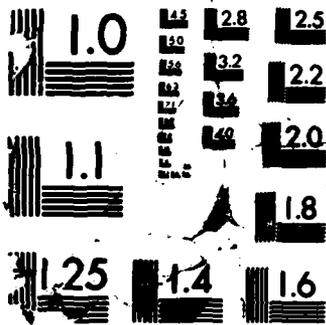


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INTERFACE EFFECTS ON ATTENUATION
AND PHASE VELOCITIES IN COMPOSITES

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ABSTRACT

Much current practical interest exists concerning wave propagation through a composite medium with a random distribution of inclusions: particles, flakes, long continuous or chopped fibers in a homogeneous matrix. Several theoretical studies report wave speeds and attenuation of coherent plane waves propagating through an elastic homogeneous medium containing reinforcing particles or fibers. All these studies assume that the interface between the matrix and the inclusion is sharply defined. Also, it is mostly assumed that the inclusion is perfectly bonded to the matrix.

In the present study, we analyze the problem of damping in metal-matrix composites when there is an interface layer through which the inclusion property changes continuously to that of the matrix.

INTRODUCTION

Determination of effective elastic moduli and damping properties of a heterogeneous material by using elastic waves (propagating or standing) is very effective. Several theoretical studies show that for long wavelengths one can calculate the effective wave speeds of plane longitudinal and shear waves through a composite material. At long wavelengths the wave speeds thus calculated are non-dispersive and hence provide the values for the static effective elastic properties. References to some of the recent theoretical and experimental works can be found in [1-12]. The scattering formulations developed in [1-8] provide a means to obtain not only the effective wave speeds but also the damping of wave amplitudes due to scattering.

In this paper we present results of some of our recent investigations of phase velocity and attenuation of plane longitudinal and shear waves in a medium with inclusions with interfacial layers. We use a wave-scattering approach together with Lax's quasicrystalline approximation to predict the phase velocity of either a longitudinal or a shear wave propagating through the medium. The scattering approach leads also to an estimation of attenuation.

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MULTIPLE SCATTERING BY A DISTRIBUTION OF SPHERICAL INCLUSIONS AT LOW FREQUENCIES

In this section we present expressions for the phase velocities and attenuation of longitudinal and shear waves in a medium containing a distribution of spherical inclusions with thin interface layers through which the elastic properties vary rapidly, but continuously, from those of the inclusions to those of the matrix. Such interface layers are often present due to processing [13,14].

In order to use the scattering approach it is necessary to calculate the scattered field due to a single spherical inclusion with an interface layer (Fig. 1).

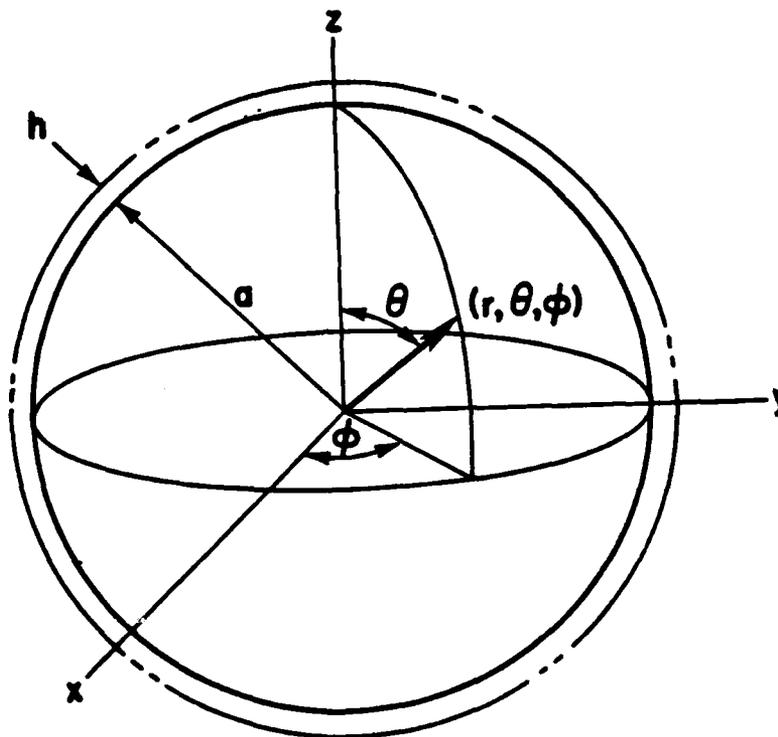


Fig. 1. Spherical inclusion with interface layer

Consider a spherical inclusion of radius a and elastic properties, λ_1, μ_1, ρ_1 embedded in an elastic matrix of material properties λ_2, μ_2 and ρ_2 . Also, let the inclusion be separated from the matrix by a thin layer of uniform thickness ($h \ll a$) but variable properties $\lambda(r), \mu(r)$, and $\rho(r)$. Here, λ, μ denote Lamé constants and ρ the density. Let $\lambda(r), \mu(r)$ be expressed as

$$\lambda(r) + 2\mu(r) = (\lambda_1' + 2\mu_1') f(r), \quad a < r < a + h \quad (1)$$

$$\mu(r) = \mu_1' g(r), \quad a < r < a + h \quad (2)$$

$f(r)$ and $g(r)$ are general functions of r . A special case arises when

$$f(a) = \frac{\lambda_1 + 2\mu_1}{\lambda_1' + 2\mu_1'}, \quad g(a) = \frac{\mu_1}{\mu_1'}$$

$$f(a+h) = \frac{\lambda_2 + 2\mu_2}{\lambda_1' + 2\mu_1'} , \quad g(a+h) = \frac{\mu_2}{\mu_1'} \quad (3)$$

with the stipulation that $f(r)$ and $g(r)$ with their first derivatives are continuous in $(a, a+h)$. Since h is assumed to be much smaller than a , it follows from (3) that $f'(a)$ can be approximated by

$$f'(a) = \frac{(\lambda_2 + 2\mu_2) - (\lambda_1 + 2\mu_1)}{h(\lambda_1' + 2\mu_1')} \quad (4)$$

$$g'(a) = \frac{\mu_2 - \mu_1}{h\mu_1'}$$

Note that λ_1' and μ_1' are Lamé constants of the interface material at some value of r ($a < r < a+h$), say at $r = a+h/2$.

Another special case would be that the interface material possesses constant properties. Then we have

$$f(r) = g(r) = 1, \quad a < r < a+h \quad (5)$$

We also make the assumption that h is very much smaller than the wavelength of the propagating wave. Then, to first order in h/λ , λ being the wavelength,

$$\begin{aligned} \tau_{rr}^t &= \tau_{rr}^s + \tau_{rr}^i, & \tau_{r\theta}^t &= \tau_{r\theta}^s + \tau_{r\theta}^i \\ \tau_{r\phi}^t &= \tau_{r\phi}^s + \tau_{r\phi}^i \end{aligned} \quad (6)$$

Here τ_{ij} is the stress tensor and superscripts t , s , and i denote the transmitted, scattered, and incident quantities, respectively. Note that τ_{ij}^t , τ_{ij}^s , and τ_{ij}^i , appearing above are calculated at $r=a$. The spherical polar coordinates r, θ, ϕ are defined in Fig. 1. Boundary conditions (6) express the fact that, to first order in h/λ , the traction components do not suffer any jump across the layer. However, the displacement components suffer jumps. Two parameters that characterize interface are:

$$K_1 = \int_0^1 \frac{dx}{f(a+hx)}, \quad K_2 = \int_0^1 \frac{dx}{g(a+hx)} \quad (7)$$

Using equations (3) and (4) in (7), it can be shown that K_1 and K_2 are given approximately by

$$K_1 = \frac{\lambda_1' + 2\mu_1'}{\lambda_2 + 2\mu_2 - (\lambda_1 + 2\mu_1)} \ln \left(1 + \frac{\lambda_2 + 2\mu_2 - (\lambda_1 + 2\mu_1)}{\lambda_1 + 2\mu_1} \right) \quad (8)$$

$$K_2 = \frac{\mu_1'}{\mu_2 - \mu_1} \ln \left(1 + \frac{\mu_2 - \mu_1}{\mu_1} \right) \quad (9)$$

On the other hand, if eq. (5) is used, then

$$K_1 = K_2 = 1 \quad (10)$$

Mal and Bose [2] studied a problem similar to the one considered here. They assumed a thin viscous third layer between the sphere and the matrix

and imposed the condition of continuity of radial displacement.

The incident wave will be assumed to be either a plane longitudinal wave propagating in the positive z-direction or a plane shear wave polarized in the x-direction and propagating in the positive z-direction. Thus,

$$u^i = e^{ik_1 z} e_z + e^{ik_2 z} e_x \quad (11)$$

where $k_1 = \omega/c_1$ and $k_2 = \omega/c_2$. ω denotes the circular frequency of the wave and c_1, c_2 denote the longitudinal and shear wave speeds in the matrix. The factor $e^{-i\omega t}$ has been suppressed.

The scattered and transmitted fields can be written as

$$u^s = \sum_{n=0}^{\infty} \sum_{m=-1}^1 [A_{mn} L_{mn}^{(3)} \delta_{m0} + B_{mn} M_{mn}^{(3)} + C_{mn} N_{mn}^{(3)}] \quad (12)$$

$$u^t = \sum_{n=0}^{\infty} \sum_{m=-1}^1 [A'_{mn} L_{mn}^{(1)} \delta_{m0} + B'_{mn} M_{mn}^{(1)} + C'_{mn} N_{mn}^{(1)}] \quad (13)$$

where the prime denotes that k_1 and k_2 are to be replaced by k_1' ($=\omega/c_1'$), and k_2' ($=\omega/c_2'$), respectively. c_1' and c_2' are the wave speeds in the inclusion. $L_{mn}^{(3)}$, $M_{mn}^{(3)}$ and $N_{mn}^{(3)}$ are defined in [15].

The constants A, B, C, A', B', C' are found by the use of conditions (6). For details, see [15].

Once A, C, and B are determined, the scattered field is then found from equation (12). Since the expressions for the field inside the inclusion will not be needed for the derivation of the dispersion equation governing the effective wave number of plane-wave propagation through the composite medium, we do not give these here. In the following we present equations governing propagation of effective plane waves through a medium composed of a random homogeneous distribution of identical spherical inclusions surrounded by the layers as discussed above.

To derive approximately the phase velocities of plane waves moving through the composite medium, we assume that wavelengths are long compared to the radius of each inclusion. In this long-wavelength limit it can be shown that, correct to $O(\epsilon^3)$,

$$A_{mn} = i\epsilon^3 [P_n \phi_{mn} + Q_n X_{mn}] \quad (14)$$

$$C_{mn} = i\tau^3 \epsilon^3 [R_n \phi_{mn} + S_n X_{mn}] \quad (15)$$

Expressions for P_n , Q_n , R_n , and S_n are defined in [15], where ϕ_{mn} and X_{mn} are also given.

Once the scattered field due to a single inclusion is known, multiple scattering due to a number of inclusions can easily be calculated. In particular, following the steps discussed before by us [8] it can be shown that effective speeds of propagation of plane longitudinal and shear waves are given by

$$\frac{k_1^{*2}}{k_1^2} = \frac{(1 + 9cP_1)(1 + 3cP_0) \left\{1 + \frac{3c}{2} P_2 (2 + 3\tau^2)\right\}}{1 - 15cP_2 (1 + 3cP_0) + \frac{3c}{2} P_2 (2 + 3\tau^2)} \quad (16)$$

$$\frac{k_2^{*2}}{k_2^2} = \frac{(1 + 9cP_1) \left\{1 + \frac{3c}{2} P_2 (2 + 3\tau^2)\right\}}{1 + \frac{3c}{4} P_2 (4 - 9\tau^2)} \quad (17)$$

$$k_1^* = \omega/c_1^*, \quad k_2^* = \omega/c_2^*$$

c_1^* and c_2^* are the effective wave speeds of plane longitudinal and shear waves, respectively. c is the volume concentration of inclusions in the matrix.

The attenuation caused by scattering (to this order of approximation) can also be calculated using equations (12) and (15). We find that the attenuation coefficients α_p and α_s are given by

$$\frac{\alpha_p}{k_1} = 3c\varepsilon^3 \left[P_0^2 + 3P_1^2 (1 + 2\tau^3) + 5P_2^2 (1 + 3\tau^5/2) \right] \quad (18)$$

$$= Q_p^{-1}$$

$$\frac{\alpha_s}{k_2} = 3c\varepsilon^3 \left[3P_1^2 (1 + \frac{1}{2}\tau^3) + \frac{15}{4} \tau^2 P_2^2 (1 + 3\tau^5/2) \right] \quad (19)$$

$$= Q_s^{-1}$$

It may be noted that good approximations to the attenuation coefficients are obtained if P_0 , P_1 and P_2 are calculated after replacing the matrix properties by the effective composite properties given by Eqs. (16) and (17).

SCATTERING CROSS SECTIONS OF A SPHERICAL INCLUSION WITH INTERFACE LAYER AT ARBITRARY FREQUENCY

Since attenuation coefficients are directly related to the scattering cross sections, in this section we present some results for scattering cross sections at finite frequencies of a sphere with interface layer.

From equation (12) the scattered field at a large distance from the sphere is given by

$$\underline{u}^{(s)} = \underline{u}^p + \underline{u}^s \quad (20)$$

where superscripts p and s denote longitudinal and shear wave components, respectively. It is easily shown that

$$\underline{u}^p = g(\theta) \frac{e^{ik_1 r}}{r} \underline{e}_r \quad (21)$$

$$\underline{u}^s = h(\theta) \frac{e^{ik_2 r}}{r} \underline{e}_\theta \quad (22)$$

where, for incident longitudinal wave corresponding to the first term in equation (11),

$$g(\theta) = \frac{1}{k_1} \sum_{n=0}^{\infty} (-i)^n A_{on}^p P_n(\cos\theta) \quad (23)$$

$$h(\theta) = \frac{1}{k_2} \sum_{n=0}^{\infty} (-i)^n C_{on}^p \frac{dP_n}{d\theta}$$

The scattering cross section is then (24)

$$\Sigma_p = \frac{4\pi}{k_1} \text{Im} \left\{ \frac{1}{k_1} \sum_{n=0}^{\infty} (-i)^n A_{on}^p \right\}$$

For incident shear wave given by the second term in Eq. (11) we get

$$\Sigma_s = \frac{4\pi}{k_2} \text{Im} \left\{ \frac{1}{k_2} \sum_{n=0}^{\infty} (-i)^n \frac{n(n+1)}{2} (C_n^s - iB_n^s) \right\} \quad (25)$$

In writing (25) we have rewritten (16) as

$$u^s = \sum_{n=0}^{\infty} [A_n^s L_{eln}^{(3)} + B_n^s M_{o1n}^{(3)} + C_n^s N_{eln}^{(3)}] \quad (26)$$

Here subscripts e and o refer to even and odd spherical wave functions (see [16]).

The coefficients appearing in (23) and (26) were calculated when the interface layer was very small ($h/a \ll 1$), and K_1 and K_2 were given by Eqs. (8) and (9). Scattering cross sections then were calculated by using equations (26) and (27). These results are discussed in the next section.

NUMERICAL RESULTS AND DISCUSSION

Attenuation coefficients for two composite materials were calculated using the procedure outlined in the previous two sections. The first composite was a SiC/Al material which was studied in detail in [8]. The constituent properties are given in Table 1. The other material was a lead/epoxy composite discussed in [17]. Table 1 lists the properties of the constituents.

Table 1. Constituent Properties of SiC/Al and Lead/Epoxy Composites

Constituents	ρ (g/cm ³)	μ (GPa)	$\lambda+2\mu$ (GPa)
SiC	3.181	188.1	474.2
Al	2.706	26.7	110.5
Lead	11.300	8.35	55.46
Epoxy	1.202	1.71	8.36

Figures 2-5 show the variations α_p/k_1 and α_s/k_2 with volume concentration of inclusions at different frequencies. Also shown in these figures is the effect of a thin interface layer with K_1 and K_2 given by Eqs. (8) and (9). For SiC/Al composite Figs. 2 and 3 show that the effect of the interface is to decrease the attenuation. This is perhaps understandable because the interface is not a sharp discontinuity. However, as shown in the case of Lead/Epoxy composite this is not always the case. These figures also show that attenuation reaches a maximum at higher and higher concentrations as the frequency increases. Behavior of attenuation vs.

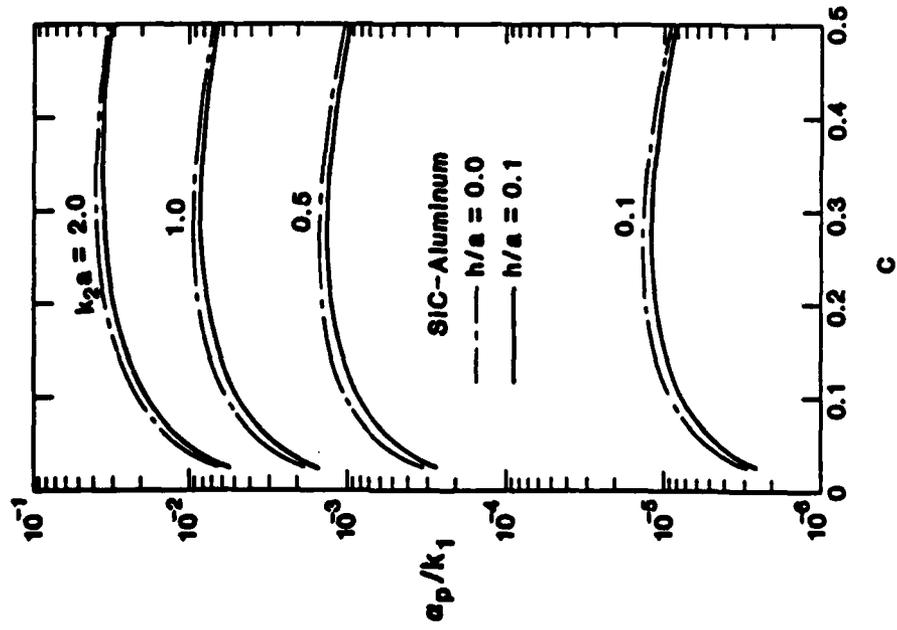


Fig. 2. Attenuation of P wave in SiC/Al Composite

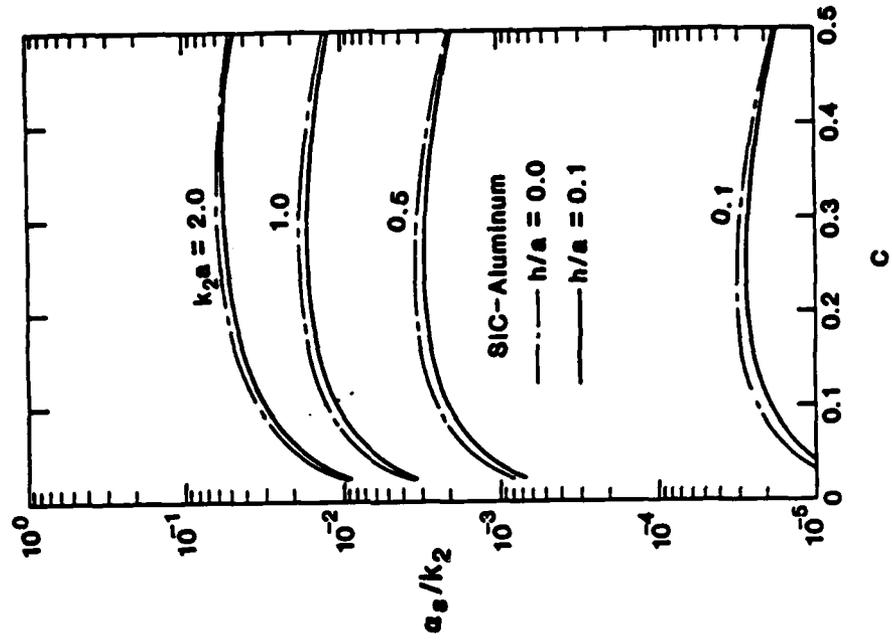


Fig. 3. Attenuation of S wave in SiC/Al Composite

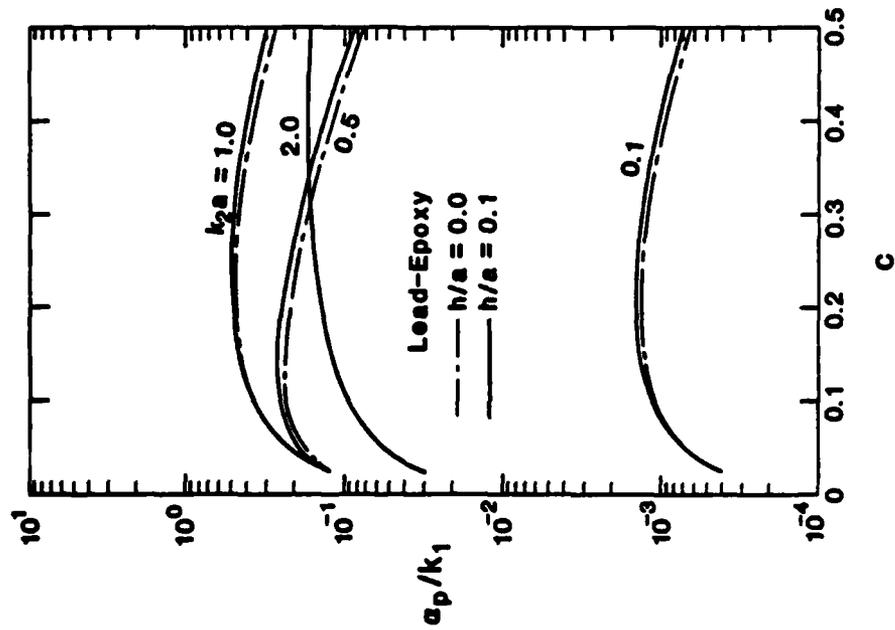


Fig. 4. Attenuation of P wave in Lead/Epoxy Composite

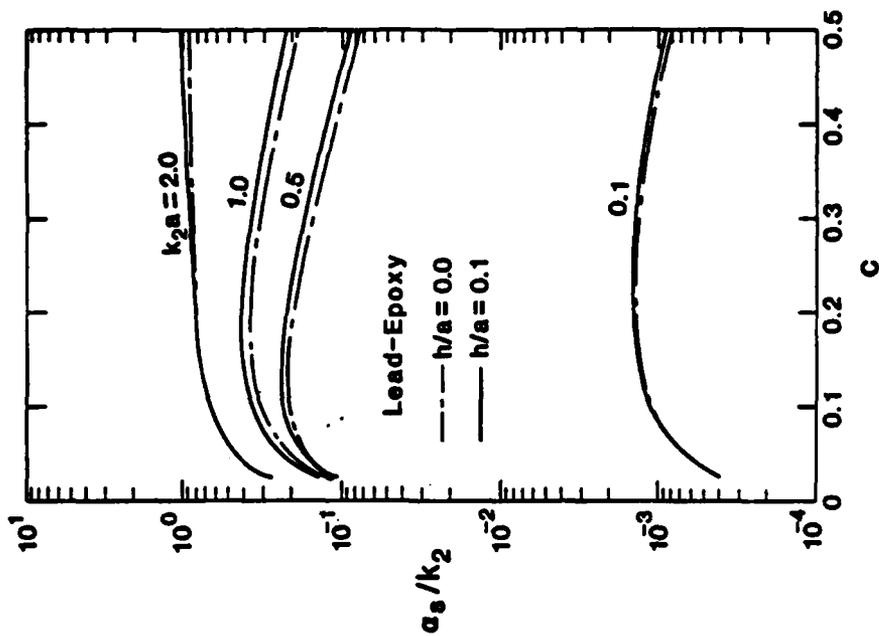


Fig. 5. Attenuation of S wave in Lead/Epoxy Composite

concentration in Lead/Epoxy composite is markedly different from the SiC/Al material. It is seen that the interface generally increases the attenuation. The peak attenuation is reached at concentrations that first increase with frequency, but then decrease and increase again at high frequencies. Figure 6 shows the P-wave attenuation vs. frequency at different concentrations. This agrees well with the results obtained in [7] using a different approach.

In conclusion, it may be stated that attenuation in a particulate composite depends crucially on the constituents and the nature of interface layers. These layers can enhance or diminish attenuation. More model studies are needed to characterize attenuation in composites with thick interface layers. This work is in progress and will be reported later.

ACKNOWLEDGMENT

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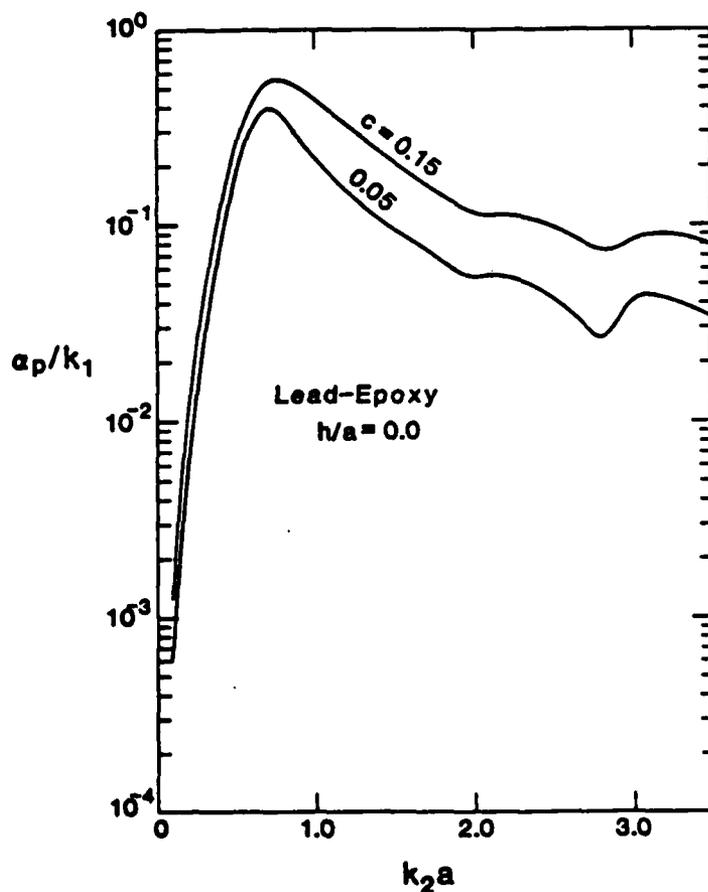


Fig. 6. Attenuation of P wave in Lead/Epoxy Composite.

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