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COMPARISON OF ACOUSTIC AND ELASTIC WAVE SCATTERING FROM 1/1  
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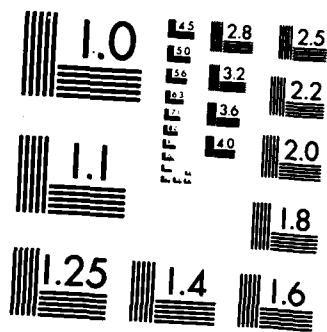
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# Comparison of Acoustic and Elastic Wave Scattering From Elliptical Shells

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May 1983, Cincinnati, Ohio

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# COMPARISON OF ACOUSTIC AND ELASTIC WAVE SCATTERING FROM ELLIPTICAL SHELLS

## INTRODUCTION

The use of the "T" matrix integral solution based on extended boundary conditions for scattering from elastic shells in an elastic medium has been discussed by Simon and Radlinski ("Elastic Wave Scattering from Elliptical Shells," J. Acoust. Soc. Am. 71 (2), 273-281, 1982). In that paper, a thin shell theory formulation was coupled to a two-dimensional surface integral formulation of scattering from elliptical surfaces which was first introduced by V. V. Varadan and Y. H. Pao ("Scattering Matrix for Elastic Waves II. Application to Elliptical Cylinders," J. Acoust. Soc. Am. 63, 1014-1024, 1978). In an elastic medium, scattering of either an incident dilatational wave (p-wave) or shear wave (sv-wave) will result in mode conversion at the scattering body. In other words, for an incident dilatational wave, there will be some conversion of the scattering energy into shear waves and correspondingly for an incident shear wave, some conversion to dilatational waves will result. The above paper on elastic shells discusses p-p, sv-sv, p-sv, and sv-p scattering as a function of aspect ratio for an ellipse.

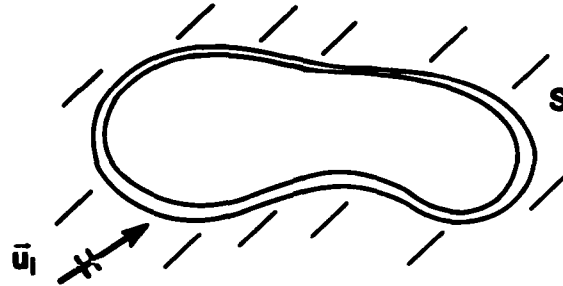
The philosophy used in this presentation treats the scattering from a shell in fluid as a special case of the solution of scattering from a shell in an elastic medium. The reduced displacement expansion used here allows shell equations which describe the force and velocity of the shell to be directly substituted into the T-matrix formulation. Comparisons of acoustic and elastic p-p scattering will indicate the effects of stiffness of an elastic material. Particular attention will be given to the effects of modal resonances on the scattering cross sections.

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## SCATTERING FROM A SHELL IN FLUID



$$\bar{u}(\vec{r}) = \bar{u}^i(\vec{r}) + \bar{u}^s(\vec{r})$$

$$\int_S \left\{ \bar{u}' \cdot \left( \hat{n}' \cdot \bar{\Sigma}' \right) - \hat{n}' \cdot \vec{r}' \cdot \bar{G} \right\} dS' = \begin{cases} \bar{u}^s, > S \\ -\bar{u}^i, < S \end{cases}$$

$$\bar{\Sigma} = \lambda_f \bar{I} \bar{\nabla} \cdot \bar{G}$$

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Figure 1.

A plane wave with displacement  $\vec{u}^i$  is shown to be incident on an arbitrary shaped shell  $S$  in an infinite fluid. The displacement vector  $\vec{u}(\vec{r})$  at any point in the fluid can be written as the sum of the incident and scattered velocities. The extended boundary integral formulation for the displacement is written in the general terms of displacement  $\vec{u}$ , the Green's stress dyadic  $\bar{\Sigma}$ , the stress tensor  $\vec{\tau}$ , and Green's dyadic  $\bar{G}$ . Note that the integral is defined both exterior and interior to the shell surface  $S$ . In the integral equation  $\hat{n}'$  is the unit outward normal to  $S$ . The Green's stress dyadic in the fluid is written in terms of  $\lambda_f$ , the bulk modulus Green's dyadic and the unit tensor  $\bar{I}$ . In the elastic medium formulation, additional terms which describe the shear motion are included.



## EXPANSION INTO BASIS FUNCTIONS

$$\bar{u}^i = \sum_{n\sigma} A_n^\sigma \operatorname{Re} \bar{\phi}_n^\sigma$$

$$\bar{u}^s = \sum_{n\sigma} \alpha_n^\sigma \bar{\phi}_n^\sigma$$

$$\bar{G}(\bar{r}|\bar{r}') = \frac{1}{4\rho\omega^2} \bar{\phi}_m^\sigma(\bar{r}_>) \operatorname{Re} \bar{\phi}_m^\sigma(\bar{r}_<)$$

$$\bar{\phi}_n^\sigma = \sqrt{\epsilon_n} \bar{V} H_n^{(1)}(k_f r) \begin{cases} \cos n\theta, & \sigma = 1; \\ \sin n\theta, & \sigma = 2; \end{cases} \quad \sqrt{\epsilon_n} = \begin{cases} 1 & n = 0 \\ \sqrt{2} & n > 0 \end{cases}$$

GIVES

$$A_n^\sigma = \frac{1}{4\rho\omega^2} \int_S [\lambda_f(\bar{v}' \cdot \bar{\phi}_m^\sigma)(\hat{n}' \cdot \bar{u}) - \hat{n}' \cdot \bar{\tau}(\hat{n}' \cdot \bar{\phi}_m)] dS'$$

$$\alpha_n^\sigma = \frac{-1}{4\rho\omega^2} \int_S [\lambda_f(\bar{v}' \cdot \operatorname{Re} \bar{\phi}_m^\sigma)(\hat{n}' \cdot \bar{u}) - \hat{n}' \cdot \bar{\tau}(\hat{n}' \cdot \operatorname{Re} \bar{\phi}_m^\sigma)] dS'$$

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Figure 2.

The incident and scattered displacements are expanded into basis functions  $\phi_n^\sigma$ . The symbol  $\operatorname{Re}$  refers to the real part of a complex quantity. The  $A_n$ 's are known coefficients and alphas ( $\alpha_n$ ) are the unknown scattering coefficients. Also, the Green's dyadic is expanded in terms of similar basis functions. For the two-dimensional cases considered here, the basis functions are the cylindrical harmonics which are functions of the fluid wavenumber  $k_f$  and the polar coordinates  $(r, \phi)$ . For the elastomer case, an additional expansion is needed to describe the shear terms. When the expansions are substituted into the extended boundary value integral, the known and unknown scattering coefficients are given in terms of integrals evaluated over the surface of the body. The terms  $\rho$  and  $\omega$  are respectively the density of the fluid and the circular frequency.



## BOUNDARY CONDITIONS

$$\bar{u}' = V \hat{n}'$$

$$\hat{n}' \cdot \bar{\tau}' = F_N \hat{n}'$$

$$F_T = 0$$

$$A_n^{\sigma} = \frac{1}{4\rho\omega^2} \int_S [V\lambda_1(\bar{\nabla}' \cdot \bar{\phi}_m^{\sigma}) - F_N(\hat{n}' \cdot \bar{\phi}_m^{\sigma})] dS'$$

$$\alpha_n^{\sigma} = -\frac{1}{4\rho\omega^2} \int_S [V\lambda_1(\bar{\nabla}' \cdot \text{Re}\bar{\phi}_m^{\sigma}) - F_N(\hat{n}' \cdot \text{Re}\bar{\phi}_m^{\sigma})] dS'$$

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Figure 3.

The boundary conditions appropriate at the shell fluid interface are the continuity of normal displacement, continuity of normal forces, and zero tangential force  $F_T$ .  $F_N$  is the normal force and  $V$  is the normal displacement of the shell. Substitution of the first two expressions into the extended boundary condition results in expressions which included the normal displacement and normal forces at the midsurface of the shell. For the elastomer case, additional terms are included for the non-zero tangential force and the tangential displacement.





## SHELL MODEL

$$F_N = \rho h \omega^2 V - \frac{Eh}{r'(1-\nu^2)} \left( \frac{dW}{ds'} + \frac{V}{r'} \right) - \frac{Eh^3}{12(1-\nu^2)} \frac{d^3}{ds'^3} \left( \frac{dV}{ds'} - \frac{W}{r'} \right)$$

$$F_T = 0 = \rho h \omega^2 W + \frac{Eh}{1-\nu^2} \frac{d}{ds'} \left( \frac{dW}{ds'} + \frac{V}{r'} \right) - \frac{Eh^3}{12r'(1-\nu^2)} \frac{d^2}{ds'^2} \left( \frac{dV}{ds'} - \frac{W}{r'} \right)$$

$$W = \sum_{n\sigma} a_n^\sigma \begin{cases} \cos k_n s', \sigma=1 \\ \sin k_n s', \sigma=2 \end{cases}$$

$$V = \sum_{n\sigma} b_n^\sigma \begin{cases} \cos k_n s', \sigma=1 \\ \sin k_n s', \sigma=2 \end{cases}$$

$$k_n = \frac{2\pi n}{s_0}$$

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Figure 4.

As was done in our previously published work on scattering from shells in an elastomer, the tangential and normal force equations are written in terms of the thin shell equations in which the first term is an inertial term, the second term describes the extensional motion, and the third term represents the bending motion of the shell. The force equations are functions of the local radius of curvature  $r'$ , the arc length  $s'$ , the normal displacement  $V$ , and tangential displacement  $W$ . Since the derivatives are in terms of the arc length, the normal and tangential velocities are expanded in terms of the arc length. Finally, the null condition in the tangential force as seen in the second equation allows one to write the coefficients  $a_n^\sigma$  for the tangential displacement in terms of the  $b_n^\sigma$  for the normal displacement. The quantity  $k_n$  represents a Fourier series component in terms of the circumference  $s_0$ . In the shell equations,  $E$  is the Young's modulus,  $\nu$  is Poisson's ratio, and  $h$  is the shell thickness.



## MATRIX EQUATIONS

$$[a] = - [R_1]^{-1} [R_2] [b]$$

$$[A] = [Q] [b]$$

$$[\alpha] = - [ReQ] [b]$$

$$[\alpha] = - [ReQ] [Q]^{-1} [A]$$

$$[\alpha] = [T] [A]$$

## NORMALIZED CROSS SECTION

$$\Omega = \frac{4}{k s_0} \sum_n |\alpha_n|^2$$

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Figure 5.

In matrix notation, the coefficients of the tangential velocity  $[a]$  are written in terms of the  $[b]$  and the  $R$  matrices which are functions of the shell parameters. Substitution of the first expression into the extended boundary results in expressions as seen in the second and third equations which relate the scattering coefficients  $A$  and  $\alpha$  to the normal velocity coefficients  $b_n^0$  of the shell by  $Q$  matrices. Finally,  $b$  is eliminated from these two equations to obtain the unknown scattering coefficients  $[\alpha]$  in terms of the known coefficients  $[A]$  and the  $T$  matrix as defined by the fourth and fifth equations.

In the following discussions, we will consider the scattering cross section per unit axial length normalized to the circumference of the scatterer  $s_0$ . The cross section is directly proportional to the sum of the square of the absolute value of the scattering coefficients.

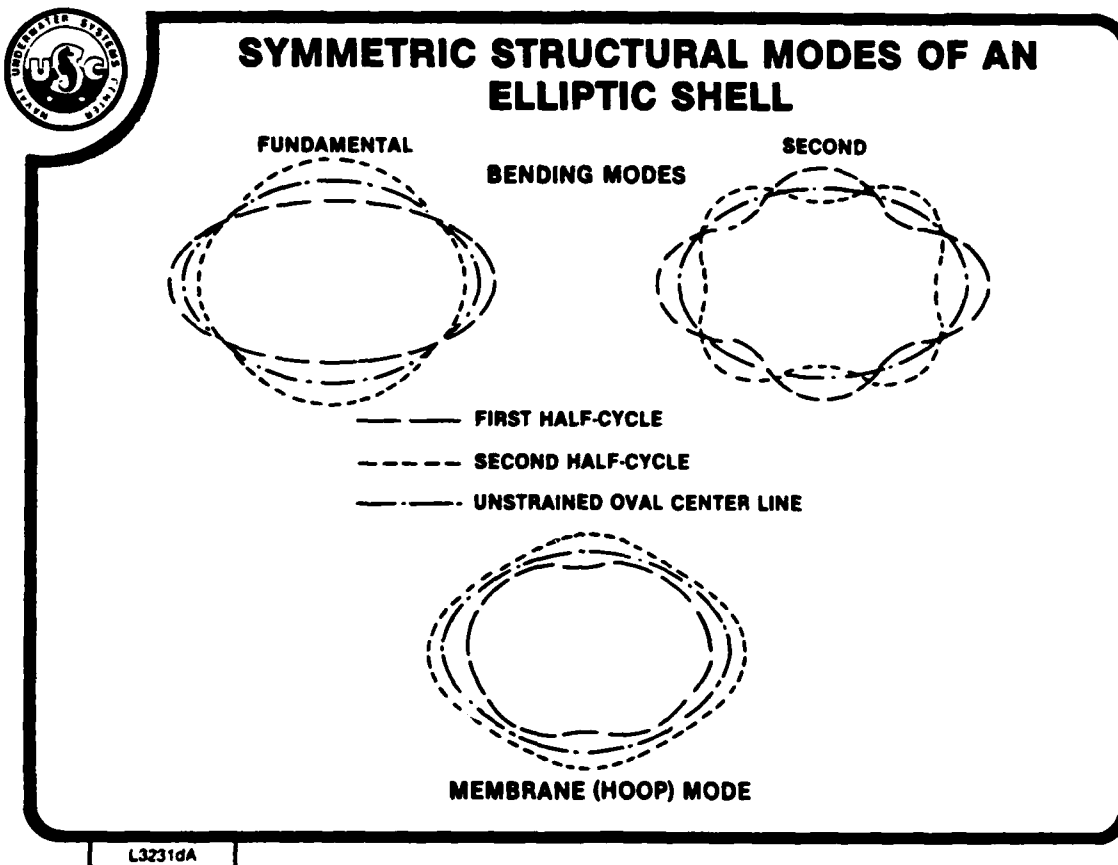
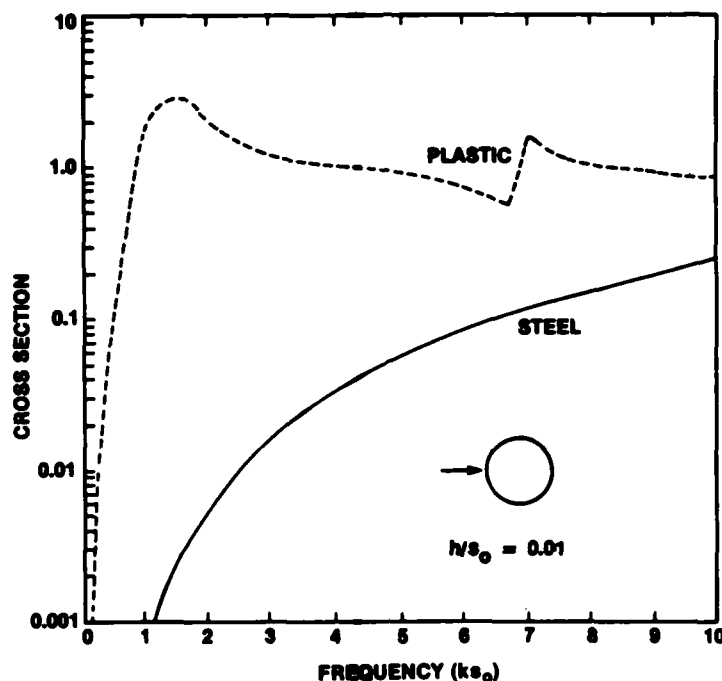


Figure 6.

In this presentation we will specialize the formulation to elliptical shells. In particular we shall investigate the effects of the inertial term, the bending modes, and the membrane or hoop modes. For an elliptical shell, the symmetrical bending modes have a net volume displacement which is not the case for the circular shell. The first and second symmetric bending modes are displayed in this case. The membrane or hoop mode, which is primarily extensional in nature, has for a given tangential strain, a maximum volume displacement for a circle and decreases with increased eccentricity of the shell.



# SCATTERING FROM PLASTIC AND STEEL CIRCULAR SHELLS IN FLUID



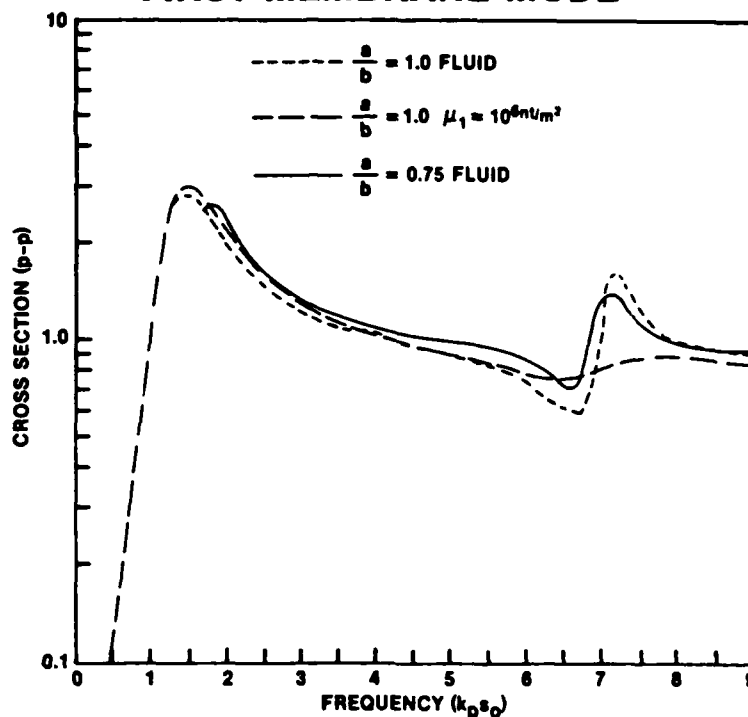
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Figure 7.

First consider total scattering cross section from circular shells. The simplest case considered is scattering from plastic and steel shells of the same radius and wall thickness. The frequency axis  $ks_0$  where  $s_0$  is the circumference of the shell and  $k$  is the fluid wave number. The sound speed in the steel material is about 3.5 times that of the plastic material which is 1.1 times that of water. The density of the plastic is about 1.2 times that of water and steel is about 8 times that of water. Because the  $n=0$  extensional resonance frequency of the steel shell is so much higher than that of the plastic shell and because the inertia of the steel shell is not as significantly affected by radiation loading as the lighter plastic shell, the scattering cross section of the steel shell is several orders of magnitude below the plastic shell at these low values of  $ks_0$ . The oscillation near  $ks_0$  of 7 is due to scattering near the first membrane or hoop mode. The in-air hoop mode frequencies for a thin circular shell are given by  $k_n s_0 = 2\pi[N^2+1]^{1/2}c_p/c_0$  where  $c_p$  is the plate velocity in the material,  $N$  is the membrane mode number and  $c_0$  is the sound velocity in fluid. The first in-air hoop mode for the plastic shell is calculated to be at  $ks_0 = 6.9$  for  $s_0 = 0.1445$  m.



## SCATTERING FROM PLASTIC SHELLS NEAR THE FIRST MEMBRANE MODE



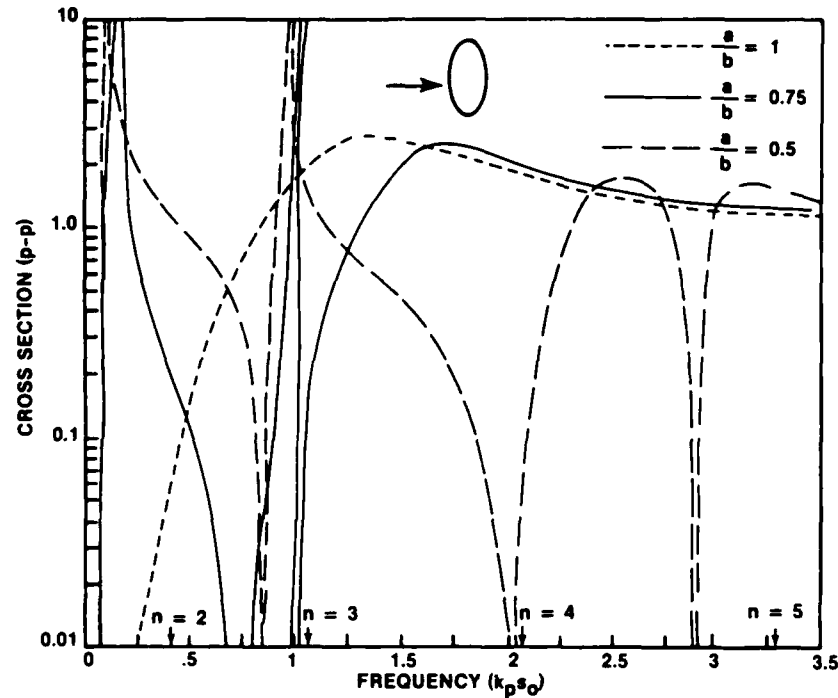
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Figure 8.

The effects of squashing the plastic shell and/or surrounding the circular shell with a low stiffness material is considered near the membrane mode from  $k_p s_0$  of 6 to 8. From the solid curve, less oscillation is seen when the aspect ratio (minor axis/major axis,  $a/b$ ) of the tube is reduced to 0.75. In general, as the aspect ratio is diminished, less volume velocity is available in the extensional modes and thus the scattering cross-sectioned variation is less for the more eccentric tube. Interestingly, an even more pronounced effect on the total cross section for p-p scattering occurs when the circular shell is in a low stiffness elastic medium. The stiffness of the medium will affect the vibrational amplitude and slightly increase the resonance frequency. Note that the low frequency behavior of the circular shell is similar in both fluid and the low stiffness elastomer. The low frequency behavior of the squashed tube in fluid is examined in the next figure.



## SCATTERING FROM A PLASTIC SHELL IN FLUID AS A FUNCTION OF SHELL ECCENTRICITY



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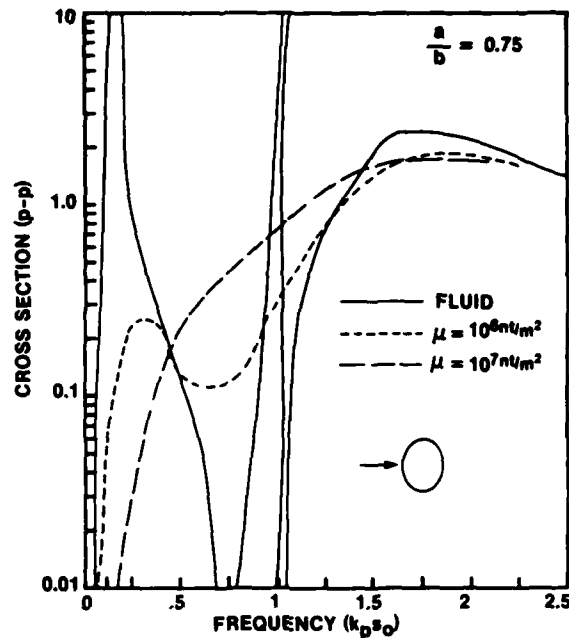
Figure 9.

Now consider the effect on the total cross-section scattering as a function of aspect ratio. As the shell is squashed from a right circular cylindrical shape, a net volume velocity is now possible at the bending modes of the shell. A net volume velocity implies that the shell becomes dynamically soft which allows for increased scattering. Significant differences are now found between the low frequency scattering from a circular shell and the elliptical shells. For a shell of  $h/s_0 = 0.01$ , the first in-air bending resonance should occur at approximately  $k_{ps_0}$  of 0.125. Although a rounder shell has a lower bending resonance for a fixed  $s_0$ , the effect of radiation loading is stronger on the flattened shells. Since the net volume velocity for bending modes go to zero more quickly for a rounder tube, there are also more higher order resonance scattering effects with the 0.5 aspect ratio tube. The in-air frequencies corresponding to the mode numbers for the lower order bending modes of the circular plastic shell are indicated on the graph. These values are given by  $ks_0 = 2\pi c_0/c_0 \times \beta[n^2(n^2-1)^2/(n^2+1)]^{1/2}$  where  $n$

is the bending wave number,  $\beta = \frac{1}{12^{1/2}} \frac{h}{a_0}$  and  $a_0$  is the radius of the shell.



## SCATTERING FROM A PLASTIC ELLIPTIC SHELL IN FLUID AND ELASTOMERS NEAR THE BENDING MODES



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Figure 10.

In this example, the change in scattering cross section for a shell of aspect ratio of 0.75 is demonstrated for elastomers of two different stiffnesses and compared with the fluid case. For the lower stiffness elastic material, an increased frequency but lower amplitude fundamental bending mode is indicated by the cross-section calculation. No indication of the second bending mode is found with the elastic matrix of shear modulus of  $10^6 \text{ nt/m}^2$ . Surrounding the thin plastic shell by a shear modulus of  $10^7 \text{ nt/m}^2$  mitigates the effects of all the bending resonances and the cross section is similar to the circular tube in fluid as previously shown in Figure 9.

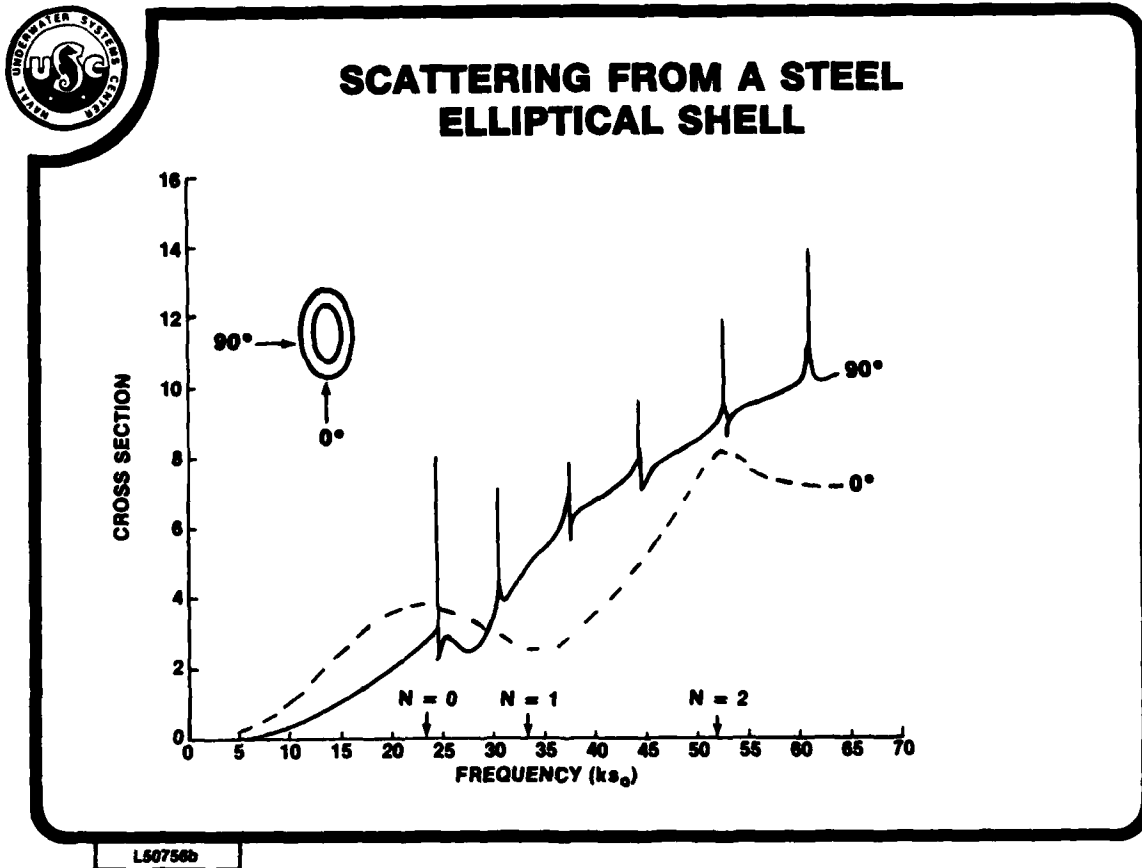


Figure 11.

This figure illustrates the effect of orientation of the shell on total cross section. High frequency scattering from a steel elliptical shell in water is presented here for an elliptical shell with an aspect ratio of 0.73 and  $h/s_0 = 0.0085$ . The resonance region below  $ks_0$  of approximately 5 has not been computed for these curves. With the plane wave incident along the minor axis of the shell, sharp discontinuities are seen in the cross section that are due to creeping waves that circumnavigate the shell. Note that maxima where the waves interfere constructively only are evident at  $ks_0$  of approximately 31 and 62. The frequency separation of creeping wave occurrences increases with increasing frequency and this characteristic corresponds to excitation of the higher order bending modes of the shell.

With the direction of the plane wave incident along the major axis of the elliptical cylinder, no evidence of the sharply defined creeping wave resonance is found for this orientation. The membrane resonances of the elliptical shell are indicated on the graph ( $N = 0, 1, 2$ ). Considering the inverse problem, if the scattering cross section is known for a wide bandwidth, the orientation could be determined from these frequency responses.





## BACKSCATTERING FROM A STEEL ELLIPTICAL SHELL

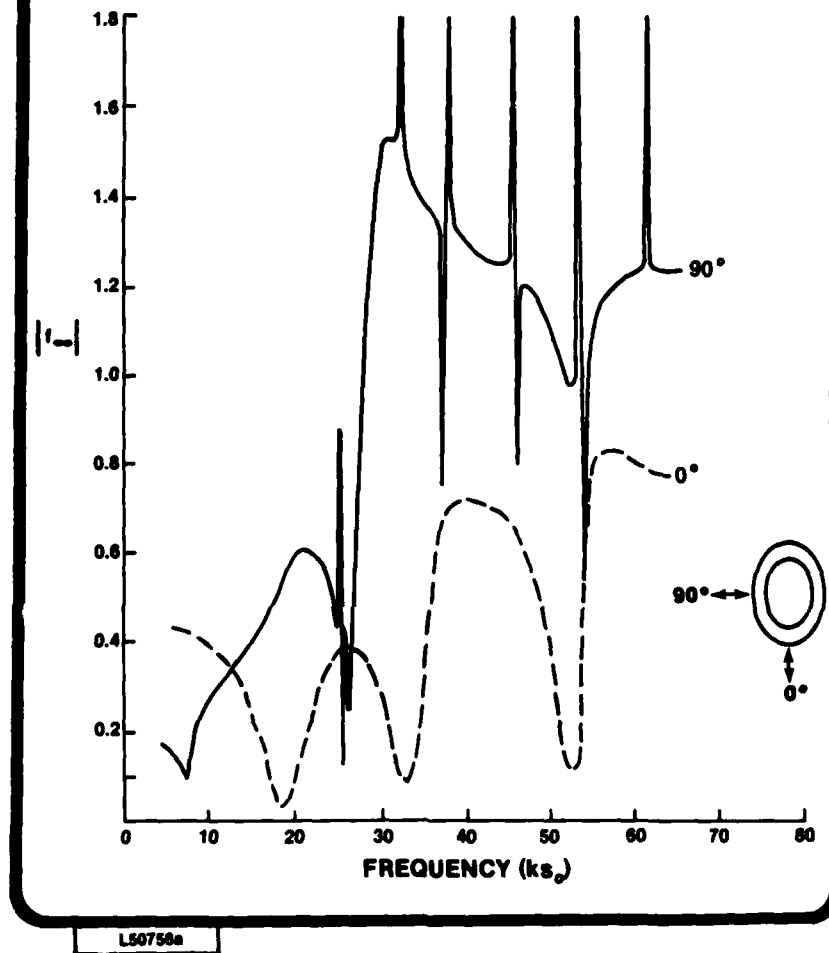


Figure 12.

High frequency backscattering for the two previously considered shell orientations is shown in Figure 12. The backscattering function  $f_\infty$  is defined here by  $f_\infty = [4/(\pi kb) f_p^2(-\theta)]^{1/2}$  where  $f_p^2(-\theta)$  is the differential cross section in the backscattered direction and  $b$  is the semimajor axis of the shell. The resonance structure due to creeping waves is again apparent for the case with the incident plane wave parallel to the minor axes of the shell. With the plane wave incident parallel to the major axes of the shell, more oscillations occur as a function of  $ks_0$  than for the corresponding curve for the scattering cross section. As might be expected from geometric considerations, at high frequencies, incidence along the direction of the minor axis produces the greater backscatter.

## CONCLUSIONS

The formulation for elastic wave scattering from elliptic cylinders has been modified to include acoustic scattering which is the limiting case of zero shear modulus. Since the shear wavelength governs series convergence for elastic wave scattering, the convergence for acoustic scattering is much faster. Also matrix manipulation is easier with acoustic scattering because of the smaller size. The low frequency acoustic cross section for scattering from plastic elliptical shells indicated strong scattering due to bending resonances but even a low stiffness elastomer mitigated this phenomenon. For acoustic waves incident in a direction that parallels the minor axis of the tube, high frequency scattering indicates the presence of creeping waves corresponding to bending resonances.

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