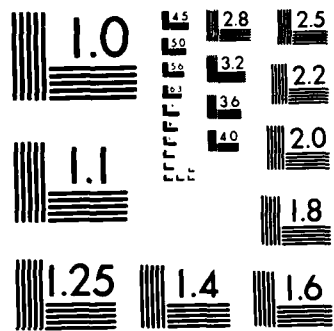MICROCOPY RESOLUTION TEST CHART

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NOTES ON THE AFSC RATE ANALYSIS TOOL AND ESTIMATOR: DEFINITIONS, ASSUMPTIONS, AND MODELING ISSUES

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August 1986

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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION	1b. RESTRICTIVE MARKINGS
UNCLASSIFIED	

2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT
	Approved for Public Release,
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	Distribution Unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION
Univ of Cincinnati and Louisiana State Univ		Air Force Business Research Mgt Center

6c. ADDRESS (City, State and ZIP Code)		7b. ADDRESS (City, State and ZIP Code)
Dept of Quantitative Analysis & Info Sys Cincinnati OH 45221-0130	Dept of Quantitative Business Analysis Baton Rouge LA 70803	AFBRMC/RDCB Wright-Patterson AFB OH 45433-6583

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AF Ofc of Scientific Research	AFOSR/XOO	F49620-85-C-0013

8c. ADDRESS (City, State and ZIP Code)	10. SOURCE OF FUNDING NOS.			
AFOSR/XOO Bolling AFB DC 20332-6600	PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
	UNKNOWN			

11. TITLE (Include Security Classification)
See BLOCK 19 BELOW

12. PERSONAL AUTHOR(S)
JEFFREY D. CAMM and THOMAS R. GULLEDGE, JR.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Final	FROM JUN 86 TO AUG 86	86 AUG	19

16. SUPPLEMENTARY NOTATION

17.	COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	Cost Analysis, Cost Model, Rate Analysis Tool and Estimator, AFSC-RATE, Analysis Tool

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

TITLE: Notes on the AFSC Rate Analysis Tool and Estimator: Definitions, Assumptions, and Modeling Issues

ABSTRACT: This paper presents a clarification of the terminology, assumptions, and modeling issues associated with the Air Force Systems Command Rate Analysis Tool and Estimator (AFSC-RATE). The primary purpose of the paper is to dispel several myths that have appeared in subsequent evaluations of the model.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
Linda K. McLaughlin	513-255-6221	AFBRMC/RDCB

DD FORM 1473, 83 APR   EDITION OF 1 JAN 73 IS OBSOLETE.

SECURITY CLASSIFICATION OF THIS PAGE

The purpose of this report is to dispel several of the myths surrounding the Air Force Systems Command Rate Analysis Tool and Estimator (AFSC-RATE). This is not a criticism of model performance, but a more precise statement of model terminology, assumptions, and other issues related to model construction.

1.0 Definitions

The primary relationship in the AFSC-RATE model is presented in [2] as

$$z = AX^B Y^C \tag{1}$$

where

z = the unit cost of the Xth item produced,

A = a constant referred to as the surface initialization point,

X = cumulative quantity produced,

B = an exponent which describes the slope of the quantity/cost curve,

Y = the production rate in effect,

C = exponent which describes the slope of the rate/cost curve.

After examing the literature it seems appropriate to note the inconsistencies in the variable definitions associated with equation (1). First consider the definition of unit cost. Smith [20, p. 37] measured cost in direct labor hour units; i.e., z = the average number of direct labor hours required to manufacture each pound of airframe. The reason for this assumption is that the different components of cost do not follow the same learning curve. This result was noted by Asher [1, p. 111]. As Asher notes, an approach that is often used is to exclude all non-recurring costs and construct a composite curve using only recurring costs. Asher [1, p. 111] also notes that "if the curves for the various elements of airframe cost differ in slope from one another, then a linear curve may not be an accurate representation of the composite curve." It does not seem reasonable to expect that airframe,

propulsion, electronics, armament, and other co. 's should follow the same learning curve, but the magnitude of the error from using a composite curve is system dependent. Smith avoided the problem by examining direct labor requirements.

Bemis [6, 7] also uses equation (1) to predict unit costs for several weapon systems programs. In these papers, z is defined as unit cost; but the exact definition of unit cost is unclear. The discussion [6, p. 85] leads one to believe that the model is used to predict system unit costs. If this is the case, the analysis is probably incorrect; that is, there is no reason to expect all units of total cost to follow the same learning curve.

Cox and Gansler [12] in their study of quantity, rate and competition, estimate unit cost as "cost to the government, which corresponds to price in classical economics." However, price includes the allocation of fixed overhead. While the fixed overhead cost per unit may decline as the number of units increases, there is no reason for that portion of cost to follow a learning curve. The allocation of overhead varies from lot to lot, which can cause upward or downward shifts in price in the presence of learning on the variable costs.

Apparently, the AFSC-RATE model projects costs at the unit recurring flyaway level. This is not correct if the variable portion of indirect costs is not included and the fixed portion is not excluded, but it is difficult to assess the error implications. Clearly, the error is dependent upon the amount of fixed indirect costs on the program of interest. Balut [3] is correct in noting that only the variable part of costs should be projected with the mathematical model. This would involve a regrouping of costs. The variable part of overhead cost should be separated from the fixed portion, and costs should be classified as fixed and variable. The variable costs should be modeled as a function of program variables, and the fixed cost should be

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considered separately. Even this approach is an approximation since this assumes that all of variable costs can be modeled with the same relationship. A more appropriate formulation would require modeling each component of variable cost (e.g., material, labor, etc.) with a separate relationship, and summing the projections to obtain an estimate for total variable costs. Note that variable costs as defined here contain direct costs plus the portion of indirect costs that are variable. A discussion of the cost separation procedure is presented in [4] and [5].

Much of the controversy surrounding the use of regression models [e.g., equation (1)] is a direct result of the use of a different dependent variable, z, from one study to another. One should expect conflicting results when in some studies z is direct man hours, in others total variable cost, in others total recurring cost, and in others price. Each of the quantities is very different. The point of this discussion is that researchers should be explicit when defining unit costs, and much additional research is needed in the area of cost aggregation.

A second definitional concern is related to the definition of production rate. Since data is not usually compiled on annual production rate, annual procurement rate is often substituted as a proxy. That is, the variable Y in equation (1) is set equal to the annual procurement lot quantity. Again, the magnitude of this error is difficult to assess since it is program dependent. However, we believe the error can be significant for those programs with long production profiles such as aircraft. More precisely, aircraft that are pro-cured in a given year are usually produced over a period of four to six years. To obtain a measure of annual production rate, the procurement lot quantity must be allocated to the years in which the actual production occurred. Smith [20, p. 41] used what he called the "lot average manufacturing rate"; the number of airframes in a production lot divided by the production time span.

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This assumes a uniform rate distribution, a result that is inconsistent with our knowledge of the production process. Still, this approach is probably better than using lot size as a proxy for production rate.

It is not clear to us how production rate is measured in the AFSC-RATE model. Production rate is not clearly defined in [2] or [8], but in [13, p. 4-13] it is stated that "lot size was used as a proxy for production rate with the exception of a few instances where it was known the quantity was produced over more than one year." No explanation is provided for the programs with production periods that exceed a year. In [8; pp. 4-5 to 4-6], the discussion leads one to believe that annual procurement lot quantity is being used as a proxy for production rate. This is confirmed in Bolton's [9] thesis where he examines equation (1). He states "the only thing it (equation 1) requires which the standard learning curve formulation does not is a production rate and it is convenient to use readily available buy schedules as a proxy for this."

We only note that this problem has been addressed [4], and research is continuing in this area. Our current research is concerned with assessing the magnitude of error introduced by the "lot quantity" proxy. Intuitively, we expect the error to be large if annual procurement quantities are fluctuating and the production period is lengthy.

We also note that the choice of an equivalent production rate measure also influences the measure of cumulative production. Cumulative annual procurement quantity is not appropriate. For example, suppose the 1986 buy quantity for some system is 400 and the production period for these 400 units is four years. Cumulative production through 1986 cannot include all of these 400 units since all 400 units have not been produced at the end of that procurement year. Some units will be produced over the remainder of the four year production period.

2.0 Parameter Estimation

It has been asserted that a major advantage of the AFSC-RATE model is that more accurate results are achieved because the model's parameters are estimated by nonlinear techniques [2, 8]. In addition, it has also been asserted that the nonlinear approach reduces the multicollinearity between cumulative output and output rate. These issues are addressed in this section.

The accuracy question only indirectly relates to the estimation technique. The problem follows from the fact that the estimation of the parameters in equation (1) is complicated by the fact that data is not available by production unit. Data is only available by procurement lot. If unit data were available, equation (1) could be estimated by linear or nonlinear regression with comparable results [17].

The problem with lot data is really not a problem of the log linear formulation. It is indeed a data problem, a problem that follows from not knowing the true lot midpoints without first knowing the learning curve slope. This problem is discussed in [14].

The AFSC-RATE model avoids the estimated midpoint bias by using the "Boeing" approximation presented in [1, pp. 35-36]. This is an approximation for total cost when the unit variable cost function is a learning curve. In the AFSC-RATE model, total cost is approximated with the following integral:

$$TLC_i = \int_{N+.5}^{N+K_i+.5} AX^B Y^C \, dx \tag{2}$$

where N = cumulative lot quantity, K_i = the number of units of lot i, and TLC_i is the total cost of lot i.

For the integration of equation (2) it is assumed that Y is constant. The resulting expression is

$$TLC_i = \frac{A}{B+1} [(N + K_i + .5)^{B+1} - (N + .5)^{B+1}]Y^C. \qquad (3)$$

This result (equation (3)) is nothing but the area under a learning curve; a learning curve with slope parameter B and first unit cost AY^C. That is, the AFSC-RATE approximation assumes that production rate proxy is constant, however, for the parameter estimation in equation (3), production rate proxy is allowed to vary.

This approximation is difficult to interpret when you consider the fact that the time required to produce a lot of aircraft may exceed four years. Since equation (3) assumes that production rate is constant over the time required to produce the lot, this implies production rate is constant over a number of years. Since the variable Y changes from year to year, an inconsistency results.

Bolton [9], apparently following earlier work by TASC [8], used the following approximation of total lot cost:

$$TLC_i = \int_{N}^{N+K_i} AX^B Y^C dX = \frac{A}{B+1} [(N+K_i)^{B+1} - N^{B+1}]Y^C. \qquad (4)$$

Unfortunately, this can be a very poor approximation. An error analysis of this unit learning curve approximation (in the absence of the production rate effect) can be found in [11]. Both of the integral approximations used in equations (2) and (4) exceed actual cost. However, the perturbed approximation used in equation (3) is a much better approximation than that used in equation (4). It is clear from Figures 1 and 2 why this is the case. In Figure 1, the shaded area represents overestimation of the actual lot cost (sum of the rectangles). The perturbed approximation is shown in Figure 2. It performs better than the unperturbed approximation because, as shown in Figure 2, the perturbed integral approximation contains some overestimation
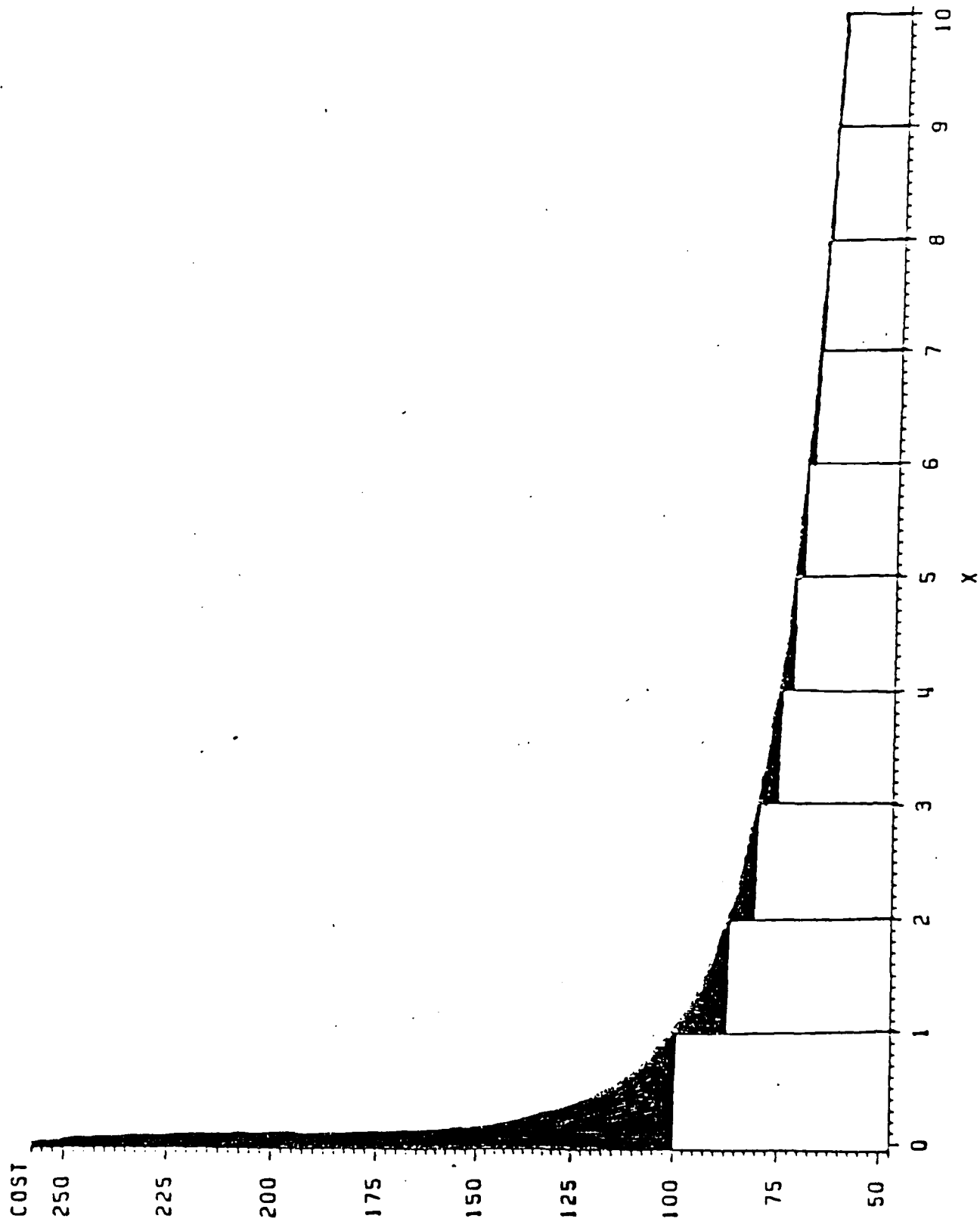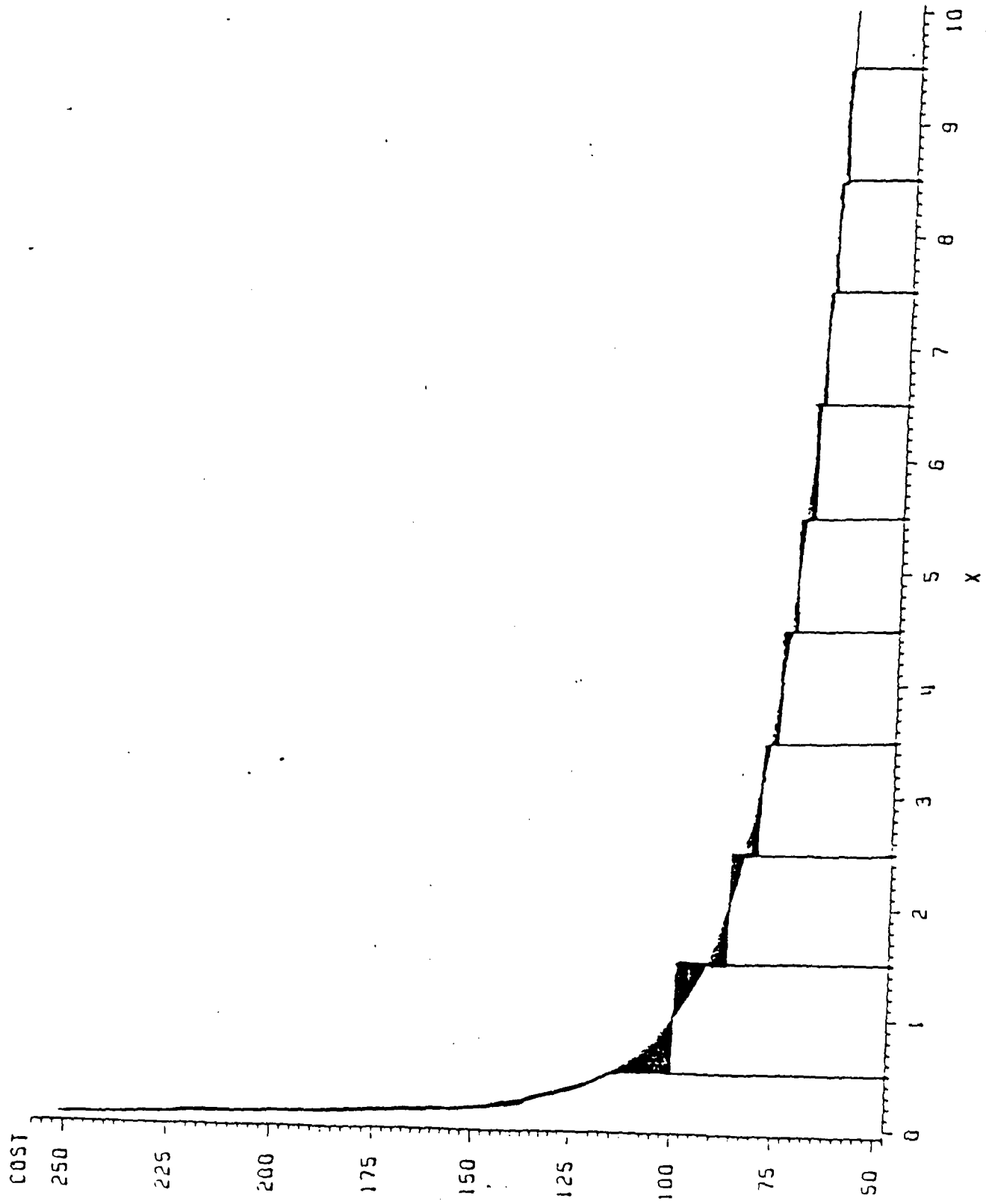
Figure 1.   The Unperturbed Approximation.

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Figure 2.   The Perturbed Approximation.

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and some underestimation for each unit in the lot. The underestimation for each unit cancels some of the overestimation yielding a net overestimation of total lot cost which is smaller than that obtained from the unperturbed approximation. In the worst case analysis presented in [11], the perturbed approximation overestimated cost by 3% while the unperturbed approximation in the same case overestimated by 290%! The degree of overestimation of the total lot cost is a function of where the lot occurs on the learning curve and the slope of the learning curve. For smaller lots early in production and steep learning curves, the unperturbed approximation can induce serious error. The reader is referred to [11] for a more detailed discussion of this error.

The functional form of the approximation used in the nonlinear least squares model can have a serious impact on the parameter estimates as shown below. Lot data for the C141 airframe program given in [21] is used to illustrate this sensitivity. We estimate the parameters of the learning curve (in the absence of the rate effect) using three different models: a direct model using the summation over all units in the lot, a model using the unperturbed integral approximation of lot cost and a model using the perturbed approximation of total lot cost:

$$\text{Model 1:} \quad \text{Min} \sum_i \left(y_i - \sum_{X=N+1}^{N+K_i} AX^B \right)^2 \tag{5}$$

$$\text{Model 2:} \quad \text{Min} \sum_i \left(y_i - \frac{A}{1+B} \left[(N+K_i)^{B+1} - N^{B+1} \right] \right)^2 \tag{6}$$

$$\text{Model 3:} \quad \text{Min} \sum_i \left\{ y_i - \frac{A}{1+B} \left[(N+K_i+.5)^{B+1} - (N+.5)^{B+1} \right] \right\}^2 \tag{7}$$

where y_i = direct labor hours for lot i and the remaining variables are as previously defined. The least squares results appear in Table 1.

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Model	$\hat{A}$	$\hat{B}$	Residual Sum of Squares (RSS) *
1	604,393	-.3897	492.2×10^9
2	521,025	-.3582	611.0×10^9
3	600,710	-.3884	492.2×10^9

*RSS were calculated after the estimation using the
estimated parameters and model 1.

Table 1

The results are exactly as anticipated given the results in [11]; that
is, the perturbed approximation is a good approximation of model 1, whereas
the unperturbed is a poor approximation. The unperturbed approximation
results in very different parameter estimates. The first unit cost, A, seems
to be particularly sensitive to the use of the unperturbed approximation used
in Model 2. This is not surprising considering Figure 1. The area under the
curve early on the learning curve contributes heavily to the overestimation of
total lot cost inherent in the approximation. Consequentially, the least
squares estimate for the first unit cost is adjusted downward to offset the
inherent overestimation of total lot cost. Furthermore, model 2 results in an
overall poorer fit to the data (the RSS increases by 24%, see Table 1).

Early TASC documents [8] use model 2 augmented for production rate,
whereas later work uses the augmented model based on model 3 [2]. Bolton [9]
apparently used an augmented model based on model 2. We suspect this may
account for his inability to duplicate TASC parameter estimates for some pro-
grams [9, p. 30].

It is not clear to us why an approximation has to be used at all. The
"true" model based on the unit learning curve is model 1. Model 1 is just as
easily augmented for production rate and programmed in SAS as model 2 or model

3. If an approximation is going to be used, it should be the perturbed approximation as presented in [2].

As for multicollinearity, it is difficult to see how an application of nonlinear least squares to equation (3) can solve the problem. We note that multicollinearity is not a problem related to statistical methodology; it is a data problem. It follows from the fact that variables that are highly correlated do not explain independent portions of the variability in the dependent variable. It is well know that for aircraft programs, production rate and cumulative quantity are usually highly correlated. Since both quantities occur as independent variables when equation (3) is used in least squares estimation, the multicollinearity is still present. In [2, p. 2-5] it is stated, "the problem of multicollinearity between cumulative quantity and production rate is avoided by the use of a nonlinear function; however, the dependency between the two variables is not assessed." The same statement is presented in [8, p. 4-5]. Also, Gardner [15] states that the issue would be considered in Bolton's [9] thesis, but we find only a statement of the problem in that work.

Our only concern with the multicollinearity is that it makes it impossible to assign a meaningful interpretation to the model's coefficients. We have addressed these problems in [10]. We are aware that if the independent variable correlation structure is the same in the prediction period as in the estimation period, the model may still provide satisfactory predictions. Of course this assumes that the joint range of the observed independent variables is not violated during the prediction period. However, in [10] we argue that these relevant ranges are probably violated.

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3.0 Hypothesis Testing

This topic is discussed since the issue is raised in several of the
documents supporting AFSC-RATE. In general there is little evidence of any
"goodness-of-fit" testing in any of the documents supporting AFSC-RATE. This
is probably appropriate since historical sample sizes for weapon system
procurement programs are usually small. Still, we clear the issue since the
avoidance of biased and consistent estimators is stated as an advantage of the
AFSC-RATE model (see, for example, [8, p. 2-4 and p. 4-2]).

The results follow from the works of Goldberger [16] and Moulenberg [19]
and apply to parameter estimation in multiplicative models, such as equation
(1). We demonstrate with a learning curve, but the same result applies to
general multiplicative functions. Assume the learning curve

$$z = AX^b \qquad (8)$$

where the variables are as previously defined. For parameter estimation in
equation (8), it is assumed that all nonquantifiable factors are contained in
a disturbance term, u, that satisfies the following assumptions:

$$E(u_i) = 0,$$

$$E(u_i^2) = \sigma^2 = \text{constant},$$

$$E(u_i u_j) = 0 \quad i \neq j,$$

$$E(X_i u_i) = 0.$$

For hypothesis testing, it is assumed that the random variable u_i are
independent and identically normally distributed. Equation (8) is usually
restated as

$$z = AX^b u, \qquad (9)$$

and logarithms of both sides are taken prior to estimation. The estimable
specification is

$$\ln z = \ln A + b \ln X + \epsilon, \tag{10}$$

where $\epsilon = \ln u$.

The parameters in equation (10) may be estimated by ordinary least squares.

If the parameters are estimated directly from equation (9), the functional form is

$$z = AX^b + u \tag{11}$$

The models in equations (9) and (11) are different because their error structures are different. If the normality assumption applies to both equations (9) and (11), then there is an inconsistency with equation (10); i.e. the disturbance term in equation (10) is lognormally distributed. Since all hypothesis tests are dependent on the normality assumption, usual tests of significance are affected.

In general, these issues are important, but we believe they are relatively unimportant in the AFSC-RATE model. There is no evidence of any hypothesis testing in any of the model documentation that we have observed, and the problem described above is only important if the model is subjected to hypothesis testing.

4.0 Alternative Formulation

In Bolton's [9] thesis, two additional production rate variation models are examined. One formulation is

$$Z = AX^B Y^C R^D \tag{12}$$

where $R = Y_i/Y_{i-1}$, D is the "production rate change parameter," and the other variables are as previously defined. Bolton states that the idea behind this model "is that the change in production rate from one lot to the next is as important in explaining the impact of production rate on cost as is the rate itself [9, p. 26]." We can only speculate on the motivation for this

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formulation. Our guess is that the model in equation (1) does not provide the proper response to changes in production rate proxy.

There are at least two reasons why equation (1) may not provide the proper response. The first reason is related to the collinearity that is typically observed in the data. Many researchers have estimated parameters in models similar to equation (1). In every case, cumulative quantity always explains most of the variation in the dependent variable, while rate has a small statistical impact. This does not mean that production rate is an unimportant variable; it is just impossible to separate the rate effect using regression analysis. Consequently, since the addition of the rate variable leads to a small reduction in the error sum of squares, the rate variable has a small impact on the forecast function.

A second reason why the model may be insensitive to the production rate proxy relates to the proxy itself. As previously mentioned, the estimation procedure assumes that production rate is constant over the procurement lot. Since a procurement lot is produced over a number of years, this assumption is equivalent to assuming that production rate is constant over a number of years. That is, the functional form for estimation assumes constant production rate over the lot and thus over time, while the data for the estimation is generated by programs where rate is varying.

Bolton's formulation [equation (12)] attempts to address these limitations by adding the change in production rate as an additional variable. If the ratio, R, is statistically significant it appears that the model is more responsive to rate changes. Unfortunately, the model in equation (12) is still plagued by the previously discussed problems, namely multicollinearity and the assumption of constant lot production rate. This observation raises

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an interesting question. Is it possible to construct an appropriate model within the format of equation (1)?

We do not really know the answer to the above question, but we do know there are methodological problems with the appraoch. The motivation for equation (12) was to modify a convenient model to accommodate the available procurement lot data. The correct approach requires constructing the theoretical model that actually generated the data, not to try to force some convenient available model on the data. Our current research is directed toward identifying a model that generates procurement lot data. That is, we believe that one of the problems with AFSC-RATE is that a simple model that was originally defined for explaining unit direct labor hours is being forced upon the more complex lot cost problem.

5.0 Conclusion

The purpose of the document is to clarify terminology and issues. We hope the paper generates discussion, but in all fairness we note two things. First, we have not made any attempt to quantify errors in predictability that follow from the issues discussed in this paper. In fact, it may be extremely difficult to quantify some of the issues. Also, our observations are only based on the documentation made available to us; the items referenced below.

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