. The defender's strategy is:

$$y_0^A = 1/3$$

$$y_j^A = 2/51 \quad , \quad 1 \leq j \leq 17 \quad .$$

As in Case II, part A) above, this is analogous to the results in Case I. (Compare Figure 4.2 with Figure 4.1d). The qualitative characteristics are the same as the other "defender thinks attacker dominates" examples. Quantitatively, v_{II} and v^* are equal at $m = m^A = 9000$. As in Case I, part C), if the attacker knows the defender's planning strategy, then all the targets are destroyed when the attack size reaches 18,000.

<u>Case III:</u> $n = 9000$

A) Let $m^A \leq 9000$.

The defender believes that he dominates. The defender's strategy is:

$$y_j^A = 1/19 \quad , \quad 0 \leq j \leq 18 \quad .$$

B) Let $m^A = 12{,}000$

The defender believes that the attacker dominates. The defender's strategy is:

$$y_0^A = 1/4$$

$$y_j^A = 3/92 \quad , \quad 0 \leq j \leq 23 \quad .$$

The qualitative results are similar to the first two cases. See Figure 4.3.

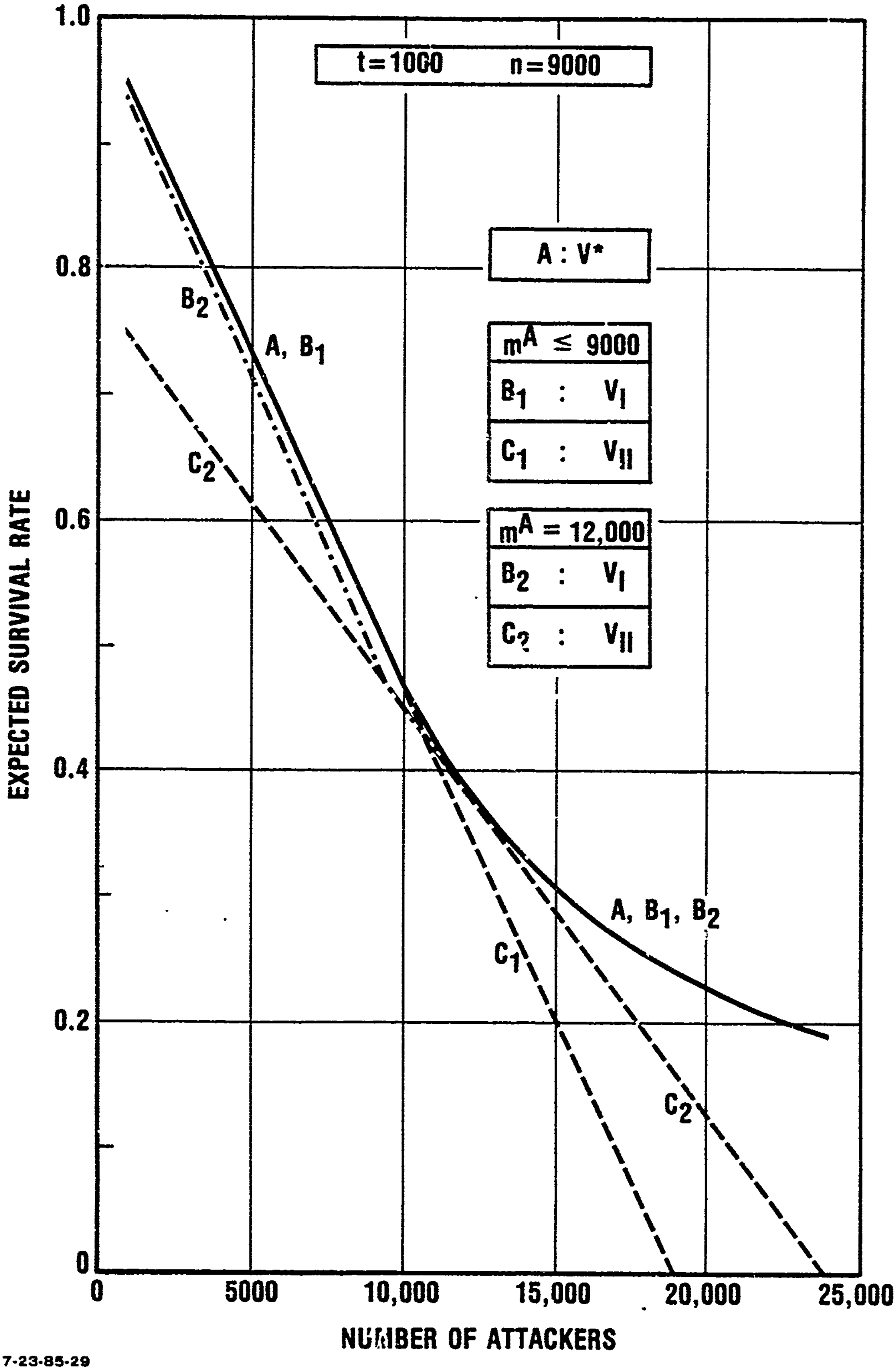

Figure 4.3. EXPECTED SURVIVAL RATES FOR n = 9000

SECTION V. COMPARISONS WITH ROBUST STRATEGIES

As shown above in Section IV, when the defender does not estimate correctly the actual attack size, Strauch's preallocated preferential defense may lead to very poor results relative to the game value. In [1], Bracken, Brooks, and Falk propose an alternate, "robust", defense that can achieve expected survival rates which are as close as possible to the game values v^* over a range of attack sizes. The criterion used for robustness is the ratio of the expected survival rate of a particular strategy y to the game value. The optimal robust defense is the strategy that maximizes the minimum of these ratios over a range of attack sizes. We assume that the attacker can discover and therefore optimize against whatever strategy the defender employs and that the defender is aware of this[1]. It turns out that this problem may be formulated as a linear program. The solution of this program yields the robust defense for the specified range of attack sizes. We denote this defense as $y^R(\overline{A})$, where $\overline{A}$ represents the range or set of attack sizes for which the defense is robust. We designate the attacker's strategy that minimizes the expected survival rate against this defense as x^R, and the expected survival rate as v_R:

$$v_R = S(x^R, y^R)$$

For each of the three defense sizes examined in Section IV we solve for two strategies y^R: one with $\overline{A}_1 = \overline{A} = \{1000, 2000, \ldots, 12{,}000\}$ and one with $\overline{A}_2 = \overline{A} = \{1000, 2000, \ldots, 18{,}000\}$. x^R is then determined against each y^R for each attack size from 1000 to 27,000.

[1]This assumption corresponds to that of Case II,II in [1].

Results for n = 3000 are given in Figure 5.1. Utilizing $y^R(\overline{A}_1)$ (see D_1 on the graph) guarantees the defender at least 65% of the game value from 1000 through 12,000 attackers. Where C_1 and C_2 go to zero quickly as attack size increases, D_1 goes to zero at about the same attack size as C_3. However, D_1 is much better than C_3 for small attacks. Utilizing $y^R(\overline{A}_2)$ (see D_2 on the graph) the defender improves considerably his chances for saving a significant number of targets against very large attack sizes, at some expense to smaller attack sizes. Below 12,000 $y^R(\overline{A}_1)$ does better, but above it $y^R(\overline{A}_2)$ does better. $y^R(\overline{A}_2)$ guarantees over $\overline{A}_2$ at least 57% of the game value and will not yield a zero expected survival rate until the attack size reaches 27,000.

As shown in Figure 5.1, the robust strategies do not perform better than those based on Strauch's defense over all attack sizes. They may not even be superior over a majority of attack sizes. (Compare C_2 and C_3 with D_1). What the robust strategy accomplishes is to provide insurance against doing very poorly relative to the game value for any attack size. If a defender is both concerned with maintaining some level of survival against large attacks and with avoiding disproportionate losses against smaller attacks, the robust strategy is preferred.

Figure 5.2 presents results for n = 6000. The qualitative behavior is similar to that for n = 3000. When the defender has 6000 interceptors at his disposal, $y^R(\overline{A}_1)$ will guarantee over $\overline{A}_1$ an expected survival rate of 83% of the game value, and $y^R(\overline{A}_2)$ will guarantee over $\overline{A}_2$ 70% of the game value. Naturally $y^R(\overline{A}_1)$ does better than $y^R(\overline{A}_2)$ for attack sizes less than or equal to 12,000, while the opposite is true for attack sizes greater than or equal to 18,000. $y^R(\overline{A}_1)$ will not yield an expected survival rate of zero until m = 18,000, thus avoiding the early collapse to zero of Strauch's defense for $m^A \leq 6000$. At the same time, while maintaining only slightly lower resistance to higher attack

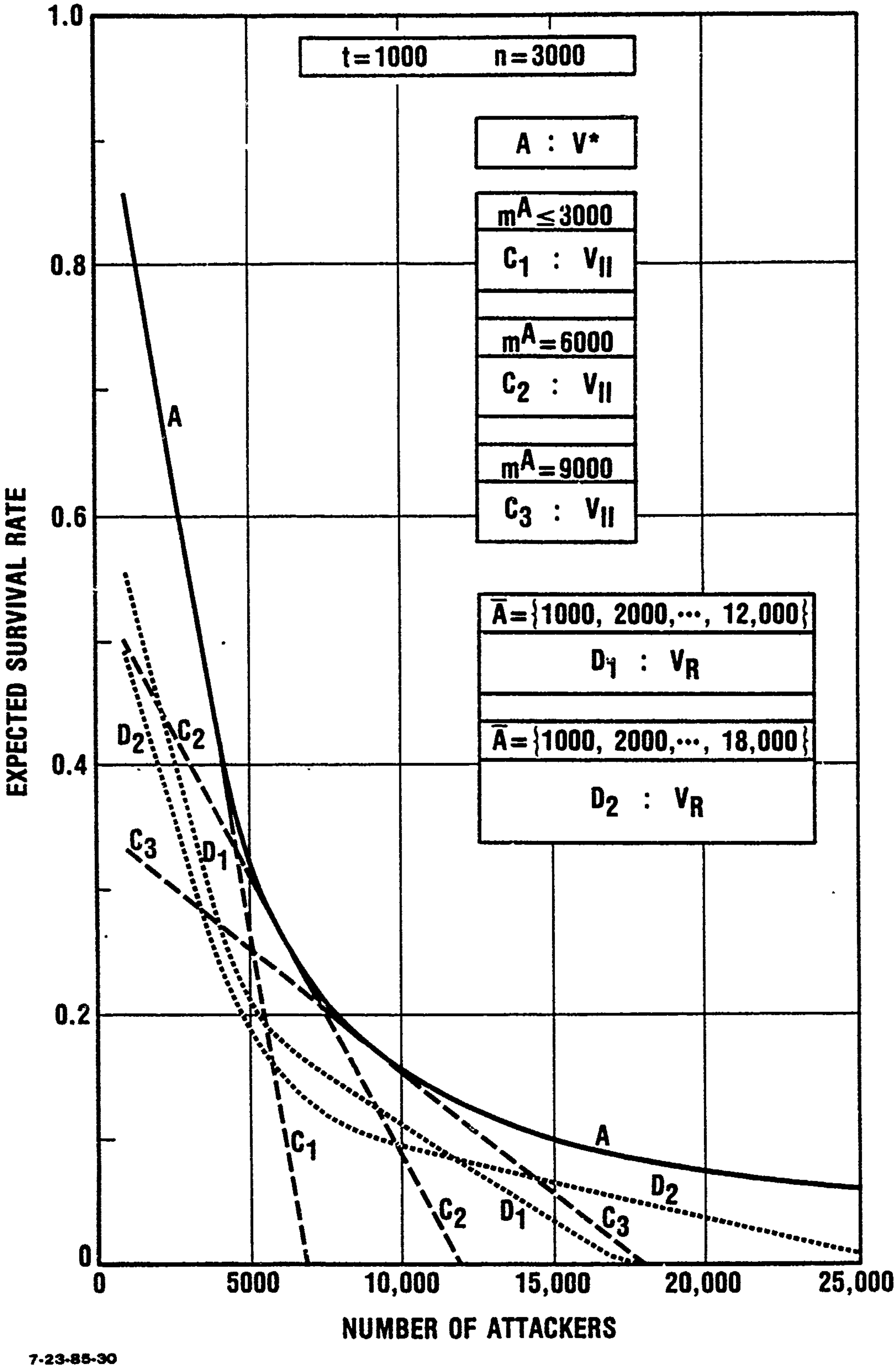

Figure 5.1. COMPARISON OF STRATEGIES FOR n = 3000

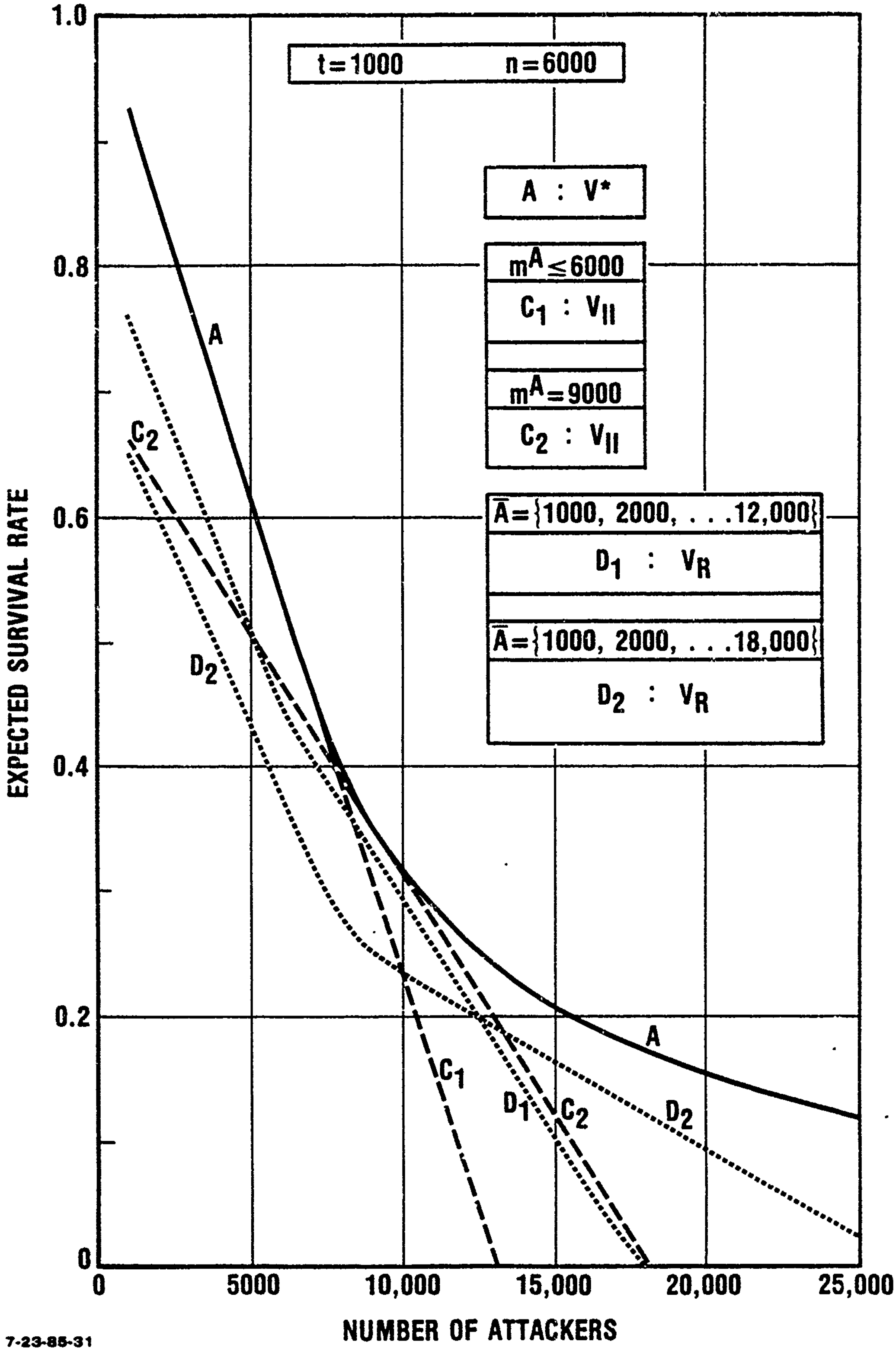

Figure 5.2 COMPARISON OF STRATEGIES FOR n = 6000

sizes (12,000 to 18,000) than Strauch's defense for $m^A = 9000$, the robust defense for $\bar{A}_1$ avoids very bad results (relative to the game value) for small attack sizes (1,000 to 3,000). $y^R(\bar{A}_2)$ gives up some of $y^R(\bar{A}_1)$'s advantages against smaller attack sizes in order to achieve greater survivability against very large attacks. Like its counterpart for $n = 3000$, $y^R(\bar{A}_1)$ will not yield an expected survival rate of zero until $m = 27,000$.

Figure 5.3 presents results for $n = 9000$. $y^R(\bar{A}_1)$ ensures over $\bar{A}_1$ a v_R which is 96% of v^* and $y^R(\bar{A}_2)$ ensures over $\bar{A}_2$ a v_R which is 82% of v^*.

As all of the examples demonstrate, using Strauch's preallocated preferential defense when the defender must protect himself against a wide range of attack sizes can lead to serious difficulties for relatively small and large attacks. A robust strategy, however, enables the defender to achieve expected survival rates "close" to the game value without doing badly at either end of the range of possible attack sizes. Indeed, even when the attack size can be narrowed to within a fairly small region, the robust preallocated preferential defense methodology is still applicable, for as $\bar{A}$ approaches a single attack size, $y^R(\bar{A})$ approaches y^* for that attack size.

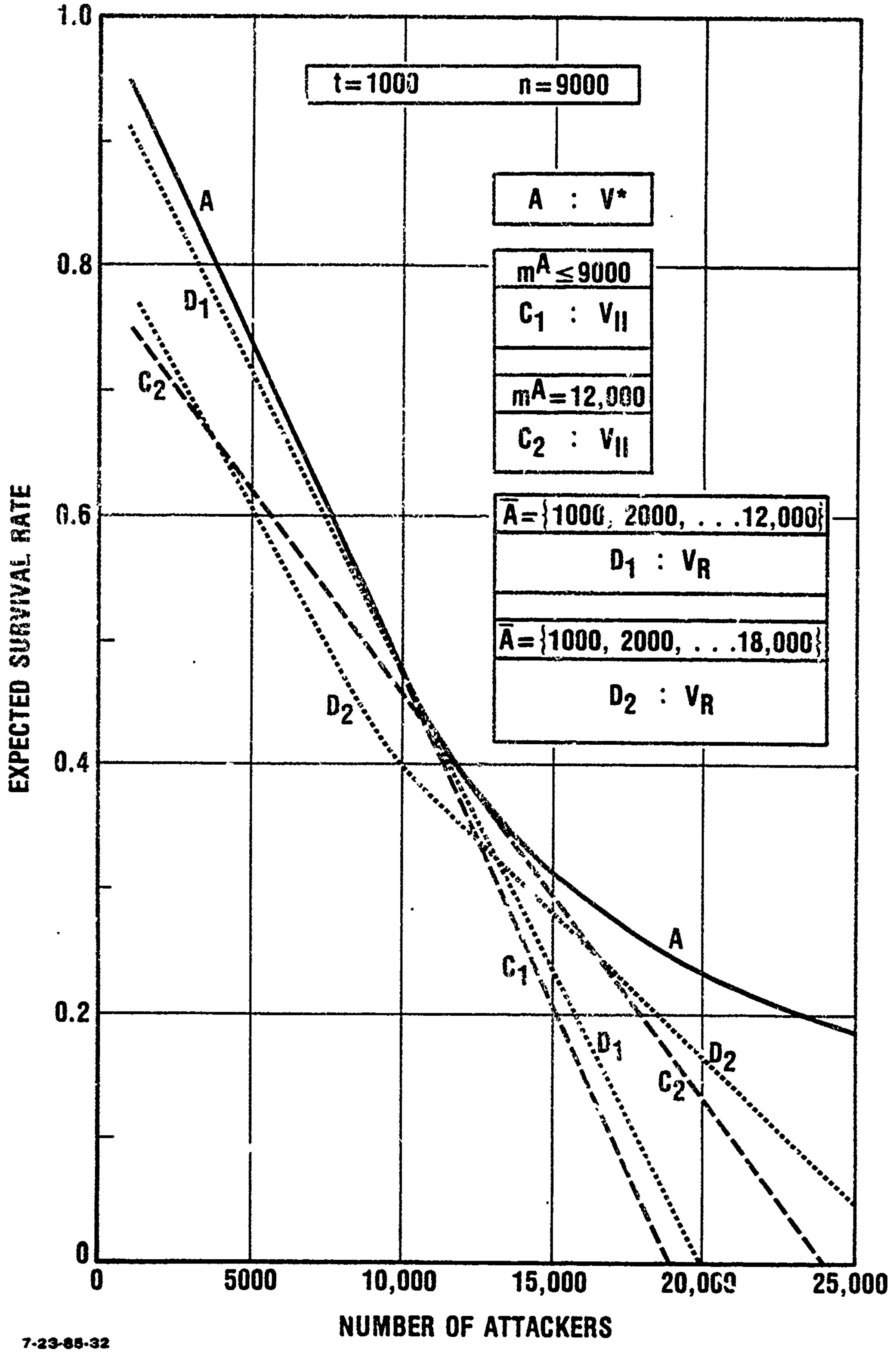

Figure 5.3. COMPARISON OF STRATEGIES FOR n = 9000

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APPENDIX

Tables 1, 2 and 3 contain the numerical values plotted in Figures 4.1, 4.2 and 4.3 of Section IV.

Table 4 contains the robust defense strategies of the examples of Section V, together with formulas for computing the optimal attack strategies. Table 5 contains the numerical values plotted in Figures 5.1, 5.2 and 5.3.

Table 1

Numerical Values for Figures 4.1a, 4.1b, 4.1c and 4.1d

$t = 1000$ $n = 3000$

v / m	$v^*(m)$	$m^A \leq 3000$		$m^A = 6000$		$m^A = 9000$	
		$v_I(m)$	$v_{II}(m)$	$v_I(m)$	$v_{II}(m)$	$v_I(m)$	$v_{II}(m)$
1000	.8571	.8571	.8571	.8247	.5000	.7955	.3333
2000	.7143	.7143	.7143	.6494	.4545	.5910	.3137
3000	.5714	.5714	.5714	.4740	.4091	.3866	.2941
4000	.4286	.4286	.4286	.3636	.3636	.2745	.2745
5000	.3333	.3333	.1429	.3182	.3182	.2549	.2549
6000	.2727	.2727	0	.2727	.2727	.2353	.2353
7000	.2308	.2308	0	.2308	.2273	.2157	.2157
8000	.2000	.2000	0	.2000	.1818	.1961	.1961
9000	.1765	.1765	0	.1765	.1364	.1765	.1765
10000	.1579	.1579	0	.1579	.0909	.1579	.1569
11000	.1429	.1429	0	.1429	.0455	.1429	.1373
12000	.1304	.1304	0	.1304	0	.1304	.1176
13000	.1200	.1200	0	.1200	0	.1200	.0980
14000	.1111	.1111	0	.1111	0	.1111	.0784
15000	.1034	.1034	0	.1034	0	.1034	.0588
16000	.0968	.0968	0	.0968	0	.0968	.0392
17000	.0909	.0909	0	.0909	0	.0909	.0196
18000	.0857	.0857	0	.0857	0	.0857	0

Table 2

Numerical Values for Figure 4.2

$t = 1000$ $n = 6000$

m \ v	$v^*(m)$	$m^A \leq 6000$		$m^A = 9000$	
		$v_I(m)$	$v_{II}(m)$	$v_I(m)$	$v_{II}(m)$
1000	.9231	.9231	.9231	.9155	.6667
2000	.8462	.8462	.8462	.8311	.6257
3000	.7692	.7692	.7692	.7466	.5882
4000	.6923	.6923	.6923	.6621	.5490
5000	.6153	.6153	.6153	.5777	.5098
6000	.5385	.5385	.5385	.4932	.4706
7000	.4615	.4615	.4615	.4314	.4314
8000	.4000	.4000	.3846	.3922	.3922
9000	.3529	.3529	.3077	.3529	.3529
10000	.3158	.3158	.2308	.3158	.3137
11000	.2857	.2857	.1538	.2857	.2745
12000	.2609	.2609	.0769	.2609	.2353
13000	.2400	.2400	0	.2400	.1961
14000	.2222	.2222	0	.2222	.1569
15000	.2069	.2069	0	.2069	.1176
16000	.1935	.1935	0	.1935	.0784
17000	.1818	.1818	0	.1818	.0392
18000	.1714	.1714	0	.1714	0

Table 3

Numerical Values for Figure 4.3

$t = 1000$ $n = 9000$

m \ v	$v^*(m)$	$m^A \leq 9000$		$m^A = 12000$	
		$v_I(m)$	$v_{II}(m)$	$v_I(m)$	$v_{II}(m)$
1000	.9474	.9474	.9474	.9445	.7500
2000	.8947	.8947	.8947	.8890	.7174
3000	.8421	.8421	.8421	.8335	.6848
4000	.7895	.7895	.7895	.7780	.6522
5000	.7368	.7368	.7368	.7225	.6196
6000	.6842	.6842	.6842	.6670	.5870
7000	.6316	.6316	.6316	.6116	.5543
8000	.5789	.5789	.5789	.5561	.5217
9000	.5263	.5263	.5263	.5006	.4891
10000	.4737	.4737	.4737	.4565	.4565
11000	.4286	.4286	.4211	.4239	.4239
12000	.3913	.3913	.3684	.3913	.3913
13000	.3600	.3600	.3158	.3600	.3587
14000	.3333	.3333	.2632	.3333	.3261
15000	.3103	.3103	.2105	.3103	.2935
16000	.2903	.2903	.1579	.2903	.2609
17000	.2727	.2727	.1053	.2727	.2283
18000	.2571	.2571	.0526	.2571	.1957
19000	.2432	.2432	0	.2432	.1630
20000	.2308	.2308	0	.2308	.1304
21000	.2195	.2195	0	.2195	.0978
22000	.2093	.2093	0	.2093	.0652
23000	.2000	.2000	0	.2000	.0326
24000	.1915	.1915	0	.1915	0

Table 4

Robust Defense Strategies for Section V

L	n = 3000		n = 6000		n = 9000	
	$y_i^R(\bar{A}_1)$	$y_i^R(\bar{A}_2)$	$y_i^R(\bar{A}_1)$	$y_i^R(\bar{A}_2)$	$y_i^R(\bar{A}_1)$	$y_i^R(\bar{A}_2)$
0	.442	.510	.234	.349	.087	.225
1	.093	.082	.064	.054	.051	.043
2	.093	.082	.064	.054	.051	.043
3	.093	.082	.064	.054	.051	.043
4	.062	.055	.064	.054	.051	.043
5	.039	.035	.064	.054	.051	.043
6	.015	.024	.038	.054	.048	.043
7	.015	.018	.038	.044	.048	.043
8	.015	.014	.038	.033	.048	.043
9	.015	.006	.038	.014	.048	.033
10	.015	.006	.038	.014	.048	.023
11	.015	.006	.038	.014	.048	.023
12	.015	.006	.038	.014	.048	.023
13	.015	.006	.038	.014	.048	.023
14	.015	.006	.038	.014	.048	.023
15	.015	.006	.038	.014	.048	.023
16	.015	.006	.038	.014	.048	.023
17	.008	.006	.025	.014	.048	.023
18	.000	.006	0	.014	.048	.023
19	.000	.006	0	.014	.044	.023
20	.000	.006	0	.014	.000	.023
21	.000	.006	0	.014	.000	.023
22	.000	.006	0	.014	.000	.023
23	.000	.006	0	.014	.000	.023
24	.000	.006	0	.014	.000	.023
25	.000	.006	0	.014	.000	.023
26	.000	.003	0	.007	.000	.023
27	.000	.000	0	.000	.000	.000

For every attack size m, let x^R be defined as follows

$$x_i^R = 1.0 \qquad i = m/t$$

$$x_j^R = 0 \qquad j \neq m/t$$

This strategy will be optimal against all six y^R listed above.

Table 5

Expected Survival Rates for Section V

m	n = 3000		n = 6000		n = 9000	
	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_1$	$\bar{A}_2$
1000	.5576	.4901	.7634	.6506	.9126	.7750
2000	.4646	.4086	.6998	.5964	.8619	.7319
3000	.3717	.3270	.6362	.5422	.8112	.6889
4000	.2787	.2453	.5726	.4880	.7605	.6458
5000	.2168	.1907	.5090	.4338	.7098	.6028
6000	.1774	.1561	.4454	.3796	.6623	.5597
7000	.1620	.1321	.4071	.3254	.6147	.5166
8000	.1465	.1145	.3689	.2819	.5672	.4736
9000	.1311	.1010	.3306	.2487	.5196	.4305
10000	.1157	.0952	.2923	.2345	.4721	.3973
11000	.1003	.0894	.2540	.2203	.4245	.3739
12000	.0848	.0837	.2158	.2061	.3769	.3505
13000	.0694	.0779	.1775	.1919	.3293	.3272
14000	.0540	.0721	.1392	.1777	.2818	.3038
15000	.0385	.0664	.1010	.1635	.2342	.2804
16000	.0231	.0606	.0627	.1492	.1867	.2570
17000	.0077	.0548	.0245	.1350	.1391	.2337
18000	.0000	.0491	.0000	.1208	.0916	.2103
19000	.0000	.0433	.0000	.1066	.0440	.1869
20000	.0000	.0375	.0000	.0924	.0000	.1635
21000	.0000	.0317	.0000	.0782	.0000	.1402
22000	.0000	.0259	.0000	.0640	.0000	.1168
23000	.0000	.0202	.0000	.0498	.0000	.0935
24000	.0000	.0144	.0000	.0356	.0000	.0701
25000	.0000	.0086	.0000	.0213	.0000	.0467
26000	.0000	.0029	.0000	.0071	.0000	.0234
27000	.0000	.0000	.0000	.0000	.0000	.0000

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