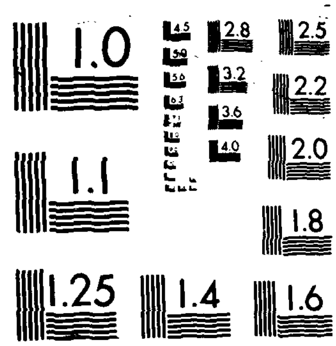
AD-A175 075 ALGEBRAIC AND COMPUTATIONAL ASPECTS OF NETWORK RELIABILITY AND PROBLEMS(U) CLEMSON UNIV SC D SHIER 15 JUL 86 AFOSR-TR-86-2115 AFOSR-84-0154 1/1

UNCLASSIFIED F/G 17/2 NL



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AFOSR-TR- 86-2115

ANNUAL TECHNICAL REPORT
Grant AFOSR-84-0154

SUMMARY

It is important to be able to assess the reliability of a complex system in terms of the reliabilities of its components. This type of problem arises with increasing frequency in the analysis of telecommunication and distribution systems, which can be represented as networks. The present research employs an underlying algebraic structure to study network reliability problems and to develop new algorithms for their solution. Iterative techniques for calculating reliability (both exactly and approximately) have been developed for both general networks and a difficult class of specialized networks. These techniques allow the solution of fairly complex networks, ones that have previously resisted analysis. In addition, the underlying structure of network reliability problems has been approached by studying the combinatorial properties of a certain polynomial defined with respect to the underlying graph topology.

RESEARCH OBJECTIVES

The focus of this research project is that of studying network reliability problems from an algebraic point of view. This perspective serves to illuminate the discrete mathematical structures underlying such reliability problems. In addition, it suggests a number of new algorithms for calculating, either exactly or approximately, certain reliability measures. The research also addresses the need for suitable data structures to implement these algorithms in a time and space efficient manner. Computational experiments are used to assess the effectiveness of the resulting implementations.

ACCOMPLISHMENTS

The algebraic approach taken here has led to a new method for calculating the s-t reliability of a directed network (namely, the probability that an operative path exists between two given nodes s and t in the network). The method developed is an iterative one, which produces a sequence of approximations to s-t reliability. A number of properties of this method have been established, including the fact that it always converges in a finite number of steps, and monotonely, to the true reliability. At any step, a lower bound on the actual reliability is obtained. Moreover, the method yields a general polynomial expression for the s-t reliability in terms of the input parameters, rather than simply producing a single numerical answer. Extensive computational testing has been conducted, using two different list structures for implementing the

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: Unclassified

1b. RESTRICTIVE MARKINGS:

2a. SECURITY CLASSIFICATION AUTHORITY:

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE:

3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribu unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S):

5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 86-2115

6a. NAME OF PERFORMING ORGANIZATION: Clemson University

6b. OFFICE SYMBOL (If applicable):

7a. NAME OF MONITORING ORGANIZATION: AFOSR/NM

6c. ADDRESS (City, State and ZIP Code): Martin Hall, Clemson, SC 29634-1907

7b. ADDRESS (City, State and ZIP Code): Bldg 410, Bolling AFB DC 20332-6448

8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR

8b. OFFICE SYMBOL (If applicable): NM

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-84-0154

8c. ADDRESS (City, State and ZIP Code): Bldg 410, Bolling AFB DC 20332-6448

10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
61103F	2304	K3	

11. TITLE (Include Security Classification): Algebraic and Computational Aspects of Network Reliability and Problems

12. PERSONAL AUTHOR(S): Professor Douglas Shier

13a. TYPE OF REPORT: Annual

13b. TIME COVERED: FROM 850615 TO 860614

14. DATE OF REPORT (Yr., Mo., Day): 860715

15. PAGE COUNT: 4

16. SUPPLEMENTARY NOTATION:

17. COSATI CODES

FIELD	GROUP	SUB. GR.

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number):

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Work progressed in three areas: (1) establishing perperties of a new method for calculating the s-t reliability of a directed network, (2) development of a new algorithm for approximating s-t reliability in acyclic networks, and (3) investigating the chromatic polymial of an undirected graph. The investigator prepared six papers for publication made five presentations, and organized a minisymposium on "Combinational Aspects of Network Reliability" at the Third SIAM Conference on Discrete Mathematics

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS ☐

21. ABSTRACT SECURITY CLASSIFICATION:

22a. NAME OF RESPONSIBLE INDIVIDUAL: Major Woodruff

22b. TELEPHONE NUMBER (Include Area Code): (202) 767-5026

22c. OFFICE SYMBOL:

method and a variety of test problems (both actual and randomly generated). Computational results show that this technique considerably expands the range of directed networks that can now be solved, either exactly or approximately. The list structure is also observed to have a significant effect on the quality of the approximations obtained, as well as on the speed of convergence. These research findings have been written up and submitted for publication to the SIAM Journal on Algebraic and Discrete Methods.

The algebraic perspective taken in this research has also led to a new algorithm for approximating s-t reliability in acyclic networks: those that do not contain directed cycles. Even though these represent fairly specialized networks (although ones important in project management and task sequencing, for example), the calculation of exact reliability for such networks is still provably intractable (i.e., NP-hard). Our research has produced an iterative technique that yields at each step both lower and upper bounds on the exact reliability value. These bounds are shown to converge to the exact answer in a finite number of steps. Thus, at each step one obtains a lower and an upper bound, guaranteed to enclose the desired value, and such bounds become progressively tighter as the algorithm proceeds. Computational results derived from several examples indicate that reasonably good bounds can be obtained, especially at higher edge reliabilities. This research work has been presented at the 250th Anniversary Conference on Graph Theory and submitted to the refereed conference proceedings.

A third area of activity has concerned the investigation of the chromatic polynomial of an undirected graph. This polynomial P(G,x) expresses the number of ways of "properly" coloring the nodes of G using at most x colors. A nonstandard representation of this polynomial (called theTutte polynomial form) reveals a number of combinatorial properties of the underlying graph, certain of which are directly relevant to the complexity of algorithms for evaluating the all-terminal reliability of G: the probability that G remains connected. The algorithmic complexity of computing the chromatic polynomial is investigated in terms of this nonstandard representation, and explicit forms for the polynomial are given for certain special types of graphs. These research findings have been presented at the 17th SE International Conference on Combinatorics, Graph Theory and Computing and submitted to the refereed proceedings of the conference.

In connection with the emphasis on discrete mathematical structures underlying network reliability problems, the investigator organized a minisymposium on the theme of "Combinatorial Aspects of Network Reliability" at the Third SIAM Conference on Discrete Mathematics (May 1986). This minisymposium, one of four such featured at the conference, brought together well-known researchers in the area (Bienstock, Provan, Satyanarayana, Wood) to provide a state-of-the-art survey of recent advances. Attendance at the session exceeded one hundred persons.

PAPERS

[1] "Iterative algorithms for calculating network reliability," *Graph Theory with Applications to Algorithms and Computer Science*, Y. Alavi et al. (eds.), John Wiley and Sons, 1985, pp. 741-752.

[2] "Algorithms for generating minimal cutsets by inversion," *IEEE Transactions on Reliability* **R-34** (1985), 314-319. With D. E. Whited.

[3] "Iterative algorithms for generating minimal cutsets in directed graphs," *Networks* **16** (1986), 133-147. With D. E. Whited.

[4] "Algebraic methods applied to network reliability problems," Clemson University Technical Report #486, August 1985. With D. E. Whited. Submitted to *SIAM Journal on Algebraic and Discrete Methods*.

[5] "The chromatic polynomial revisited," Clemson University Technical Report #507, April 1986. With S. Frank. Submitted to Proceedings of the 17th SE International Conference on Combinatorics, Graph Theory and Computing.

[6] "Approximating network reliability," Clemson University Technical Report #510, May 1986. With D. Whited. Submitted to Proceedings of the 250th Anniversary Conference on Graph Theory.

PRESENTATIONS

- "Algebraic methods applied to network reliability problems," Expository Seminar Series, National Bureau of Standards, October 1985. (INVITED)

- "Network reliability and algebraic structures," Clemson University, Seminar Series, December 1985.

- "The chromatic polynomial revisited," 17th SE International Conference on Combinatorics, Graph Theory and Computing, Boca Raton, Florida, February 1986.

- "Approximating network reliability," 250th Anniversary Conference on Graph Theory, Ft. Wayne, Indiana, March 1986.

- "Algebraic aspects of computing network reliability," Third SIAM Conference on Discrete Mathematics, Clemson University, May 1986. (INVITED)

ADDITIONAL CONSULTATIONS

- Organized minisymposium (one of four such) at the Third SIAM Conference on Discrete Mathematics, entitled "Combinatorial Aspects of Network Reliability."

PERSONNEL

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(2) David E. Whited, Research Assistant, Ph. D. Candidate, degree expected in August, 1986. Thesis title: "Calculation of s-t Reliability in Planar Graphs"

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