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Inverse perspective of a road from a single image

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PREFACE

This document was prepared by the Computer Vision Laboratory, Center for Automation Research, University of Maryland, College Park, Maryland, under contract number DACA76-84-C-0004 for the U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia, and the Defense Advanced Research Projects Agency, Arlington, Virginia. The Contracting Officer's Representative was Ms. Rosalene M. Holecheck.

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INVERSE PERSPECTIVE OF A ROAD FROM A SINGLE IMAGE

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ABSTRACT

A method is presented for reconstructing the geometry of a road from a single image of the road. This problem has an infinity of solutions unless restrictive hypotheses about geometric characteristics of this road are assumed. The road is modeled as a space ribbon defined by a spine (centerline) and generators which are horizontal line segments cutting the spine at their midpoint at a normal angle. Properties of two neighboring generators of such a ribbon are examined, and it is found that if a generator is known, a neighbor is completely defined if one of its ends is known. The proposed method uses this property to reconstruct the visible part of the world road, by iteratively finding a series of generators. For validation of this method, a road making an S on a hill or in a valley is defined analytically and graphically, and its perspective image is obtained; from this image, algorithms to be tested reconstruct a ribbon which is compared to the original world model by superposition, in top view and side view. The proposed method is tested against a simple method which assumes that the ground is flat ("flat earth assumption"), and against another method which uses vanishing points.

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1. Introduction

In a vehicle able to follow a road without human intervention, one of the tasks to perform is the construction of a world model of the road from the images given by the vehicle's sensors [1, 2]. Sensors being currently tested for this task are:

- A single camera capturing images of the scene.
- A pair of cameras for stereo triangulation between two images of the scene.
- A laser ranger giving range data by measuring the phase lag between transmitted and reflected pulses of a laser beam scanning the scene.

We will not discuss the relative strengths and weaknesses of these techniques here, but will simply remark that a camera will most probably be present in the final designs of autonomous navigation systems, in association with another camera and/or a laser ranger. An autonomous system will apply a multiple-expert strategy, obtaining its clues from several groups of algorithms, each group processing specifically filtered pieces of the total information received by the different sensors. One of these experts could focus on the information obtained by a single camera. In this paper we present a method which such an expert system could invoke to build geometric models of the world, in a limited world of roads and highways.

Papers such as [1], [3], [4] also consider the problem of road inverse perspective, and the methods proposed there are based on finding pairs of line segments which are images of parallel segments in the scene. For such pairs, the direction from the focal point of the camera to the vanishing point of the pair gives the direction of the parallel segments in space (Figure 1).

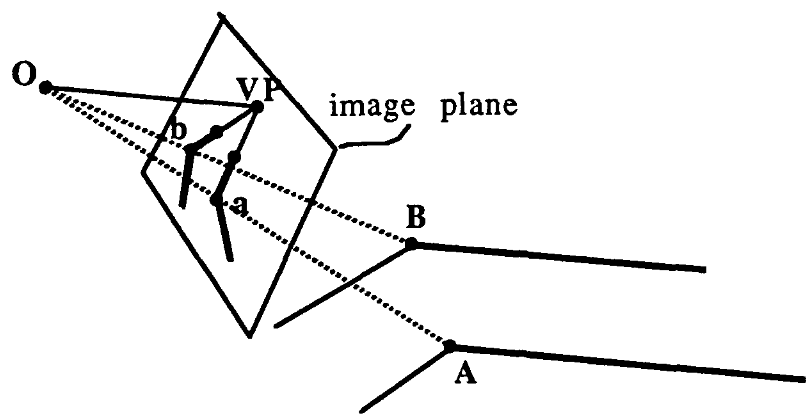

Figure 1

Furthermore, the intersections of these parallel segments with the segments processed at the previous iteration can be found as the projections of the corresponding image intersections onto the plane of the previous segments. These parallel segments are then fully defined in space.

This type of procedure relies on the assumption that points which are intersections of the edges with a plane normal to the centerline ("opposite points") are points at which the tangents to the edges are parallel. It also assumes that the line segments found by image processing around the images of these points are images of parallel line segments ("opposite segments"). One problem with this method is that the test for opposite points and opposite segments can be applied only in 3-D. Thus, checking whether two segments in the image are images of opposite segments in space would require knowing the 3-D structure of the world road that we are trying to calculate.

A second problem, which occurs in images of road turns, is that the set of line segments found by image processing to approximate the images of the edges of the road is a discontinuous representation of continuous curves, and it is probable that none of the possible pairs formed by picking a segment on each edge is the image of a pair of parallel segments. If Figure 2 shows the correct inverse perspective world segments of the segments in an image (no pair of parallel segments can be found), then a method assuming that parallel pairs of world segments can be found will give the wrong inverse perspective segments and an erroneous world road.

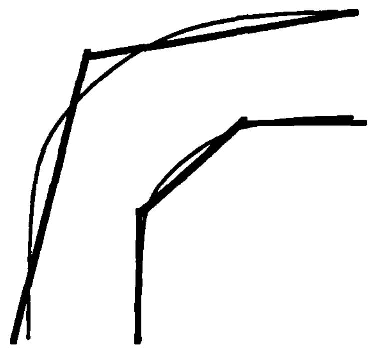

Figure 2

In this case a pair of parallel world segments built from any pair of image segments will diverge from the real road.

A third problem is that a road reconstructed using parallel pairs of edge segments is developable because it is a chain of planar patches. Such a model is acceptable for flat roads or straight roads on a changing slope, but not when a turn occurs on a slope. In such combinations of turn and slope, the premise that, for opposite points, tangents to the road edges are parallel - the very basis for the vanishing point methods - is wrong. In such cases, road patches made from opposite segments are warped, and trying to model them with planar patches gives a reconstructed road which does not match the real-world road.

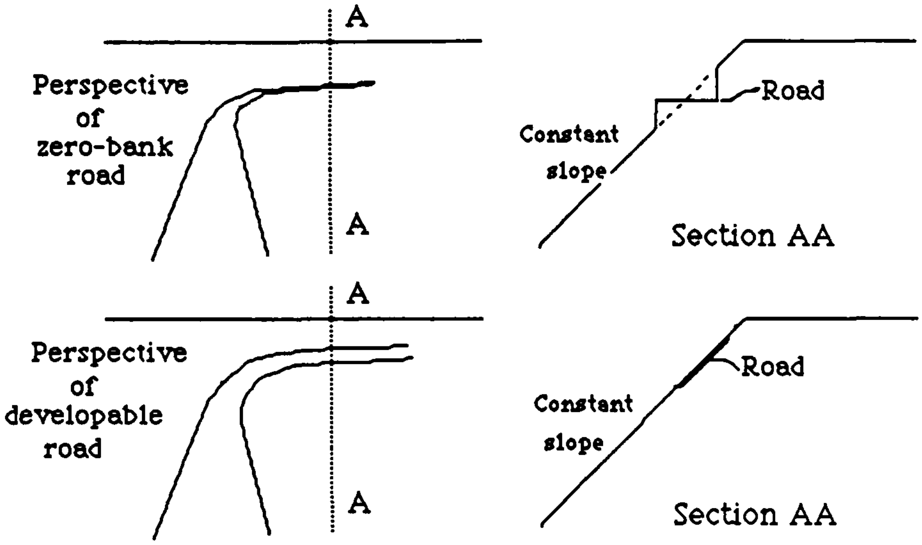

Figure 3

This last point is illustrated in Figure 3. A road climbs a constant slope, and takes a turn while its centerline stays in the plane of the slope. For a real world road, a section of the hill would look like the top right drawing.

The only possible developable road taking a turn on a constant slope is a planar road which stays on the surface of the hill, and takes an unrealistic bank equal to the slope of the hill (bottom drawing). The image of such a road would be much wider after the turn than the image of a non-developable road (top); therefore an inverse perspective program using the top perspective view and reconstructing the road with planar patches will not find a road on the slope of the hill. From the narrowing of the road after the turn in the top perspective view of the figure, such a program will assume that the road after the turn is much further away than it actually is.

The previous comments have shown the importance of a geometric model of road ribbons flexible enough to represent the situations encountered in the real world. The following section summarizes the proposed modeling scheme.

2. Summary of the world road modeling scheme

In the inverse perspective algorithm presented in this paper, the world road ribbon is defined by a spine and a sweeping line segment which slides along the spine at its centerpoint, while remaining normal to this spine. Furthermore, the generating segment is assumed to remain horizontal. This road model was originally suggested by Ozawa and Rozenfeld in [5].

This definition of the road ribbon is continuous. In practice, however, only a discrete number of neighboring cross-segments are computed, and we discuss in Sections 3 and 4 the geometric properties of two successive cross-segments in a space road ribbon. In the proposed algorithm, the calculation of a new cross-segment is iterative, requiring knowledge of the previous cross-segment. The geometric relations between two successive cross-segments are used to deduce the new cross-segment. These are summarized in the four following characteristics describing a new cross-segment:

1. The new cross-segment is normal to the road centerline. In a discrete representation of nearby cross-segments, this can be expressed by saying that the average direction between the previous cross-segment direction and the new cross-segment direction is normal to the vector joining the midpoints of these two segments.
2. The new cross-segment has the same length as the previous one.
3. The new cross-segment is horizontal.
4. The images of the ends of the new cross-segment are on the images of the road edges.

These four constraints usually yield more than one possible new cross-segment. Among these, the cross-segment which gives the minimum change of road slope is chosen.

Sections 3 and 4 show that the first two constraints constrain a pair of neighboring cross-segments to form a warped isosceles trapezoid. In Section 5, details of the calculation are given for combining this warped isosceles trapezoid approximation with constraints 3 and 4 and finding a new horizontal cross-segment from the knowledge of the road image and a previous cross-segment. Section 6 completes the description of this road reconstruction algorithm by presenting a method to obtain an initial cross-segment. Section 7 applies this algorithm to the perspective image of a road defined analytically which makes an S on a hill or in a valley. The results are compared to the reconstructions obtained by a method that assumes that the ground is flat, and by another method that tries to find images of parallel segments in the perspective image.

3. Geometry of a road of constant width

3.1.Planar road

The only reason for first considering a planar road is to introduce the concepts which will be applied to a non-planar road. The properties of a space ribbon will of course remain appropriate for modeling a planar road.

Consider a planar road of constant width (Figure 4). It has a ribbon shape which can be generated in the followi-; way:

a. A planar spine defines the trajectory of the road in the plane.

b. A line segment of constant length has its midpoint on the spine, and is required to stay normal to the spine as it moves along the spine. The ribbon generated by this process is a Brooks ribbon of constant width [6].

Such a ribbon makes a reasonable model of a planar road of constant width:

1. The sides of the road are the loci of the end points of the generating segment.

2. The spine is the centerline of the road.

3. The constant length of the generators (cross segments) is the road width. The end points of a cross-segment are opposite points of the road.

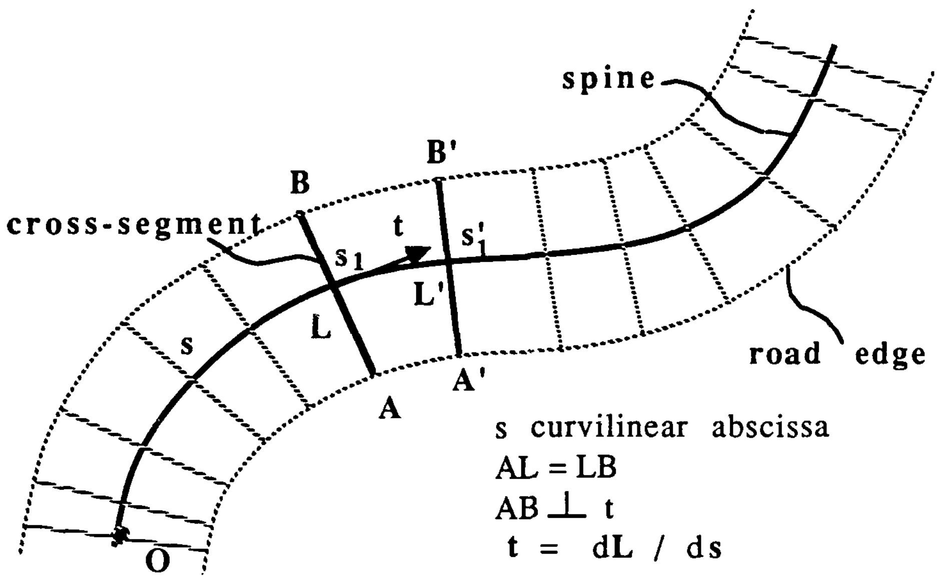

Figure 4

The coordinates of a point L moving on the spine can be expressed as functions of the curvilinear abscissa s. The spine is then defined by the coordinates xL(s) and yL(s) of the vector function **OL**(s), or, omitting the origin O, **L**(s). (We use bold face for points, vectors and vector functions, and plain face for scalars). The unit vector tangent to the spine at the abscissa s_1 is the derivative d**L**/ ds for the value s_1. If AB is the cross-segment at this point, we can express the fact that AB is normal to the spine by the equation

$$\mathbf{AB} \cdot d\,\mathbf{L}/ d\,s = 0 \quad \text{for} \quad s = s_1 \qquad (1)$$

Instead of dealing with an infinity of contiguous cross-segments, we can capture the general form of the road ribbon from a discrete set of closely spaced cross-segments, in the same way as a railroad can be defined by its ties.

This discrete approximation is more useful for iterative computation purposes than a continuous representation; therefore we will examine some of its properties.

Consider two successive cross-segments AB and A'B', and their midpoints L and L' (Figures 4, 5). We are looking for a discrete equivalent of the property that cross-segments are normal to the centerline. An appropriate approximation is

$$1/2\ (\mathbf{AB} + \mathbf{A'B'}) \cdot \mathbf{LL'} = 0 \qquad (2)$$

Indeed, if we knew the curvilinear s_1 and s'_1 for L and L', we would write the previous equation as

$$1/2\ (\mathbf{AB} + \mathbf{A'B'}) \cdot (\mathbf{L'} - \mathbf{L}) / (s'_1 - s_1) = 0 \qquad (3)$$

and this equation tends to equation (1) when the point L' tends toward the point L, which supports our choice of equation (2) in the discrete approximation of the road (Other choices which do not give symmetrical roles to AB and A'B', or use more than two cross-segments, are possible). This equation simply states that the average direction of the road, **LL'**, is normal to the average direction (**AB** + **A'B'**) / 2 of the two cross-segments **AB** and **A'B'** located at L and L'.

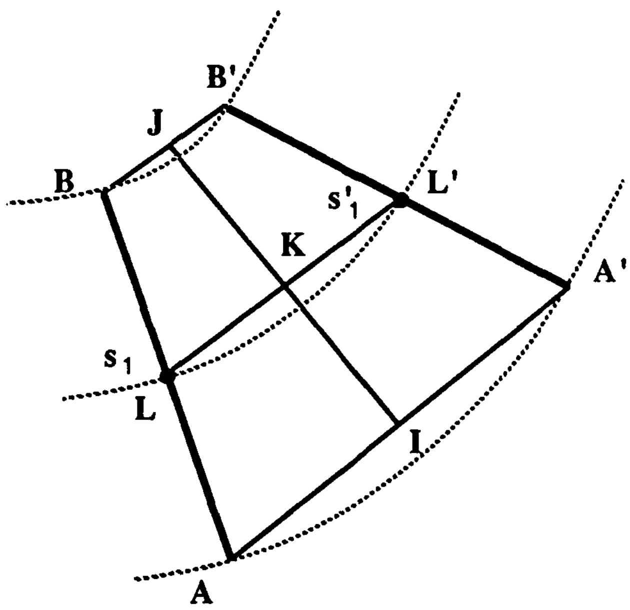

Figure 5

Consider the line segment IJ joining the midpoints I and J of the line segments AA' and BB'. The vector IJ is equal to (**AB** + **A'B'**)/2:

$$(\mathbf{AB} + \mathbf{A'B'})/2 = 1/2(\mathbf{AI} + \mathbf{IJ} + \mathbf{JB} + \mathbf{A'I} + \mathbf{IJ} + \mathbf{JB'}) = \mathbf{IJ} \qquad (4)$$

Thus equation (2) also states that IJ is normal to LL'.

Consider the line segment joining A and A', and the angles between AB and AA' and between A'B' and AA'. The cosines of these angles are obtained from the dot products **AB•AA'** and **A'B'•A'A**:

$$\mathbf{AB} \cdot \mathbf{AA'} - \mathbf{A'B'} \cdot \mathbf{A'A} = \mathbf{AA'} \cdot (\mathbf{AB} + \mathbf{A'B'})$$

$$= (\mathbf{AL} + \mathbf{LL'} + \mathbf{L'A'}) \cdot (\mathbf{AB} + \mathbf{A'B'})$$

Since **LL'** • (**AB** + **A'B'**) = 0 and **AL** = **AB**/2 and **L'A'** = - **AB**/2, this simplifies to

$$\mathbf{AB} \cdot \mathbf{AA'} - \mathbf{A'B'} \cdot \mathbf{A'A} = 1/2(\mathbf{AB} - \mathbf{A'B'}) \cdot (\mathbf{AB} + \mathbf{A'B'})$$

$$\mathbf{AB} \cdot \mathbf{AA'} - \mathbf{A'B'} \cdot \mathbf{A'A} = 1/2\,(\mathbf{AB}^2 - \mathbf{A'B'}^2)$$

$$\mathbf{AB} \cdot \mathbf{AA'} - \mathbf{A'B'} \cdot \mathbf{A'A} = 0 \qquad (5)$$

Thus the angles (AB, AA') and (A'B', A'A) are equal, and the quadrilateral ABB'A' is a planar isosceles trapezoid. Therefore the line segment IJ is normal to AA' and BB' (and LL', as already found). We will find this property to be conserved in the case of a non-planar road, despite the fact that in that case AA', LL', and BB' are not generally coplanar.

3.2. Non-planar road

In the analytic representation of a non-planar road, we shall retain the modeling by a ribbon such as a Brooks-type ribbon; a spine will define the meandering up and down and sideways of the road, while a generating line segment of constant length will give some flesh to that spine by sweeping along the spine. The sliding point of the generating segment is its midpoint, and the segment is restricted to be normal to the spine at this sliding point. For a spine in space, however, this last restriction only constrains the segment to be in a plane normal to the spine at the sliding point, and does not define a unique ribbon: the generating segments could take any angles within these normal planes (Figure 6). The intersection of the normal plane with a horizontal plane can be used as a reference for measuring these angles. This reference line is therefore both horizontal and normal to the spine. The angle of the generating segment with this horizontal line in the normal plane is called the bank or superelevation of the road. Its role in real-world roads is to eliminate rain water from the road, and to give a favorable horizontal road reaction component on vehicles in turns. The tangent of the bank is ordinarily 3 %, and at most 10 % [5] .

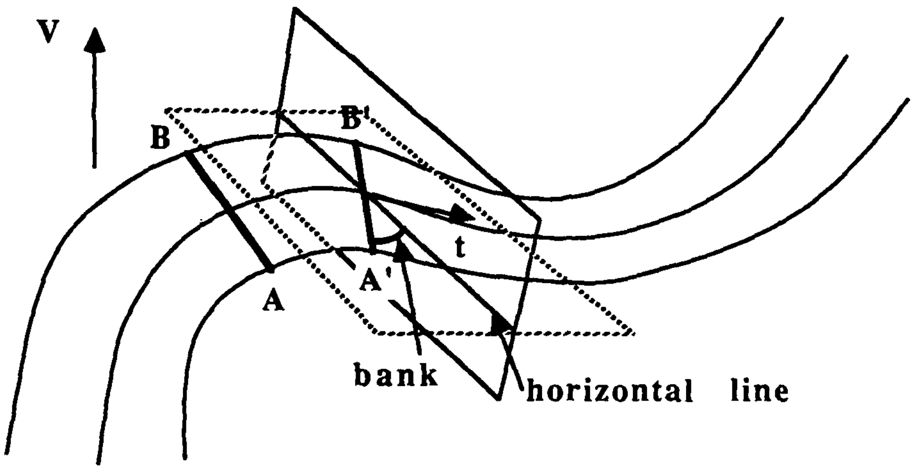

Figure 6

Let us assume first that some rule is used to define a unique bank for each generating segment as function of the local characteristics of the spine, and that a unique ribbon is therefore generated for any given spine, in which the generating segments are normal to the spine at their midpoints. In a development identical to that followed for a planar road, a sliding point L on the spine can be defined as a vectorial function L(s) of a curvilinear abscissa s. The cross-segment AB, whose midpoint is L, is normal to the spine in L. This is expressed by equation (1):

$$\mathbf{AB} \cdot d\mathbf{L}/ds = 0 \qquad (1)$$

If we now want to define the space road ribbon by a discrete number of generating segments, the same approximation as for the planar road applies for two nearby generating segments AB and A'B' (Figure 7):

$$1/2\ (\mathbf{AB} + \mathbf{A'B'}) \cdot \mathbf{LL'} = 0 \qquad (2)$$

We showed that for a planar road this relation constrains the four points A, B, B', A' to define a trapezoid. In the case of a space road, the segments AB and A'B' are most likely to be non-coplanar due to combinations of slope and turns, even if the bank is assumed to be constant or null. Consider for instance the ribbon generated by the edges of the steps in a spiral stair case. The spine is a cylindrical helicoid. All the generating step edges are horizontal (zero bank), but two consecutive step edges AB and A'B' are non-coplanar, because the line segment BB' on the inside of the turn is much more vertical than the segment AA' on the outside of the turn; the quadrilaterals formed by two consecutive step edges are warped. A similar situation occurs when a change of altitude occurs in a road turn. In other words, such a ribbon is not a developable surface, and efforts to model it with non-triangular planar patches would be a waste of time.

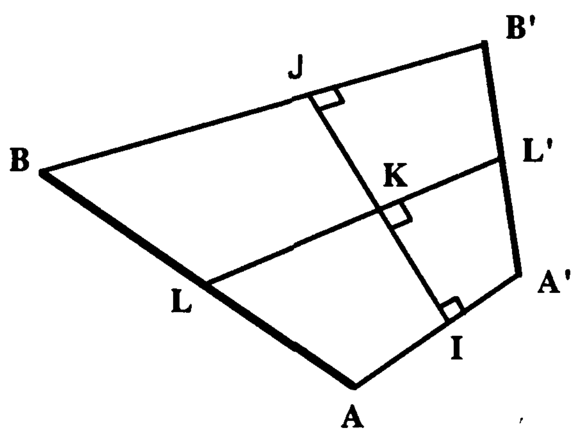

Figure 7

Let us examine some geometrical properties of a warped quadrilateral ABB'A' of two nearby generating segments AB and A'B'.

The angles (AB,AA') and (A'A,A'B') are found to be equal, using the previous development (5) to show that the quantity **AB•AA' - A'B'•A'A** is null. The angles (BA,BB') and (B'B,B'A') can also be shown to be equal in a similar way. Thus we will call the quadrilateral ABB'A' a warped isosceles trapezoid. Note that such a figure is <u>not</u> obtained by folding a planar isosceles trapezoid along one of its diagonals, since this procedure would decrease the values of the two angles at the diagonal, while leaving the other two angles the same, and the angle equalities would be lost. Instead, fold a parallelogram ABA'B' along one diagonal, for instance AA' (Figure 8).

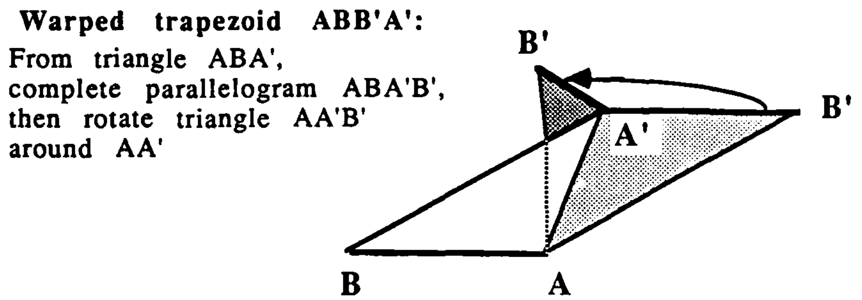

Figure 8

The maximum folding gives a planar isosceles trapezoid ABB'A' (the order of the points A' and B' is reversed because we now see the back half of the parallelogram). For all incomplete foldings, ABB'A' is a warped isosceles trapezoid (including the starting parallelogram itself).

The midpoints of AB and A'B' are L and L' (Figure 7). Calling I and J the midpoints of AA' and BB', K the midpoint of IJ and K' the midpoint of LL', one can find four expressions for KK' and add them to show that K and K' are confounded. Therefore, in a warped isosceles trapezoid, I, J, L, L', and K are coplanar (we will not use this property in this paper).

We determine that IJ is normal to LL' by the same equalities leading to equation (4) used in the planar case.

Next we want to show that IJ is also normal to AA' and BB', even though AA', BB', and LL' are not parallel in the general case:

$$\mathbf{AA'} \cdot \mathbf{IJ} = (\mathbf{AL} + \mathbf{LL'} + \mathbf{L'A'}) \cdot \mathbf{IJ}$$

$$\mathbf{AA'} \cdot \mathbf{IJ} = (\mathbf{AL} + \mathbf{L'A'}) \cdot \mathbf{IJ} \quad \text{since } \mathbf{IJ} \text{ is normal to } \mathbf{LL'}$$

$$\mathbf{AA'} \cdot \mathbf{IJ} = (\mathbf{AL} - \mathbf{A'L'}) \cdot (\mathbf{AL} + \mathbf{A'L'}) \quad \text{from eq.(4) for IJ.}$$

$$\mathbf{AA'} \cdot \mathbf{IJ} = \mathbf{AL}^2 - \mathbf{A'L'}^2 = 0 \tag{6}$$

4. Warped isosceles trapezoids in zero-bank space ribbons

In the following, we will assume the bank of all cross-segments to be null (Figure 9). Incorporating a non-zero bank in the geometric modeling of the road would be possible, but it does not seem that rigid rules are used for bank in road construction, in part because an optimal bank in a turn depends not only on turn radius, but also on the expected speeds of the vehicles.

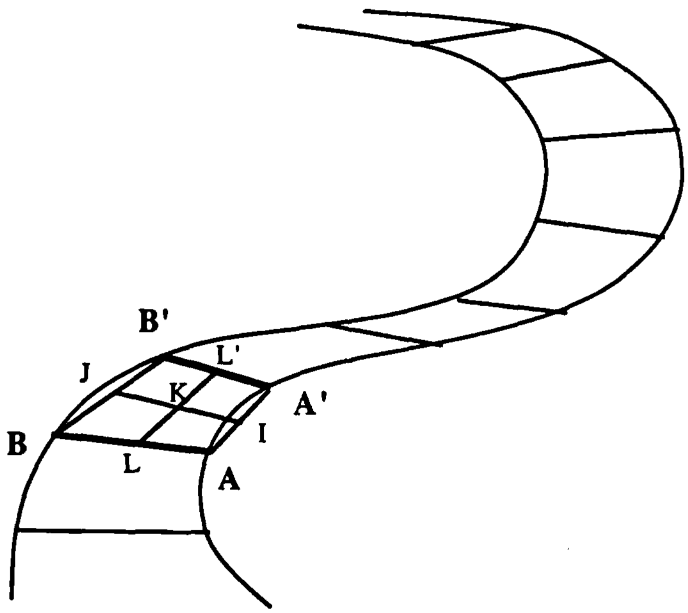

Figure 9

With the hypothesis of zero bank for a ribbon defined by a continuum of generating segments, for any given point L of the road spine, the generating segment is at the intersection of the plane normal to the spine at this point L, and of the horizontal plane through this point. An equivalent statement, using vectors only, is more useful for computations. We define a vertical vector **V**. The tangent unit vector to the spine at L is **dL/ds** (using the same definitions and notations as for the planar horizontal road). The direction of a generating segment at L for the zero bank space road is then given by the cross product **V** x d **L/ds**.

In the case of a definition of the ribbon from discrete cross-segments, such as AB and A'B', equation (4) implies that if AB and A'B' are horizontal, then IJ is horizontal too:

$$\mathbf{V} \cdot \mathbf{IJ} = 1/2\ \mathbf{V} \cdot (\mathbf{AB} + \mathbf{A'B'}) = 0 \qquad (7)$$

Therefore IJ is normal to both V and AA', so that its direction is given by the cross product **VxAA'**.

Its length is also known: since IJ is normal to AA' and BB' (equation 6), I and J are the projections of A and B on the line in the direction just found going through I; thus the length of IJ is defined by the dot product of AB by the unit vector in that direction.

This unit vector is $(\mathbf{V} \times \mathbf{AA'})/((\mathbf{V} \times \mathbf{AA'})^2)^{1/2}$. Therefore the expression for IJ is

$$\mathbf{IJ} = [(\mathbf{AB} \cdot (\mathbf{V} \times \mathbf{AA'})/ (\mathbf{V} \times \mathbf{AA'})^2]\ (\mathbf{V} \times \mathbf{AA'}) \quad (8)$$

where the part in brackets is a scalar resulting from the calculation of the length of IJ and the normalization of (**V** x **AA'**); the last parenthesis is the vector (**V** x **AA'**) which gives the direction of IJ.

Now that IJ is known, B' is uniquely defined from **AB**, A', and **IJ**, by equation (4), which gives

$$\mathbf{B'} = \mathbf{A'} - \mathbf{AB} + 2\ [(\mathbf{AB} \cdot (\mathbf{V} \times \mathbf{AA'})/ (\mathbf{V} \times \mathbf{AA'})^2]\ (\mathbf{V} \times \mathbf{AA'}) \qquad (9)$$

This relation uniquely defines the end B' of a cross-segment A'B' when the end A' is given and a neighboring cross-segment AB is known. It will allow us to find A'B' from AB in the following section.

5. Inverse perspective zero-bank algorithm

The processing of the persective image of the road is assumed to output two ordered lists of points, one for each side of the road image (for details on image processing procedures, see [7], [8]). These are inputs to the algorithm. The output of the algorithm is an ordered list of neighboring cross-segments of the world road.

Since the algorithm reconstructs a world road cross-segment from a previously calculated cross-segment, an initial cross-segment is needed to start the iteration. Methods for obtaining the initial cross-segment are discussed in the next section. We assume for now that such a cross-segment has been found, and we are going to look at one intermediate step of the iteration.

The two edges of the road image are used in a different way in the search for a new cross-segment:

- From one edge of the road image, we pick a point, and we say that this point is the image of one end of our yet unknown cross-segment; this end is somewhere on the line between the camera focal point and the chosen image point.
- From the other side of the road image we consider the segments joining consecutive points of the road side image. The image of the other end of the world road cross-segment must belong to one of these image segments: we constrain this end to belong to one of the planar sectors defined by the focal point and the image segments.

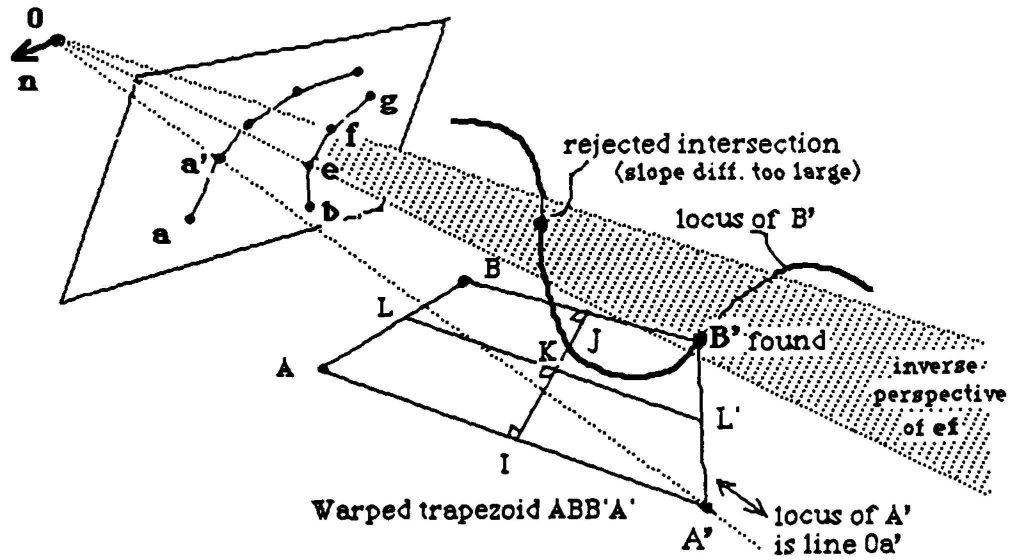

Figure 10

The inverse perspective of an image point a' of one road image edge (Figure 10) is the line Oa' (O is the focal point). The inverse perspective of a segment ef of the other road image edge is the planar facet between the lines Oe and Of, and the inverse perspective of the whole edge is the polyhedral cone defined from the facets corresponding to each segment. One end, A', of the road cross-segment A'B' belongs to the line Oa'; the other end, B', has to belong to the polyhedral cone.

If A' belongs to line Oa', vector **A'** can be expressed as a function of a single parameter m:

$$\mathbf{A'} = m\,\mathbf{a'} \qquad (10)$$

The previous section showed that vector **B'** is defined as a function of **A'** by

$$\mathbf{B'} = \mathbf{A'} - \mathbf{AB} + 2\,[(\mathbf{AB} \cdot (\mathbf{V} \times \mathbf{AA'}) / (\mathbf{V} \times \mathbf{AA'})^2]\,(\mathbf{V} \times \mathbf{AA'}) \qquad (9)$$

Here A and B are the end points of the previous cross-segments and have known coordinates, and **V** is the unit vertical vector. Thus **B'** is a function of a single parameter, m, and the locus of the point B' is a space curve. Among the points of this curve, the points which are acceptable as possible ends of the new cross-segment are the points whose images are on the road edge image. An acceptable point B' is an intersection of the space curve locus and one of the planar facets defined by the segments of the edge image and the camera focal point. We do not need to calculate this locus; we need only to calculate the parameter m corresponding to these intersections. This can be done as follows:

If B' belongs to a planar facet such as defined by the image segment ef, we can write that the vector **B'** is normal to the normal to a normal **n** to this plane:

$$\mathbf{n} = \mathbf{e} \times \mathbf{f}, \qquad \mathbf{B'} \cdot \mathbf{n} = 0, \qquad (11)$$

We must specify that B' is not only in the plane, but on the limited sector between the lines Oe and Of, and in front of the camera. These two conditions are expressed by

$$(\mathbf{B'} \times \mathbf{e}) \cdot (\mathbf{B'} \times \mathbf{f}) < 0, \quad (\mathbf{B'} \cdot \mathbf{e}) > 0 \qquad (12)$$

When both sides of equation (9) are multiplied by **n** (dot product), a scalar equation is obtained, and the left end side is 0, since **B'** . **n** = 0 (eq. 11).

Replacing **A'** by m**a'**, a cubic equation in the parameter m is obtained:

$$a_3 m^3 + a_2 m^2 + a_1 m + a_0 = 0 \qquad (13)$$

and the four coefficients are found to be

$$a_3 = (\mathbf{a'} \cdot \mathbf{n})(\mathbf{V} \times \mathbf{a'})^2$$

$$a_2 = -2(\mathbf{a'} \cdot \mathbf{n})((\mathbf{V} \times \mathbf{a'}) \cdot (\mathbf{V} \times \mathbf{A})) - (\mathbf{AB} \cdot \mathbf{n})(\mathbf{V} \times \mathbf{a'})^2 + 2(\mathbf{n} \cdot (\mathbf{V} \times \mathbf{a'}))(\mathbf{AB} \cdot (\mathbf{V} \times \mathbf{a'}))$$

$$a_1 = (\mathbf{a'} \cdot \mathbf{n})(\mathbf{V} \times \mathbf{A})^2 + 2(\mathbf{AB} \cdot \mathbf{n})((\mathbf{V} \times \mathbf{a'}) \cdot (\mathbf{V} \times \mathbf{A})) - 2(\mathbf{n} \cdot (\mathbf{V} \times \mathbf{a'}))(\mathbf{AB} \cdot (\mathbf{V} \times \mathbf{A})) - 2(\mathbf{n} \cdot (\mathbf{V} \times \mathbf{A}))(\mathbf{AB} \cdot (\mathbf{V} \times \mathbf{a'}))$$

$$a_0 = (\mathbf{AB} \cdot \mathbf{n})(\mathbf{V} \times \mathbf{A})^2 + 2(\mathbf{n} \cdot (\mathbf{V} \times \mathbf{A}))(\mathbf{AB} \cdot (\mathbf{V} \times \mathbf{A})) \qquad (14)$$

The real roots (one or three) of the equation are easily obtained analytically. These values of m define the possible positions of the point A' on line Oa' (from condition (10)) such that B' satisfies condition (9) and condition (11). Once A' is found, B' is uniquely determined by (9). For the pair(s) (A'B') given by the root(s) m of (14), only the pair(s) for which B' satisfies (12) is (are) valid. If more than one point B' satisfies this criterion (12), only the point B' such that the corresponding road direction has the minimum reasonable slope difference with respect to the previous road direction is kept.

These conditions reduce to one or none the number of acceptable solutions for A'B' for the image segment ef being considered. If no acceptable solution is found with the image segment ef, the next image segment, fg, is used. Only a small number of such image segments (2 to 4) usually need to be considered. If, after a limited number of segments are examined, no solution is found, this may be due to the fact that the inverse perspective of a road image with polygonal sides cannot have a smooth constant width. The length of A'B' is increased by a small percentage, and the search process is started again with the same segments.

6. Acquisition of an initial cross-segment

Since the process described above is iterative and uses a previously calculated cross-segment, an initial cross-segment is needed to start the algorithm for each new image frame. This initial cross-segment can be calculated from a "flat earth assumption". In most autonomous systems, the position of the camera is known with respect to the road patch above which it operates; this initial road patch is assumed to be planar, and the equation of its plane can be calculated in the camera coordinate system as a function of the camera height and tilt (it is invariant to camera pan). With the proper camera tilt and camera field of view, the line segments of the image road edges at the bottom of the image correspond to world road edges which are close to the camera, and belong to this plane. These world segments are obtained by projecting the bottom image segments of the left and right road edge image on this plane (Figure 11).

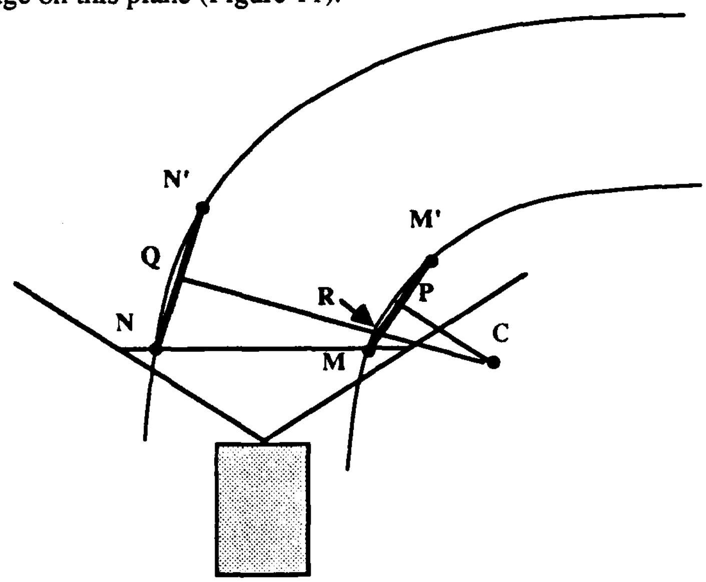

Figure 11

MM' and NN' are the two world segments corresponding to the lowest image segments of the image. Their midpoints are P and Q. One of the ends of the initial cross-segment is taken to be one of these two midpoints, here Q, and its direction is the normal to the corresponding edge segment, NN', in the ground plane. If MM' and NN' are found to be parallel (within acceptable limits), the other end of the initial cross-segment is taken to be the intersection R of this normal with the other edge segment, MM'. If the two edge segments are not parallel, as can happen in a turn (Figure 11), a kind of center of curvature C of the turn is found as the intersection of the normals to the midpoints of the two segments, and the width w of the road is calculated as the difference between CQ and CP. Then the endpoint R of the initial cross-segment is taken at a distance w from its endpoint Q.

7. Experimental evaluation of the proposed zero-bank algorithm

For experimental evaluation of the proposed method, a world road is given analytically. The centerline describes an "S" on a hill or in a valley, and the road surface is defined by horizontal cross-segments cutting the centerline at their midpoints. The analytic expressions for the edges are derived from the following specifications:

The road centerline projects on the horizontal plane as a straight part of length 0.5 units, followed by a 90 degree right turn with radius 1, then a 90 degree left turn with radius 1. The road centerline is on a hill or in a valley described by one period, trough to trough or peak to peak, of a *sinusoidal cylinder.* The road edges are deduced from the centerline by taking distances of 0.2 units on each sides of horizontal normals to the centerline curve. The slope starts at the beginning of the road(straight part) and changes sign when the road is at 90 degrees from its starting direction. On Figures 12 to 16, a top view (center) and a side view (right) of this road in orthogonal projection are drawn on the right.

Five slope configurations were tested:

a. Horizontal road.

b. Hill height 0.1.

c. Hill height 0.2.

d. Valley depression 0.1.

e. Valley depression 0.2.

The slope changes can be seen on the side view of the road, at the extreme right of Figures 12 to 16.

The perspective image of this road is computed, assuming that the image plane is vertical, and that the focal point is at a height 1, at a horizontal distance 0.2 from the start of the road. It is shown on the left of Figures 12 to 16.

The algorithm was run on this perspective image, and the world road that the algorithm reconstructed is shown in top view and side view for comparison with the actual road. For debugging purposes, the images of the inverse perspectives of the cross-segments are also shown on the road image (their end points must be on the road edge images).

Three algorithms were tested:

- The flat earth assumption algorithm
- A vanishing point method
- The proposed zero-bank road algorithm

The flat earth assumption algorithm is the simplest. The camera is assumed to look at a flat world. In the case of a camera on a vehicle, the plane of the world is an extension of the planar patch supporting the vehicle at the time of image acquisition, and is calculated from the camera height and tilt. The central projection of the edges of the road image on this plane is assumed to give the edges of the world road.

A vanishing point method was briefly described in [1]. In the introduction, we described how such a method can construct two parallel segments in space from two image segments (Figure1), and we mentioned that one of the problems was the choice of segments in the persective image which are images of opposite segments (segments containing opposite points of the road). The implementation tested here was based on [1] and on discussions with Eli Liang. It uses the world segments calculated at the previous iteration step to deduce the positions of the images of opposite segments at the new step. At an intermediate iteration step, once two parallel world segments are found, a plane normal to these segments is calculated so that it goes through the far end of one segment and cuts the other segment. The end of one segment and the intersection of the normal plane with the other are assumed to form a pair of opposite points, and the segments in the image which follow the images of these two points are assumed to be images of parallel segments and are used in the next iteration step. This approximation therefore takes opposite points where they can be calculated (from the road element just calculated) and assumes that the segments coming just after the images of these opposite points on the perspective image are images of parallel segments. As discussed in the next section, this approximation is not very satisfactory.

8. Qualitative evaluation of the results

The flat earth assumption algorithm performs better than the proposed method on flat ground (Figure 12). For hill or valley configurations (Figures 13 to 16), the proposed zero-bank road algorithm gives a much more correct inverse perspective of the road than the other two algorithms. The drawings also show how far from the truth the flat earth assumption can be as soon as there are some slope changes in front of the camera.

The vanishing point method attempts to construct a developable ribbon. As a result it underestimates the first turn. Further down the road, the catastrophic deterioration of its reconstruction is due to the added errors from the approximation for finding the images of parallel segments .

9. Concluding remarks

Modeling roads as Brooks ribbons in space with horizontal generators of constant length, we have shown that two neighboring generators form approximately a warped isosceles trapezoid. It follows that when one generator is known, the next generator can be recovered completely if one of its ends is given. On the basis of this observation, we proposed an iterative algorithm for reconstructing the a world road from its perspective image. At an intermediate step of the iteration, a previous generator is known, and a point on one road edge image is chosen as the image of one end of the next generator. This end has only one degree of freedom along the line inverse perspective of its image. From our previous observation, the other end also has only one degree of freedom. Therefore its locus is an arc and depends on one parameter. We calculate the parameters such that this other end has its image in the image of the other edge of the road, as solutions of cubic equations. When more than one solution occurs, we pick the appropriate generator on the basis of minimal change of slope of the resulting road.

We tested this algorithm on a road image synthesized from an analytic description of a road describing a "S" , on a hill and in a valley. With this algorithm the results of the reconstruction are much closer to the original road than with two other algorithms chosen for comparison, an algorithm based on a flat earth assumption, and an algorithm based on vanishing points. In the introduction, we summarized the limitations of vanishing point methods for road inverse perspective.

Our next goal is to test this algorithm on images of real roads and to check whether in natural conditions warped isosceles trapezoids perform better than planar patches.

Acknowledgements

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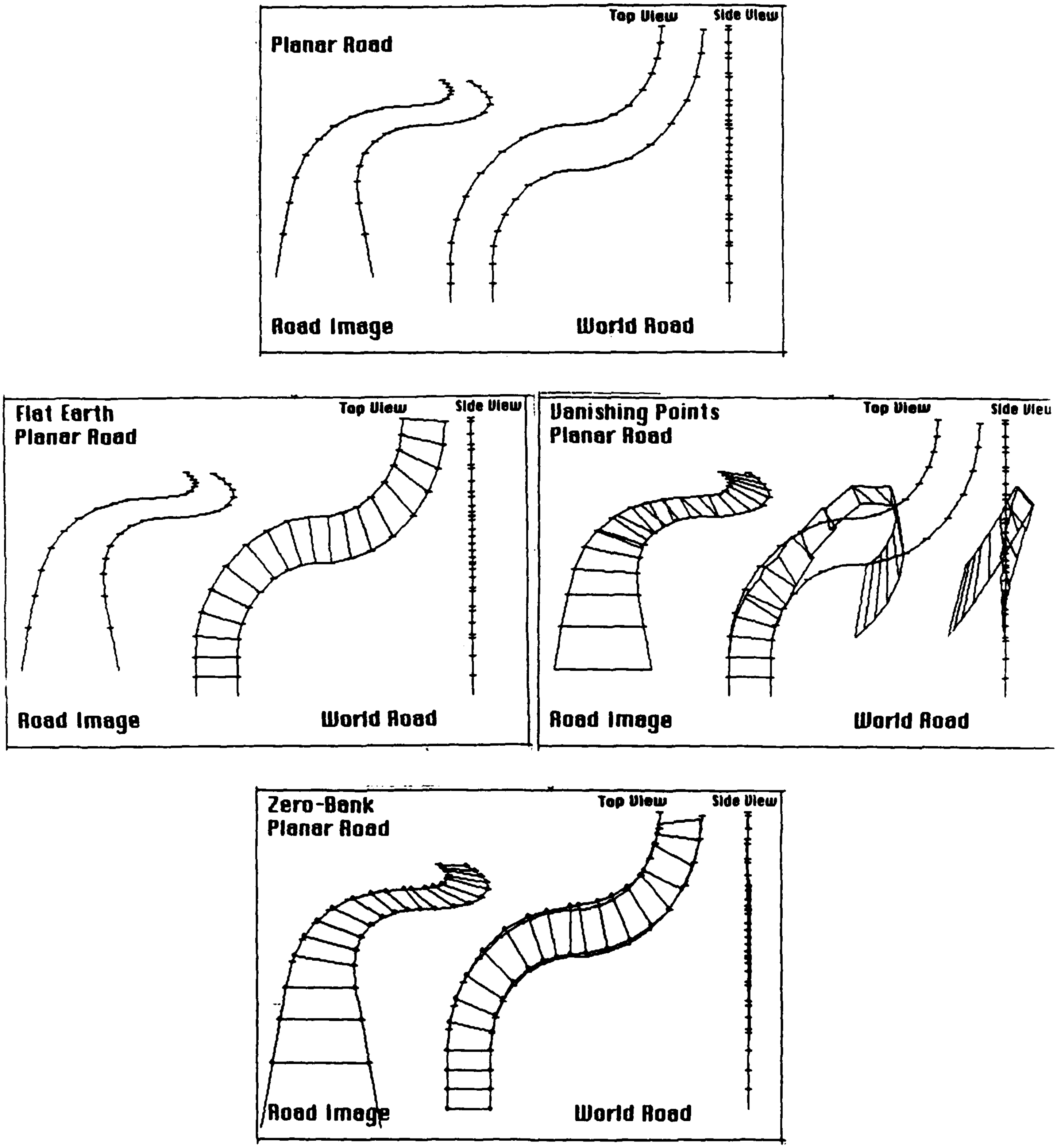

Planar Road
Top View
Side View
Road Image
World Road
Flat Earth
Planar Road
Top View
Side View
Road Image
World Road
Vanishing Points
Planar Road
Top View
Side Vieu
Road Image
World Road
Zero-Bank
Planar Road
Top View
Side View
Road Image
World Road

Figure 12

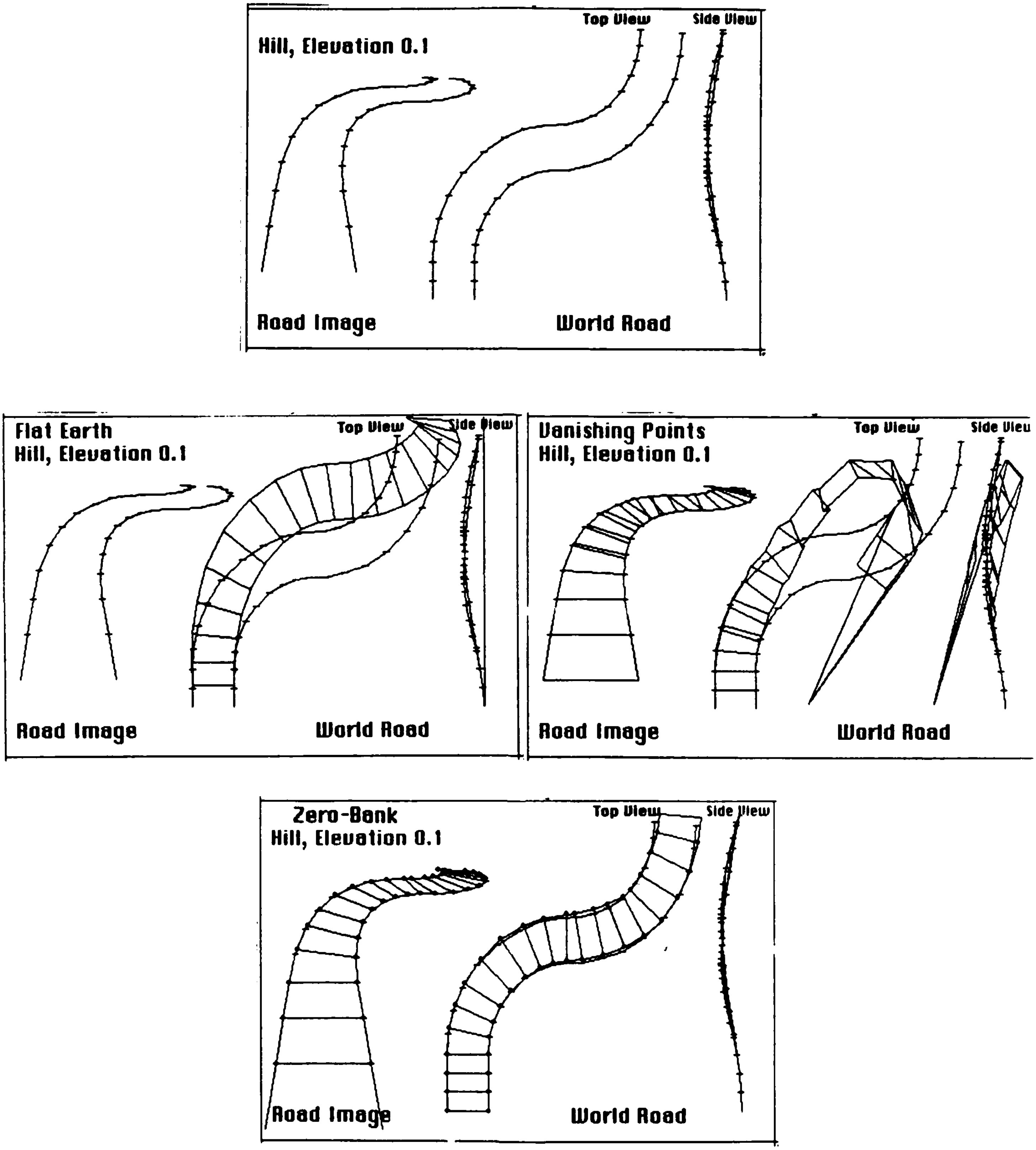
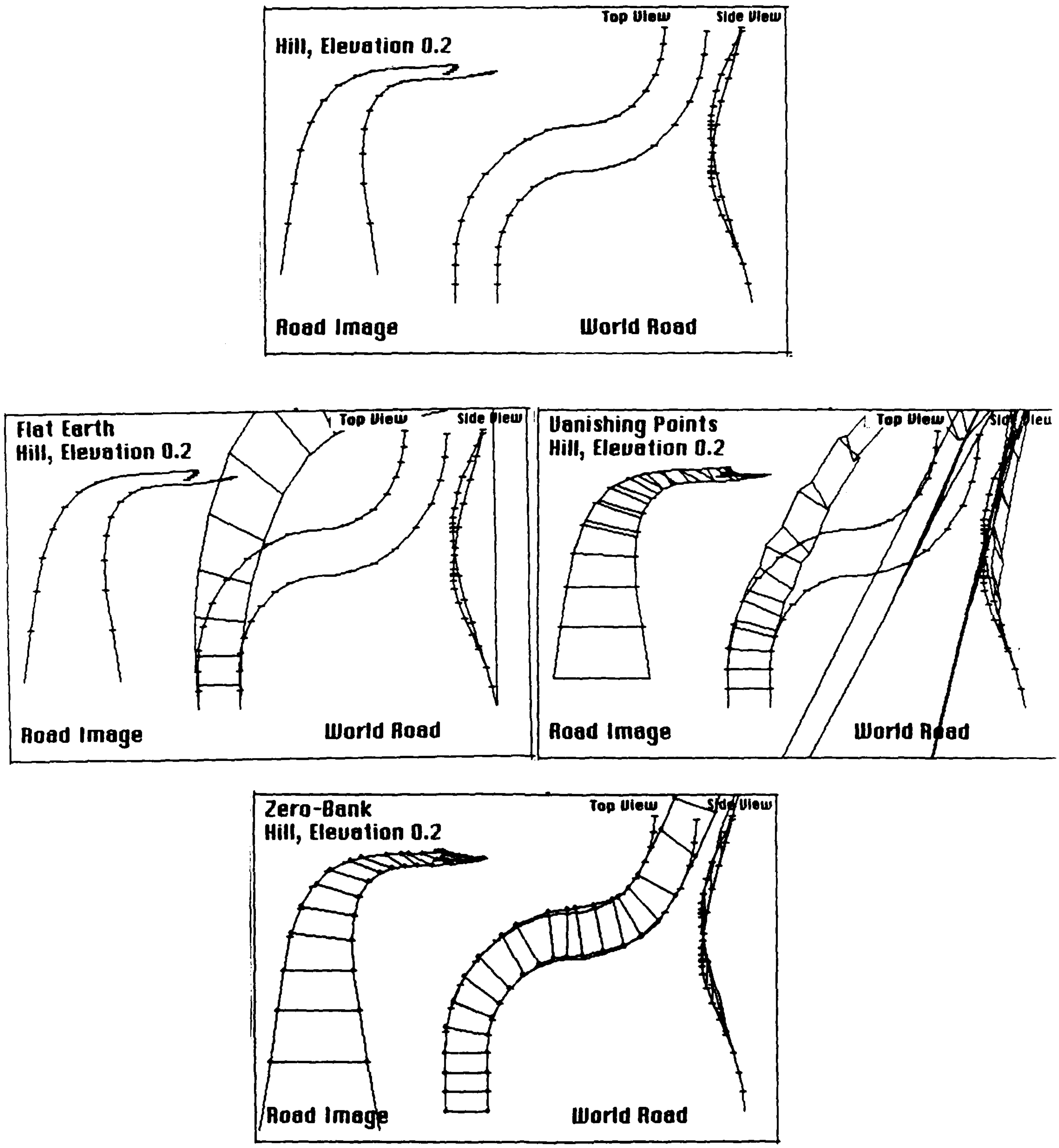

Hill, Elevation 0.1
Top View
Side View
Road Image
World Road
Flat Earth
Hill, Elevation 0.1
Top View
Side View
Road Image
World Road
Vanishing Points
Hill, Elevation 0.1
Top View
Side View
Road Image
World Road
Zero-Bank
Hill, Elevation 0.1
Top View
Side View
Road Image
World Road

Figure 13



Top View
Side View
Hill, Elevation 0.2
Road Image
World Road
Flat Earth
Hill, Elevation 0.2
Top View
Side View
Road Image
World Road
Vanishing Points
Hill, Elevation 0.2
Top View
Side View
Road Image
World Road
Zero-Bank
Hill, Elevation 0.2
Top View
Side View
Road Image
World Road

Figure 14

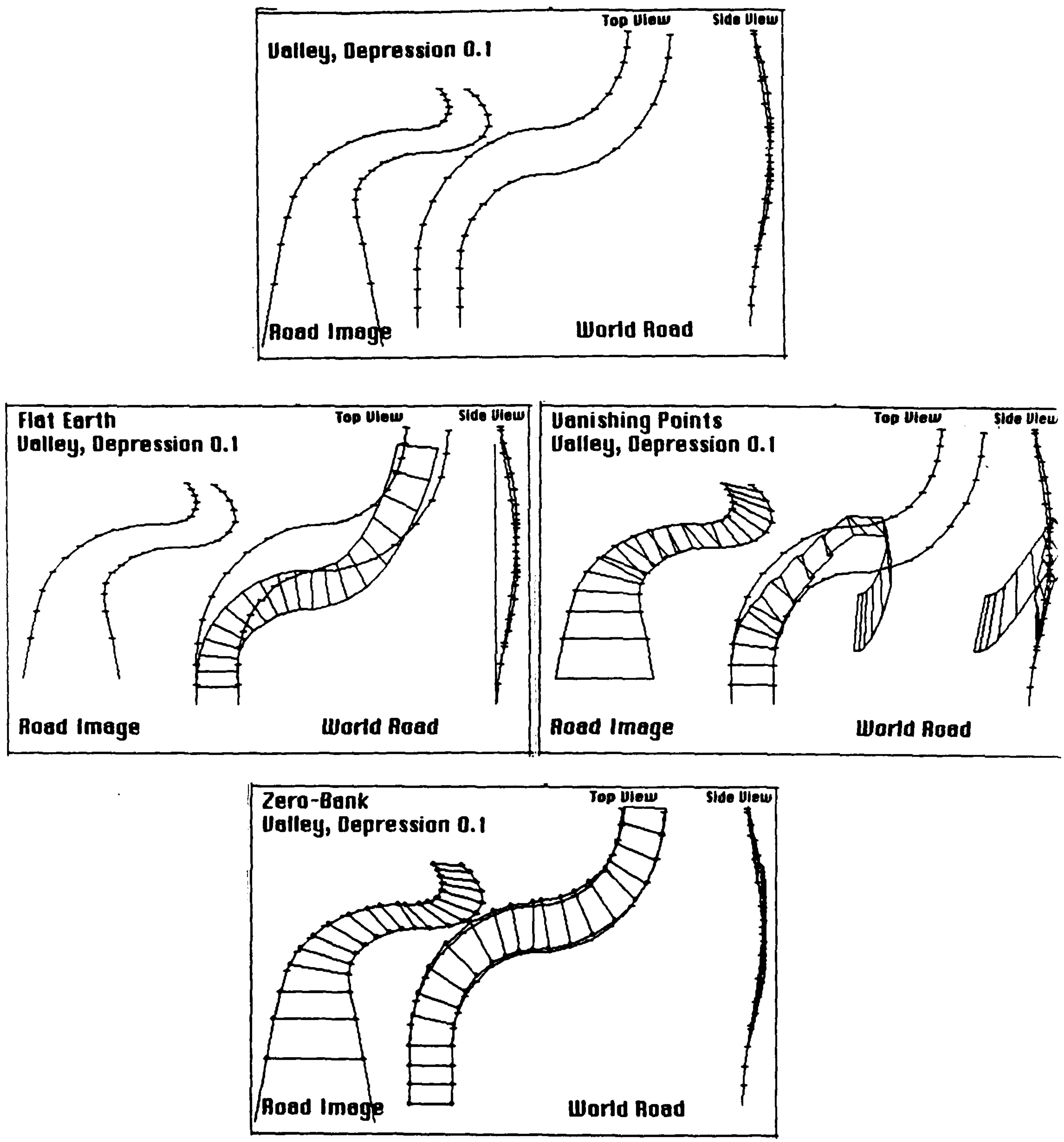

Valley, Depression 0.1
Top View
Side View
Road Image
World Road
Flat Earth
Valley, Depression 0.1
Top View
Side View
Road Image
World Road
Vanishing Points
Valley, Depression 0.1
Top View
Side View
Road Image
World Road
Zero-Bank
Valley, Depression 0.1
Top View
Side View
Road Image
World Road

Figure 15

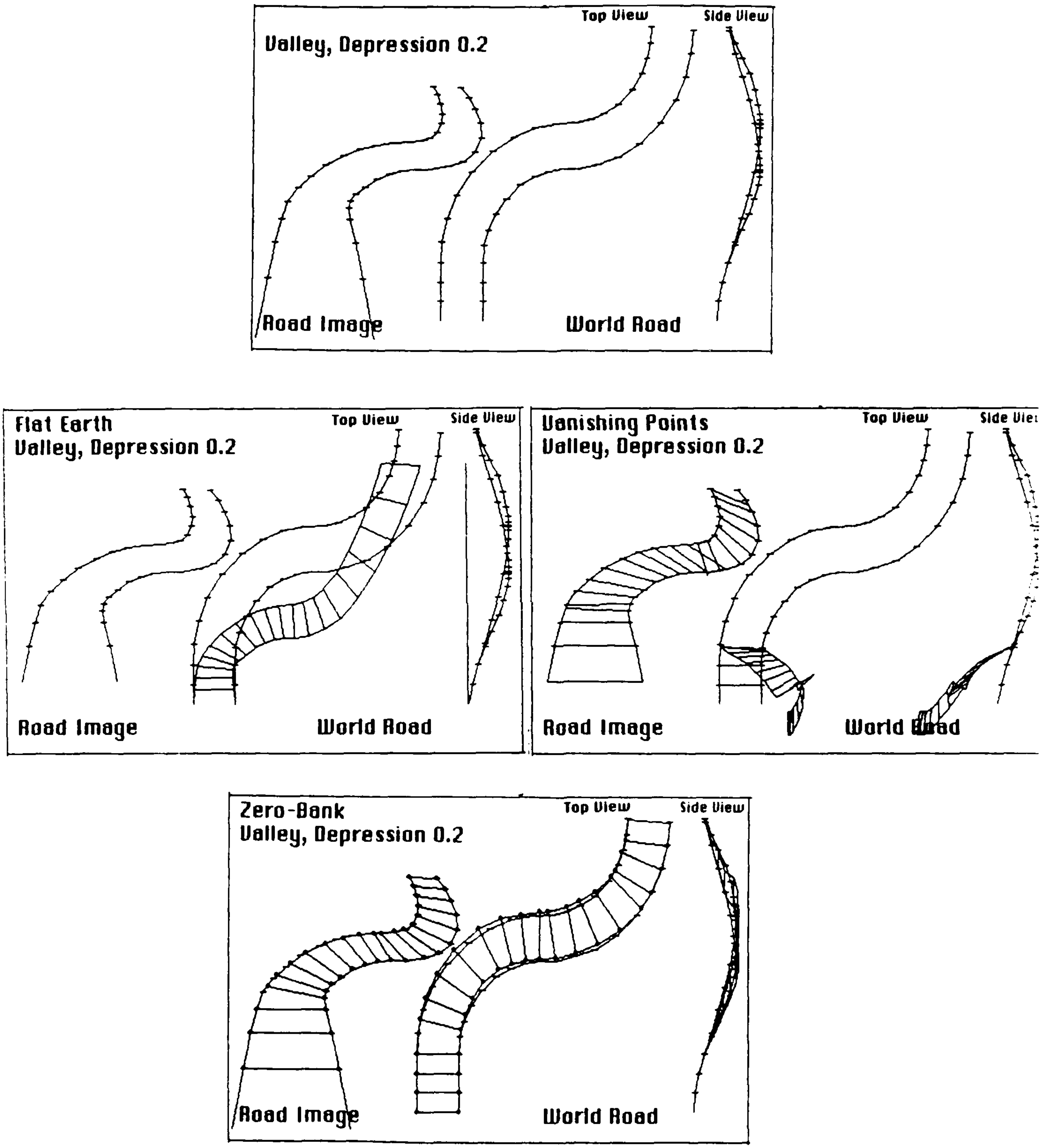

Valley, Depression 0.2
Top View
Side View
Road Image
World Road
Flat Earth
Valley, Depression 0.2
Top View
Side View
Road Image
World Road
Vanishing Points
Valley, Depression 0.2
Top View
Road Image
Zero-Bank
Valley, Depression 0.2
Top View
Side View
Road Image
World Road

Figure 16