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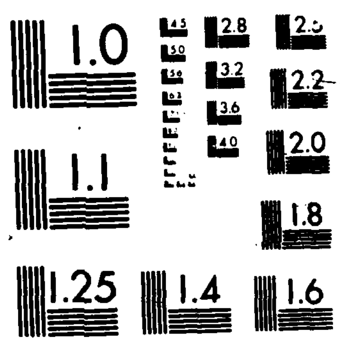
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A Bäcklund transformation and linearization for an instance of the Hénon–Heiles system is examined. This provides a special form of solution depending on the three parameters. In addition, a direct formulation in terms of the schwarzian derivative is defined for the Hénon–Heiles system and second Painlevé transcendent. This provides (1) a classification of the Hénon–Heiles system as equations of "Novikov" type, and; (2) a simple method for deriving the Bäcklund transformations and special solutions of the second Painlevé transcendent. As equations of Novikov type the integrable occurrences of the Hénon–Heiles system can be completely integrated by known methods.

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THE ANALYTIC STRUCTURES OF DYNAMICAL SYSTEMS

This report is on work supported by the AFOSR in the period from June 1984 to December 1985. During this time we have developed several new methods for "solving" nonlinear ordinary and partial differential equations. Specific results include: 1) the classification of integrable systems through a "direct Schwarzian" formulation. 2) the "uniformization" of integrable systems with multiple-valued singularities. The solutions are expressed in uniform (single-valued) variables that satisfy equations with the "Painlevé property". 3) the relationship of integrable systems with or without Bäcklund transformations to the Painlevé property. 4) the definition and investigation of the "periodic fixed points" of Bäcklund transformations. This leads to a new method for both defining and solving integrable systems.

Previous to June 1984 we have proposed a method of Bäcklund transformation and modified equations for the study of equations possessing the Painlevé property [1]. That is, from the Bäcklund transformation, equations are found that are formulated in terms of the Schwarzian derivative. With these "modified" equations there are found Miura transformations (from modified to "original" equation) which linearize into the Lax Pair for the original system. To apply this method the original equation should both possess the Painlevé property and have a Bäcklund transformation. However, not all integrable systems directly possess the Painlevé property or have Bäcklund transformations. We address these points in the work described herein.

In references [2] we began the study of ordinary differential equations by the Bäcklund method. In ref. [3] we find that the solutions so obtained depend on fewer parameters than the general form of solution. However, by a "direct Schwarzian" formulation of the system of odes we identify the equation as instances of "Novikov" systems for which the general solution can be readily found. This procedure is applied to the Henon-Heiles system and to the second Painlevé transcendent.

In reference [4] the Bäcklund method is applied to the KP and Hirota-Satsuma systems. For both we find the appropriate modified equations and Miura transformations. By linearizing the (Ricati-type) Miura transformations the Lax pairs are calculated. Consideration of the several distinct Bäcklund transformations of the modified equations obtains a method for the iterative construction of the rational solutions.

In reference [5] we show that the Bullough-Dodd equation, which is completely integrable, possesses the Painlevé property and does not allow a Bäcklund transformation. By a direct formulation in terms of the Schwarzian derivative this equation is shown to be a specialization of the "minus-one" equation of the Caudrey-Dodd-Gibbon sequence. The "positive" equations of the CDG sequence are shown to have non-trivial Bäcklund transformations. On the other hand, the Sine-Gordon equation is found to be the minus-one equation of the Korteweg-de Vries (KdV) sequence and to have a Bäcklund transformation. Within a certain class of scalar evolution equations the KdV and CDG sequences are shown to be the unique, integrable equations.

Also, in reference [5] the (completely integrable) Harry Dym sequence, which does not directly possess the Painlevé property, is found to have a "uniformization" within the KdV sequence. Uniformization means a simultaneous representation of dependent and independent variables by "meromorphic" (Painlevé) functions of the uniform variables. A connection with the automorphic functions is examined. Bäcklund transformations, Lax Pairs and a Hamiltonian formulation are found for the Harry Dym sequence.

Finally, in reference [6] we present a new method for studying integrable systems based on the "periodic fixed points" of Bäcklund transformations. Normally, the BT maps an "old" solution into a "new" solution and requires a known "seed" solution to get started. It can also be difficult to qualitatively classify the result of applying the BT several times to a known solution. By studying the periodic fixed points of the BT (regarded as a nonlinear map in a function space), we obtain integrable systems of equations of finite degree (equal to the order of the fixed point), and a method for the systematic classification of the solutions of the original system.

The publications produced with AFOSR support during this contract period are refs. [3, 4, 5, 6]. The title pages and abstracts are contained in the Appendix.

References

1. John Weiss, Final Report for Air Force grant AFOSR-83-0095, "The Analytic Structure of ordinary and partial differential equations", (1984)

2. John Weiss, "Bäcklund transformation and linearizations of the Henon-Heiles system", Phys. Letts. 102A, 329 (1984)

3. John Weiss, "Bäcklund transformation and the Henon-Heiles system", Phys. Letts. 105A, 387 (1984)

4. John Weiss, "Modified equations, rational solutions, and the Painlevé property for the Kadomtsev-Petviashvili and Hirota-Satsuma equations", J. Math. Phys. 26 2174 (1985)

5. John Weiss, "Bäcklund transformation and the Painlevé property", to appear. J. Math. Phys. (1985)

6. John Weiss, "Periodic fixed points of Bäcklund transformations and the Korteweg-de Vries equation", sub. to J. Math. Phys. (1985)

Appendix

Publications with AFOSR support during contract period, June 1984 to December 1985.

BÄCKLUND TRANSFORMATION AND THE HENON–HEILES SYSTEM ☆

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A Bäcklund transformation and linearization for an instance of the Hénon–Heiles system is examined. This provides a special form of solution depending on the three parameters. In addition, a direct formulation in terms of the schwarzian derivative is defined for the Hénon–Heiles system and second Painlevé transcendent. This provides (1) a classification of the Hénon–Heiles system as equations of "Novikov" type, and; (2) a simple method for deriving the Bäcklund transformations and special solutions of the second Painlevé transcendent. As equations of Novikov type the integrable occurrences of the Hénon–Heiles system can be completely integrated by known methods.

In ref. [1] Bäcklund transformations and the consequent linearizations of the Hénon–Heiles system were found using the methods of refs. [2,3]. It can be shown that the solution found by this method is "special" in that it will depend on three (not four) arbitrary parameters. To find the complete solution a direct formulation in terms of the schwarzian derivative is presented. Herein we present this result and in addition, show how a "direct" formulation can provide a different form of Bäcklund transformation. Bäcklund transforms are derived for the second Painlevé transcendent, and it is found that the three integrable instances of the Hénon–Heiles system can be transformed into a "canonical" class of "Novikov" equations [4–6] considered in ref. [3].

We consider the Hénon–Heiles system:

$$\ddot{X} = -AX - 2dXY, \quad \ddot{Y} = -BY + cY^2 - dX^2, \tag{1}$$

with hamiltonian

$$H = \tfrac{1}{2}(\dot{X}^2 + \dot{Y}^2 + AX^2 + BY^2) + dX^2Y - \tfrac{1}{3}cY^3. \tag{2}$$

This sytems is known to be integrable [7] when:

(i) $d/c = -1, \quad B = A$,

(ii) $d/c = -\tfrac{1}{6}$,

(iii) $d/c = -\tfrac{1}{16}, \quad B = 16A$. (3)

In ref. [1] we found the Bäcklund transformations for cases (ii) and (iii). Case (i) is separable [7]. The BT of ref. [1] for case (ii) may be "improved" to the extent that the BT will depend on an additional arbitrary parameter. Therefore we first present this result. For case (ii),

$$d/c = -\tfrac{1}{6}, \tag{4}$$

the solutions (X, Y) of eqs. (1) have (meromorphic) expansions of the form:

$$X = \varphi^{-1} \sum_{j=0}^{\infty} X_j \varphi^j, \quad Y = \varphi^{-2} \sum_{j=0}^{\infty} Y_j \varphi^j. \tag{5}$$

As explained in ref. [1], to define the BT we let

$$X = X_0\varphi^{-1} + X_1, \quad Y = Y_0\varphi^{-2} + Y_1\varphi^{-1} + Y_2, \tag{6}$$

where $(\varphi, X_0, X, Y_0, Y_1, Y_2)$ are functions of t.

Their results, after evaluation:

$$Y_0 = -\varphi_t^2, \quad Y_1 = \varphi_{tt},$$

$$Y_2 = \tfrac{1}{12}(4\lambda - B - 3V - 3\varphi_{tt}^2/\varphi_t^2),$$

$$X_0^2 = \varphi_t^2 V, \quad X_1 = -\tfrac{1}{2}(V_t/V + \varphi_{tt}/\varphi_t)V^{1/2}, \tag{7}$$

☆ Work supported by Department of Energy contract DOE DE-AC03-81ER10923 and U.S. Air Grant No. AFOSR 83-0095.

Modified equations, rational solutions, and the Painlevé property for the Kadomtsev–Petviashvili and Hirota–Satsuma equations

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We propose a method for finding the Lax pairs and rational solutions of integrable partial differential equations. That is, when an equation possesses the Painlevé property, a Bäcklund transformation is defined in terms of an expansion about the singular manifold. This Bäcklund transformation obtains (1) a type of modified equation that is formulated in terms of Schwarzian derivatives and (2) a Miura transformation from the modified to the original equation. By linearizing the (Ricati-type) Miura transformation the Lax pair is found. On the other hand, consideration of the (distinct) Bäcklund transformations of the modified equations provides a method for the iterative construction of rational solutions. This also obtains the Lax pairs for the modified equations. In this paper we apply this method to the Kadomtsev–Petviashvili equation and the Hirota–Satsuma equations.

I. INTRODUCTION

In Ref. 1 we have formulated a procedure for calculating the Lax pair and rational solutions of partial differential equations that possess the Painlevé property. That is, for an equation with the Painlevé property, a Bäcklund transformation is defined in terms of an expansion about the "singular manifold." This Bäcklund transformation obtains (1) a type of "modified equation" that can be expressed in terms of Schwarzian derivatives and (2) a Miura transformation from the modified to the original equation. By linearizing the Ricati-type Miura transformation (and the modified equations), the Lax pair is found. Then, further consideration of the Bäcklund transformations for the modified equations provides a method for the iterative construction of "rational" solutions, and finds the Lax pair for the modified equations as well.

We recall that the partial differential equation is said to possess the Painlevé property[2–7] when the solutions of the partial differential equation (pde) are "single valued" about the movable, singularity manifold and the singularity manifold is "noncharacteristic." To be precise, if the singularity manifold is determined by

$$\varphi(z_1,z_2,\dots,z_n)=0\,, \tag{1.1}$$

and $u=u(z_1,\dots,z_n)$ is a solution of the pde, then we require that

$$u=\varphi^{\alpha}\sum_{j=0}^{\infty}u_j\varphi^{j}\,, \tag{1.2}$$

where $u_0\neq 0$, $\varphi=\varphi(z_1,\dots,z_n)$, and $u_j=u_j(z_1,\dots,z_n)$ are analytic functions of (z_j) in a neighborhood of the manifold (1.1) and α (the leading-order exponent) is a (negative) rational number. The requirement that the manifold (1.1) be noncharacteristic (for the pde) insures that the expansion (1.2) will be well defined, in the sense of the Cauchy–Kovalevskaya theorem.[8] Substitution of (1.2) into the pde determines that value(s) of α, and defines the recursion relations for u_j, $j=0,1,2,\dots$. When the expansion (1.2) is well defined and contains the maximum number of arbitrary functions allowed at the "resonances,"[2,9,10] the pde is said to possess the Painlevé property and is conjectured to be integrable. Informally, the resonances are the values of j for which the u_j are not "fixed" by the recursion relations (i.e., are arbitrary).

The Bäcklund transformation is defined by truncating the expansion (1.2) at the constant level term. That is, we set

$$u=u_0\varphi^{-n}+u_1\varphi^{-n+1}+\cdots+u_n\,, \tag{1.3}$$

and find, from the recursion relations for u_j and the condition that u_j vanish for $j>n$, a system of equations for $(\varphi, u_j, j=0,1,\dots,n)$, where u_n will satisfy the (original) pde. This system of equations will, in general (depending on the values of the resonances), be overdetermined. Upon solving this system, it is found, for those equations considered, the φ satisfies an equation formulated in terms of Schwarzian derivatives[3]:

$$\{\varphi;x\}=\frac{\partial}{\partial x}\left(\frac{\varphi_{xx}}{\varphi_x}\right)-\frac{1}{2}\left(\frac{\varphi_{xx}}{\varphi_x}\right)^2. \tag{1.4}$$

This equation, or system of equations, we regard as a type of modified equation. By the invariance of (1.4) under the Moebius group,

$$\varphi=(a\psi+b)/(c\psi+d),\quad \{\varphi;x\}=\{\psi;x\}\,, \tag{1.5}$$

the "modified" equations allow the Bäcklund transformation (1.5).

The above procedure may now be reapplied to the "modified" (or equivalent) equations to find different forms of Bäcklund transformations. These Bäcklund transformations may take the form of discrete symmetries,[1,5,6] reductions,[1] or, as we shall see, more complicated structures. The group of Bäcklund transformations for the modified equations may be conveniently employed to iteratively construct sequences of rational solutions. Also, by linearizing the Miura transformation from modified to original equation we propose to calculate the Lax pair.[1,6]

In this paper we consider the Kadomtsev–Petviashvili

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Bäcklund transformation and the Painlevé property

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ABSTRACT. When a differential equation possesses the Painlevé property it is possible (for specific equations) to define a Bäcklund transformation (by truncating an expansion about the "singular" manifold at the constant level term). From the Bäcklund transformation it is then possible to derive the Lax-Pair, modified equations and Miura transformations associated with the "completely integrable" system under consideration. In this paper we consider completely integrable systems for which Bäcklund transformations (as defined above) may not be directly defined. These systems are of two classes.

The first class consists of equations of Toda-Lattice type (e.g. Sine-Gordon, Bullough-Dodd equations). We find that these equations can be realised as the "minus-one" equation of sequences of integrable systems. Although the "Bäcklund transformation" may or may not exist for the "minus-one" equation, we show, for specific sequences, that the Bäcklund transformation does exist for the "positive" equations of the sequence. This, in turn, allows the derivation of Lax-Pairs and the recursion operation for the entire sequence.

The second class of equations consists of sequences of "Harry Dym" type. These equations have branch point singularities and, thus, do not directly possess the Painlevé property. Yet, by a process similar to the "uniformization" of algebraic curves, their solutions may be parametrically" represented by "meromorphic" functions. For specific systems, this is shown to provide a natural extension of the Painlevé property.

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Table of Contents

Periodic fixed points of Bäcklund transformations and the Korteweg-de Vries equation

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Abstract

We present a new method for studying integrable systems based on the "periodic fixed points" of Bäcklund transformations. Normally, the BT maps an "old" solution into a "new" solution and requires a known "seed" solution to get started. Besides this limitation, it can also be difficult to qualitatively classify the result of applying the BT several times to a known solution. By studying the periodic fixed points of the BT (regarded as a nonlinear map in a function space), we obtain integrable systems of equations of finite degree (equal to the order of the fixed point), and a method for the systematic classification of the solutions of the original system.

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