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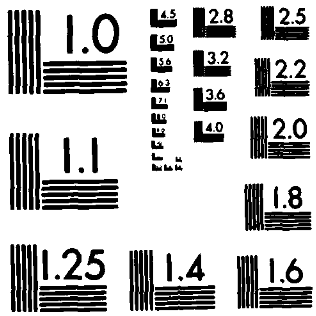
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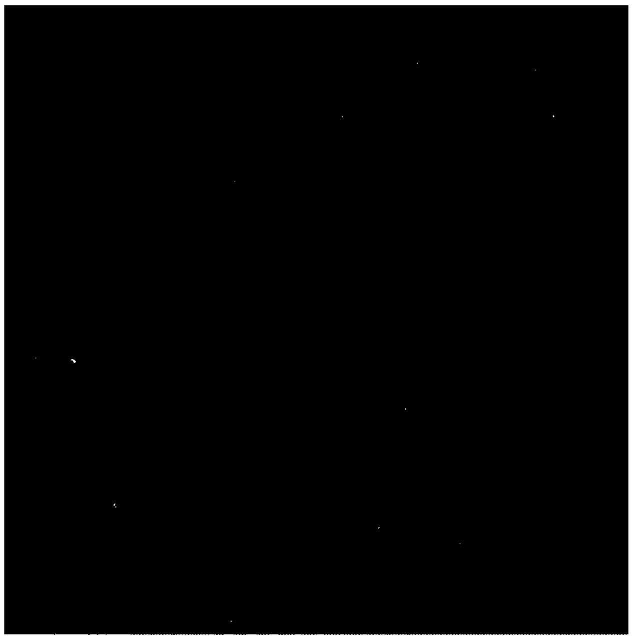

ROYAL SIGNALS AND RADAR ESTABLISHMENT

Memorandum 3922

TITLE: ON SUPER-RESOLUTION AND THE MUSIC ALGORITHM

AUTHOR: G D de Villiers

DATE: May 1985

SUMMARY

Simulation results for phased array signal processing using the MUSIC algorithm are presented. The model used is more realistic than previous ones and it gives an indication as to how the algorithm would perform for the case of low signal/noise ratio. Strengths and shortcomings of the method are discussed.

CONTENTS

ON SUPER-RESOLUTION AND THE MUSIC ALGORITHM

1. INTRODUCTION

At present there is a considerable amount of interest in "high-resolution" algorithms in radar signal processing. One of the more popular of these algorithms is the MUSIC algorithm of Schmidt [1,2]. In this paper simulation results for this algorithm are presented and discussed. Curves of "resolution" against number of snapshots for varying signal to noise ratios are shown. Shortcomings of the method are also displayed.

Super-resolution in radar signal processing is normally taken to mean the following: If one can distinguish two signals within a beamwidth (as defined by half the distance between the nulls either side of the central maximum) then super-resolution has taken place. Strictly speaking the word super-resolution in the above sense should not be applied to cases with multiple snapshots as the classical resolution problem is only phrased for a single snapshot. With one snapshot from a phased array only classical resolution is possible but with many snapshots high "resolution" may be achieved if the signals have different statistical behaviour in the time domain. In this sense we shall see that one may distinguish two point sources well within a beamwidth.

The problem of resolving two point sources with a phased array radar is a notoriously complicated one. This can be seen from the wealth of papers in the literature all containing different results. The complexity is due to the large number of variables. The most important ones are:-

(1) Shape of array - one or two dimensional
(2) Number of elements in array
(3) Spacing of elements in array
(4) Form of array response
(5) Direction and type (random or deterministic) of incoming signals
(6) Form of noise (whether correlated, what distribution it comes from)
(7) Number of samples in time domain
(8) Signal/noise ratio
(9) Spacing of samples in time domain.

In this paper most of these variables have been fixed in order to demonstrate some important features of the method. The following variables have been fixed: the array shape and response, the form of the incoming signals in the time domain (pure sinusoids of variable frequency), the noise distribution (Rayleigh) and the spacing of samples in the time domain. Clearly there could be many interesting points from varying these variables but time dictates that a choice must be made.

In the following section the model used is discussed in detail.

Section 3 is concerned with the MUSIC algorithm and the way in which it was implemented.

Section 4 contains the results of the simulation and a discussion thereof.

Finally Section 5 contains a discussion of previous work in the field and conclusions.

The appendix contains some comments on the concept of resolution.

2. THE MODEL

The model array we use is an eight element linear equally spaced array with inter-element spacing of $d = \lambda/2$, λ being the carrier wavelength. The array response we have chosen is of the form

$$a(\theta_i) = \begin{pmatrix} \exp i1 \frac{2\pi d}{\lambda} \sin\theta_i \\ \exp i2 \frac{2\pi d}{\lambda} \sin\theta_i \\ \vdots \\ \exp i8 \frac{2\pi d}{\lambda} \sin\theta_i \end{pmatrix}$$

which, for $d = \lambda/2$ becomes

$$a(\theta_i) = \begin{pmatrix} \exp i\pi \sin\theta_i \\ \exp i2\pi\sin\theta_i \\ \vdots \\ \exp i8\pi\sin\theta_i \end{pmatrix}$$

Here θ is the angle of arrival of the signal with respect to a horizontal normal to the plane containing the array. For signals we have chosen pure complex sinusoids of equal strength but differing phase and frequencies

$$F_j(t) = \sigma e^{i\nu_j t} e^{i\phi_j}$$

where σ is a real constant for all signals.

The frequencies were all chosen to lie within a few percent of each other. For demonstrating the effect of multipath two of the frequencies were made equal. The justification for using sinusoids for incoming signals was that this was a major simplification from using stochastic signals which would hopefully make the results easier to understand. We chose a time sampling rate of about one sample per hundred cycles of the modulating wave for one of the signals. This ensured that (if one had thought of the process as stochastic) the signals could be statistically separated by forming covariance matrices, for a suitable number of time snapshots.

In the way we have set up the problem the noise is sampled independently of the signals plus noise so as to obtain an estimate of the noise covariance matrix. This is necessary if one is to use the MUSIC algorithm although in the past papers have appeared in which this was not done.

The noise was taken to be additive Rayleigh noise given by

$$Wi = \sqrt{(1 - \text{random1})}\, e^{i2\pi(\text{random2})}$$

Where randoms 1 and 2 are different calls of random number generator, producing a uniform distribution over [0, 1]. The above form for the noise has a mean of $1/\sqrt{2}$.

For part of the work we also incorporated a jammer into the problem. This was taken to be a point source with random time behaviour. The situation for which the MUSIC algorithm would be used would consist of signals being transmitted in pulses with periods of no signal in between. The noise only covariance matrix would be sampled during these quiet periods. However a jammer would transmit during the quiet periods and hence would be seen as correlated noise, which is how it was modelled.

3. METHOD

The algorithm was implemented on the ICL 1906 in Fortran 77 using various NAG Library routines. The relevant part of Schmidt's algorithm for direction finding and resolution is as follows:-

The data is modelled by

$$\begin{pmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_8(t) \end{pmatrix} = \left(a(\theta_1) \,\vdots\, a(\theta_2) \,\vdots \cdots\cdots \vdots\, a(\theta_D) \right) \begin{pmatrix} F_1(t) \\ F_2(t) \\ \vdots \\ F_D(t) \end{pmatrix} + \begin{pmatrix} W_1(t) \\ W_2(t) \\ \vdots \\ W_8(t) \end{pmatrix}$$

Here $a(\theta)$ is given by the expression in the previous section, D is the number of incoming signals, $F_j(t)$ is the jth incoming signal as given in the previous section and $W_i(t)$ is Rayleigh noise of unit mean, the random numbers being generated by the NAG Fortran 77 library routine GØ5CAF. We define the signal/noise ratio to mean

$$\frac{\sqrt{\text{mean (signal squared)}}}{\sqrt{\text{mean (noise squared)}}}$$

We choose to keep this as a simple ratio rather than expressing it in decibels. From the forms of signal and noise we may see that the signal/noise ratio is of the order of $D\sigma$ where D is the number of incoming signals. The graphs in the results section are all labelled by σ. This is easily converted, as above, into a rough signal/noise ratio.

Writing out $X_k(t)$ explicitly

$$X_k(t) = W_k(t) + \sigma \sum_{\substack{\text{sources} \\ j}} e^{i\nu_j t} e^{i\phi_j} e^{ik\pi \sin\Theta_j}$$

Discretising the time $t=t_n$ and representing $X_k(t_n)$ (the data at the kth array element at time t_n) by X_k^n we may form the covariance matrix

$$S_{ij} \stackrel{def}{=} \frac{1}{\text{No of time samples N}} \left(\sum X_i^n X_j^n \right)$$

In our case the noise and signals are asymptotically uncorrelated so we may write

$$\underset{\sim}{S} = \underset{\sim}{S}' + \underset{\sim}{S}_o'$$

in the asymptotic limit, where $\underset{\sim}{S}'$ corresponds to the signals and $\underset{\sim}{S}_o'$ to the noise. As stated before, crucial to the MUSIC algorithm is that one must be able to sample the noise independently of the signals plus noise in order to form an estimate of the noise covariance matrix $\underset{\sim}{S}_o'$.

Following Schimdt we then solve the generalised eigenvector/eigenvalue problem:

$$\underset{\sim}{S} \vec{e}_i = \lambda_i \underset{\sim}{S}_o \vec{e}_i$$

where $\underset{\sim}{S}_o \stackrel{def}{=} \frac{8}{TrS_o'} \times \underset{\sim}{S}_o'$

is the noise covariance matrix normalised such that its trace is equal to the number of array elements.

Once we have obtained the eigenvalues λ_j we may look at their relative magnitudes in order to decide on the number of incoming signals. As will be seen from the results this is impossible if multipath is present. However, in the absence of multipath one selects the largest eigenvalues to correspond to signals and the remaining eigenvalues are chosen to correspond to noise. The justification for this may be found in Schmidt [1], [2]. In most of the cases dealt with in this paper the separation of the eignevalues into two classes was easy as the signal eigenvalues were an order of magnitude higher than the noise ones.

The solution of the generalised eigenvalue problem was obtained using the NAG Fortran library routine FO/2GJF. This routine uses the QZ algorithm to solve

$$\underset{\sim}{A}x = \lambda \underset{\sim}{B}x$$

Where $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are complex square matrices. Unfortunately there were no routines available for solving the problem using Hermitian matrices but the above routine performed more than adequately as shown by the fact the imaginary parts of the eigenvalues so found were roughly a factor of 10^{-10} down on the real parts.

The final part of Schmidt's algorithm is to project a(Θ) against the matrix of noise eigenvectors to find the zeroes. These correspond to a(Θ) lying in the signal subspace.

4. RESULTS

In Figure 1 are plotted graphs of resolution against number of snapshots for various values of σ - the signal strength as defined earlier. The values of σ chosen are smaller than one normally finds in studies of such algorithms. This is because the process is essentially statistical and a high signal/noise ratio will not highlight the random nature of the reconstruction. The curves have been drawn in the way shown to indicate the best resolution there is evidence for in the data. The points lying below their corresponding curves are due to large values of noise at around forty snapshots. This was unfortunate but it does emphasize an important flaw with simulations. This is whether the noise realisation for a given run is in any sense typical or not. Clearly the best way to produce such curves is to do a whole set of runs with different seeds for the random number generator and then to average the results. This was not done because of the prohibitive amount of computing time required. Error bars were omitted from the points to make the graph easier to understand. If one takes the fourth or fifth point on each curve, treats its distance from the curve as the error at that point and then observes that there is an effective $1/\sqrt{N}$ noise reduction with number of snapshots N, one may be not too far from the truth.

These curves demonstrate that resolution below a beamwidth (= .125 radians for an 8 element $\lambda/2$ spaced linear array) is possible. The beamwidth is indicated in the figure.

An important feature to note is the rapid rise of the curves for small numbers of snapshots. This indicates that for very small numbers of snapshots the resolution is essentially classical but as statistical separation of the signals in the time domain becomes more effective the resolution rises sharply and then flattens out. It would seem that for these signal strengths 25-30 snapshots are all that are required for this particular scenario.

Figure 2 shows a typical plot used to obtain the curves in Figure 1. It consists of $| a^H(\Theta).E_n |^2$ plotted against Θ (in radians) and normalised so that the highest value of the function is unity. For this plot the values of Θ were $\Theta=\pm 0.038$ radians, σ was 3.0 and 40 snapshots were used. In producing the curves of Figure 1 an arbitrary depth of valley between the peaks was selected. For all points in

Figure 1 this depth was very close to that in Figure 2.

Figure 3 shows a typical plot for two sources at $\theta = \pm 0.05$ radians, $\sigma=3.0$ and 50 snapshots. This example is used in the remaining figures to demonstrate the effect of jammers and multipath.

Figure 4 has the same basic scenario as Figure 3 but has an additional signal of equal strength at $\theta=0.15$. It can be seen that the resolution of Figure 3 has been lost completely. This behaviour is typical. With suitable choices of the θ_i's one may reconstruct up to six sources with progressively worse results as each source is added. However as they all need to lie outside a beamwidth from each other this was not thought to be terribly realistic and hence was not included.

Figure 5 consists of the same set up as in Figure 3 but with the time behaviour of the two signals identical. This corresponds to multipath. One may observe that only one peak results. In a handwaving way one may say that no new information about the signals is gained from taking multiple snapshots and the only improvement is due to the effective $1/\sqrt{N}$ noise reduction.

The remaining Figures (6 to 8) again show the scenario for Figure 3 but with a jammer at $\theta=0.5$ radians of strength 1.0, 3.0 and 10.0 respectively. Two things are to be noted from these results.

(i) The presence of the jammer does affect resolution (compare with Figure 3) but not too badly.

(ii) Although the signal for the jammer lies in the "noise subspace" its position is indicated with increasing clarity as its power increases. As yet the author is unaware of a theoretical explanation for this interesting point.

5. CONCLUSIONS

The literature is full of claims that "resolution" of the two point sources within one hundredth of a beamwidth of each other is possible. There is a fallacy here which should be pointed out. These claims all rely on knowing that the object consists of a very small number of point sources. If one possesses this knowledge then the MUSIC algorithm is a good way of finding the positions of these sources. Clearly for arbitrarily small noise one can find these positions with increasing accuracy to the point where one is limited by computer rounding errors. Of course this situation is totally unrealistic and should be treated with contempt.

Another flaw which has crept into the literature is that one may treat the noise covariance matrix as the identity matrix multiplied by a constant (the mean noise power). This can not really be justified if one is attempting to simulate a noise covariance matrix constructed from a few snapshots. Similarly simulated noise must be added to the signals for the signal plus noise covariance matrix otherwise one is back to computer rounding error limited resolution!

In this paper simulation results have been presented which indicate the kind of results one can expect from using the MUSIC algorithm with low signal/noise ratio. These high apparent resolutions result from knowing that there are two point targets. Clearly a very detailed analysis of a set of DFTs of the incoming data would enable one to arrive at the same conclusion as to the position of these sources. However, the MUSIC algorithm presents the data in such a way that interpretation of the result is very easy. It also manages to separate out the effects of a jammer remarkably well. For a discussion on resolution see appendix.

The major drawback if one does not know how many sources are in the field of view is that an estimate of the number must be made from the covariance matrix. This is mainly a psychological problem and if it is not immediately clear that there are m "high" eigenvalues and n "low" ones then one is in trouble. Consequently, in a situation where one knows how many targets there are the MUSIC algorithm is very useful but where there is any doubt the results can be very misleading.

APPENDIX - ON THE MEANING OF RESOLUTION

1. INTRODUCTION

The purpose of this appendix is to clear up some widely held misconceptions concerning the nature of resolution. The literature is full of claims that certain algorithms possess the property of super-resolution. In order to understand these claims it is essential to understand what is meant by "ordinary" resolution. The standard definition of super-resolution is as follows: if one can resolve two point sources which are so close that they cannot be resolved according to the Rayleigh resolution criterion then super-resolution has occurred. A natural starting point, then, is to consider the definition of the Rayleigh resolution criterion (henceforth abbreviated to RRC). [3]

RRC: Given an exact instrument point spread function (psf), two point sources of equal power may be distinguished from one by eye at a glance if the null of the psf corresponding to one source aligns with the central maximum of the psf corresponding to the other source, provided no noise is present on the image.

There are several points arising from this definition.

(1) One must know precisely the psf of the instrument before applying the RRC.

(2) The instrument must have a linear response - ie the resultant image due to two point sources is the sum of the images due to each source individually.

(3) It is difficult to quantify "at a glance" but clearly the more closely an observer examines the image the easier it will be to distinguish two sources from one. Hence if (1) is true one can beat Rayleigh resolution just by closer inspection! Does one then say that the eye can super-resolve?

(4) In order to apply the RRC one must have psf's which have a large central maximum and nulls either side. Sidelobes are not necessary, though they often occur.

(5) The question of noise being present is a difficult one as in a sense the inability to resolve two sources from one source is due to "noise" in the eye/brain. There will always be noise of a thermodynamic nature even in a correctly designed instrument, hence the RRC is rather an idealisation.

The precise way of viewing the RRC is as a hypothesis test between one point source and two point sources of equal power given the knowledge that there are only these two possibilities. Knowledge of the exact psf is also required. The problem of whether one can know that there are only one or two sources present is considered later on.

A better definition of resolution could be the following one:- Given two point sources of equal amplitude with various parameters (positions, frequencies or any other distinguishing characteristics), if all of the individual parameters for the two sources can be demonstrated (by the best parameter estimation method available) to be different then one should say that the sources are resolved. Should any of the individual parameters turn out to be the same then one should say that the sources are not resolved with respect to that particular parameter. If the individual parameters are all identical for the sources then one should say that the sources are not resolvable with respect to any parameter.

In order to understand how the RRC should be applied it is necessary to have a clear view as to what noise means for the problem at hand. Hence this will be considered next.

2. NOISE

For the case of resolution noise is fairly difficult to define. It is clear it should be of a random nature. One can attempt to define it by saying that for a given separation of the two sources noise is any measure of how difficult it is to say that there are two sources - Definition 1. This definition implies that a casual glance at the image by the observer contains more noise than a long observation. This is clearly true but not useful. A better definition is the following one: There is a mean image with random fluctuations around it. These random fluctuations constitute the noise - Definition 2. The "noise" in the eye/brain is then only important as quantifying a suitable resolution criterion. In practice there will always be a certain amount of noise in the sense of definition 2 as uncertainties in the image due to thermodynamical variations will occur. Hence there will always be a physical limit of resolution when two point sources cannot be distinguished from one by any means. This limit, however, is way below the RRC. There is noise (in the sense of definition 1) quantifying how uncertain our knowledge of the psf is and also noise (in the sense of definition 2) due to physical variations in the psf. In what follows we shall adopt definition 2. We now consider problems related to the number of sources present.

3. NUMBERS OF SOURCES

It is a valid question as to when, given the exact psf, one is entitled to view the problem as a hypothesis test between one or two point sources. In the case of noiseless data a single psf could be viewed as arising from an indeterminate number of point sources positioned at the same point. How does this viewpoint affect the resolution problem? Some light may be shed on this by considering an analogous problem of a person confronted by an apple. On being asked how many apples were present they could say there were as indeterminate positive integer number of identical apples present whose centres were coincident. A perverse person could say that they were not even sure that an integer number of apples were present. Clearly the common sense reply would be that one apple was present. In this way the problem can be linked to the problem of defining the number one. In the above case the common sense view is the same as that in the work of Frege [4], Russell and Whitehead [5].

Hence it can be argued that one psf should be thought of as due to one point source. For the case of noisy images if one has several point sources which, given the image, are not in principle resolvable then one is entitled to say, from the previous argument, that there is only one point source present. The phrase "not in principle resolvable" here means that from the image there is no way of telling that there is more than one point source present. These concepts of numbers of sources introduced here seem the simplest and certainly most useful ones.

4. SUPER-RESOLUTION

From the foregoing it should be clear what is meant by the term resolution. It is then possible to consider what is meant by super-resolution. Super-resolution is normally defined to be better than the RRC. Hence, as pointed out before, the eye is capable of super-resolution if one looks at an image closely enough. Resolution is related to how well the brain can recognise a single psf as apposed to two nearly superimposed. In the various "high resolution" algorithms the brain is helped by a computer "improving" the image. Thus resolution is, in a real sense, improved by simply squaring the intensity in an image. Kay and Demeure [6] have shown that the two spectral estimators Principal component Bartlett and MUSIC are very simply related even though one appears to have far better resolution than the other. Clearly what this says is that, for a relatively noise free situation resolution is governed by the ability of the mind to interpret correctly what it sees. This is certainly in line with Rayleigh's original work on the subject.

ACKNOWLEDGEMENTS

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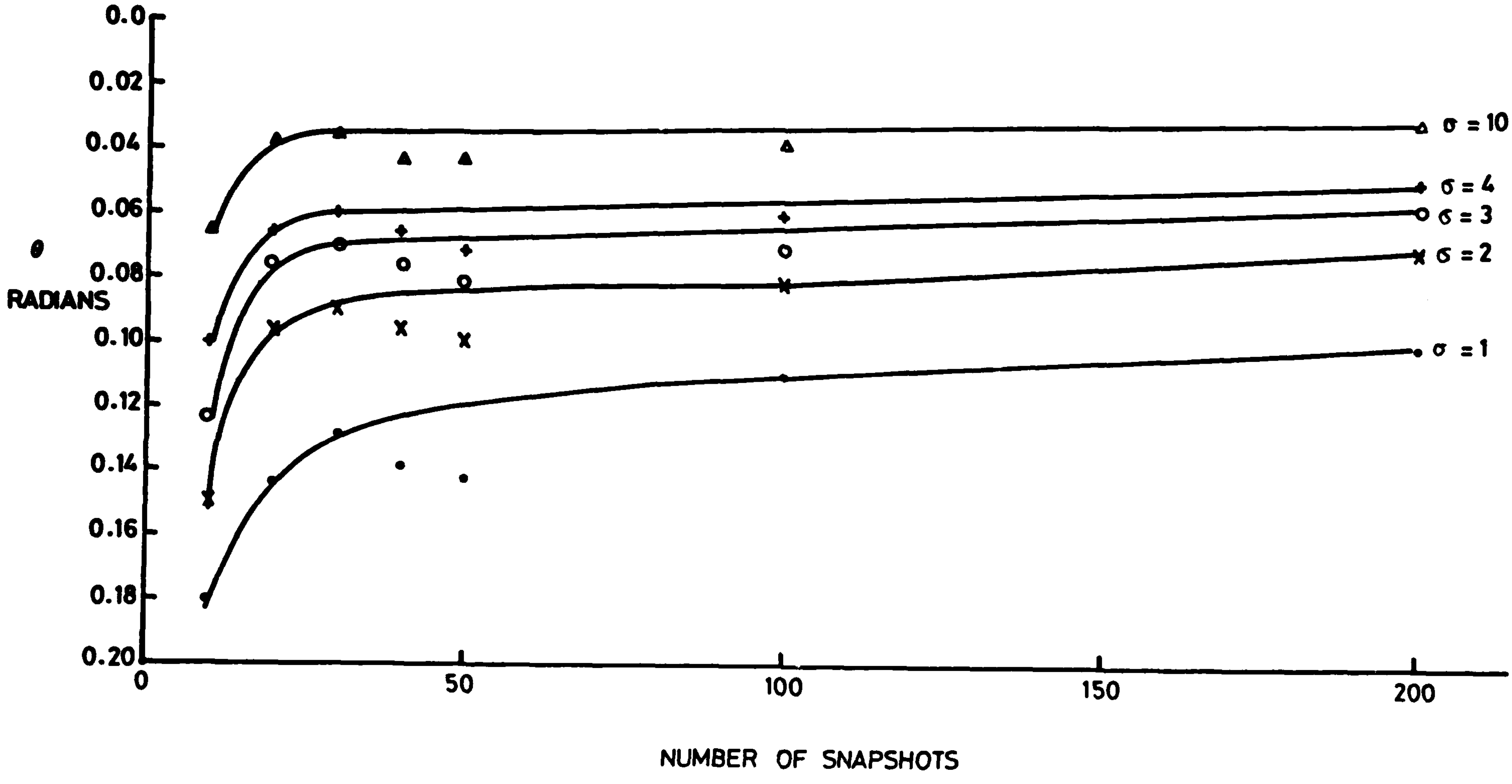
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RADIANS
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NUMBER OF SNAPSHOTS
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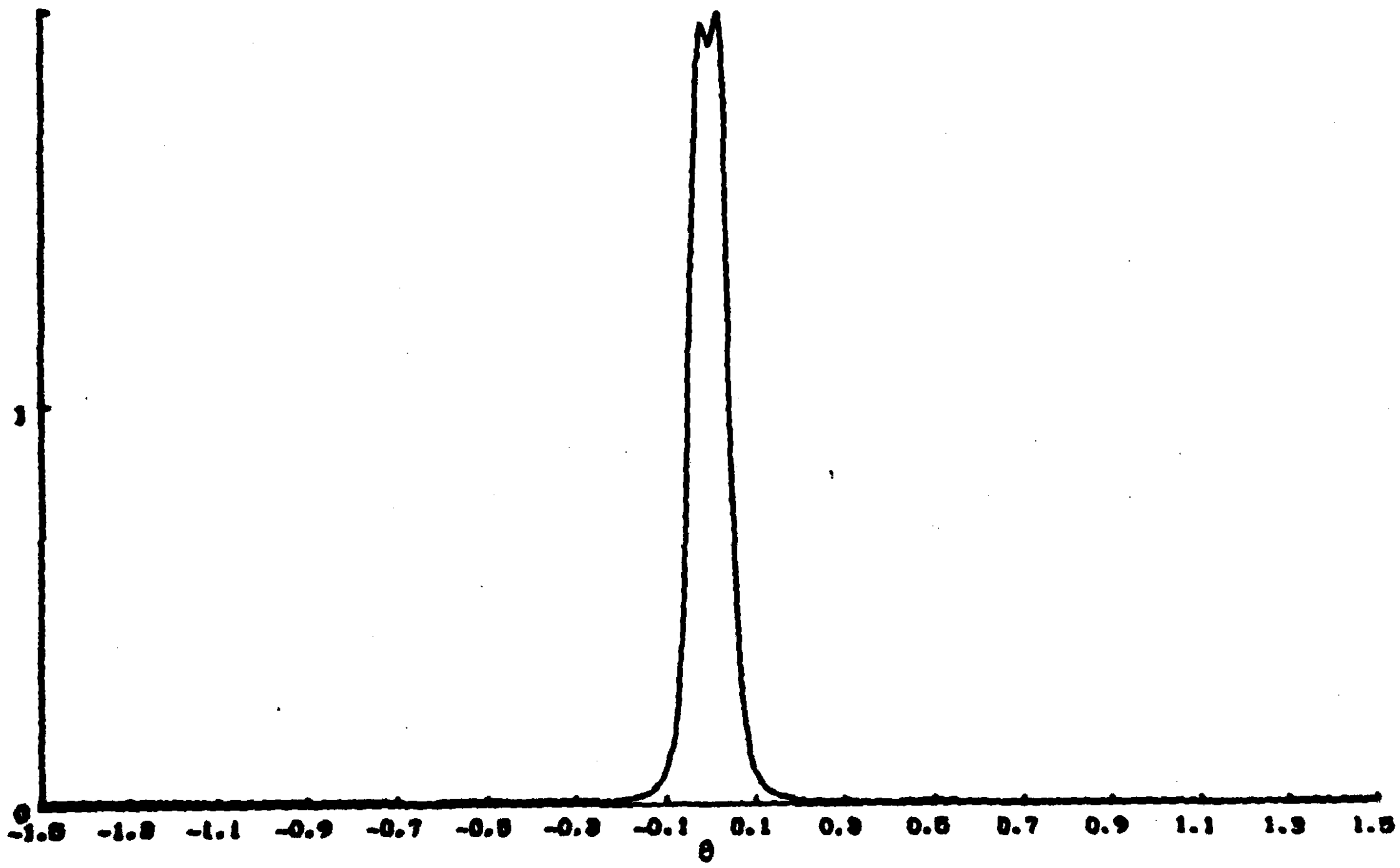
FIGURE 1



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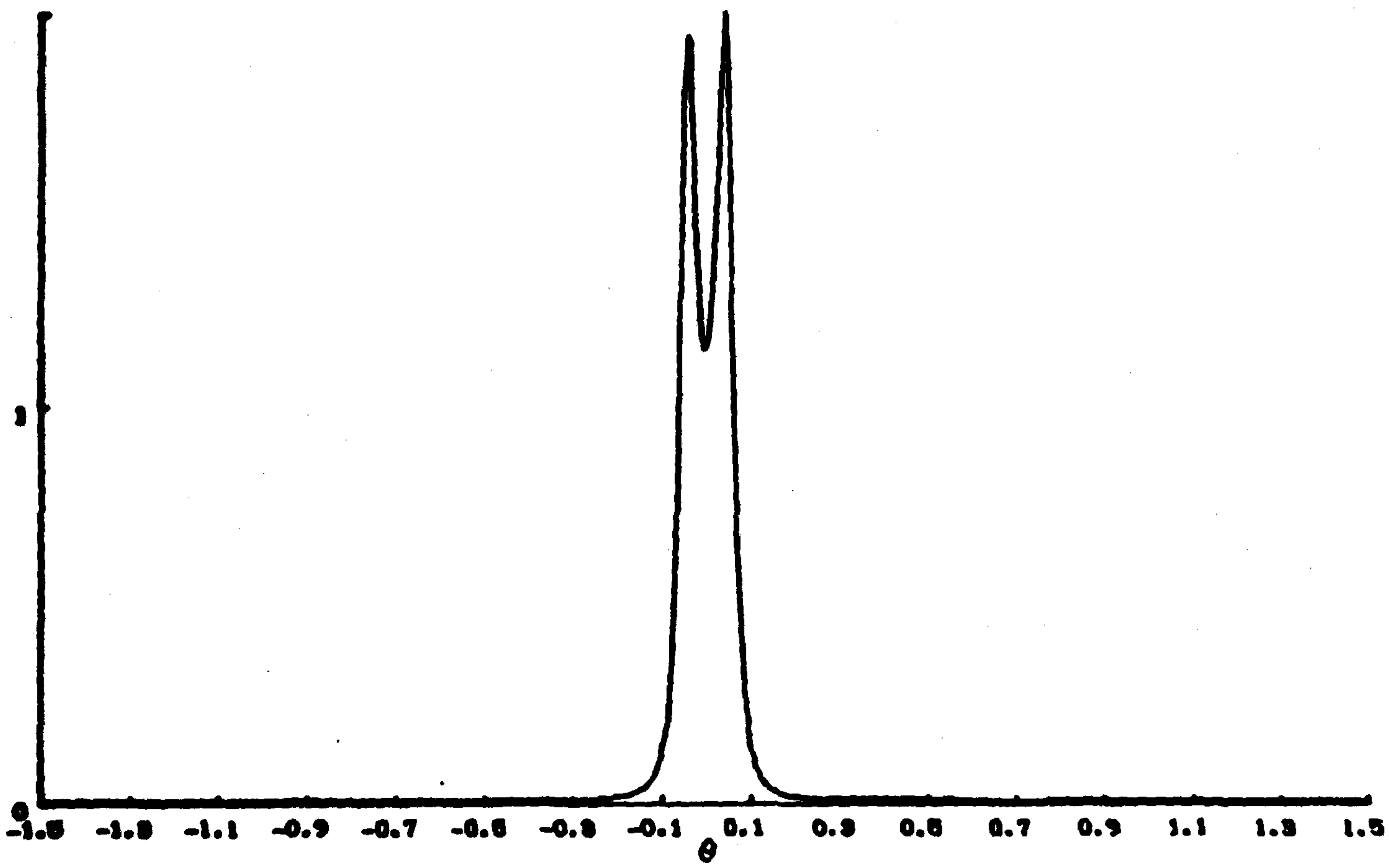

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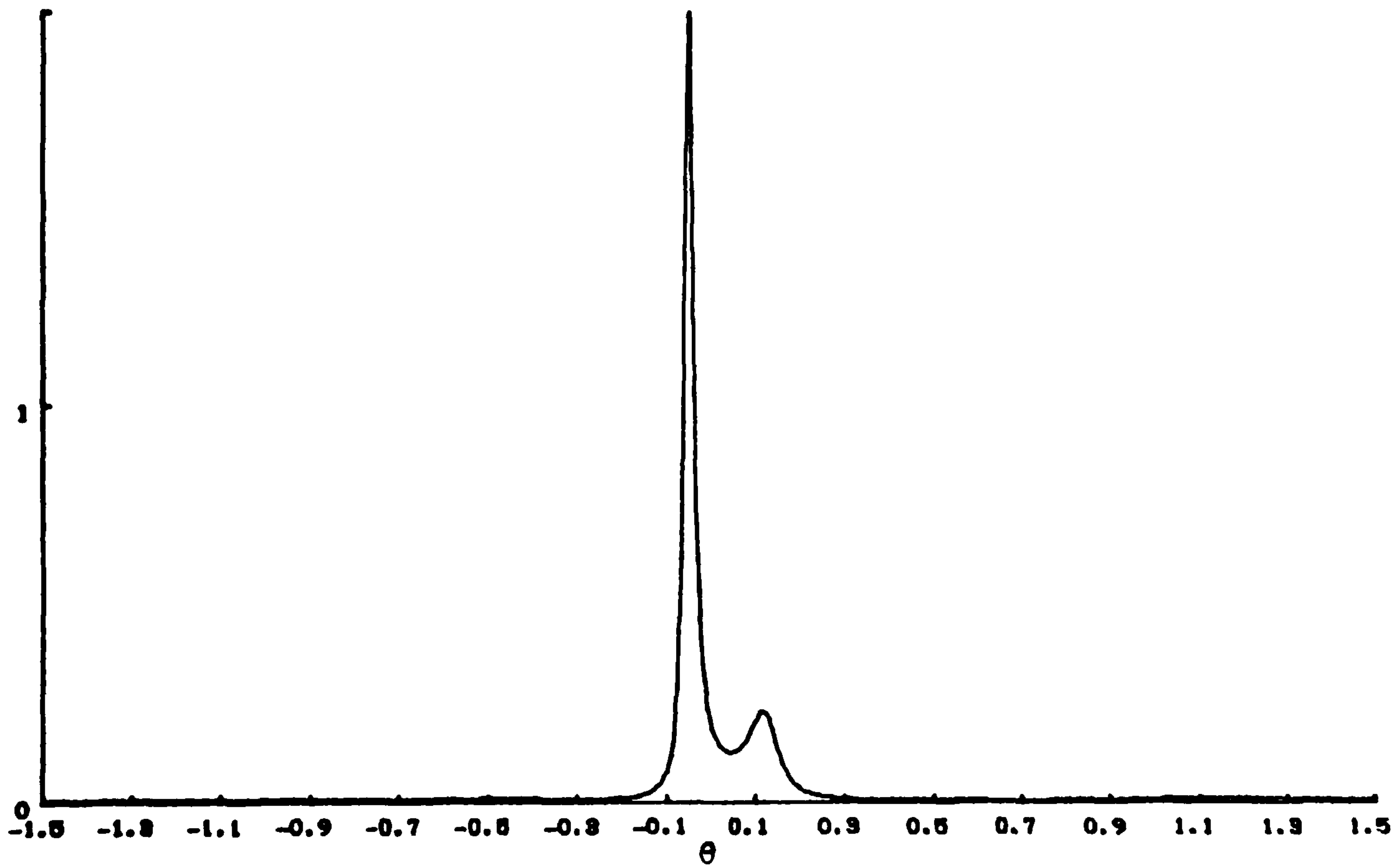

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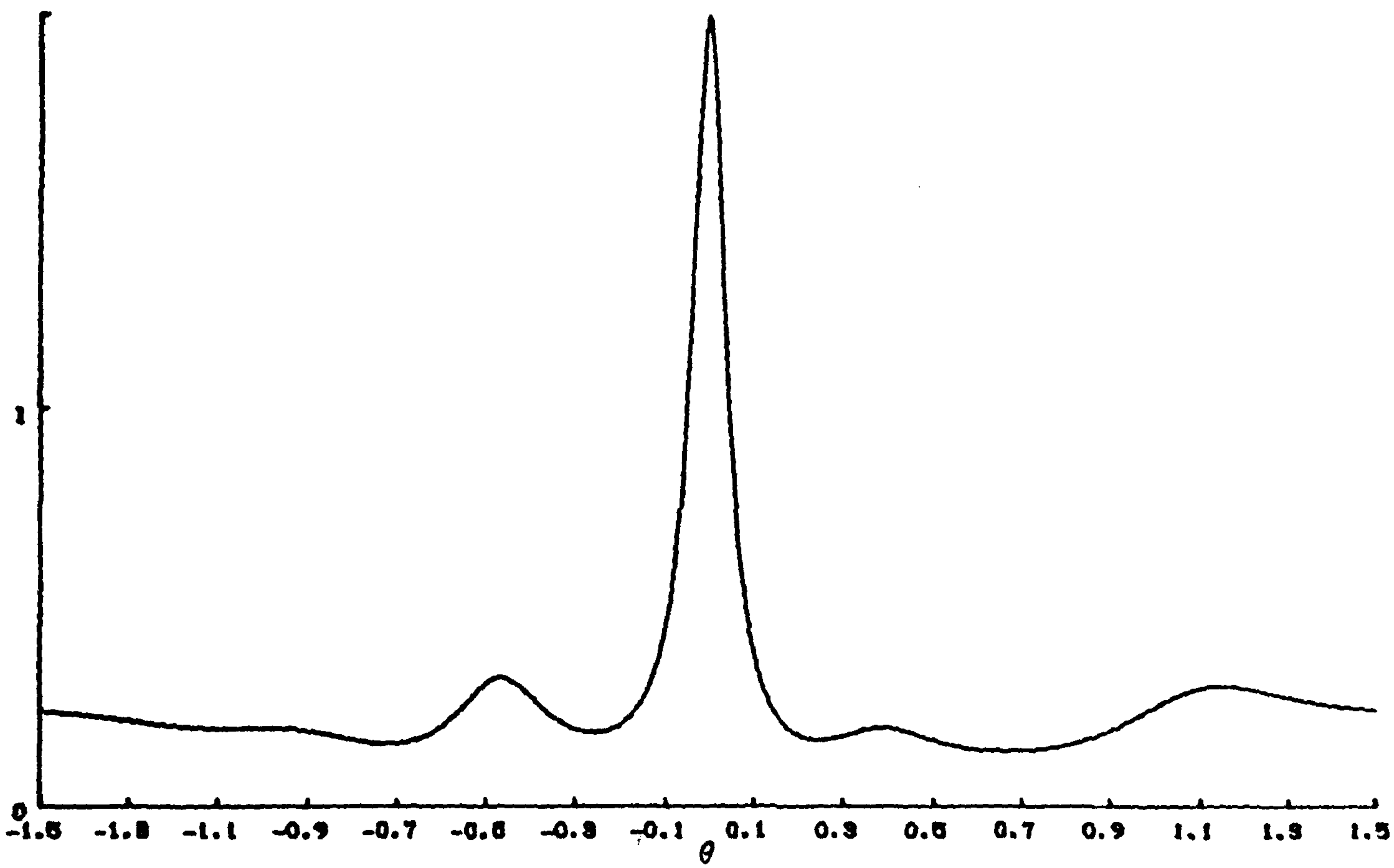
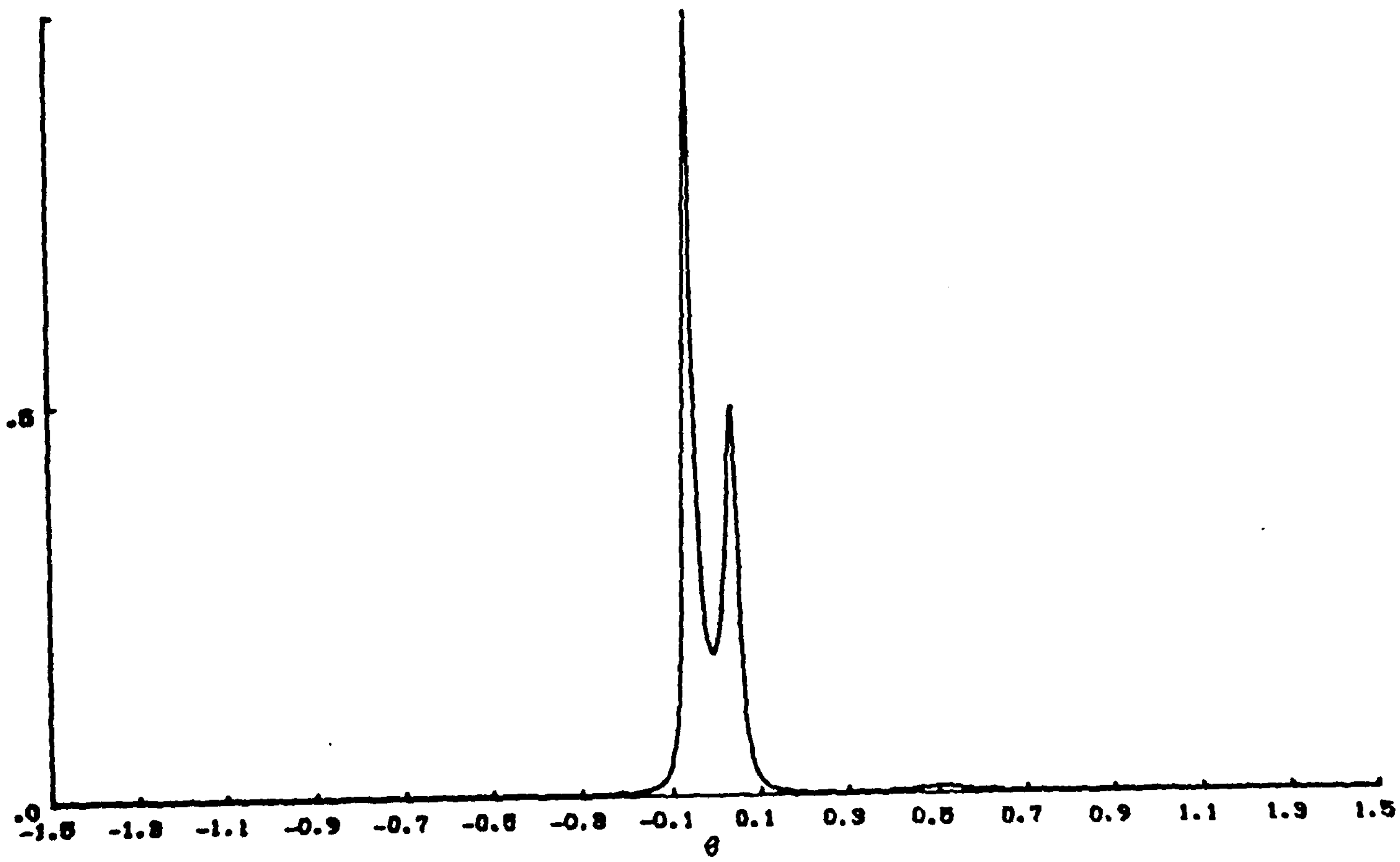


FIGURE 5



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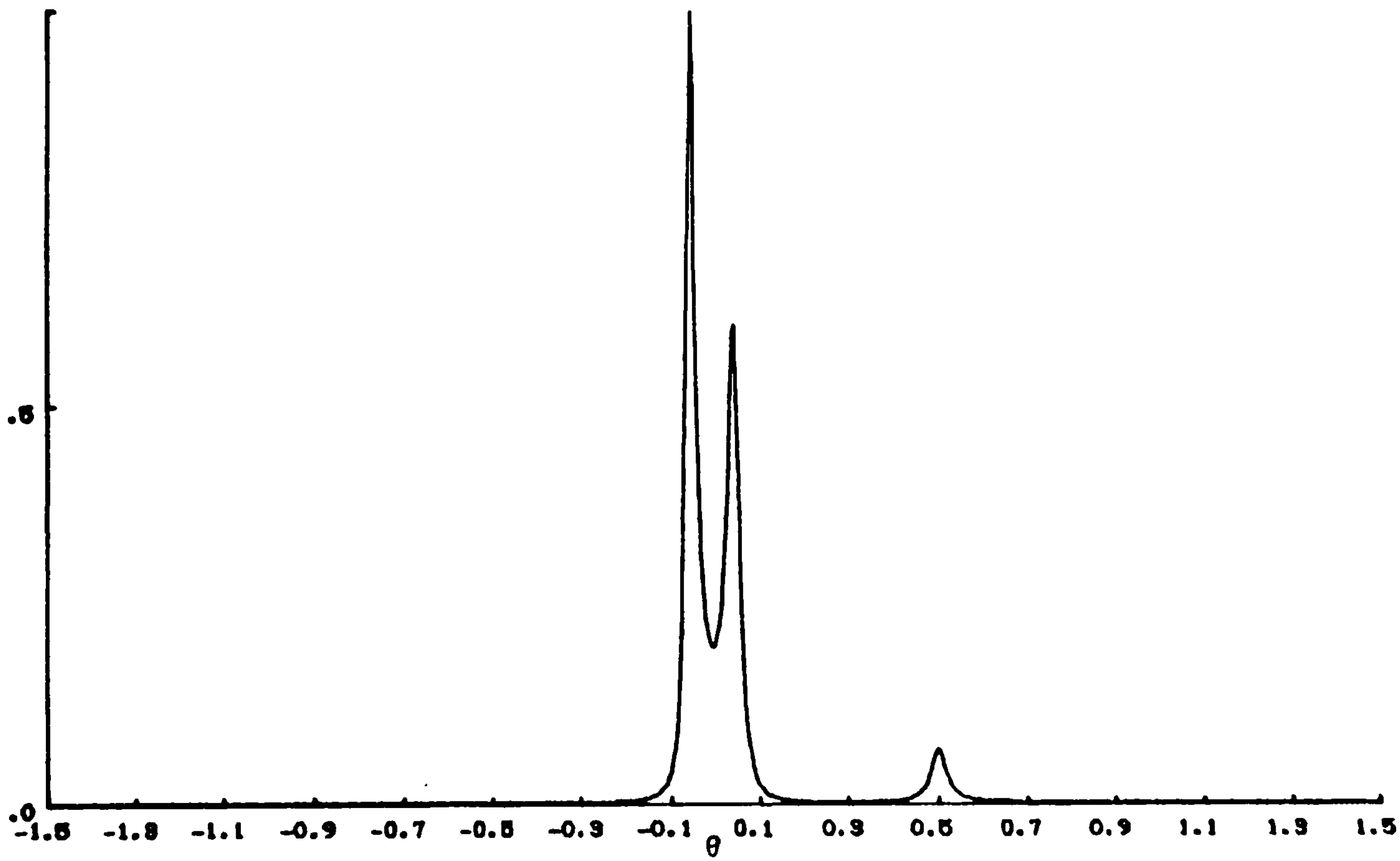

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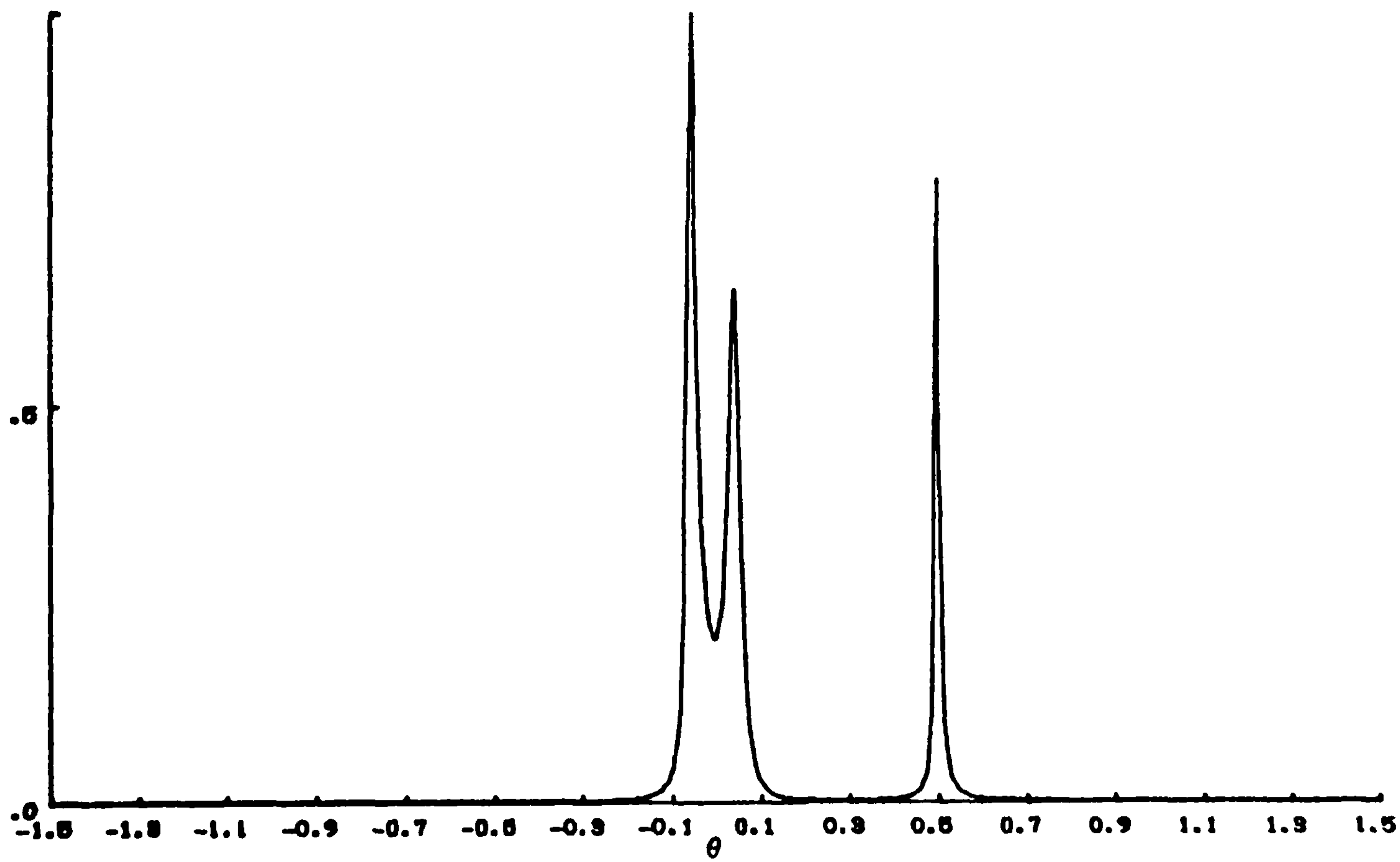

FIGURE 8

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Abstract

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