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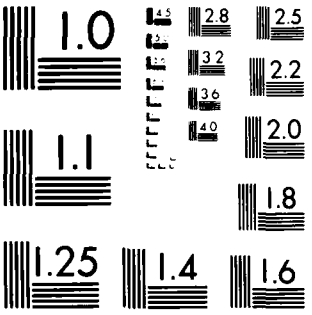

OPTIMAL REPLACEMENT AGE IN AN IMPERFECT INSPECTION MODEL

By

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ABSTRACT

A device is maintained under an age replacement policy. The status of the device (functioning or failed) is known only by inspection at some fixed interval k. With probability q, an inspection error may be made, and a functioning unit will be declared to have failed and be replaced by a new unit. On the contrary, when a failed unit is inspected, it is assumed that no inspection error will be made. Assuming that the cost of replacing a failed unit (actually failed or believed failed) is greater than the cost of replacing a functioning unit, we show that we can obtain an optimum replacement age which minimizes L(T), the long-run expected cost per unit of time. We find a lower bound for the optimal replacement age and obtain asymptotic and monotonicity properties for L(T).

1. INTRODUCTION

Most of the recent work on maintenance models takes into account additional real-world factors like "minimal" or "imperfect" repair actions. Another relevant factor, imperfect inspection, has not been considered extensively in the literature.

Brown and Proschan (1983) propose several different imperfect maintenance and imperfect inspection models in [3] and study properties of the distribution of the mean time between perfect repairs in [4]. Fontenot and Proschan (1984) then develop optimal policies for several maintenance models based on the imperfect repair model in [4].

The objective of this paper is to develop an optimal replacement policy for an imperfect inspection model. Derman and Sacks (1960) solved the problem of choosing an optimal replacement rule for deteriorating equipment when the amount of deterioration is observed periodically. Perfect inspection was assumed. Barlow, Hunter, and Proschan (1963) showed how to obtain optimum inspection schedules for a broad class of failure distributions in [1]. For an unknown failure distribution, Derman (1961) obtained the minimax schedule assuming that failure has probability $p > 0$ of being detected by an inspection, and in the absence of failure the unit is replaced at some specified time T. More recently, Taylor (1975), assuming a cumulative damage model for system failure, found an optimal replacement strategy that minimizes the long-run expected cost per unit of time.

In this paper we are concerned with a device subject to an age replacement policy. We assume that all repair (replacement) actions are perfect and repair time is negligible. The state (functioning or failed) of the device is determined by periodic inspection at a specified interval k. The failure of a unit remains undetected until the unit is actually inspected. Due to an inspection error

occurring with probability q, a functioning unit is declared to have failed and is replaced by a new unit. On the contrary, when a failed unit is inspected, no error in inspection occurs.

An inspection error may be due to human error or malfunction of a detection device. Inspection policies are frequently used for safety or security devices. One example is a fire extinguisher which is checked periodically. If the pressure gauge malfunctions, it may erroneously indicate the extinguisher is empty and so the extinguisher is unnecessarily replaced.

It is apparent that an age replacement policy is inappropriate if the underlying failure rate is decreasing. Thus we will assume that the failure rate $r(t) = f(t)[\bar{F}(t)]^{-1}$ is increasing. We assume that the life distribution F of the device is absolutely continuous with density f and $F(0) = 0$.

We use the notation $\bar{F} = 1 - F$, $\bar{F}_i = \bar{F}(i)$, and $r_i = r(i)$.

2. FORMULATION AND SOLUTION OF MODEL.

A device has life length X with distribution F. The device is installed at time 0, and is inspected at successive times k, 2k, ... When the device is functioning, the probability of no error in inspection is $p = 1 - q$. The device is replaced by a new unit at age T, which is a positive integer multiple of k, or at the first inspection following failure, whichever comes first. This process is continued indefinitely. Clearly the process renews itself at times of replacement.

The probability that the device is replaced after time $t = ik$ is $P[Y > ik] = \bar{F}_{ik} p^i$ for $i = 0, 1, 2, \ldots$, since the device must survive to time ik and pass i inspections. Thus Y is the observed life length of the device. Note that

$EY = k \sum_{i=0}^{\infty} \bar{F}_{ik} p^i$. Let Z be the elapsed time between replacements; then $Z = \min\{Y,T\}$.

Let c_1 be the cost of an unscheduled replacement (actual or believed failure) and c_2 be the cost of a scheduled (age) replacement. Assume $c_1 < c_2$. Our objective is to find the replacement age T which minimizes L(T), the long-run expected cost per unit of time. In Theorem 2.1 we give a formula for L(T) and in Theorem 2.2 we describe the optimal value of T. These results are analogous to those found by Fontenot and Proschan (1984) for a modified age replacement model with imperfect repair.

Theorem 2.1

$$L(T) = \frac{c_1 - (c_1-c_2)\bar{F}_{T-k} p^{\frac{T}{k}-1}}{k \sum_{i=0}^{\frac{T}{k}-1} \bar{F}_{ik} p^i} \quad .$$

Proof: The times of replacement are renewal times in a renewal-reward process whose interarrival distribution is that of $Z = \min\{Y,T\}$. From renewal theory, we have that

$$L(T) = \frac{C(T)}{D(T)} \quad ,$$

where C(T) is the expected cost per renewal cycle and D(T) is the expected duration of a renewal cycle. Denote the distribution function of Y by G. Then

$$\bar{G}(t) = P[Y>t] = \bar{F}_{(i-1)k} p^{i-1} \quad \text{for} \quad (i-1)k \le t < ik \quad \text{and } i = 1, 2, \ldots$$

Then the expected length of a cycle is

$$D(T) = E(Z) = \int_0^T \bar{G}(t)\,dt = \sum_{i=1}^{T/k} \int_{(i-1)k}^{ik} \bar{G}(t)\,dt$$

$$= \sum_{i=1}^{T/k} \bar{F}_{(i-1)k}\, p^{i-1} \int_{(i-1)k}^{ik} dt = k \sum_{i=0}^{\frac{T}{k}-1} \bar{F}_{ik}\, p^i .$$

With probability $\bar{F}_{T-k}\, p^{\frac{T}{k}-1}$, the device is replaced after time $T-k$ and a cost of c_2 is incurred. Otherwise, it is replaced at or before time $T-k$ and a cost of c_1 is incurred. Therefore the expected cost per cycle is

$$C(T) = c_1(1-\bar{F}_{T-k}\, p^{\frac{T}{k}-1}) + c_2\, \bar{F}_{T-k}\, p^{\frac{T}{k}-1} . \qquad \square$$

Theorem 2.2. In addition to the assumptions about F in Section 1, assume that r is differentiable. Then

a. $L'(T) = \bar{F}_{T-k}\, p^{\frac{T}{k}-1} [k \sum_{i=0}^{\frac{T}{k}-1} \bar{F}_{ik}\, p^i]^{-2} H(T)$, where

$$H(T) = (c_1-c_2)[(kr_{T-k} - \log p) \sum_{i=0}^{\frac{T}{k}-1} \bar{F}_{ik}\, p^i + \bar{F}_{T-k}\, p^{\frac{T}{k}-1}] - c_1.$$

b. An optimal replacement age T^* exists (T^* may be infinite) and $T^* = k$ if and only if $p \le \exp(\frac{c_2}{c_2-c_1} + kr_0)$.

c. T^* is finite if and only if $r_\infty > \frac{c_1}{(c_1-c_2)EY} + \frac{1}{k}\log p$.

d. $\frac{EY}{k} < \frac{c_1}{c_2}$ implies that $T^* < \infty$.

e. A lower bound for T^* is $\frac{c_2}{c_1} EY$.

Proof: a. Differentiate.

b. Given $c_1 > c_2$ note that H is increasing since r is increasing and

$$H'(T) = (c_1-c_2)\, k\, r'_{T-k} \sum_{i=0}^{\frac{T}{k}-1} \bar{F}_{ik} p^i .$$

Then $T^* = k \iff L$ is increasing $\iff H(k) \geq 0 \iff (c_1-c_2)(kr_0 - \log p + 1) - c_1 \geq 0$ $\iff p \leq \exp(\frac{c_2}{c_2-c_1} + kr_0)$.

If $p > \exp(\frac{c_2}{c_2-c_1} + kr_0)$, then $H(k) < 0$. Thus if we can find $T_0 = \min\{T: H(T) \geq 0\}$, then L' is negative on $(0, T_0-k)$ and positive on (T_0, ∞). So we choose T^* such that $L(T^*) = \min\{L(T_0-k), L(T_0)\}$. Otherwise, if there is no T such that $H(T) \geq 0$, then L is decreasing and $T^* = \infty$.

c. The inequality involving r_∞ implies $\lim_{T\to\infty} H(T) > 0$. Thus the equation $T_0 = \min\{T: H(T) \geq 0\}$ has a finite solution.

d. Note that $L(T^*) \leq L(k) = {}^{c_2}/_{k}$. If $T^* = \infty$, then $L(T^*) = {}^{c_1}/_{EY} \leq {}^{c_2}/_{k}$. By the contrapositive, ${}^{EY}/_{k} < {}^{c_1}/_{c_2}$ implies that $T^* < \infty$.

e. It is easy to show that $L(T) \geq \frac{c_2}{T}$. Thus (LT) must cross $L(\infty) = \frac{c_1}{EY}$ (if it crosses at all) to the right of the value at which $\frac{c_2}{T}$ crosses $\frac{c_1}{EY}$; this value is $T = \frac{c_2}{c_1} EY$. Therefore, we should schedule replacement at an age greater than $\frac{c_2}{c_1} EY$. □

3. Asymptotic and monotonicity properties of L(T).

If we let $p = 1$, we get a desired asymptotic result for L(T) as the length of the inspection interval, k, goes to zero. In this case our model reduces to the simple age replacement model for which the optimization problem was solved by Barlow and Proschan (1965), pp. 85 - 90. Since the denominator of L(T) is a

Rieman sum, it converges to the integral form given in Barlow and Proschan (1965) p. 88, if $\bar{F}$ is Riemann integrable.

$$\lim_{k\to 0} L(T) = \lim_{k\to 0} \frac{c_2 + (c_1-c_2)F_{T-k}}{k\sum_{i=0}^{\frac{T}{k}-1} \bar{F}_{ik}} = \frac{c_2 + (c_1-c_2)F_T}{\int_0^T \bar{F}(x)dx}.$$

If $0 < p < 1$ we will show that $L(T) \to \infty$ as $k \to 0$. This result is expected since more frequent inspections will lead to more units being erroneously replaced.

Case 1. If $T = k$, then $\lim_{k\to 0} L(k) = \lim_{k\to 0} \frac{c_2}{k} = \infty$.

Case 2. If $T = \infty$, then $\lim_{k\to 0} L(\infty) = \lim_{k\to 0} \frac{c_1}{EY} = \lim_{k\to 0} \frac{c_1}{k\sum_{i=0}^{\infty} \bar{F}_{ik}p^i} > \lim_{k\to 0} \frac{c_1}{k\sum_{i=0}^{\infty} p^i}$

$$= \lim_{k\to 0} \frac{c_1}{k\left(\frac{1}{1-p}\right)} = \infty.$$

Case 3. If $k < T < \infty$, then $\lim_{k\to 0} \frac{c_1 - (c_1-c_2)\bar{F}_{T-k}p^{\frac{T}{k}-1}}{k\sum_{i=0}^{\frac{T}{k}-1} \bar{F}_{ik}p^i} > \lim_{k\to 0} \frac{c_1 - (c_1-c_2)p^{\frac{T}{k}-1}}{k\sum_{i=0}^{\frac{T}{k}-1} p^i}$

$$= \lim_{k\to 0} \frac{c_1 - (c_1-c_2)p^{\frac{T}{k}-1}}{k\left(\frac{1-p^{T/k}}{1-p}\right)} = \infty.$$

For a particular value p_1 of p, we denote $L_{p_1}(T) = [c_1 - (c_1-c_2)\bar{F}_{T-k}p_1^{\frac{T}{k}-1}]$ $/k\sum_{i=0}^{\frac{T}{k}-1} \bar{F}_{ik}p_1^i$. Then it is easily seen that $Lp(T)$ increases as $p \to 0$ as follows.

a. Note that $\bar{F}_{ik}p_1^i \le \bar{F}_{ik}p_2^i$ for $p_1 < p_2$, $i = 0, 1, \ldots, \frac{T}{k} - 1$.

Thus $1/[k\sum_{i=0}^{\frac{T}{k}-1}\bar{F}_{ik}p_1^i] \geq 1/[k\sum_{i=0}^{\frac{T}{k}-1}\bar{F}_{ik}p_2^i]$ for $T = k, 2k, \ldots$. When $T > k$, we have strict inequality above.

b. Note that $\bar{F}_{T-k}\, p_1^{\frac{T}{k}-1} \leq \bar{F}_{T-k}\, p_2^{\frac{T}{k}-1}$ for $p_1 < p_2$ and $T = k, 2k, \ldots$. When $T > k$ we have strict inequality. Thus $c_1 - (c_1-c_2)\bar{F}_{T-k}\, p_1^{\frac{T}{k}-1} \geq c_1 - (c_1-c_2)\bar{F}_{T-k}\, p_2^{\frac{T}{k}-1}$.

From a and b we have $L_{p_1}(T) = L_{p_2}(T)$ for $T = k$ and $L_{p_1}(T) > L_{p_2}(T)$ for $p_1 < p_2$ and $T = 2k, 3k, \ldots$. This is intuitively reasonable since more mistakes in inspection lead to an increase in unnecessary replacements. This can be seen in Figure 4.10 in the next section. Clearly $\lim_{p\downarrow 0} L_p(T) = c_1/k$.

Since $L_p(T)$ is a decreasing function of p, we obtain a sharp lower bound for L(T) by taking the limit as p increases to 1. Thus

$$L(T) \geq \lim_{p\uparrow 1} L_p(T) = \frac{c_1 - (c_1-c_2)\bar{F}_{T-k}}{k\sum_{i=0}^{\frac{T}{k}-1}\bar{F}_{ik}} = L_1(T) .$$

This bound is shown in Figure 4.10.

We denote the optimal value of T for a particular p as T_p^*. Then the optimal long-run expected cost per unit of time is $L_p(T_p^*) = \min_T L_p(T)$. Since for all T we have $L_{p_1}(T) \geq L_{p_2}(T)$ for $p_1 < p_2$, it follows that $L_{p_1}(T_{p_1}^*) = \min_T L_{p_1}(T) \geq \min_T L_{p_2}(T) = L_{p_2}(T_{p_2}^*)$. Thus $L_p(T_p^*)$ is also a decreasing function of p. This can be seen in figure 4.10.

4. APPLICATIONS

Our model makes sense only for a failure rate which is increasing. We see from Theorem 2.1 that the denominator of L(T) is a sum which can be explicitly solved for an exponential distribution F. Although it is clear that the optimal

policy is to replace only at failure times, because replacing an exponential component by a new component does not change the residual life, we will show that Theorem 2.2 confirms that $T^* = \infty$. Let $\bar{F}(x) = e^{-\lambda x}$, $\lambda > 0$, and $k = 1$. Then $EY = \sum_{i=0}^{\infty} \left(\frac{p}{e^{\lambda}}\right)^i = \frac{1}{1 - p/e^{\lambda}}$. Setting $L'(T) = 0$ and solving for T we get

$$T = 1 + \log \left[\frac{\left(\frac{c_1}{c_1 - c_2}\right)\left(1 - \frac{p}{e^{\lambda}}\right) + \log \frac{p}{e^{\lambda}}}{1 + \frac{p}{e^{\lambda}}\left(\log \frac{p}{e^{\lambda}} - 1\right)} \right] \Big/ \log \frac{p}{e^{\lambda}} .$$

Note that $1 + \frac{p}{e^{\lambda}}\left(\log \frac{p}{e^{\lambda}} - 1\right) > 0$ since it reduces to the form $\log x < x - 1$, which is true for $x = \frac{e^{\lambda}}{p} > 1$. Thus for T to exist, we need $\left(\frac{c_1}{c_1 - c_2}\right)\left(1 - \frac{p}{e^{\lambda}}\right) + \log \frac{p}{e^{\lambda}} > 0$, which implies $\frac{c_1}{c_1 - c_2} > \frac{\lambda - \log p}{1 - p/e^{\lambda}}$, which in turn implies $r_{\infty} = \lambda < \frac{c_1}{(c_1 - c_2)EY} + \log p$. Thus from Theorem 2.2c. we see that $T^* = \infty$.

Next we will consider an application of our model to the truncated normal distribution so that we can compare our results to those in Example 1, p. 90, of Barlow and Proschan (1965). "Example 1. Many types of electron tubes used in commercial airline communication equipment and elsewhere tend to have a truncated normal failure distribution (Aeronautical Radio, Inc., 1958). A certain tube used in commercial equipment has a truncated normal failure distribution with a mean life of 9080 hours and a standard deviation σ of 3027. Suppose c_1 = \$1100 and c_2 = \$100, so that $c_1/c_2 = 11$."

The density f(x) of the truncated normal distribution may be written as

$$f(x) = \begin{cases} \frac{1}{b\sigma}\,\psi\left(\frac{x-\mu}{\sigma}\right), & x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

where $\psi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ and $b = \frac{1}{\sigma}\int_0^\infty \psi\left(\frac{x-\mu}{\sigma}\right) dx$. From $L'(T) = 0$ we get

$$(2.1) \qquad (kr_{T-k} - \log p) \sum_{i=0}^{\frac{T}{k}-1} \bar{F}_{ik} p^i + \bar{F}_{T-k} p^{\frac{T}{k}-1} = \frac{c_1}{c_1-c_2} .$$

Making the change of variable $y_0 = \frac{T-k-\mu}{\sigma}$ and defining $r_N(x) = \frac{\psi(x)}{\int_x^\infty \psi(t)dt}$, we have from (2.1) for a truncated normal distribution F,

$$(2.2) \qquad K(y_0) = \left(\frac{k}{\sigma} r_N(y_0) - \log p\right) \sum_{i=0}^{\frac{\sigma y_0+\mu}{k}} p^i \int_{\frac{ik-\mu}{\sigma}}^\infty \psi(v)dv + p^{\frac{\sigma y_0+\mu}{k}} \int_{y_0}^\infty \psi(v)dv$$

$$= \frac{bc_1}{c_1-c_2} = b + \frac{bc_2}{c_1-c_2} .$$

If $\frac{\mu}{\sigma} \geq 3$, then f(x) is very close to the density of a normal distribution with mean μ and standard deviation σ.

In Example 1, Barlow and Proschan estimate b by 1 and use a graph of $K(y_0) = K(y_0) + b$ for $\frac{\mu}{\sigma} = 3$ to find y_0. To obtain more accurate results for comparison with our model, we compute $b = .9987$ and $K(y_0) = \frac{bc_2}{c_1-c_2} = .09987$. Then $y_0 = -1.63$ and the optimal replacement age is 4146 hours with an associated minimum cost of $L(4146) = \$.036$.

For our model we consider several examples with various values for k, the length of the inspection interval, and p, the the probability of no error in inspecting a functioning unit. Note that for the truncated normal distribution, $r_\infty = \infty$, so that we know by Theorem 2.2c. that T* is finite. Using (2.2) we wish to find the value of T such that $K(\frac{T-k-\mu}{\sigma})$ is nearest to $K(y_0) = \frac{bc_1}{c_1-c_2} = 1.0986$.

Example 2: $k = 1000$, $p = .95$.

Using (2.2) we compute the following values:

T	y	K(y)
1000	-3.00	1.050
2000	-2.67	1.051
3000	-2.34	1.060
4000	-2.01	1.088
5000	-1.68	1.149

FIGURE 4.1: Values of K(y) for $k = 1000$, $p = .95$.

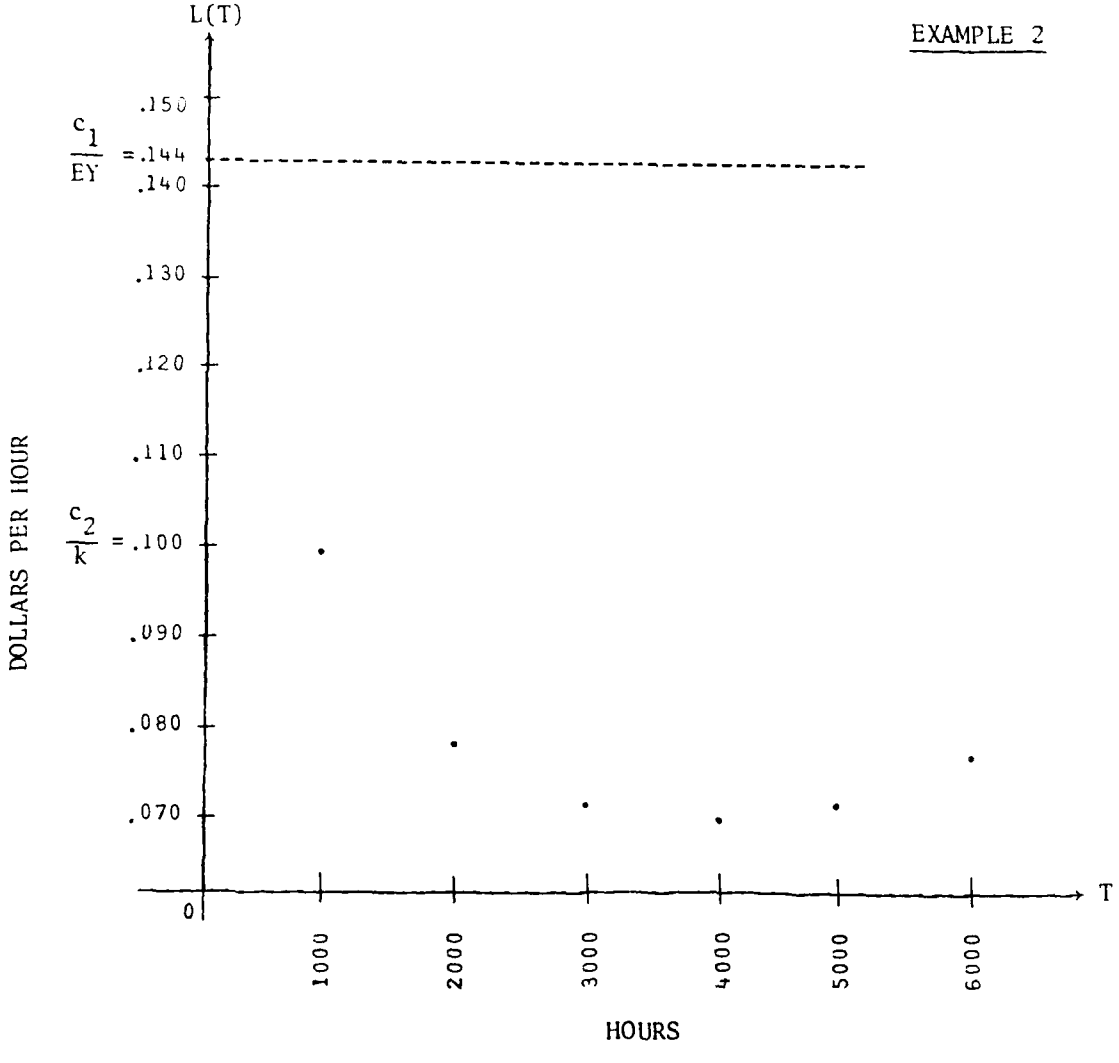

FIGURE 4.1: Values of L(T) for $k = 1000$, $p = .95$.

We see from Figure 4.1 that T* = 4000 is the optimal value, which gives us the minimum cost L(T*) = \$.0710 as shown in Figure 4.2. We compute EY = 7629. Recall from the proof of Theorem 2.2e that $c_1/EY = .144$ is the horizontal asymptote for L(T). Also $\frac{c_2}{c_1} EY = 694$ is a lower bound for T*.

We see from Figure 4.3 that T* = 4000 is the optimal value and L(T*) = \$.1271 shown in Figure 4.4. We compute EY = 5979. Then $\frac{c_2}{c_1} EY = 544$ is a lower bound for T*. Notice in Figure 3.4 that $\frac{c_2}{k} > \frac{c_1}{EY}$, thus the implication in Theorem 2.2d cannot be reversed.

Comparing Examples 2 and 3, it is clear that more frequent inspections result in a higher optimal cost L(T*).

We see from Figure 4.5 that the graph of $K(y)$ does not cross $K(y_0) = 1.0986$. Thus there is no solution to $L'(T) = 0$, so that $L' > 0$, and the cost is strictly increasing, as shown in Figure 4.6. Then the minimum cost is \$.10 = L(1000) at T* = k. This is the expected result from Theorem 2.2b since $.5 = p \le \exp\left(\frac{c_2}{c_2 - c_1} + kr_0\right) = \exp[-.1 + 1000(.0000015)] \approx .9062$. Since EY = 1980, $\frac{c_2}{c_1} EY = 180$ is a lower bound for T*.

Just as in Example 4 we see from Figure 4.7 that K does not cross $K(y_0) = 1.0986$, so that cost is strictly increasing and T* = k as expected from Theorem 2.2b since $.5 = p \le \exp\left(\frac{c_2}{c_2 - c_1} + kr_0\right) = \exp[-.1 + 500(.0000015)] \approx .9055$.

A lower bound for T* is $\frac{c_2}{c_1} EY = \frac{100}{1100}(998) = 91$.

Example 3: k = 500, p = .95

T	y	K(y)
3500	-2.01	1.0700
4000	-1.84	1.0916
4500	-1.68	1.1205
5000	-1.51	1.1605

FIGURE 4.3: Values of K(y) for k = 500, p = .95 .

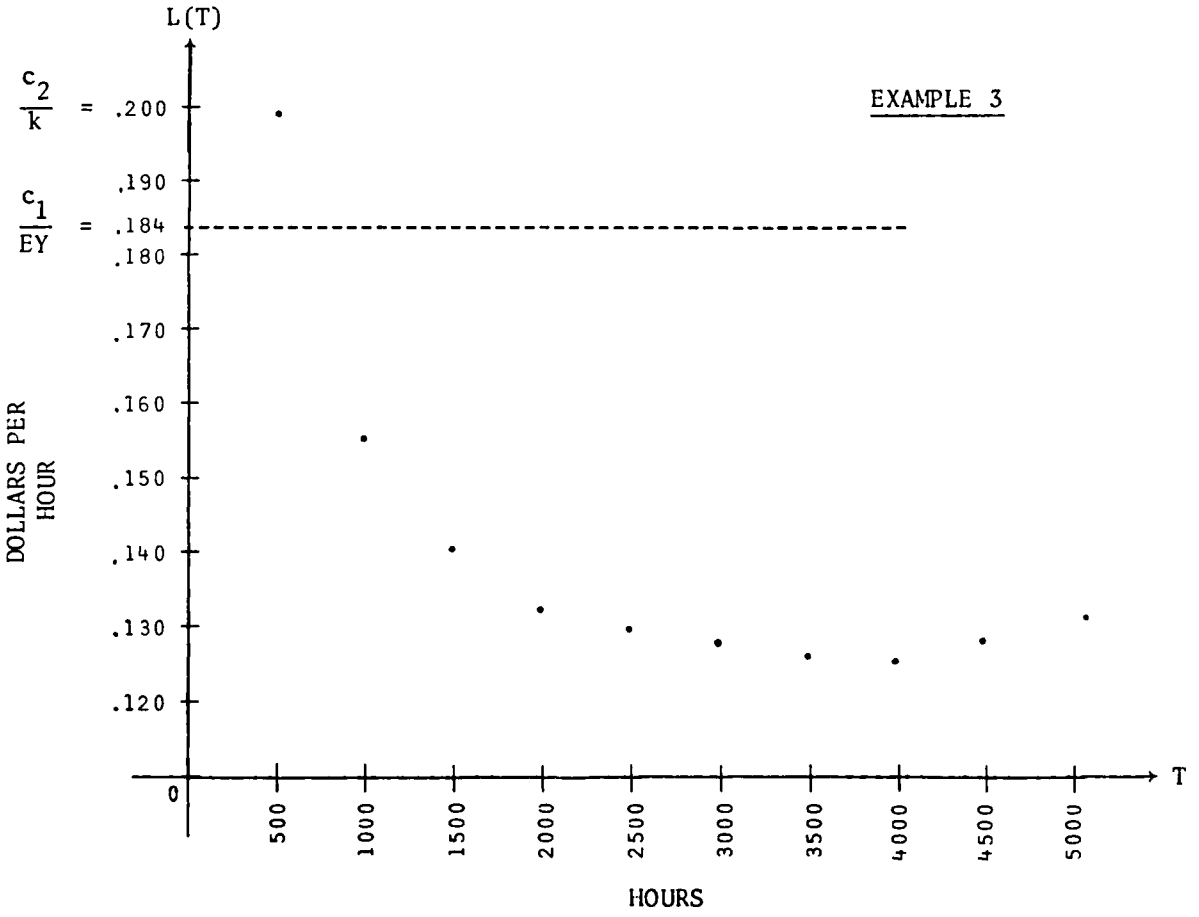

FIGURE 4.4: Values of L(T) for k = 500, p = .95 .

Example 4: k = 1000, p = .5

T	y	K(y)
1000	-3.00	1.6923
2000	-2.67	1.5410
3000	-2.34	1.4710
4000	-2.01	1.4490
5000	-1.68	1.4590
6000	-1.35	1.4969

FIGURE 4.5: Values of K(y) for k = 1000, p = .5.

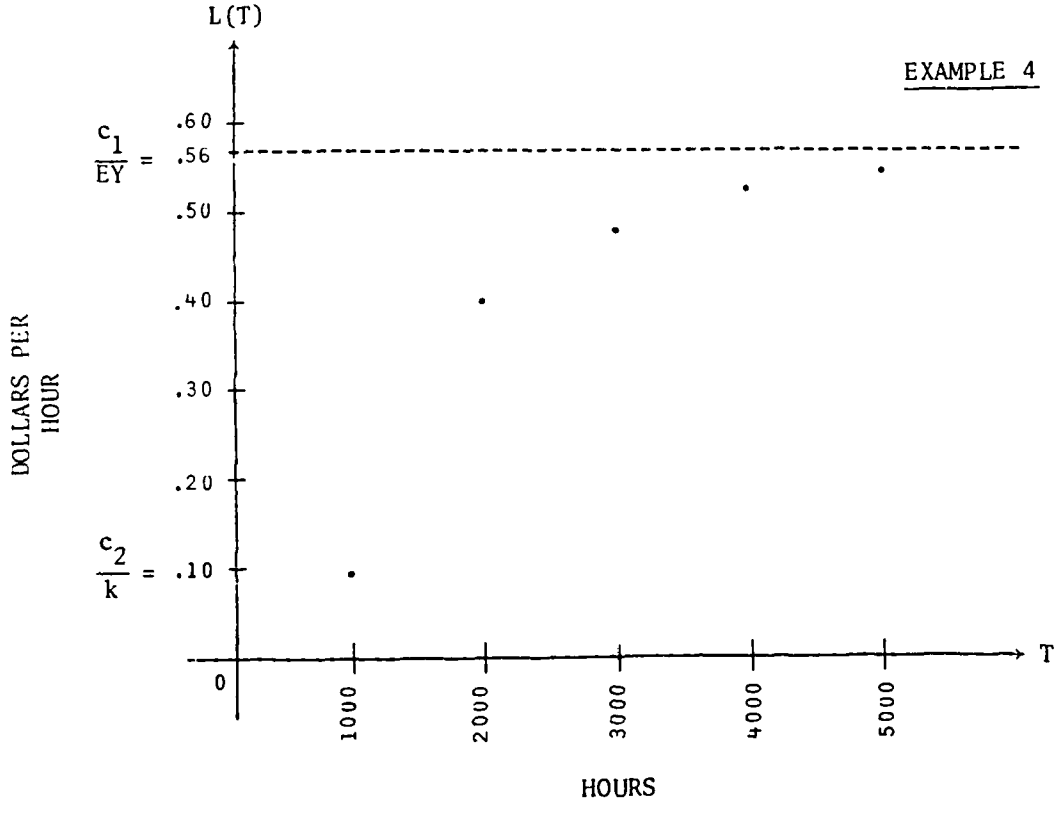

FIGURE 4.6: Values of L(T) for k = 1000, p = .5.

Example 5: k = 500, p = .5

T	y	K(y)
500	-3.00	1.6915
1000	-2.83	1.5384
1500	-2.67	1.4630
2000	-2.50	1.4260
2500	-2.34	1.4096
3000	-2.17	1.4040
3500	-2.01	1.4042
4000	-1.84	1.4089
4500	-1.68	1.4164

FIGURE 4.7: Values of K(y) for k = 500, p = .5.

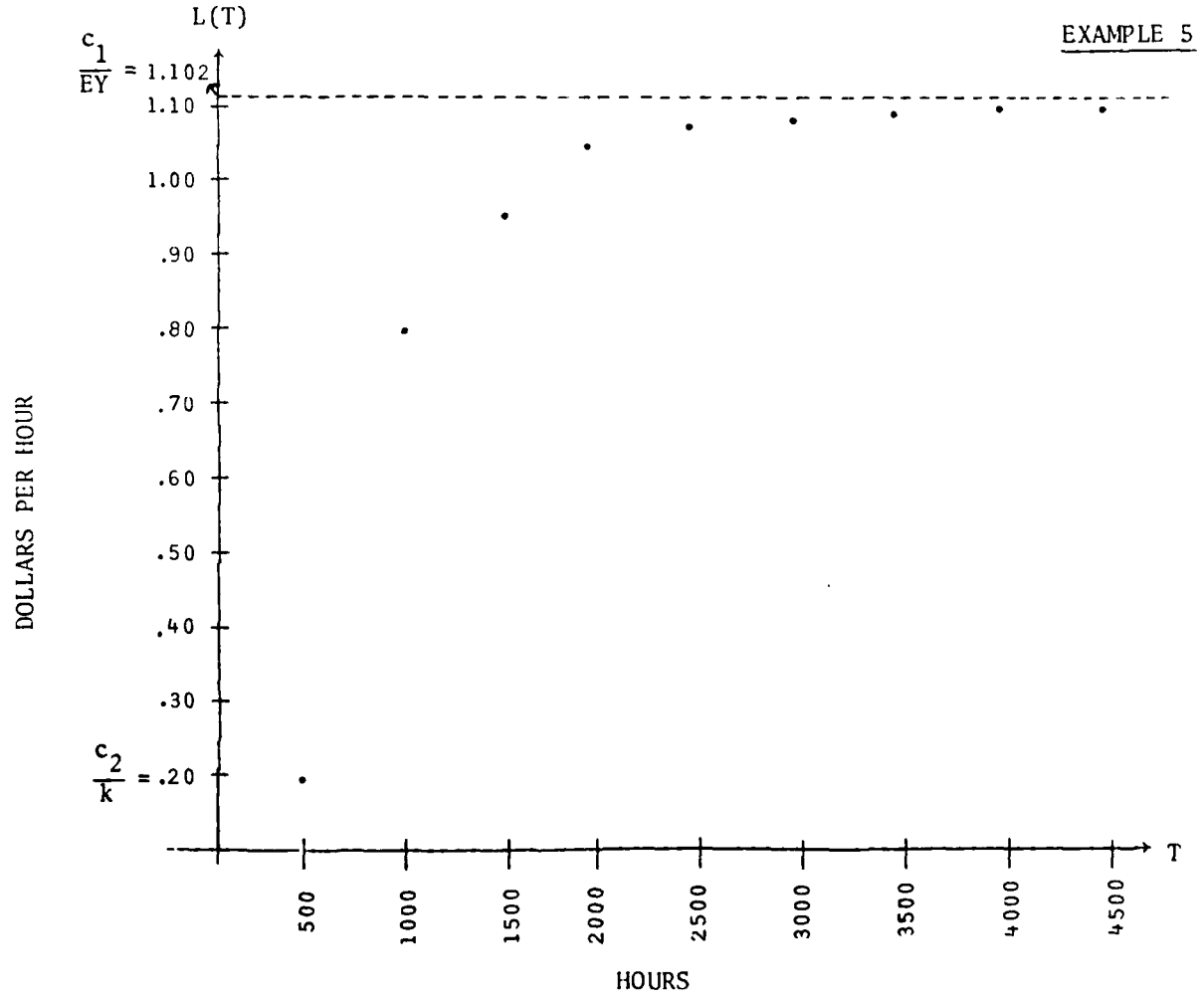

FIGURE 4.8: Values of L(T) for k = 500, p = .5 .

We summarize the results as follows:

EXAMPLE	p	k	$\frac{c_2}{c_1}EY_p$	T_p^*	EY_p	$L_p(T_p^*)$
1	1.00	0	$\frac{c_2}{c_1}\mu=825$	4146	$\mu=9080$	\$.036
2	.95	1000	694	4000	7629	\$.071
3	.95	500	544	4000	5979	\$.127
4	.50	1000	180	1000	1980	\$.100
5	.50	500	91	500	998	\$.200

Table 4.9: Summary of Examples.

Asymptotic and monotonicity properties of $L_p(T)$ are shown in Figure 4.10 for $k = 1000$. Optimal values $L_p(T_p^*)$ are circled. For $p = .5$, .7, and .9 we have $T_p^* = k = 1000$. Clearly $L_p(T_p^*)$ and $L_p(T)$ are decreasing in p. $L_1(T)$ is a sharp lower bound. Horizontal asymptotes are labeled $L_p(\infty)$. Lines connecting values of $L_p(T)$ for a given value of p were drawn to illustrate the trend of $L_p(T)$ as $T \to \infty$.

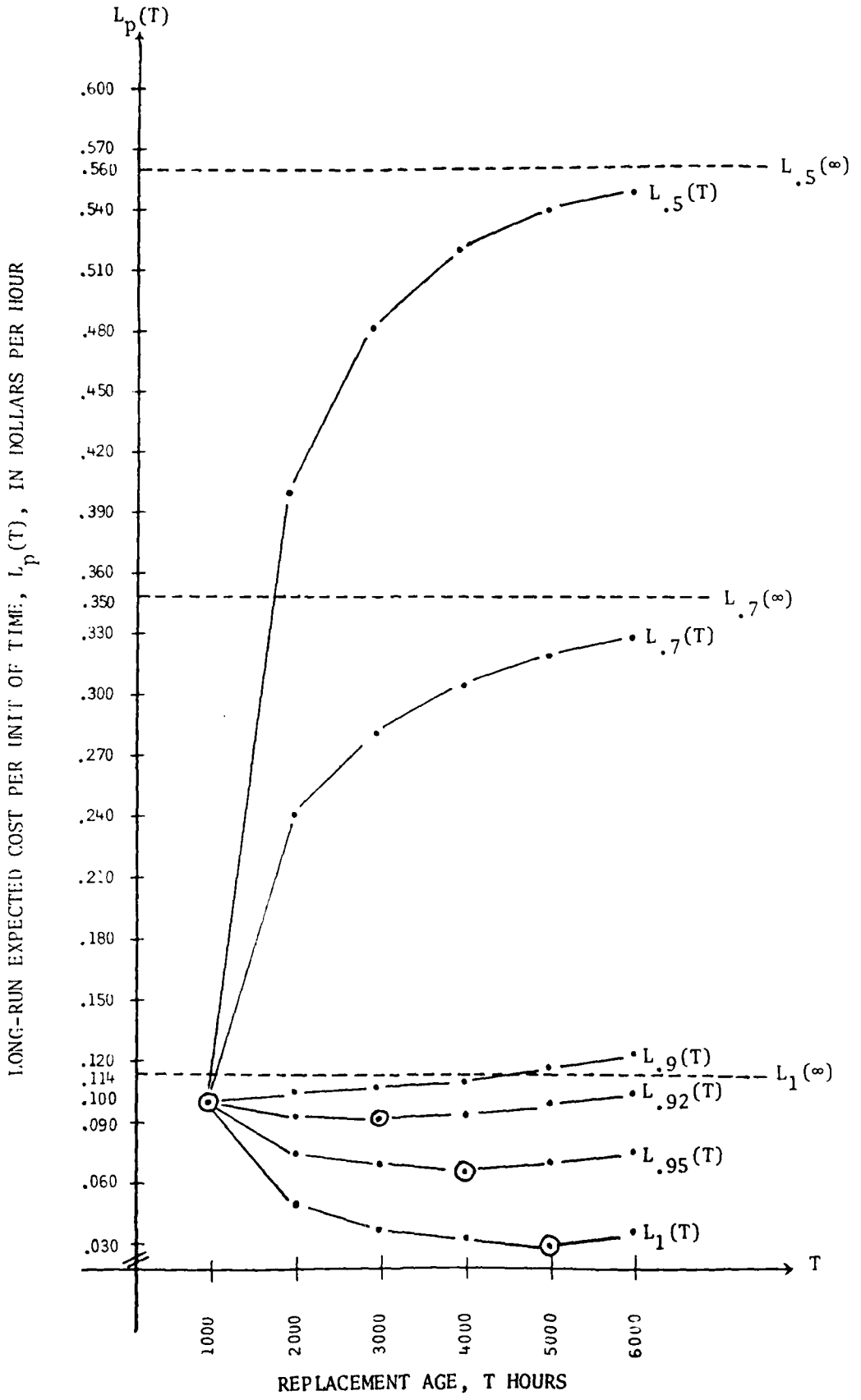

FIGURE 4.10: Asymptotic and monotonicity properties of $L_p(T)$ for an inspection interval k = 1000 hours, for a truncated normal life distribution.

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A device is maintained under an age replacement policy. The status of the device (functioning or failed) is known only by inspection at some fixed interval k. With probability q, an inspection error may be made, and a functioning unit will be declared to have failed and be replaced by a new unit. On the contrary, when a failed unit is inspected, it is assumed that no inspection error will be made. Assuming that the cost of replacing a failed unit (actually failed or believed failed) is greater than the cost of replacing a functioning unit, we show that we can obtain an optimum replacement age which minimizes L(T), the long-run expected cost per unit of time. We find a lower bound for the optimal replacement age and obtain asymptotic and monotonicity properties for L(T).

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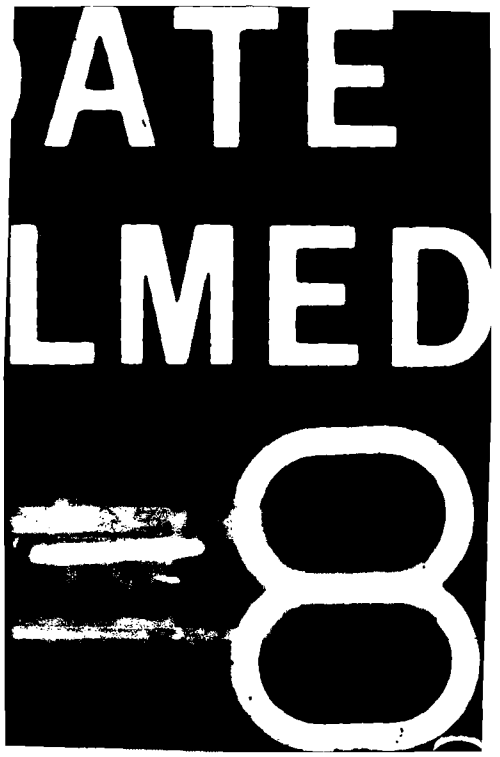
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