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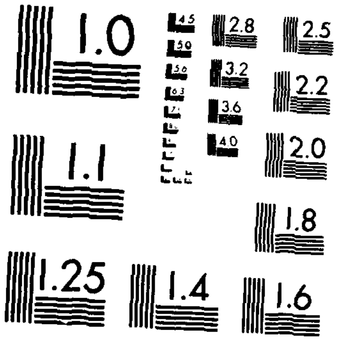

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IDA MEMORANDUM REPORT M-227

REDUCTION IN COVERAGE OF KEW BOOST-PHASE-INTERCEPT SYSTEM DUE TO DECREASED BOOSTER BURN TIME AND INCREASED COMMITMENT DELAY

Reinald G. Finke

July 1986

Prepared for

Strategic Defense Initiative Organization
and
Deputy Under Secretary of Defense
for Strategic and Theater Nuclear Forces

IDA

INSTITUTE FOR DEFENSE ANALYSES
1801 N. Beauregard Street, Alexandria, Virginia 22311

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Shortened booster turn time is calculated to be a very effective countermeasure against a KEW BPI system. Reduction to about 120 seconds negates a KEW system that has an interceptor velocity of 5 km/sec and a platform altitude of 500 km. Reduction to 150 seconds requires a factor of more than five increase in number of platforms to compensate. Increasing the interceptor velocity is a more effective counter-countermeasure than lowering the platform altitude. However, increasing the interceptor velocity to 7 km/sec does not reduce below 100 sec the booster bum time to negate the system.

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ABSTRACT

A simplified model of a space-based Kinetic-Energy-Weapon (KEW) Boost-Phase-Intercept (BPI) System has been constructed to evaluate the effects on geometrical leakage (incomplete coverage) from countermeasures and responses to countermeasures. The reduction in KEW coverage due to decrease in ballistic-missile burn time and increase in commitment delay time is balanced against system responses that shorten the time taken by the KEW interceptor to reach the ballistic-missile booster: increasing interceptor velocity or shortening the distance to be traveled by the interceptor by lowering the altitude or increasing the number of KEW satellite platforms.

Shortened booster burn time is calculated to be a very effective countermeasure against a KEW BPI system. Reduction to about 120 seconds negates a KEW system that has an interceptor velocity of 5 km/sec and a platform altitude of 500 km. Reduction to 150 seconds requires a factor of more than five increase in number of platforms to compensate. Increasing the interceptor velocity is a more effective counter-countermeasure than lowering the platform altitude. However, increasing the interceptor velocity to 7 km/sec does not reduce below 100 sec the booster burn time to negate the system.

CONTENTS

REDUCTION IN COVERAGE OF KEW BOOST-PHASE-INTERCEPT SYSTEM DUE TO DECREASED BOOSTER BURN TIME AND INCREASED COMMITMENT DELAY

I. BACKGROUND

In discussions in early November 1985 in preparation for the HEDS II (High Endoatmospheric Defense System, Phase II) Red Team Interim Report scheduled for mid-December, a requirement was identified to provide an analysis of the leakage of an upstream Kinetic Energy Weapon (KEW) boost-phase-intercept (BPI) layer as a function of those engagement parameters under the control of the offensive system, as well as those under control of the defensive system. The principal parameters of the offensive system influencing the engagement are booster burn time and system commitment delay; the key engagement parameters of the defensive system for counter-countermeasuring are velocity of the Kinetic Energy Kill Vehicle (KKV), altitude of the KEW satellite platforms and number of platforms. The introductory analysis in this paper linking these parameters through the effectiveness criterion, leakage, was presented to the Red Team meeting at Titan Systems on December 3, 1985 and to representatives of the Blue and Omega teams as part of the HEDS II Red Team Interim Report at Systems Planning Corporation on December 17, 1985. This Memorandum Report documents this analysis.

II. METHODOLOGY

Leakage of a KEW BPI system will occur because of system limitations such as imperfect detection-sensor performance, imperfect terminal-homing performance, imperfect KKV-booster reliability, and inability of the KEW interceptor to reach a ballistic-missile booster before it burns out. The approach here is to assume that all components of the system perform as designed in order to isolate the effects on leakage of an inability of the interceptor to reach a booster in time due to countermeasures taken by the offensive system. The offensive system can decrease the vulnerability time of its ballistic missile by changing to a faster burning design and can increase the delay time from booster ignition to defensive-system commitment by launching under a cloud cover or by introducing confusion in the defensive discrimination function through booster decoys (e.g., proliferated lower stages with inert upper stages, or spurious radiant-energy sources). The defensive system can attempt to thwart these countermeasures by shortening the time taken by the KKV to reach the booster through increasing the KKV velocity or through shortening the distance to be traveled by the KKV by lowering the altitude or increasing the number of KEW satellite platforms.

Each satellite platform in an orbital constellation commands an area enclosing all points that are closer to it than they are to adjacent platforms. That area varies in size with the latitude of the ballistic-missile-silo fields, and the distribution of the ballistic-missile silos within that area depends on the longitudes at which the orbit tracks cross the latitude of the missile fields at the moment the missiles are launched (simultaneously). If the exact launch time cannot be predetermined and if the characteristics of the KKVs and the orbital array of their platforms are to be treated in general, the exact distribution of the silos with respect to each satellite cannot be defined and the silos must be assumed to be uniformly distributed. With that assumption, the geometrical component of leakage is just given by the reduction in coverage.

The "leakage" of a KEW BPI system is therefore defined here as the fraction of that area over the earth's surface commanded by a KEW satellite platform that cannot be reached by a KKV from that platform in the time before burnout of a ballistic-missile booster. Within that definition are requirements to define (1) the coverage assigned to a satellite platform, (2) the reach of the KKV interceptor during the time between interceptor commitment and booster burnout (i.e., burn time minus commitment delay) and (3) the altitude attained by the ballistic-missile booster at burnout.

Figure 1 (from IDA Paper P-1200) shows a simplified earth-coverage geometry at the equator for a satellite constellation with a maximum required radius of coverage, r_c. Simplifying assumptions for this introductory analysis are (1) the area reachable by an interceptor in a given time is a circle, i.e., the effect of gravity on the interceptor's trajectory is neglected, (2) r_c is small enough in comparison with the earth's radius R_e so that the orbit rings can be considered to be parallel straight lines and the earth's surface can be considered to be locally flat (allowing representation with plane triangles), and (3) the orbits are polar (inclination = 90 deg). Within these assumptions, the smallest unit of area generally representative of the division of coverage between two adjacent satellites (in adjacent rings) is the cross-hatched triangle* (replicated four times per satellite), and the total number of satellites, n_T, required to maintain continuous coverage at the equator (the most difficult latitude to cover) is

$$n_T = \left(\frac{\pi R_e}{1.5 r_c}\right)^2 \sqrt{3}$$

*At the minimum latitude for full coverage, the satellite geometry becomes simply a hexagonal array, and this triangle is made up of three smaller equal 30/60/90 deg triangles.

or, conversely, if the number of satellites is given, the maximum radius of coverage, r_{cm}, required is

$$r_{cm} = \frac{\pi R_e}{1.5} \sqrt{\frac{\sqrt{3}}{n_T}} \ .$$

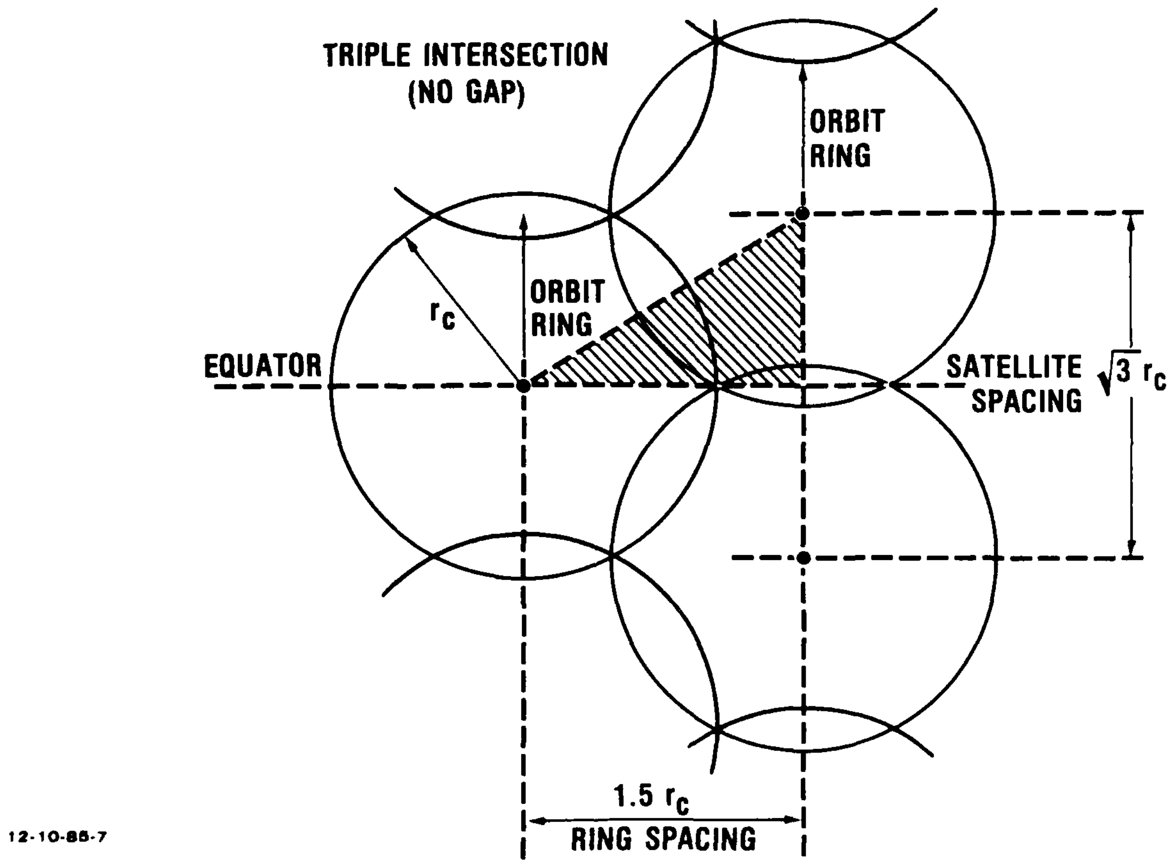

FIGURE 1. Sample Earth-Coverage Geometry at the Equator
(Polar Orbits)

A simplified depiction of the interceptor reach is shown in Figure 2. Besides taking the locally-flat-earth assumption, this representation assumes that the interceptor path is a straight line (the effect of neglecting gravity) and that the interceptor velocity is constant along the path. So the maximum slant range, r, for interception is just that distance the KKV interceptor can fly at a velocity V_{kkv} in a time interval between the commitment delay, t_{delay}, and the ballistic-missile burn time, t_{burn},
or

$$r = V_{kkv} \, (t_{burn} - t_{delay})$$

If the interceptor must fly down from a satellite platform altitude h_s to a booster burnout altitude h_b while traversing r, the projection of the interceptor reach on the earth's surface is

$$r_{ci} = \sqrt{r^2 - (h_s - h_b)^2}$$

The burnout altitude, h_b, attained by the booster in its burn time, t_{burn}, increases with burn time. Burnout altitudes for several representative three-stage boosters following gravity-turn trajectories with each stage burning one-third of the total burn time are plotted as a function of total burn time in Figure 3*; the variation is close enough to a straight line on this log-log plot to be described by the curve-fit

$$h_b(\text{km}) = (8/3) \cdot [t_{burn} \, (\text{sec})]^{0.87506}$$

*This curve is representative only; a greater apportionment (than one-third) of the total burn time to the first stage gives lower burnout altitudes, for example.

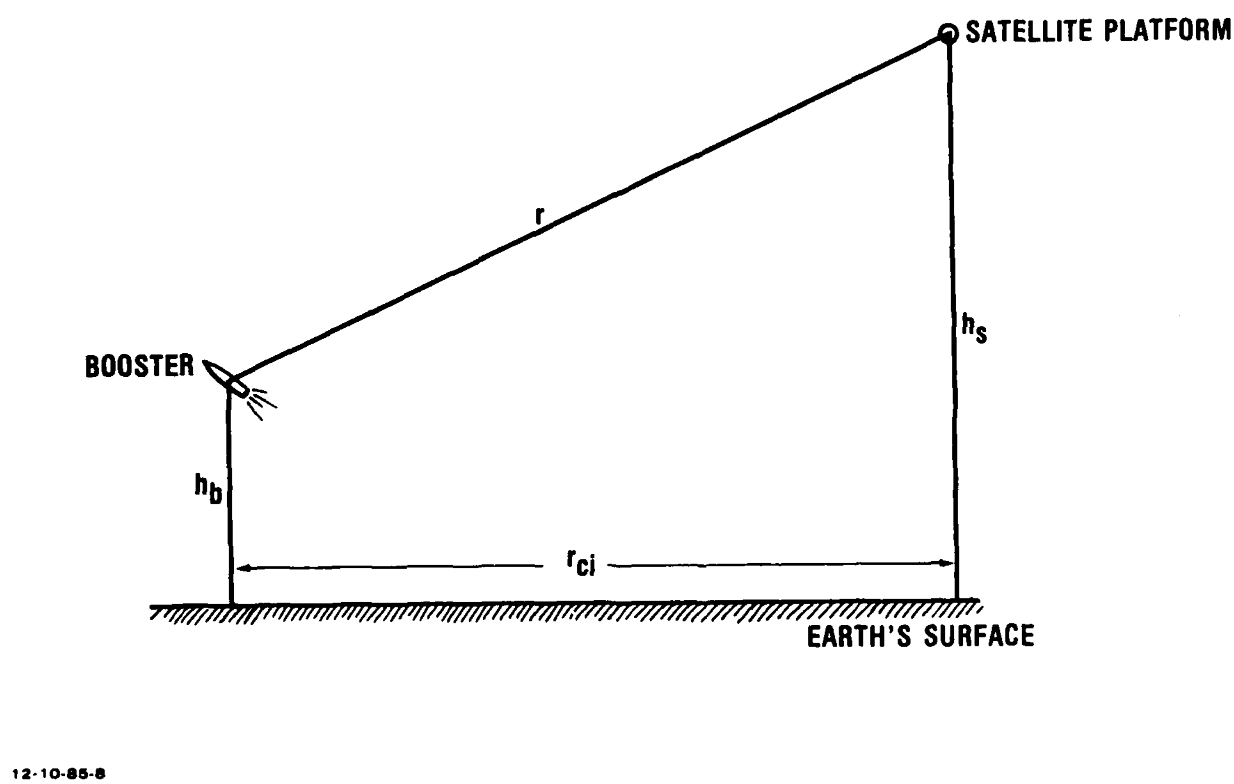

FIGURE 2. Geometry of KEW Intercept

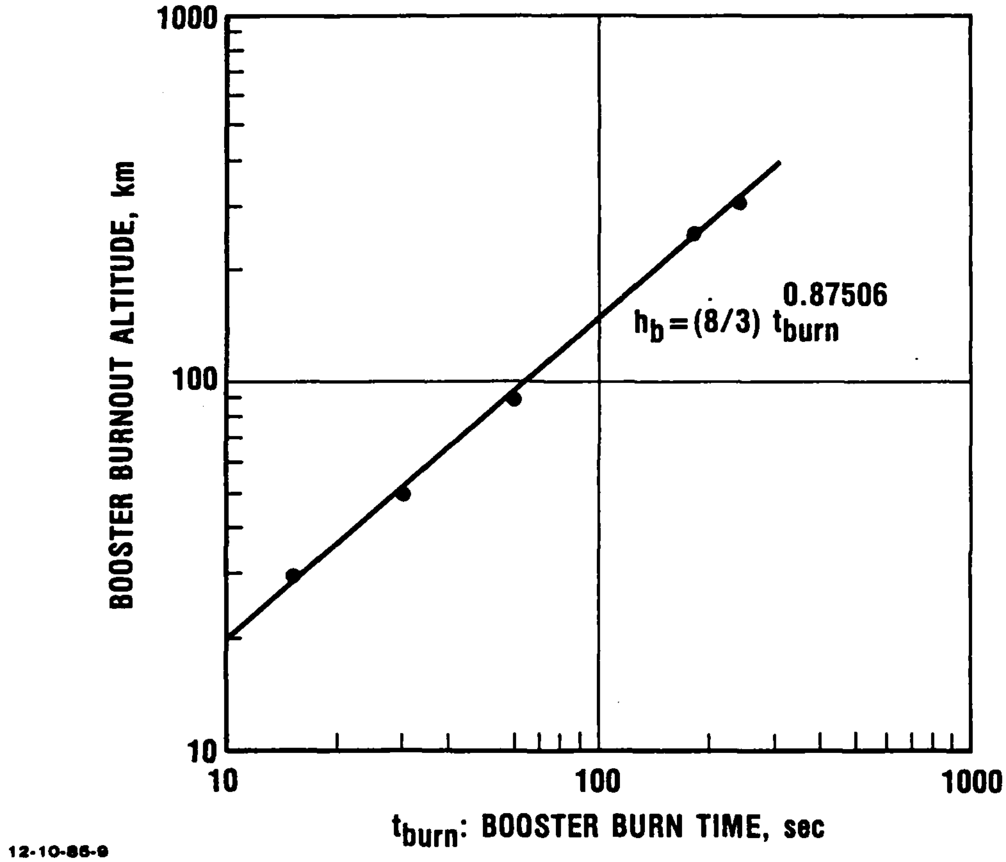

FIGURE 3. Curve-Fit to Dependence of Booster Burnout Altitude on Burn Time

The triangular area of coverage of Figure 1, divided on the perpendicular bisector of the line connecting the satellites in adjacent rings (the hypotenuse) and rotated around the bisection point until the two halves of the hypotenuse touch, becomes the trapezoidal area of Figure 4 commanded by one satellite. Figure 4 shows the variation in shape and size of this trapezoidal quadrant of coverage for different spacings of the rings, equivalent to latitudes greater than the latitude λ_1 for full coverage. The total area, A_T, of this quadrant required to be commanded by the satellite is

$$A_T = \frac{3\sqrt{3}}{8} \cos\lambda \; r_{cm}^2$$

A generalized quadrant of interceptor coverage shown in Figure 5 can be described by the area above the x and y axes within boundaries set, in part, by the three straight lines

$x = 1.5 r_{cm} \cos\lambda$ (the line dividing the space between rings),

$y = \frac{\sqrt{3}}{2} r_{cm}$ (the line dividing the space between satellites in a ring), and

$y = y_o - \sqrt{3}\, x \cos\lambda$ (the perpendicular bisector of the line connecting satellites in adjacent rings),

and otherwise by intersecting interceptor-coverage circular arcs of radius r_{ci}. The operative variables in the derivation of the area of the generalized quadrant are identified on Figure 5 (with the "leakage" depicted by the shaded areas).

The area of this generalized quadrant of interceptor coverage, aggregated from the successive wedges between increasing angles, is

$$A = \frac{1}{2} r_{ci} r_{cm} \left[1.5 \cos\lambda \sin\theta_1 + \frac{r_{ci}}{r_{cm}} \left\{ \theta_2 - \theta_1 + \sin(\alpha-\theta_2)\cos(\alpha-\theta_2) + \sin(\theta_3-\alpha)\cos(\theta_3-\alpha) + \theta_4 - \theta_3 \right\} + \frac{\sqrt{3}}{2} \cos\theta_4 \right].$$

The leakage, the fraction of A_T not reachable by the KKV interceptor with radius of coverage r_{ci}, is then just

$$\text{Leakage} = 1 - A/A_T$$

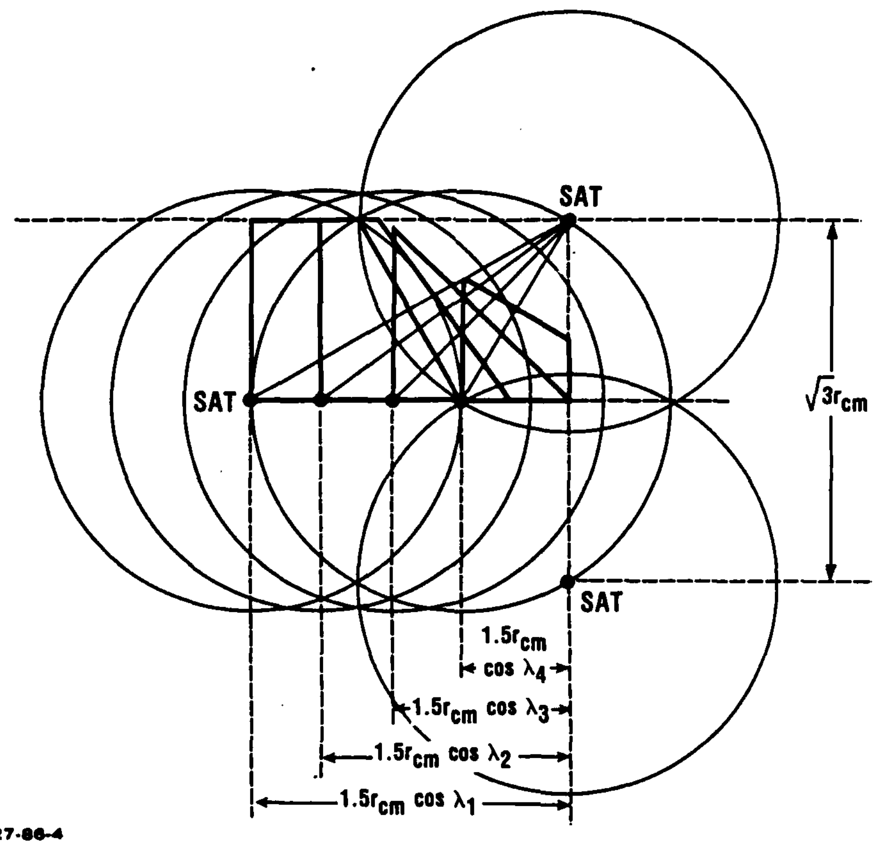

FIGURE 4. Variation of Shape of Quadrant of Coverage With Latitude ($\lambda_4 > \lambda_3 > \lambda_2 > \lambda_1$ = Minimum Latitude For Full Coverage)

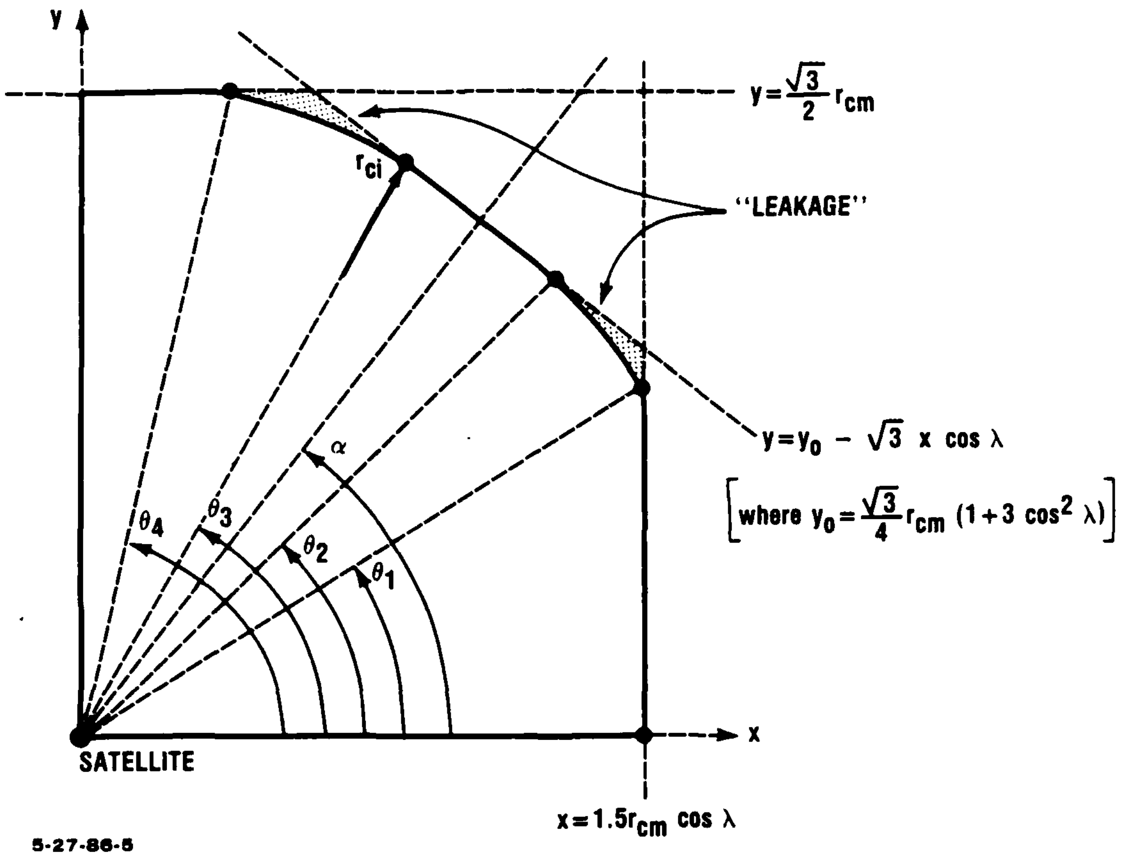

FIGURE 5. Generalized Quadrant of Coverage for Definitions (not all boundaries occur at same time in actuality)

III. RESULTS

A representative plot of the growth in fractional area coverage, A/A_T, as the interceptor reach grows toward the maximum required for coverage at the latitude of the ballistic-missile launch sites, is shown in Figure 6. The rate of change of the area coverage with the interceptor reach goes to zero near the maximum required reach; the fractional area coverage is still 90 percent at an interceptor reach of 80 percent of the maximum required. It can be observed, therefore, that for small departures (i.e., less than 10 percent, say) in the interceptor reach from the maximum required, the leakage is very small (i.e., less than 5 percent); the KEW BPI system suffers "graceful" degradation near full coverage.

The leakage for decreased booster burn time and increased commitment delay was calculated for a baseline KEW system giving full coverage at a 55-deg target latitude, with platforms at a 500-km altitude and KKVs with a 5-km/sec velocity, against ballistic missiles with a burn time of 240 sec and a commitment delay of 60 sec. The number of satellite platforms calculated to be required was 286. The calculated increase in leakage as the burn time was decreased below the design value of 240 sec and as the commitment delay was increased above the design value of 60 sec is shown in Fig. 7. As the booster burn time is decreased, with commitment delay kept at 60 sec, the leakage increases and reaches 1.0 at a booster burn time of about 124 sec, designated as $t_{burn}(1)$, i.e., the maximum burn time for full leakage. As the commitment delay is increased, with burn time kept at 240 sec, the leakage increases and reaches 1.0 at a commitment delay of 204 sec, designated as $t_{delay}(1)$, i.e., the minimum delay time for full leakage. While the $t_{burn}(1)$ of 124 sec, e.g., three stages of about 41-sec burn time per stage, is not short enough to be in the class of a "fast burn" booster, the $t_{delay}(1)$ of 204 sec, i.e., the

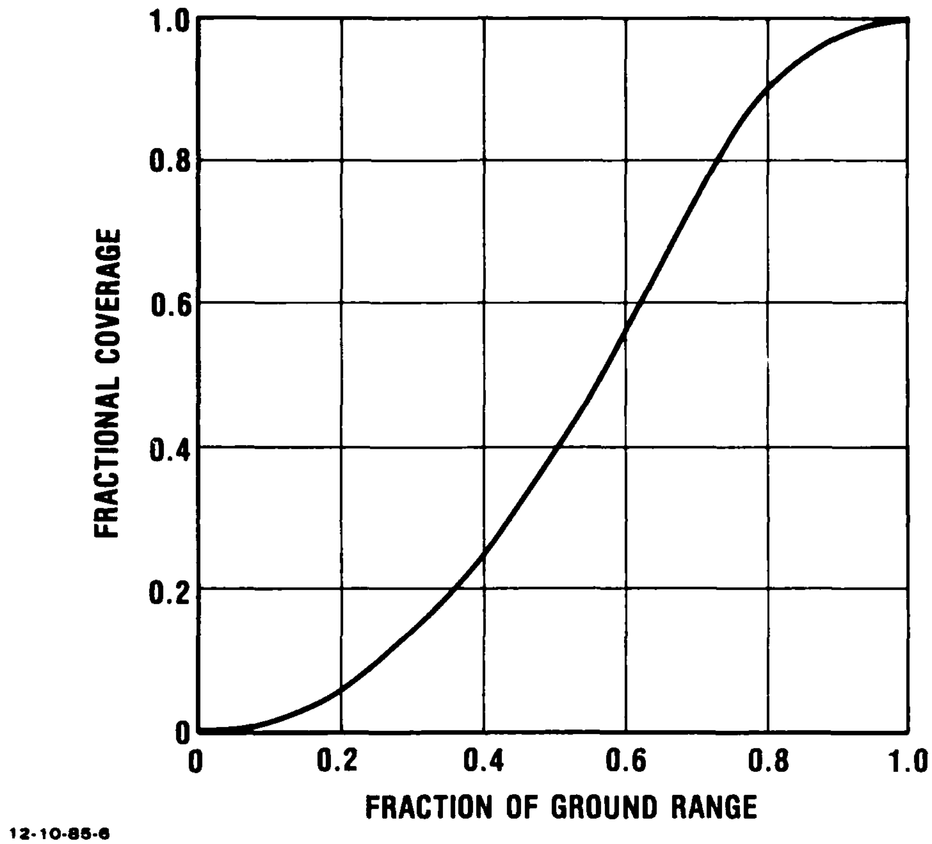

FIGURE 6. KEW Platform Area Coverage versus Ground Range

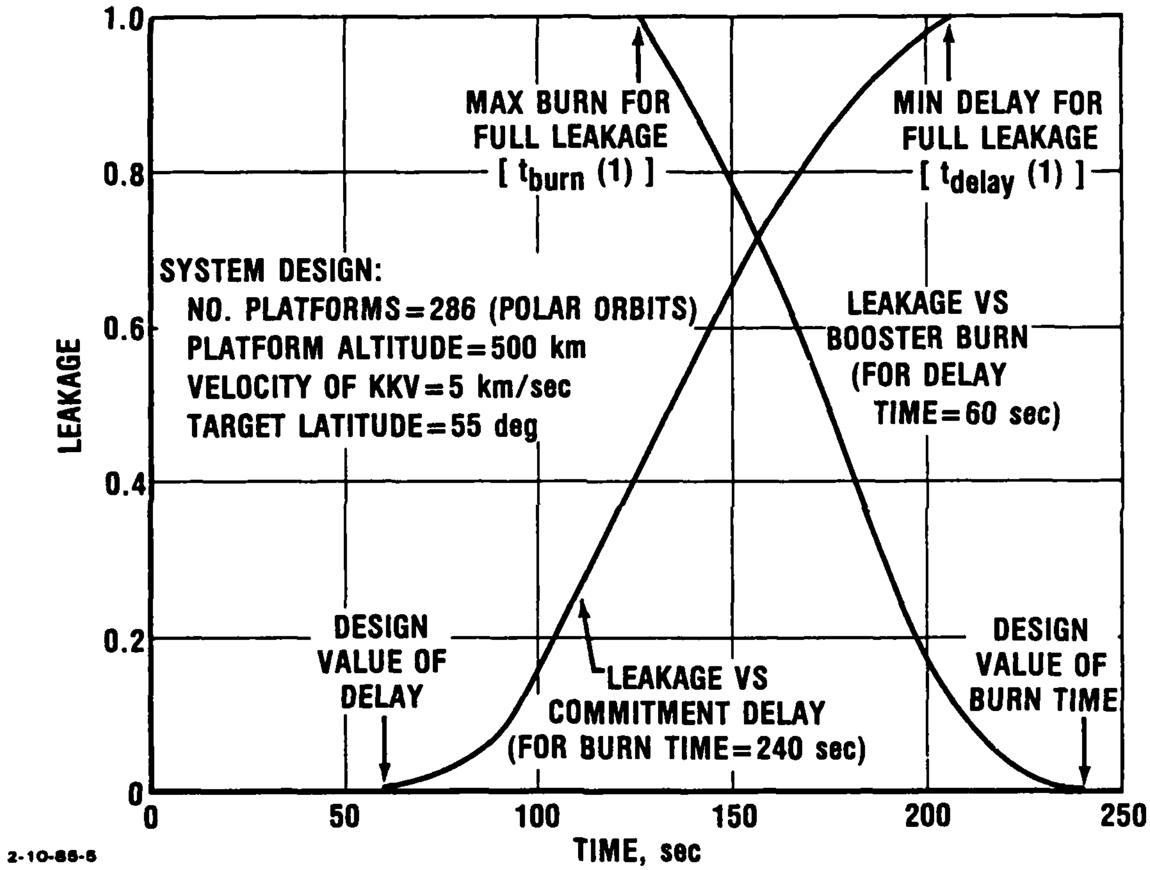

FIGURE 7. Leakage of KEW Boost-Phase-Intercept System Due to Decreased Booster Burn Time and Increased Commitment Delay

time from booster ignition until the signal to fire defensive weapons is given, seems to be long compared with delays projected for available conceptual deceptive launch tactics.

To counteract the increase in leakage as the booster burn time is shortened, or as the commitment delay is lengthened, increasing the number of satellite platforms would be an option more readily available after the defensive system is deployed than system design changes, such as increased KKV velocity or reduced satellite altitude. Figure 8 shows the calculated increase in the number of platforms required to maintain zero leakage as booster burn time is decreased (for constant commitment delay), or as commitment delay is increased (for constant burn time).

As the design booster burn time is shortened and the required numbers of satellites increase accordingly, the maximum burn time for full leakage, $t_{burn}(1)$, does not change from 124 sec (because the time to reach a booster directly below a satellite is independent of the number of satellites), but the minimum delay time for full leakage, $t_{delay}(1)$, decreases by about a factor of two from its 204 sec for a 240-sec burn time while the required number of satellites is increasing by about a factor of five. This large change in the required number of satellites to maintain zero leakage takes place while the design booster burn time is shortened from 240 sec to about 155 sec.

As the design commitment delay time is lengthened and the required number of satellites increases accordingly, the minimum delay time for full leakage, $t_{delay}(1)$, does not change from 204 sec, but the maximum burn time for full leakage, $t_{burn}(1)$, increases by less than a factor of two from its 124 sec for a 60-sec delay time while the required number of satellites is increasing by about a factor of five. This large change in the required number of satellites to maintain zero leakage

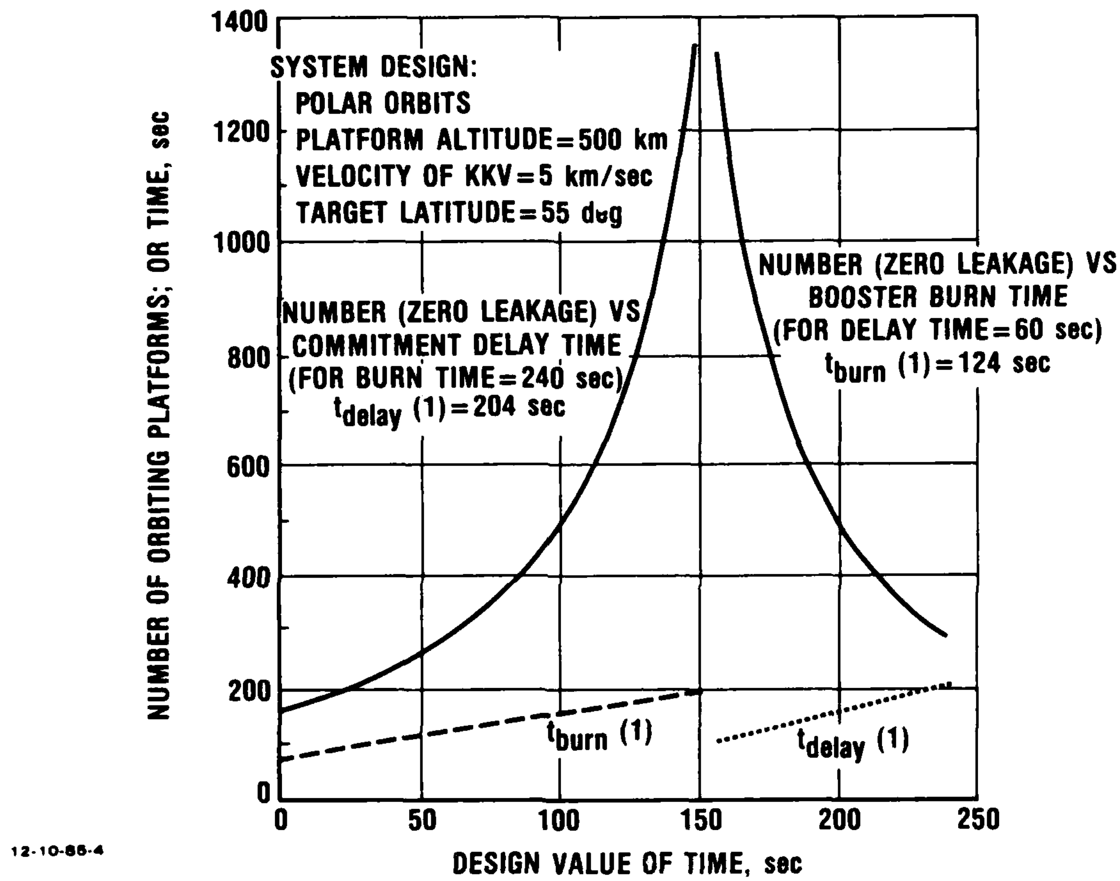

FIGURE 8. Increase in number of Platforms Required To Achieve Zero Leakage -
With Decrease in Design Booster Burn Time
With Increase in Design Commitment Delay Time

takes place while the design commitment delay time is lengthened from 60 sec to about 150 sec.

The two remaining principal parameters of the defensive system, the platform altitude and the KKV velocity, can each be varied (while other parameters of the system are held fixed) to determine their individual effects on the number of satellites required to give zero leakage. Figure 9 shows the weak dependence of the number of satellites on the choice of platform altitude in the range from 400 km to 700 km; the required number of satellites for zero leakage increases by only about 20 percent as the altitude is increased over that range. Shown also on the figure as a measure of the dependence of the shape of the curve of leakage on platform altitude are the dependences of the maximum burn time for full leakage, $t_{burn}(1)$, and the minimum delay time for full leakage, $t_{delay}(1)$. The maximum burn time for full leakage increases linearly with platform altitude by only about 40 percent (from 108 sec to 156 sec) and the minimum delay time for full leakage decreases by only about 30 percent over the 400-700 km altitude range; the behavior of the curves of leakage is therefore not significantly different from that shown in Figure 7.

The strong dependence on KKV velocity of the number of satellites for zero leakage is shown in Fig. 10; the required number of satellites decreases by more than a factor of four as the KKV velocity is increased by a factor of two from 3.5 to 7.0 km/sec. In this velocity range, however, the $t_{burn}(1)$ and the $t_{delay}(1)$ vary very little; the maximum burn time for full leakage decreases from 144 sec to 109 sec and the minimum delay time for full leakage increases from 189 sec to 214 sec. The leakage curves would therefore not show an appreciable change in behavior with KKV velocity from those in Fig. 7.

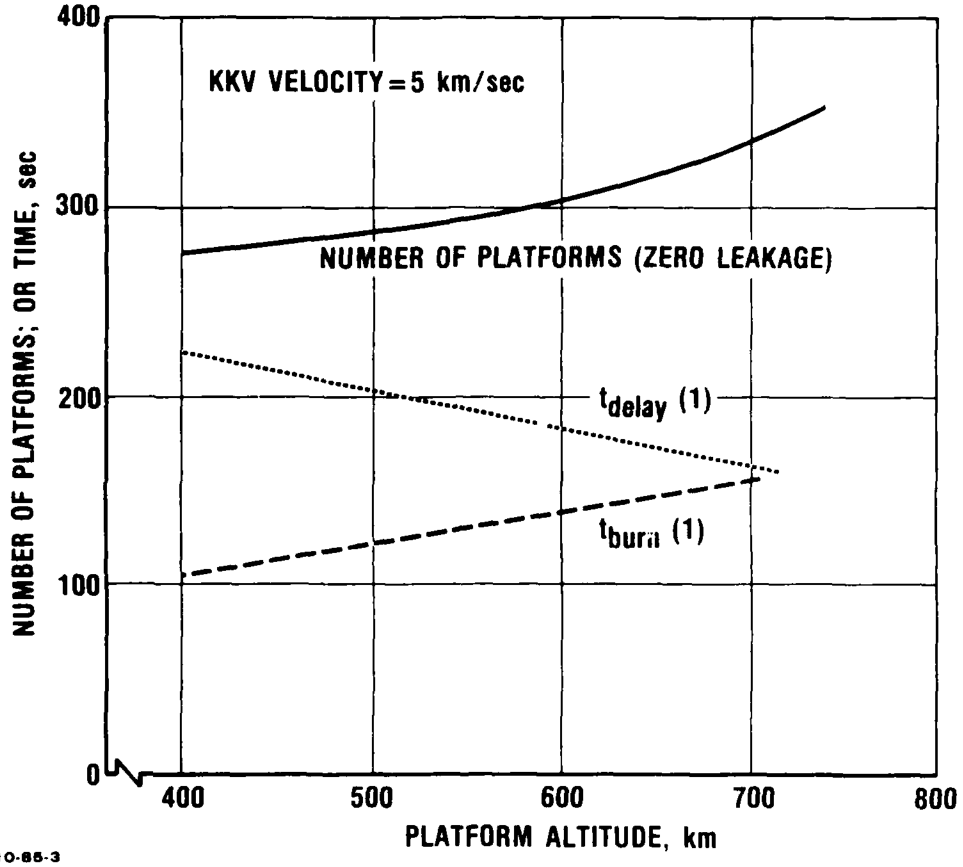

FIGURE 9. Dependence on Platform Altitude of -
Number of Platforms to Give Zero Leakage For Selected Systems Parameters
Maximum Burn Time For Full Leakage For Selected Parameters = [t_{burn} (1)]
Minimum Delay Time For Full Leakage For Selected System Parameters = [t_{delay} (1)]

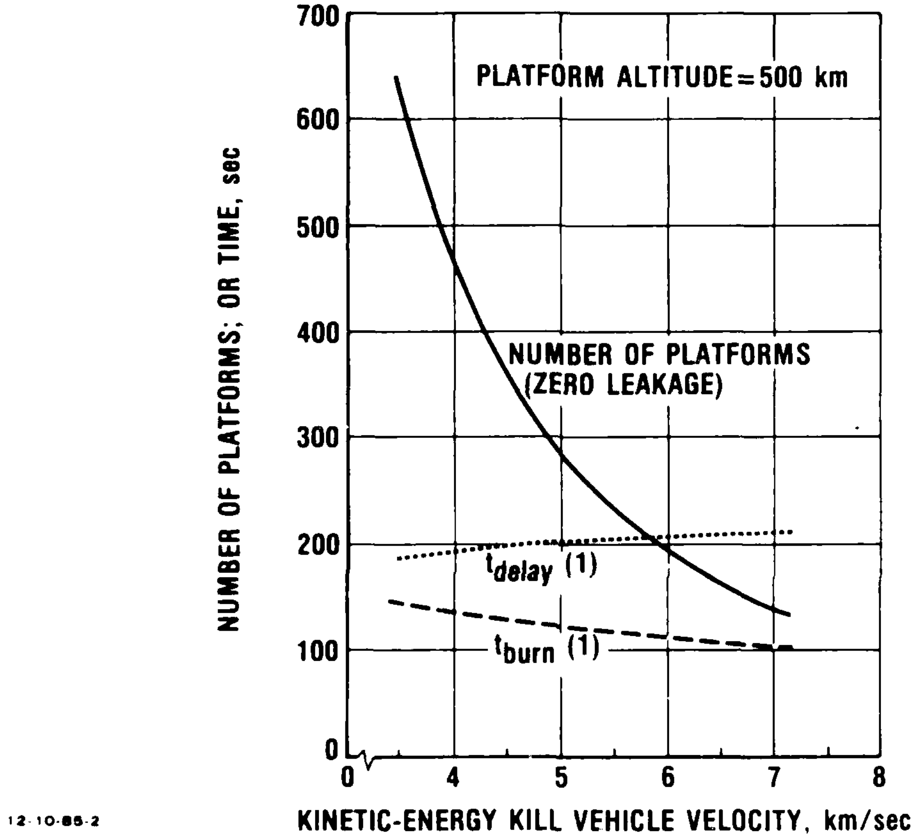

FIGURE 10. Dependence on Kinetic-Energy Kill Vehicle Velocity of-
Number of Platforms to Give Zero Leakage For Selected System Parameters
Maximum Burn Time For Full Leakage For Selected System Parameters = [t_{burn} (1)]
Minimum Delay Time For Full Leakage For Selected System Parameters = [t_{delay} (1)]

OBSERVATIONS

The small rate of reduction in area coverage as a KEW interceptor's reach is reduced from the maximum required indicates that the KEW system fails gracefully just off design.

The required number of KEW platforms for full coverage at the launch latitude of targeted ballistic missiles is much more dependent on the KKV velocity than on the platform altitude. Higher velocities and lower altitudes yield smaller requirements for numbers of satellite platforms.

Shortened booster burn time is a very effective countermeasure against a KEW BPI system; reduction to about 120 sec negates the example system. Major increases in the number of KEW platforms are required to compensate for moderate shortening of booster burn time, and the response to shortened burn time of leakage for a fixed system size does not change markedly over ranges of a factor of two in system altitude or KKV velocity.

Increased commitment delay is a less effective countermeasure; an improbable lengthening to over 200 sec is required to negate the example KEW system. If the commitment delay time were lengthened from 60 sec to 120 sec, however, the required number of platforms to enforce zero leakage would double. The nature of the response to lengthened delay time of leakage for a fixed system size is, as with the shortened burn time, insensitive to system altitude or KKV velocity.

The FORTRAN computer program to calculate KEW BPI system leakage is given in the appendix following.

APPENDIX--FORTRAN COMPUTER PROGRAM

```
      PROGRAM KKLEAK
C------------------------------------------------------------------------
100   FORMAT(2X, 18H NO. SATELLITES = , F7.1)
101   FORMAT(2X,8HREL. RAD,7H RADIUS,2X,5H AREA)
102   FORMAT (F10.3,F7.2,F7.3)
103   FORMAT (2X, 21H INPUT XLAT, XNT, TB )
C------------------------------------------------------------------------
      RE = 6378.149
      PI = 3.1415926536
      RAD = 180./PI
      SR3 = SQRT(3.)
      HS = 500.
2     TYPE 103
      READ(5,*) XLAT,XNT,TB
      IF(TB .EQ. 0.) GO TO 99
      CLAT = COS(XLAT/RAD)
      TPBV = 0.
      TDEL = 60.
      GBO = 21. + 31.* EXP(-TB/50.)
      HB = (8./3.) * TB ** 0.87506 + 7.5 * TPBV * SIN (GBO/RAD)
      TVUL = TB + TPBV -TDEL
      VKKV = 5.
      RCI = SQRT (VKKV*VKKV*TVUL*TVUL - (HS-HB)**2)
      RCM = RCI
      IF(XNT .NE. 0.) RCM = (PI*RE/1.5)* SQRT(SR3/XNT)
      TBETA = SR3*CLAT
      CBETA = 1./SQRT(1.+TBETA*TBETA)
      ALPHA = ASIN(CBETA)
      XN1 = 2.*PI*RE/(1.5*RCM)
      XNT = XN1*XN1 *SR3/4.
      TYPE 100, XNT
      TYPE 101
      AT= 3.* SR3 * RCM * RCM * CLAT/8.
C------------------------------------------------------------------------
      CTH1 = 1.5 * RCM * CLAT/RCI
      IF(ABS(CTH1) .GT. 1.) CTH1=SIGN(1.,CTH1)
      TH1 = ACOS(CTH1)
      STH1 = SQRT(1.-CTH1*CTH1)
C
      CDTH = (SR3 * RCM/(4.*RCI*CBETA))
      IF(ABS(CDTH) .GT. 1.) CDTH=SIGN(1.,CDTH)
      DTH = ACOS(CDTH)
      TH2 = ALPHA - DTH
      IF (TH2 .LT. TH1) TH2 = TH1
C
      STH4 = (0.5 * SR3 *RCM/RCI)
      IF(ABS(STH4) .GT. 1.) STH4=SIGN(1.,STH4)
      TH4 = ASIN(STH4)
      TH3 = ALPHA + DTH
      IF (TH3 .GT. TH4) TH3 = TH4
      CTH4 = SQRT (1. - STH4 * STH4)
C
      A = 0.5*RCI*RCM*(1.5*STH1*CLAT+RCI*(TH4-TH3+TH2-TH1+
     &    SIN(TH3-ALPHA)*COS(TH3-ALPHA)+SIN(ALPHA-TH2)*
     &    COS(ALPHA-TH2))/RCM+0.5*SR3*CTH4)
      AO = A/AT
      RP = RCI
      RF = RP/RCM

98    TYPE 102, RF, RP, AO
      GO TO 2
99    STOP
      END
```

END

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