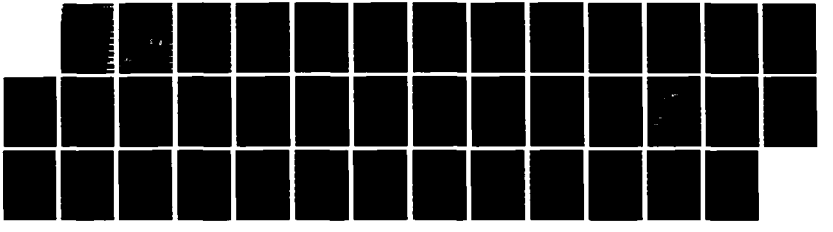
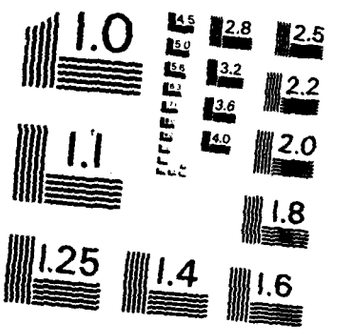
AD-A170 857 SCATTERING COEFFICIENTS AND THE ABSORPTION EDGE OF LONGITUDINAL COHERENT..(U) NAVAL SURFACE WEAPONS CENTER SILVER SPRING MD K P SCHARNHORST ET AL. 28 JUN 85 1/1

UNCLASSIFIED NSWC/TR-85-196 F/G 20/1 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS - 1963 - A

(12)

NSWC TR 85-196

SCATTERING COEFFICIENTS AND THE ABSORPTION EDGE OF LONGITUDINAL COHERENT SOUND WAVES IN SELECTED INHOMOGENEOUS MATERIALS

BY K. P. SCHARNHORST W. M. MADIGOSKY E. BALIZER

RESEARCH AND TECHNOLOGY DEPARTMENT

28 JUNE 1985

DTIC
ELECTE
AUG 0 5 1986
S D
D

Approved for public release; distribution is unlimited.

AD-A170 857

DTIC FILE COPY

NAVAL SURFACE WEAPONS CENTER

Dahlgren, Virginia 22448-5000 • Silver Spring, Maryland 20903-5000

86 8 4 035

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC/TR 85-196	2. GOVT ACCESSION NO. AD-A170857	3. RECIPIENT'S CATALOG NUMBER
4. TITLE *(and Subtitle)* SCATTERING COEFFICIENTS AND THE ABSORPTION EDGE OF LONGITUDINAL COHERENT SOUND WAVES IN SELECTED INHOMOGENEOUS MATERIALS		5. TYPE OF REPORT & PERIOD COVERED Interim Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Kurt P. Scharnhorst W. M. Madigosky E. Balizer		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center 10901 New Hampshire Ave. Silver Spring, MD 20903-5000		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS ZR01108
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE<br~~30 September 1985~~ 6/28/85 13. NUMBER OF PAGES 38
14. MONITORING AGENCY NAME & ADDRESS*(if different from Controlling Office)*		15. SECURITY CLASS. *(of this report)* UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution is unlimited.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Sound Absorption
Acoustical Scattering

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

The first three terms in the far field forward scattering amplitude for longitudinal waves impinging on spherical objects embedded in various matrix materials were studied as functions of frequency. The major resonances in these terms were used to obtain rough estimates of the position of the absorption edge in corresponding elastic and viscoelastic inhomogeneous materials. It has been generally known that in relatively compressible inhomogeneous materials the lowest resonance frequency is that of a monopole. It is achieved by means of highly compressible fluid inclusions in soft rubbery media. Not

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

20. Abstract (Cont.)

as widely known has been the fact that in incompressible materials the lowest achievable resonance frequency is that of the dipole. It occurs in relatively soft, rubbery media with heavy solid inclusions. By ordering our materials with respect to the lowest frequency at which significant scattering occurs, we put these observations into perspective. ~~We also give~~ an estimate of the absorption edge for every material combination in this study. A special effort is made to trace the development of the first three resonances in elastic solids as functions of the material properties of the host and the inclusion. We find ~~that~~ the monopole quickly moves to higher frequencies than the dipole as the compressibility of the inclusion is decreased, and that as a rule, these two terms interfere unless the mass density of the inclusion is much greater than that of the host.

FOREWORD

This work was carried out in the Nonmetallic Materials Branch of the Materials Division of the Research and Technology Department, partly under an ONR contract (ACSAS) and partly under an NSWC Independent Research Grant. The task area number is: ZR01108.

Approved by:

Jack R. Dixon

JACK R. DIXON, Head
Materials Division

Accesion For
NTIS CRA&I ☑
DTIC TAB ☐
Unannounced ☐
Justification .....
By .....
Distribution /
Availability Codes
Dist | Avail and / or Special
A1

Report date on Cover of 6/28/85 correct per J.C phone call.

QUALITY INSPECTED 4

CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS (Cont.)

ILLUSTRATIONS (Cont.)

TABLES

CHAPTER 1

INTRODUCTION

In this study we examine the imaginary parts of the first three terms in the far field forward scattering amplitude for longitudinal plane waves impinging on spherical scatterers. A wide variety of different matrix materials is considered. The objective was to obtain an idea of the approximate location of the absorption edge of coherent sound waves due to resonant scattering from inclusions in inhomogeneous materials containing randomly distributed spherical scatterers at any concentration.

Evidently it would be best to calculate absorption directly. However, at all but the lowest concentrations of inclusions that would be a formidable task. On the other hand, if the basic resonances of the inclusions which show up in the imaginary part of the scattering amplitude, f(0), are known, the approximate location of the absorption edge may be inferred. It is clear on physical grounds that the lowest frequencies at which this edge can possibly occur are those at which resonant scattering of the individual inclusions sets in. In any given situation, measurable absorption may occur at somewhat higher frequencies in an inhomogeneous material, but it certainly cannot occur at significantly lower ones. If there is a shift to lower frequencies, it should be due to volume concentration effects; due to the number of single and multiple scattering events per unit volume. As these events multiply with the concentration of inclusions, the effects of individual resonances add and increase absorption. Of course, scattering in directions other than forward and backward should also be considered. But the resonance frequencies do not depend on angular variables; hence, the scattering coefficients in f(0) contain all the information we need to locate the absorption edge. The forward scattering amplitude provides a convenient conceptual framework.

In this sense we are studying the absorption edge independently of any particular theory. The edge is of great interest to the materials designer whose task it may be to push it to the lowest frequency he can possibly reach within given material constraints.

As an example illustrating how the coefficients of the longitudinal scattering amplitude might enter into an expression for the absorption length, we quote the results of the Waterman-Truell theory[1] in the following section. In certain limiting situations their expression for the longitudinal effective wave vector yields the classical first order result.[2]

CHAPTER 2

SCATTERING COEFFICIENTS AND ABSORPTION IN INHOMOGENEOUS MATERIALS; AN EXAMPLE

The Waterman-Truell theory[1] of propagation of coherent longitudinal sound waves in inhomogeneous materials with randomly distributed scatterers yields the following rather simple looking expression for the effective wave vector:

$$K_{eff} \equiv \frac{\omega}{C_{eff}} + i\alpha_{eff} = K\left\{1 + \left(\frac{3\phi}{K^2A^3}\right) f(0) + \frac{1}{4}\left(\frac{3\phi}{K^3A^3}\right)^2 (f^2(0) - f^2(\pi))\right\}^{1/2} \tag{1}$$

where $K = K(C_{d_1})$ is the wave vector of the matrix material, A the radius of the inclusion, ϕ the volume fraction of the inclusions and $f(0)$ and $f(\pi)$ are the plane wave forward and back scattering amplitudes respectively, as calculated from the scalar potential function of the problem. C_{eff} is the effective wave speed and $1/\alpha_{eff}$ the effective absorption length.

At low concentrations of inclusions, or whenever $[f^2(0) - f^2(\pi)] << 1$, the quadratic term may be neglected, the square root expanded to first order and one obtains:

$$\alpha_{eff} = Im(K_{eff}) \simeq Im\left[K\left\{1 + \frac{1}{2}\left(\frac{3\phi}{K^2A^3}\right) f(0)\right\}\right] \tag{2}$$

This is the classical first order result.[2]

In terms of the coefficients of the longitudinal scattering potential one finds that:

$$f(0) = \frac{1}{iK}\sum_{n=0}^{\infty} i^{-n} a_n \equiv f_e(0) + f_o(0) \tag{3}$$

and:

$$f(\pi) = \frac{1}{iK}\sum_{n=0}^{\infty} (-1)^{-n} a_n \equiv f_e(0) - f_o(0) \tag{4}$$

where $f_e(0)$ and $f_o(0)$ are sums over even and odd n in f(0) respectively. Substituting into Eq. 1, one finds:

$$K_{eff} = K\left\{1 + \left(\frac{3\phi}{K^2A^3}\right) f(0) + \left(\frac{3\phi}{K^2A^3}\right)^2 f_e(0) f_o(0)\right\}^{1/2} \tag{5}$$

or:

$$K_{eff} = K\left\{1 + (X_0 - X_1 + X_2 \pm \ldots) + (X_0X_1 - X_1X_2 \pm \ldots)\right\}^{1/2} \tag{6}$$

up to n = 2, where:

$$X_0 = \frac{3i\phi a_0}{(KA)^3}$$

$$X_1 = \frac{3\phi a_1}{(KA)^3} \tag{7}$$

$$X_2 = \frac{3i\phi a_2}{(KA)^3}$$

The imaginary parts of these expressions are the three functions studied in this paper.

Evidently, in elastic host materials the inverse absorption length in Eq. 1, α_{eff}, is simply related to the imaginary parts of quantities like X_i and $X_j X_k$ in Eq. 6, where j = even and k = odd. When these are small, attenuation of coherent waves approaches zero. When they are large, or more accurately when the appropriate sums of the imaginary parts in the brackets in Eq. 6 become significantly different from zero, absorption becomes important. By studying these coefficients we therefore obtain a first impression as to where to expect the absorption edge in various combinations of matrix materials and inclusions.

We focus on the first three coefficients, the monopole (n = 0), the dipole (n = 1), and the quadrupole (n = 2). We find that in most cases either the monopole or the dipole yields the lowest resonance frequency. Often the quadrupole is near one or the other of these. Assuming that still higher order resonances are weak or occur well above the first three, the approximate frequency region of the absorption edge may be read directly off our plots of $Im(X_i)$, i = 0,1,2.

We have plotted only $Im(X_i)$, which is all we need to do in order to understand the lowest order theory, Eq. 2. In the case of the Waterman-Truell theory we could have obtained a more complete picture of α_{eff}, if we had also plotted $Re(X_i)$, since the cross-terms, $Im\ (X_j\ X_k)$, use both of these components. But clearly, for our purpose, which is to detect the lowest frequency at which the edge could possibly occur in any theory, the study of $Im(X_i)$, which contains the typical resonance peaks is entirely sufficient.

CHAPTER 3

GENERAL REMARKS

We have collected the data in several groups of related material characteristics; Tables 1 and 2. Listed are the first Lamé parameter, $\lambda^* = \lambda(1-i\eta_\lambda)$, the shear modulus, $\mu^* = \mu(1-i\eta_\mu)$ and the mass density, ρ, of the host (1) and the inclusion (2). Both λ and μ have been reduced by the modulus of water. Some of the materials in these groups are on the verge of being unrealistic, such as porous metallic inclusions and certain materials with rather high relaxation absorption. These have been included in order to study the outer limits of material parameter combinations.

The moduli were assumed to be independent of frequency. This is a potential problem if the size of the resonator is assumed to be such that the resonances appear in the relaxation region of the shear modulus. The present work might be extended in that direction.

The parameters of foamed rubber were generated with the aid of the Kerner (quasi-static) theory,[3] using reasonable rubber parameters as input and 40% air. Here we assume that the individual air pockets in the resulting foam are small, hence far from the resonances of the inclusions.

We have plotted the imaginary parts of $-X_0$, $-X_1$, and X_2 in all of our figures, since X_0 and X_1 are negative. Also, many of the minor resonances contain scaling factors. The dominant resonance is approximately full scale. A scaling factor of -5 for instance means that $\mathrm{Im}(X_1)$ has been multiplied by -5. This would be indicated in the figure caption as $X_1(-5)$. The reader is cautioned to check these numbers before judging the relative strengths of the resonances.

All of our data is plotted versus the real part of $K(C_{d_1})xA$ of the matrix material, which is obtained when the loss tangents of λ_1^* and μ_1^* are equated to zero. Note that the result is not the same as $\mathrm{Real}\,(K(C_{d_1})xA)$. But it allows the reader to compare otherwise similar elastic and viscoelastic materials directly. At the left hand end of the abscissa we also indicate the corresponding frequency, assuming 2mm for the radius, A, of the inclusion, thus enabling the reader to shift the data on the frequency axis when the radius is changed.

The frequency in the leading edge of the lowest frequency major resonance in each figure, f_Γ, at which Im(X) reaches one half its maximum, is a measure of the frequencies at which reasonably strong absorption might be expected in each

TABLE 1. MATERIAL PARAMETERS OF ELASTIC MATERIALS; FIGURES 1-6 AND 9-21

Fig. #	ELASTIC MATERIALS: MATRIX MATERIAL λ_1	$\eta_{\lambda 1}$	μ_1	$\eta_{\mu 1}$	ϱ_1	MATERIAL	INCLUSION λ_2	$\eta_{\lambda 2}$	μ_2	$\eta_{\mu 2}$	ϱ_2	MATERIAL
AIR FILLED CAVITIES IN ELASTIC SOLIDS												
1	0.975	0.	0.0044	0.	1.13	Rubber	6.31×10^{-5}	0.	0.	0.	0.00121	Air
2	1.25	0.	0.05	0.	1.19	Hard Rubber	6.31×10^{-5}	0.	0.	0.	0.00121	Air
3	0.0048	0.	0.0017	0.	0.50	Foamed Rubber	6.31×10^{-5}	0.	0.	0.	0.00121	Air
4	2.048	0.	0.70	0.	1.18	Plastic	6.31×10^{-5}	0.	0.	0.	0.00121	Air
5	0.854	0.	0.689	0.	0.689	Syntactic Foam	6.31×10^{-5}	0.	0.	0.	0.00121	Air
6	55.97	0.	33.486	0.	7.84	Steel	6.31×10^{-5}	0.	0.	0.	0.00121	Air
ELASTIC MATERIALS IN PLASTIC AND HARD RUBBER MATRICES												
9	2.048	0.	0.70	0.	1.18	Plastic	40.09	0.	11.6	0.	2.0	Metallic Material
10	2.048	0.	0.70	0.	1.18	Plastic	8.05	0.	11.6	0.	2.49	Glass
11	2.048	0.	0.70	0.	1.18	Plastic	40.09	0.	11.6	0.	0.249	Porous Metal
12	1.545	0.	0.0266	0.	1.11	Hard Rubber	0.604	0.	1.422×10^{-3}	0.	2.0	Heavy Silicone Rubber
13	2.048	0.	0.70	0.	1.18	Plastic	55.97	0.	33.486	0.	7.84	Steel
ELASTIC MATERIALS IN RUBBER MATRICES												
14	0.975	0.	0.0044	0.	1.13	Rubber	43.29	0.	10.0	0.	2.49	Metallic Material
15	0.975	0.	0.0044	0.	1.13	Rubber	43.29	0.	10.0	0.	0.40	Porous Metal
16	0.972	0.	0.0044	0.	1.18	Rubber	55.97	0.	33.486	0.	7.84	Steel
17	0.972	0.	4.4×10^{-5}	0.	1.18	Very Soft Rubber	55.97	0.	33.486	0.	7.84	Steel
18	0.531	0.	3.45×10^{-4}	0.	1.02	Silicone Rubber	0.527	0.	4.028×10^{-3}	0.	1.71	Heavy Silicone Rubber
ELASTIC MATERIALS IN SYNTACTIC FOAM												
19	0.854	0.	.689	0.	0.689	Syntactic Foam	0.606	0.	1.422×10^{-4}	0.	2.0	Heavy Silicone Rubber
20	0.854	0.	.689	0.	0.689	Syntactic Foam	0.604	0.	1.422×10^{-3}	0.	2.0	Heavy Silicone Rubber
21	0.854	0.	.689	0.	0.689	Syntactic Foam	0.150	0.	1.422×10^{-3}	0.	4.0	Heavy, Loaded Silicone Rubber

TABLE 2. MATERIAL PARAMETERS OF VISCOELASTIC MATERIALS; FIGURES 22-33

Fig. #	VISCOELASTIC MATERIALS: MATRIX MATERIAL λ_1	$\eta_{\lambda 1}$	μ_1	$\eta_{\mu 1}$	ϱ_1	MATERIAL	INCLUSION λ_2	$\eta_{\lambda 2}$	μ_2	$\eta_{\mu 2}$	ϱ_2	MATERIAL
VISCOELASTIC MATERIALS IN PLASTIC AND HARD RUBBER MATRICES												
22	1.545	0.	2.655×10^{-2}	0.45	1.11	Hard Rubber	0.604	0.	1.422×10^{-3}	0.16	2.0	Heavy Silicone Rubber
23	0.4228	.0014	0.282	0.0098	1.985	Heavy Plastic	0.159	0.	2.84×10^{-4}	0.16	4.0	Heavy, Loaded Silicone Rubber
24	0.4186	.0015	0.282	0.0109	1.985	Heavy Plastic	0.154	0.	2.84×10^{-3}	0.16	4.0	Heavy, Loaded Silicone Rubber
25	1.028	.0006	0.282	0.0109	1.985	Heavy Plastic	1.208	0.	2.84×10^{-3}	0.16	4.0	Heavy, Loaded Rubber
26	0.824	.0006	0.2094	0.01	1.213	Plastic	0.604	0.	1.422×10^{-3}	0.16	2.0	Heavy Silicone Rubber
VISCOELASTIC MATERIALS IN SILICONE RUBBER AND FOAMED RUBBER MATRICES												
27	0.444	0.	4.44×10^{-3}	1.0	1.0	Silicone Rubber	1.334	0.	0.444	0.5	1.0	Plastic
28	0.444	0.	4.44×10^{-3}	1.0	1.0	Silicone Rubber	1.334	0.	0.444	0.5	4.0	Heavy, Loaded Plastic
29	0.531	0.	3.45×10^{-4}	0.12	1.02	Silicone Rubber	0.527	0.	4.028×10^{-3}	0.09	1.71	Heavy Silicone Rubber
VISCOELASTIC MATERIALS IN SYNTACTIC FOAM												
30	0.854	0.	.689	0.01	0.689	Syntactic Foam	0.604	0.	1.422×10^{-3}	0.16	2.0	Heavy Silicone Rubber
31	0.854	0.	.689	0.01	0.689	Syntactic Foam	0.606	0.	1.422×10^{-4}	0.16	2.0	Heavy Silicone Rubber
32	0.854	0.	.689	0.01	0.689	Syntactic Foam	0.150	0.	1.422×10^{-3}	0.16	4.0	Heavy Loaded Silicone Rubber
33	0.854	0.	.689	0.01	0.689	Syntactic Foam	1.334	0.	0.444	0.50	4.0	Heavy Loaded Plastic

system. Note that this frequency is independent of the assumed volume fraction of inclusions. f_Γ is given in each figure caption and all of these frequencies are summarized in Table 3.

We have also done a more systematic study of the displacements of the first three multipoles with changes in the material parameters. The results are collected in Table 4 and Fig. 8. Again some unrealistic materials are included, this time in order to achieve a continuous sequence of parameter combinations. Only elastic materials were considered. Some of these are also included in Table 1. Again we have studied $\mathrm{Im}(X_1)$ rather than a_1. The frequencies of the major resonance peaks in each of these materials are displayed in Fig. 8. Some idea of the widths of the peaks may be obtained by referring to the explicit plots of the resonances in elastic material combinations which are common to both parts of this study.

Because of the inclusion of viscoelastic materials in this study it might have been more appropriate to consider $i^{-(n+1)}\ a_n/K^2$ rather than $i^{-(n+1)}a_n/K^3$ in order to estimate the position of the absorption edge, since the factor K in front of the curly bracket in Eq. 2 mixes the real and imaginary parts of the terms inside whenever viscoelastic materials are considered. We have checked this effect in the case of example #27 in Table 2; i.e., for a plastic in silicone rubber. f_Γ increased by 10% and the quadrupole (main) resonance frequency increased by 15%.

Calculations were performed in two ways: On the basis of the classical solutions of these problems as given by Yamakawa (1962)[4] as well as on the basis of the T-matrix solution given by Waterman (1976)[5] and Pao (1978).[6] These are two completely independent approaches to these problems. Our numerical calculations give identical results. Numerical difficulties with the classical solution, due to extremely large and extremely small elements in the same matrix (2x2 when $n = 0$, 4x4 when $n > 0$) were overcome in the frequency intervals treated here, by appropriately scaling the columns and/or the rows. Such problems occurred only occasionally and then only at the highest or the lowest frequencies, never in the resonance region itself. With the T-matrix method, no computational problems at all were encountered.

Whenever asymptotic formulas were applicable, such as in the case of air cavities in rubber and heavy inclusions in rubbery matrices, the results so obtained agreed very well with our numerical calculations.

The concentration of inclusion, ϕ, in Eq. 7 was arbitrarily assumed to be 1%.

TABLE 3. EDGE FREQUENCIES, f_Γ, OF VARIOUS MATERIAL COMBINATIONS

#	FIG. #	f_Γ (kHz)	HOST MATERIAL	INCLUSION
1	17	0.44	Very Soft Rubber	Steel
2	29	2.20	Silicone Rubber	Heavy Silicone Rubber
3	18	2.30	Silicone Rubber	Heavy Silicone Rubber
4	16	4.20	Rubber	Steel
5	28	5.60	Silicone Rubber	Heavy Loaded Plastic
6	14	6.70	Rubber	Metal
7	3	9.6	Foamed Rubber	Air
8	15	12.8	Rubber	Porous Metal
9	1	13.0	Rubber	Air
10	24	33.4	Heavy Plastic	Heavy, Loaded Silicone Rubber
11	23	34.2	Heavy Plastic	Heavy, Loaded Silicone Rubber
12	32	39.6	Syntactic Foam	Heavy, Loaded Silicone Rubber
13	21	40.2	Syntactic Foam	Heavy, Loaded Silicone Rubber
14	2	42.2	Hard Rubber	Air
15	25	42.6	Heavy Plastic	Heavy, Loaded Rubber
16	26	51.5	Plastic	Heavy Silicone Rubber
17	13	56.2	Plastic	Steel
18	33	67.9	Syntactic Foam	Heavy, Loaded Plastic
19	30	81.7	Syntactic Foam	Heavy Silicone Rubber
20	31	81.7	Syntactic Foam	Heavy Silicone Rubber
21	20	82.8	Syntactic Foam	Heavy Silicone Rubber
22	10	83.6	Plastic	Glass
23	19	85.6	Syntactic Foam	Heavy Silicone Rubber
24	27	96.6	Silicone Rubber	Plastic
25	22	113.0	Hard Rubber	Heavy Silicone Rubber
26	12	116.8	Hard Rubber	Heavy Silicone Rubber
27	4	125.9	Plastic	Air
28	11	141.3	Plastic	Porous Metal
29	5	149.6	Syntactic Foam	Air
30	9	168.2	Plastic	Metal
31	6	316.2	Steel	Air

TABLE 4. MATERIAL PARAMETERS OF ELASTIC MATERIALS; FIGURE 8

	MATRIX MATERIAL				INCLUSION		
#	λ_1	μ_1	ϱ_1		λ_2	μ_2	ϱ_2
1	55.97	33.49	7.84		6.33×10^{-5}	0.	0.00121
2	2.048	0.7	1.18		6.33×10^{-5}	0.	0.00121
3	0.972	0.0044	1.13		6.33×10^{-5}	0.	0.00121
4	0.972	0.0044	1.13		6.33×10^{-3}	0.	0.00121
5	0.972	0.0044	1.13		6.33×10^{-2}	0.	0.00121
6	0.972	0.0044	1.13		6.33×10^{-2}	10^{-6}	0.00121
7	0.972	0.0044	1.13		63.3	10^{-6}	0.00121
8	0.972	0.0044	1.13		63.3	10^{-4}	0.00121
9	0.972	0.0044	1.13		63.3	10^{-2}	0.00121
10	0.972	0.0044	1.13		62.3	0.5	0.00121
11	0.972	0.0044	1.13		43.3	10.0	0.00121
12	0.972	0.0044	1.13		43.3	10.0	0.12
13	0.972	0.0044	1.13		43.3	10.0	0.30
14	0.972	0.0044	1.13		43.3	10.0	0.40
15	0.972	0.0044	1.13		43.3	10.0	0.50
16	0.972	0.0044	1.13		43.3	10.0	1.00
17	0.972	0.0044	1.13		43.3	10.0	1.75
18	0.972	0.0044	1.13		43.3	10.0	2.49
19	3.44	0.0044	1.13		43.3	10.0	0.12
20	3.31	0.07	1.18		43.3	10.0	0.12
21	2.048	0.7	1.18		43.3	10.0	0.12
22	2.048	0.7	1.18		40.09	11.6	0.12
23	2.048	0.7	1.18		40.09	11.6	0.12
24	2.048	0.7	1.18		40.09	11.6	0.249
25	2.048	0.7	1.18		40.09	11.6	1.00
26	2.048	0.7	1.18		40.09	11.6	1.50
27	2.048	0.7	1.18		40.09	11.6	2.00
28	2.048	0.7	1.18		40.09	11.6	2.49
29	2.048	0.7	1.18		8.05	11.6	0.249
30	3.43	0.0077	1.18		8.05	11.6	0.249
31	3.43	0.0077	1.18		8.05	11.6	2.49
32	2.048	0.70	1.18		8.05	11.6	2.49
33	2.048	0.7	0.50		8.05	11.6	2.49
34	2.048	0.7	1.18		55.97	33.49	7.84

CHAPTER 4

RESULTS AND DISCUSSION

ELASTIC MATERIALS

Air Filled Cavities in Elastic Solids

The most conspicuous feature here is that the monopole always occurs at a lower frequency than the dipole. However, usually the two resonances overlap and the quadrupole also mixes in. But in one case, that of an air cavity in rubber, Figs. 1 and 2, the monopole is the only important resonance. In that case, $K(C_{d_1})xA$ is small and the resonance frequency, f_o, as well as the amplitude, $X_0(f_o)$ and the width Γ of the resonance in X_0 may be calculated from the resonance of the first term in Eq. 3.[7,8] Substituting into Eq. 7, we find:

$$f_o = \frac{C_{s_1}}{\pi A} \text{ or } K(C_{d_1})\ A = \frac{1}{\alpha}; \alpha = C_{d_1}/2\,C_{s_1} \tag{8}$$

$$X_0(f_o) = -\,3\phi\alpha^3 \tag{9}$$

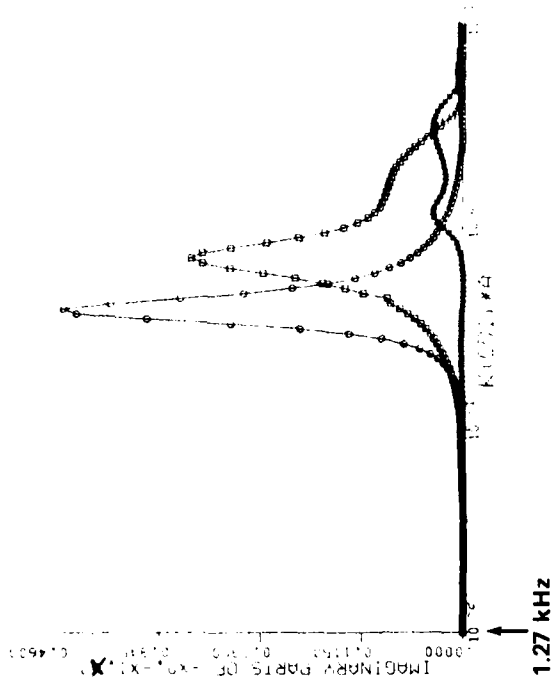
$$\Gamma = 1/\alpha^2 \tag{10}$$

Note how much larger and narrower this resonance is compared to all of the others in this group.

In the other examples, Figs. 3-6, Eqs. 8-10 should no longer apply, since $K(C_{d_1})xA$ is no longer small. However, we find that Eq. 8 still yields the resonance frequency with reasonable accuracy. In Fig. 7 we have plotted the halfwidths of the major resonance peaks in Figs. 1-6 versus the resonance frequency. Note that at large values of $K(C_{d_1})xA$ where Eq. 8 no longer applies, the halfwidths seem to depend approximately linearly on the resonance frequencies. When Eq. 8 applies, the relationship between f_o and Γ follows from Eqs. 8 and 10:

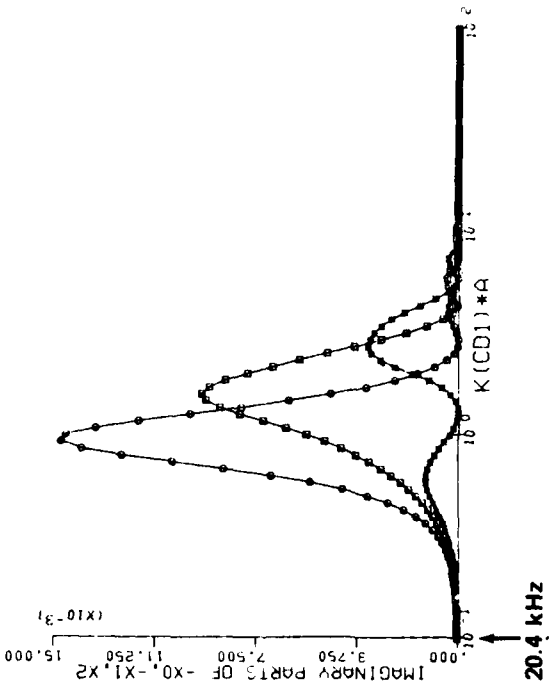
$$f_o = \frac{C_{d_1}}{2\pi A}\,\Gamma^{1/2}$$

Hence in materials with giant monopoles and equal longitudinal wave speeds, the halfwidth of the monopole resonance depends quadratically on f_o.



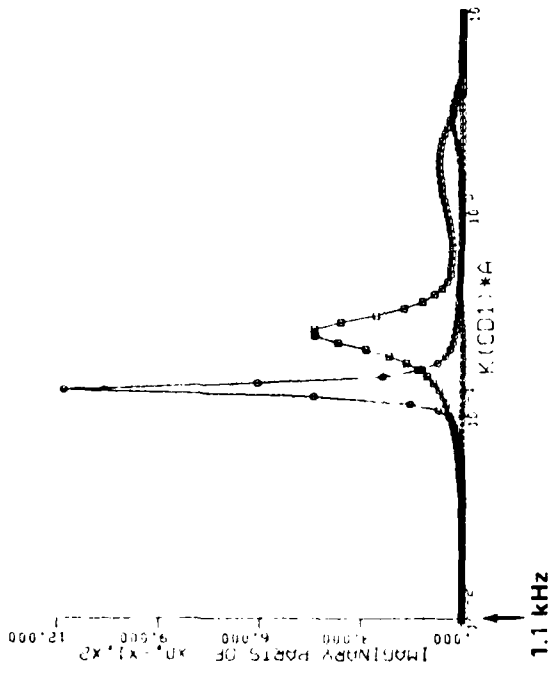
1.27 kHz

FIGURE 2. AIR FILLED CAVITY IN HARD RUBBER



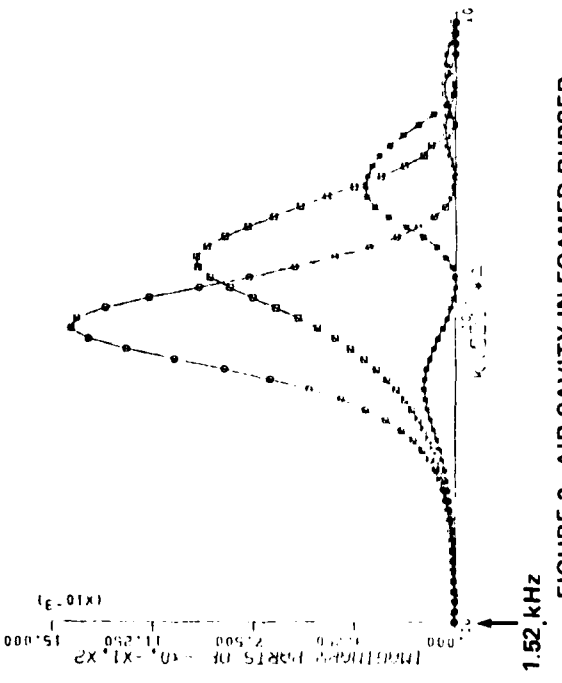
20.4 kHz

FIGURE 4. AIR CAVITY IN A PLASTIC MATERIAL



1.1 kHz

FIGURE 1. AIR FILLED CAVITY IN RUBBER



1.52 kHz

FIGURE 3. AIR CAVITY IN FOAMED RUBBER

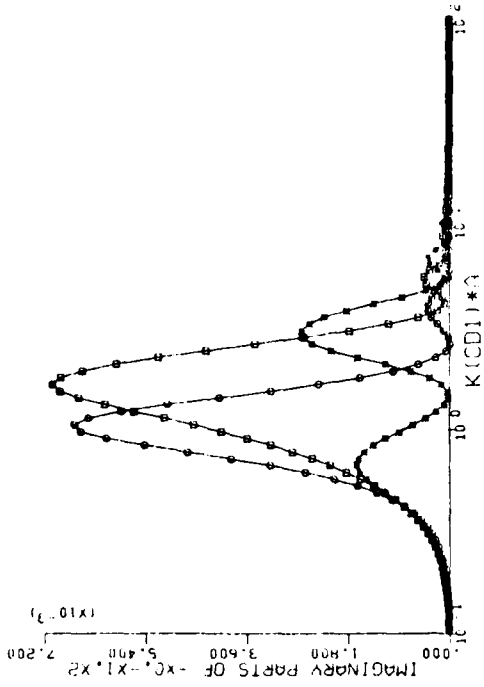


FIGURE 6. AIR CAVITY IN STEEL

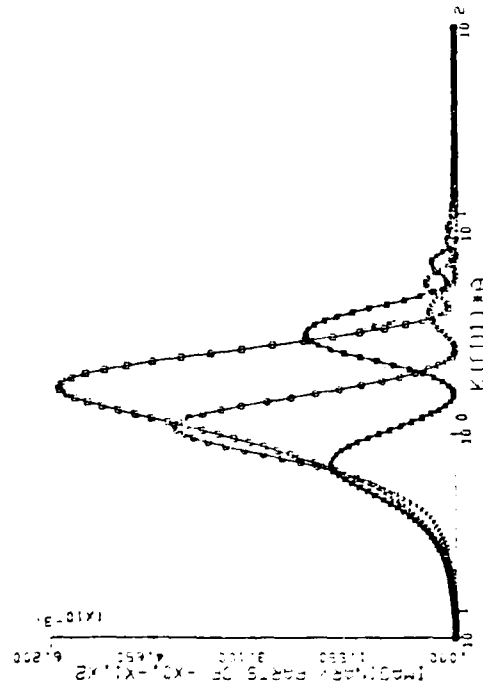


FIGURE 5. AIR CAVITY IN SYNTACTIC FOAM

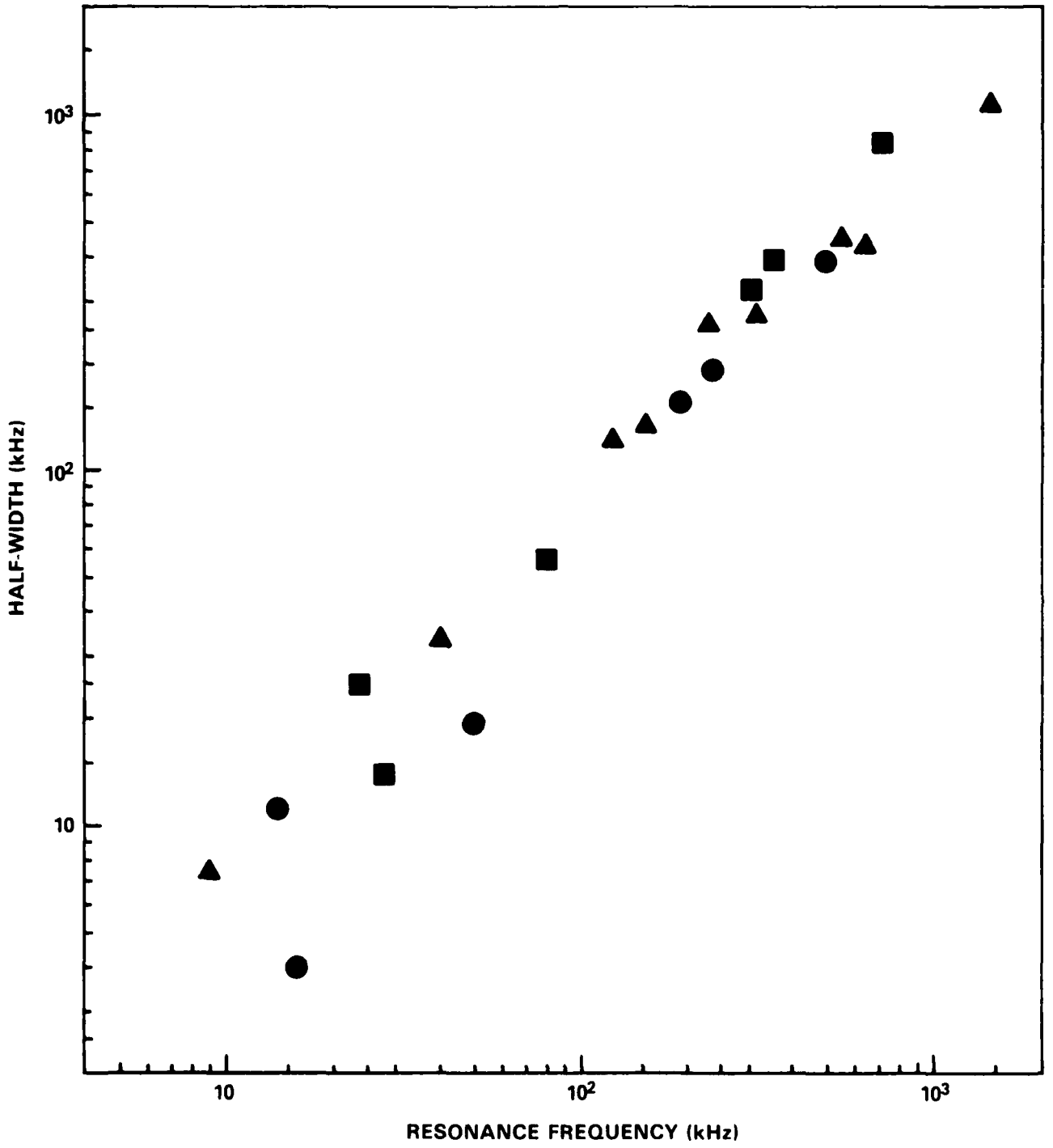


FIGURE 7. MONOPOLE (●), DIPOLE (■) AND QUADRUPOLE (▲) HALF WIDTHS AND RESONANCE FREQUENCIES OF AIR CAVITIES ON VARIOUS MATERIALS; TABLE 1 AND FIGURES 1-6

It has been argued in the past[9] that a cavity in steel does not resonate in the monopole mode (or, more accurately, that the monopole resonance frequency is imaginary), essentially because the condition $(2C_{s_1}/C_{d_1}) \gg 1$, which is satisfied in that case, removes the resonance from the denominator of a_o. Referring to Fig. 6 we would like to draw particular attention to the fact that not only does this resonance exist, but it is in fact rather prominent.

On the basis of our more systematic studies of elastic materials, Table 4 and Fig. 8, we may also make the following general statement about the first two multipoles in hard and soft rubber materials: Starting from an air filled cavity, as the compressibility of the inclusion material is decreased, the monopole moves to higher frequencies and has a strong tendency to pass the dipole (see the sequences 3 to 18 and 2, 21 to 29 in Table 4 and Fig. 8).

Elastic Materials in Plastic and Hard Rubber Matrices

The quadrupole tends to be a strong resonance in some of these examples and the dipole tends to occur at lower frequencies than the monopole. The latter is in fact the rule for solid inclusions in solids, as will become clear from the remainder of this study.

In the case of steel in plastic the dipole is the only significant resonance. It may be shown that in this case the resonance frequency of a_1 may be calculated from:[10]

$$f_o = \frac{3C_{s_1}}{2\pi A} \sqrt{\frac{\rho_1}{2\rho_2 + \rho_1}} \tag{11}$$

or

$$K(C_{d_1}) * A = 3 \sqrt{\frac{\mu_1 \rho_1}{(\lambda_1 + 2\mu_1)(2\rho_2 + \rho_1)}} \tag{12}$$

The narrower this resonance, the more accurately Eq. 11 yields the resonance frequency of X_1.

We would also like to point out that the example in Fig. 10 has recently been used to illustrate a theory of the dynamic properties of inhomogeneous materials with tenuous random distributions of spherical inclusions.[11] It was assumed quite generally that the monopole is the lowest resonance in solids with solid inclusions, which is clearly not the case in this particular example. Even if the monopole happens to be the lowest resonance, we have never observed it to be well separated from the dipole. An example of that type of situation is that of heavy silicone rubber in hard rubber, Fig. 12.

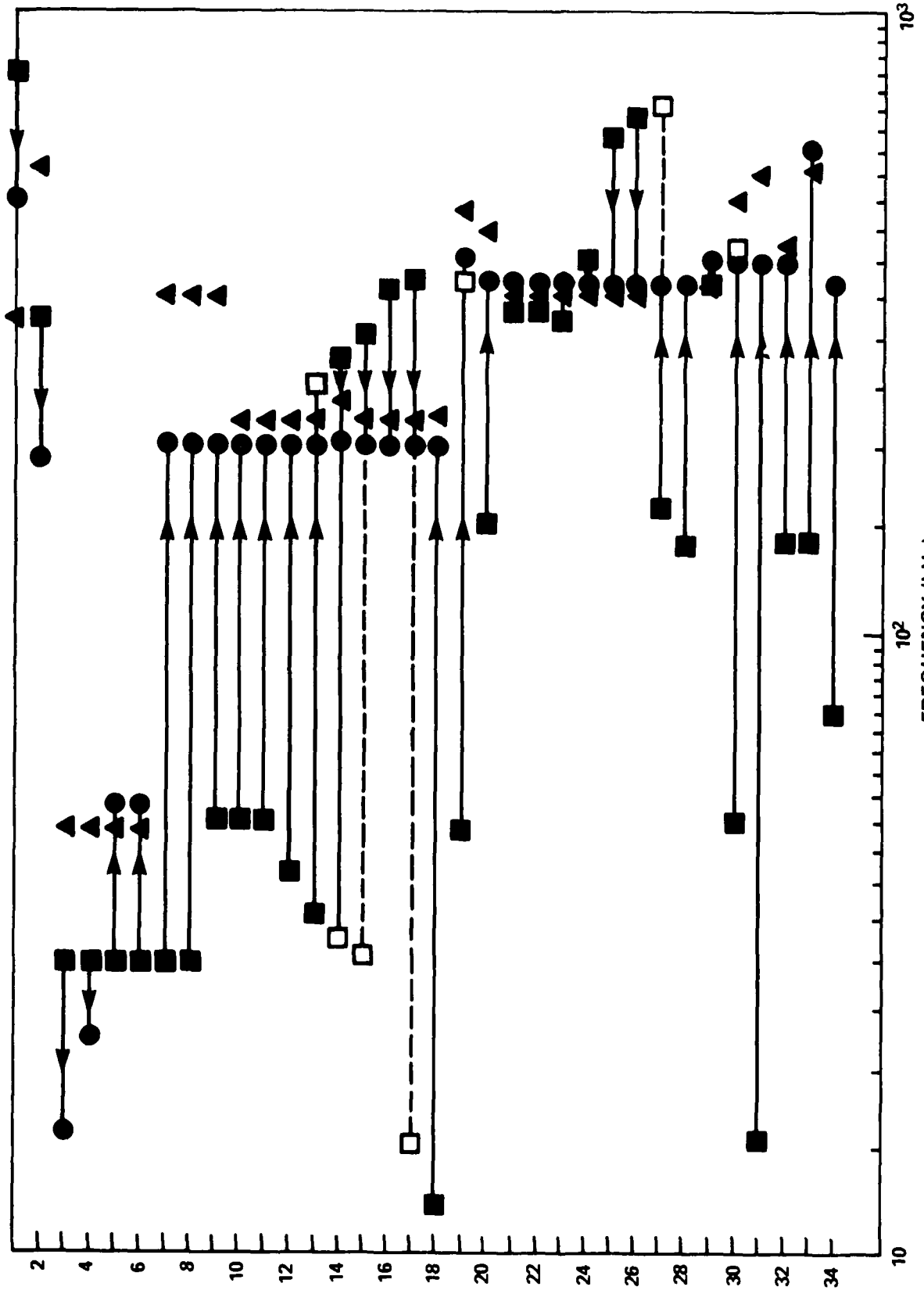
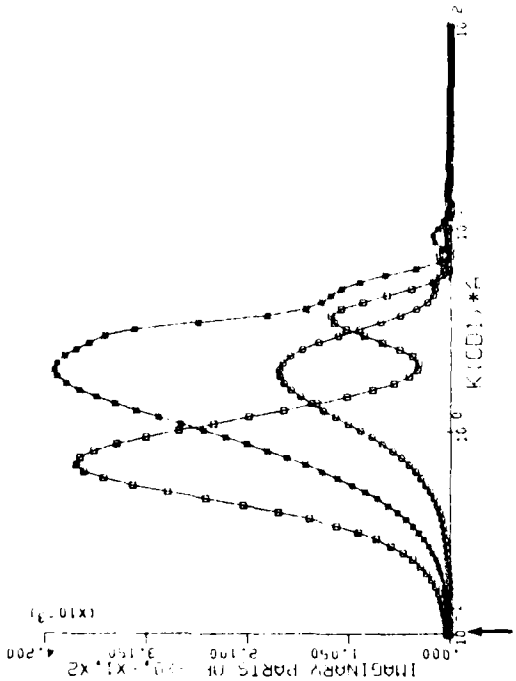
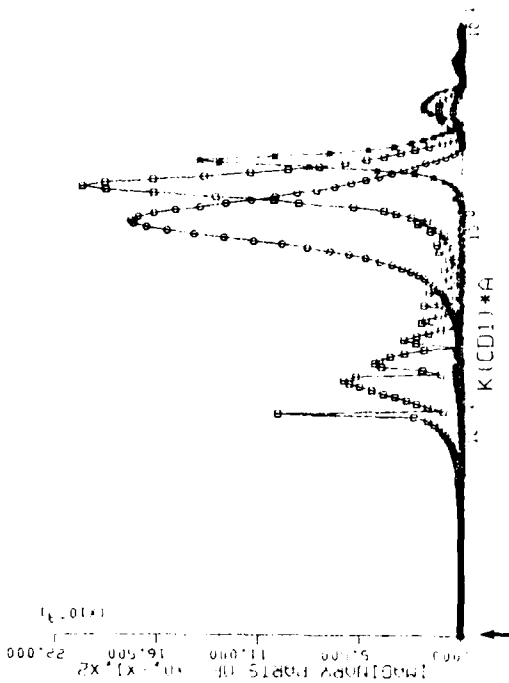


FIGURE 8. RESONANCE FREQUENCIES OF THE MONOPOLE(●), DIPOLE (■) AND QUADRUPOLE (▲) OF THE ELASTIC MATERIAL COMBINATIONS SHOWN IN TABLE 4. (OPEN SQUARES CORRESPOND TO MINOR DIPOLE RESONANCES. THE ARROWS POINT FROM THE STRONGEST DIPOLE TO THE MONOPOLE)



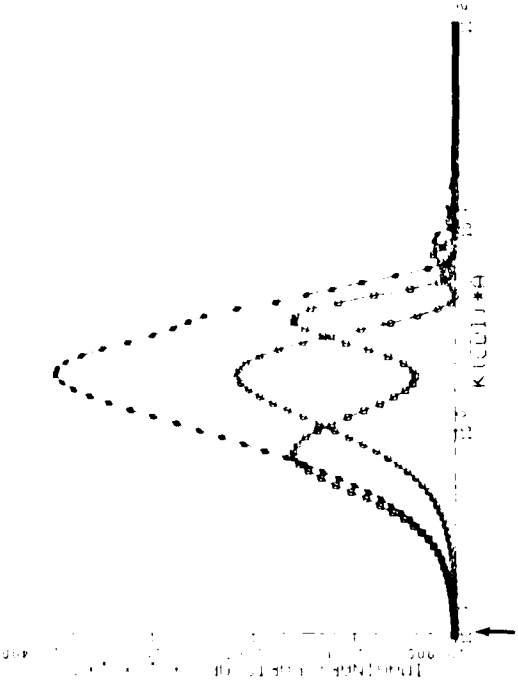
20.4 kHz

FIGURE 10. GLASS INCLUSION IN A PLASTIC MATERIAL



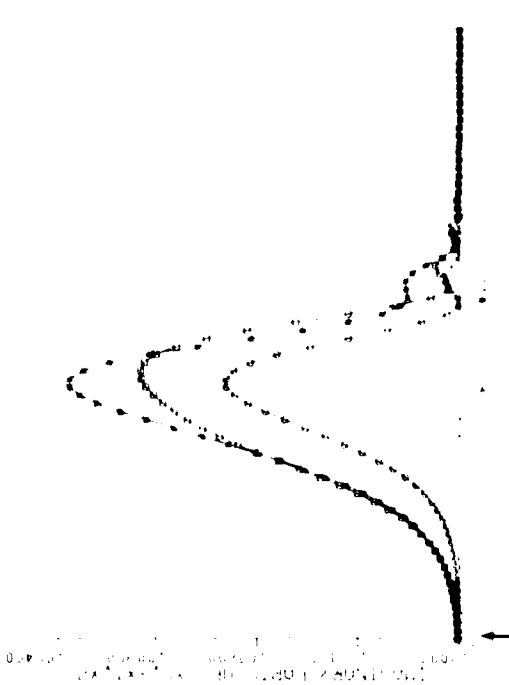
1.43 kHz

FIGURE 12. HEAVY SILICONE RUBBER INCLUSION IN HARD RUBBER



20.4 kHz

FIGURE 9. METAL INCLUSION IN A PLASTIC MATERIAL



20.4 kHz

FIGURE 11. POROUS METAL INCLUSION IN A PLASTIC MATERIAL

Note that the host materials in Figs. 4, 9 and 13 are identical.

Elastic Materials in Rubber Matrices

The dipole is clearly the lowest resonance in this group. It virtually dominates the spectrum at low frequencies. It can be moved to lower frequencies by increasing the mass of the inclusion and/or by decreasing the shear modulus of the host. The dipole resonance in Fig. 17 for instance (the lowest resonance observed in this study) is at $K(C_{d_1})*A = 5.34 \times 10^{-3}$ or $f_o = 580$ Hz. In all examples, except that for porous metallic inclusions in rubber, it is possible to calculate the resonance frequency of the dipoles from Eq. 11.

Note that the host materials in Figs. 1, 14, 15, and 16 are essentially the same.

Also note that in going from Fig. 14 to Fig. 15, we are merely decreasing the mass of the inclusion. In Fig. 15 we observe a dipole double resonance. In going to Fig. 14, the low frequency peak of this resonance increases at the expense of the high frequency one. Had we decreased the mass, the high frequency peak would have increased at the expense of the low frequency one. We have observed this effect several times in other material combinations (see Table 4 and Fig. 8, sequences 11 to 18 and 23 to 29). Hence the motion of the dipole is not an outright motion of a single resonance, but rather an exchange of emphasis on two distinct resonance frequencies. But, as mentioned before, although the low frequency dipole resonance (heavy inclusion) may occur well below the monopole, the high frequency one (light weight inclusion), although above the monopole in frequency, remains close to it.

Combining these observations with our remarks on air filled cavities in elastic solids and drawing on our more detailed studies, Table 4 and Fig. 8, we may make the following general statement about the movements of the first two multipoles in rubber and plastic matrix materials: Starting with a light weight, highly compressible, low shear inclusion (air) in an elastic solid, the monopole is below the dipole. As we decrease the compressibility of the inclusion, regardless of its shear rigidity, while keeping its mass constant, the monopole resonance has a tendency to move to higher frequencies than the dipole. Here we are of course leaving the domain of real substances at ordinary temperatures (extremely lightweight but fairly incompressible). In order to return to real substance, we now increase the mass of the inclusion. To be specific, let us also assume that we have increased its shear rigidity. Hence we are dealing with a solid inclusion. The dipole first moves through the effect described above, until its major resonance occurs above that of the monopole, although close to it (entries 12 to 16 in Table 4 and Fig. 8). If we now keep increasing the mass of the inclusion, the same effect will bring the dipole back down below the monopole (see entries 16 to 18 in Table 4 and Fig. 8).

So far the host material has remained unchanged. As mentioned before, decreasing the shear rigidity of the host while increasing the mass of the

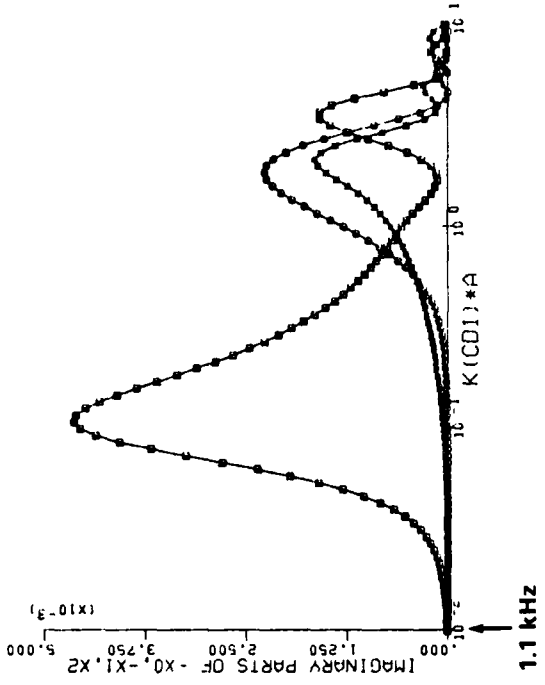


FIGURE 14. METALLIC INCLUSION IN RUBBER

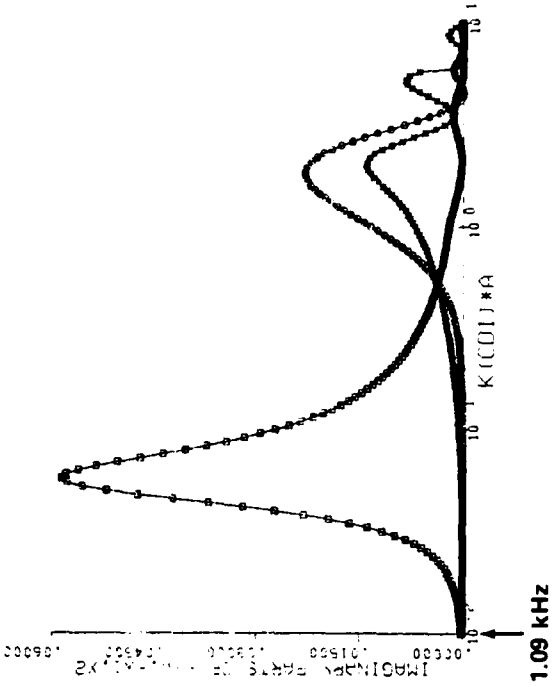


FIGURE 16. STEEL INCLUSION IN RUBBER

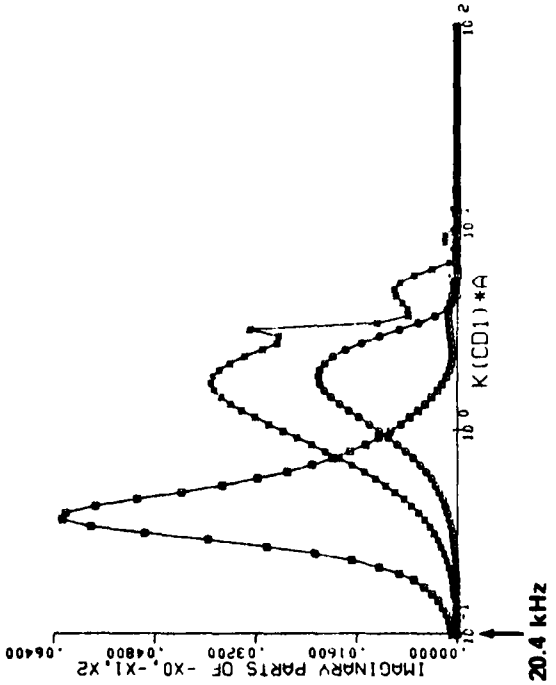


FIGURE 13. STEEL INCLUSION IN A PLASTIC MATERIAL

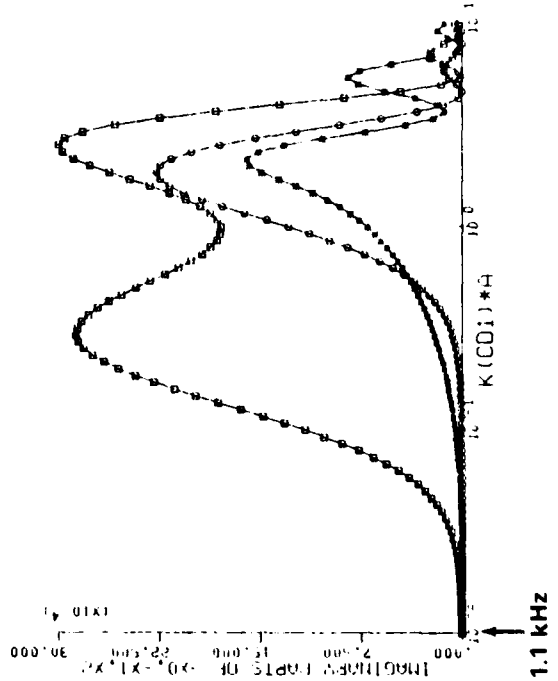
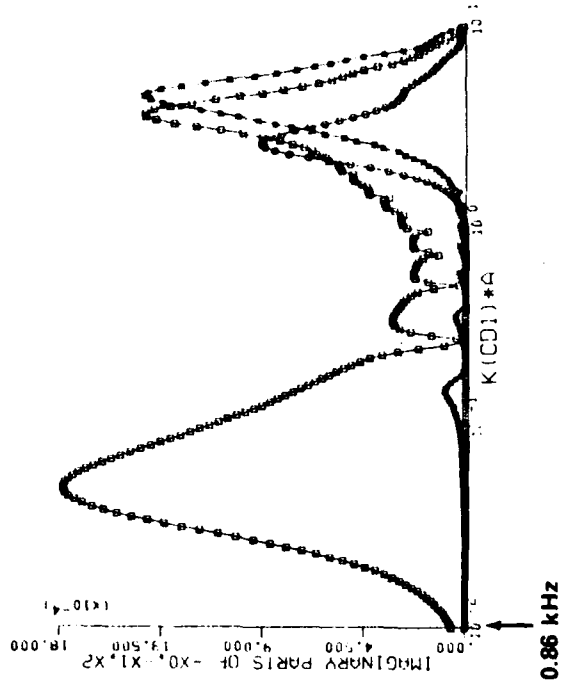
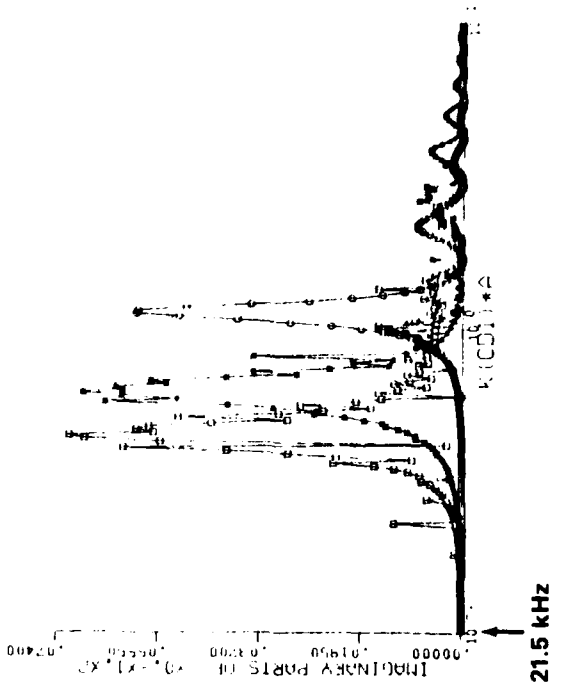


FIGURE 15. POROUS METAL INCLUSION IN RUBBER



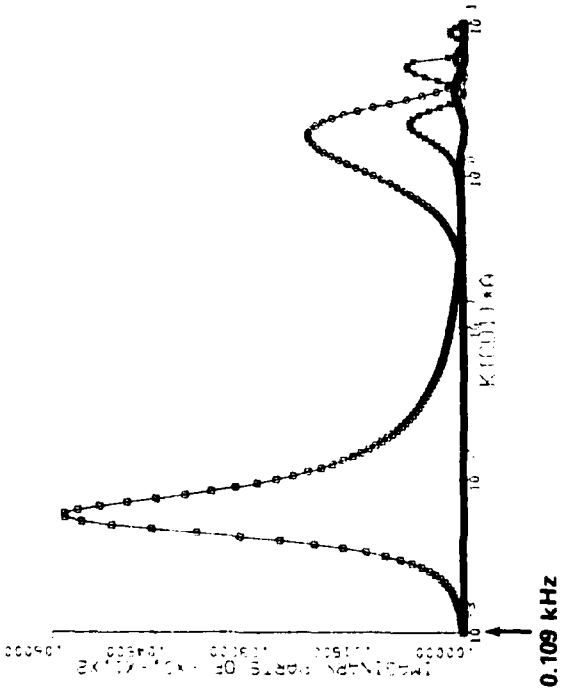
0.86 kHz

FIGURE 18. HEAVY SILICONE RUBBER INCLUSIONS IN SILICONE RUBBER



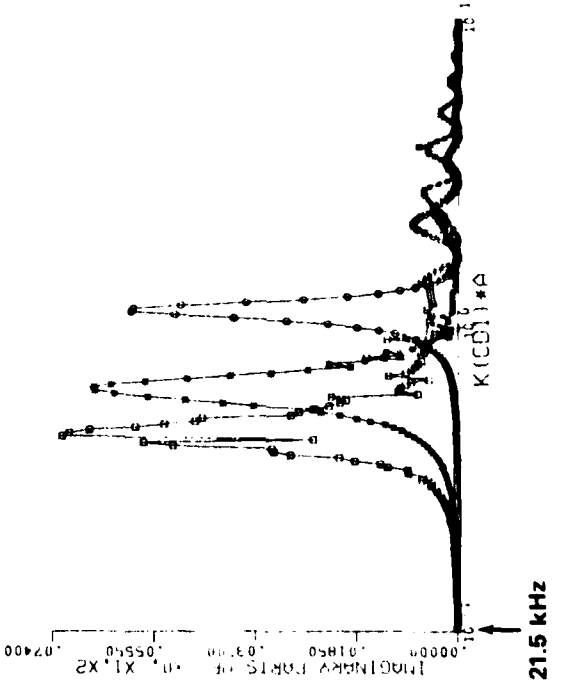
21.5 kHz

FIGURE 20. HEAVY SILICONE RUBBER INCLUSION IN SYNTACTIC FOAM



0.109 kHz

FIGURE 17. STEEL INCLUSION IN VERY LOW SHEAR, SOFT RUBBER



21.5 kHz

FIGURE 19. HEAVY SILICONE RUBBER INCLUSIONS IN SYNTACTIC FOAM

inclusion will yield the lowest possible dipole frequencies. More generally, as a rule, the more waterlike the host medium, i.e., the smaller C_{s_1}/C_{d_1}, the lower the resonance frequencies of the dipole and the monopole. The general domains of the resonance frequencies of the two primary matrix materials in Table 4 and Fig. 8 illustrate this point. The matrix materials of examples 3 to 18, as well as of 30 and 31 are waterlike; those of examples 21 to 29, 32 to 34 as well as those of 1 and 2 are not. Examples 19 and 20 form a bridge between these sequences.

Elastic Materials in Syntactic Foam

Heavy elastic silicone rubber materials in elastic syntactic foams have a tendency to yield spectra in which the principal resonances are broken up into many minor ones. Our best example of this effect is shown in Fig. 21. If absorption is introduced, particularly into the rubber inclusions, yielding more realistic materials, the resonances become smooth again (see below).

Note that the host material in Fig. 5 is the same as the host material of the present group.

VISCOELASTIC MATERIALS

Viscoelastic Materials in Plastics and Hard Rubber Matrices

As before, the dipole is below the monopole in these solid-solid combinations, or if not, at least it overlaps it considerably. The latter is true for the example shown in Fig. 22, which is the example of Fig. 12, except that absorption was added.

At low frequencies in Fig. 22 the dipole and quadrupole coefficients are negative. This does not mean of course that we are dealing with "negative absorption." Absorption must be calculated by combining all of the $\mathrm{Im}(X_1)$ in some way; at the lowest concentrations, essentially by summing them. But the extra factor of $K(C_{d_1})$ in front of the bracket in Eq. 2, which is complex valued now, must also be taken into account.

Viscoelastic Materials in Silicone and Foamed Rubber Matrices

This group contains an example of a plastic material in silicone rubber, Fig. 27, in which the quadrupole dominates. The monopole and the dipole overlap each other.

When heavy inclusions are considered, the usual dipole predominance is conspicuous again (see Figs. 28 and 29). Note also that except for relaxation absorption, the host materials in Figs. 18 and 29 are identical.

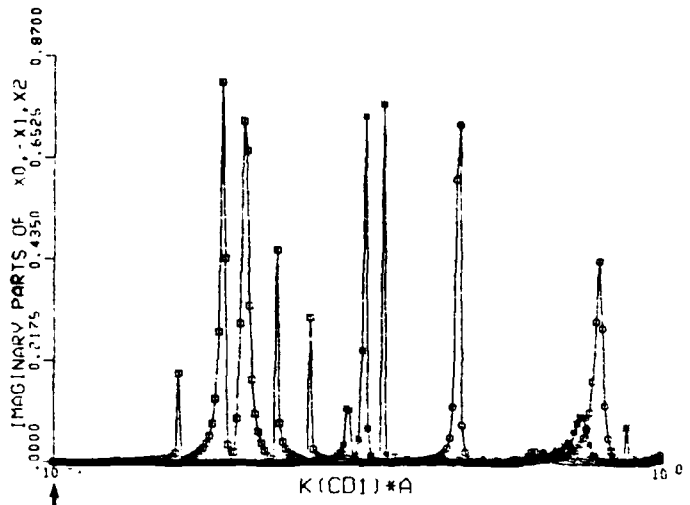


FIGURE 21. HEAVY LOADED SILICONE RUBBER INCLUSIONS IN SYNTACTIC FOAM

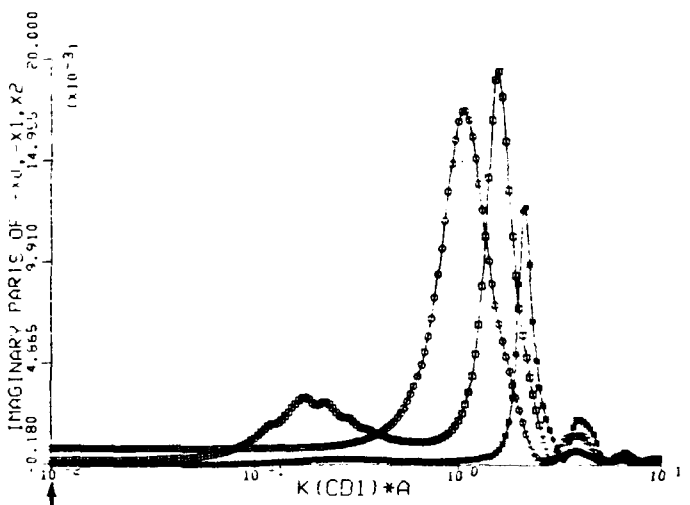


FIGURE 22. HEAVY SILICONE RUBBER INCLUSION IN HARD RUBBER

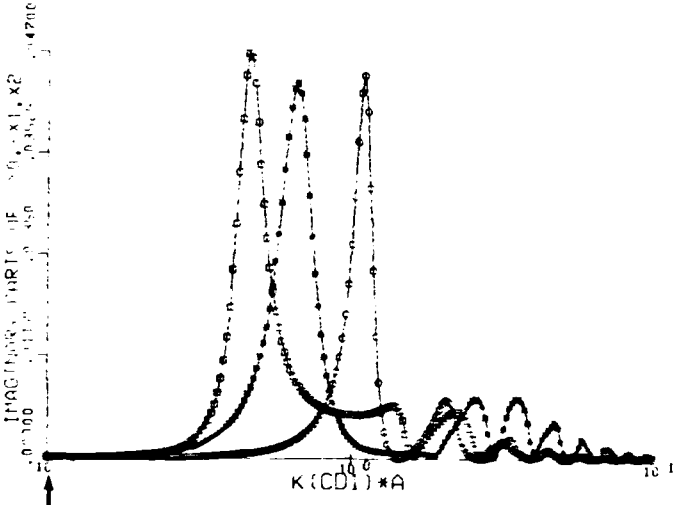


FIGURE 23. HEAVY, LOADED SILICONE RUBBER INCLUSION IN A HEAVY PLASTIC MATERIAL

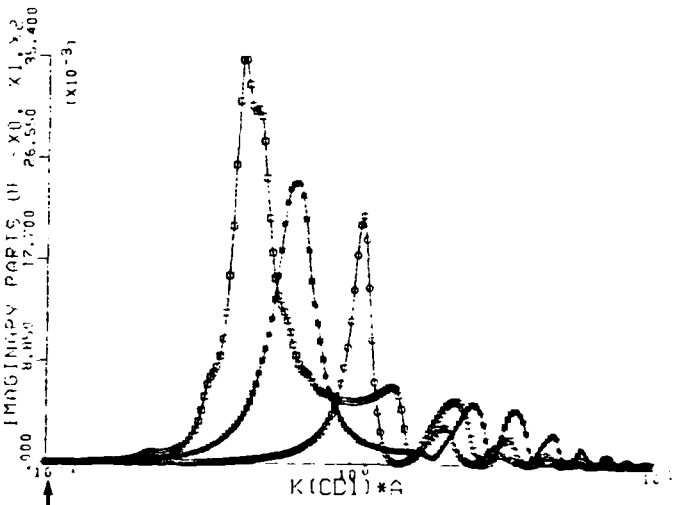
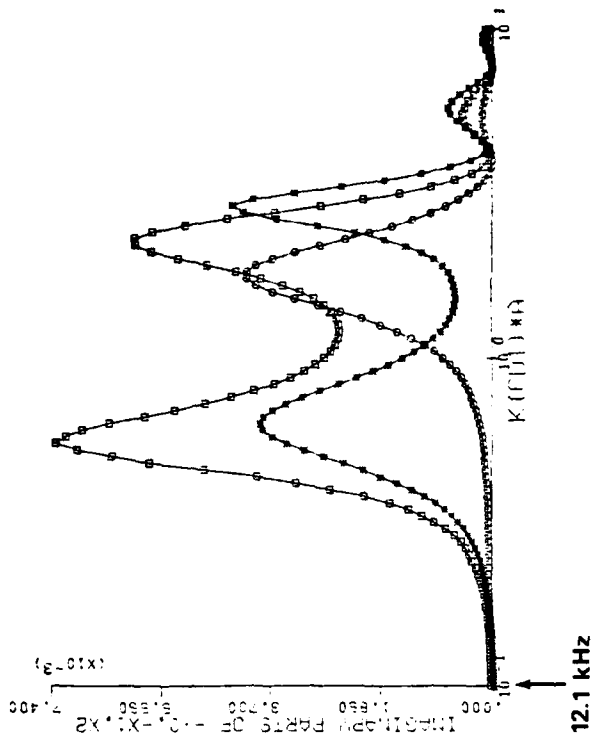
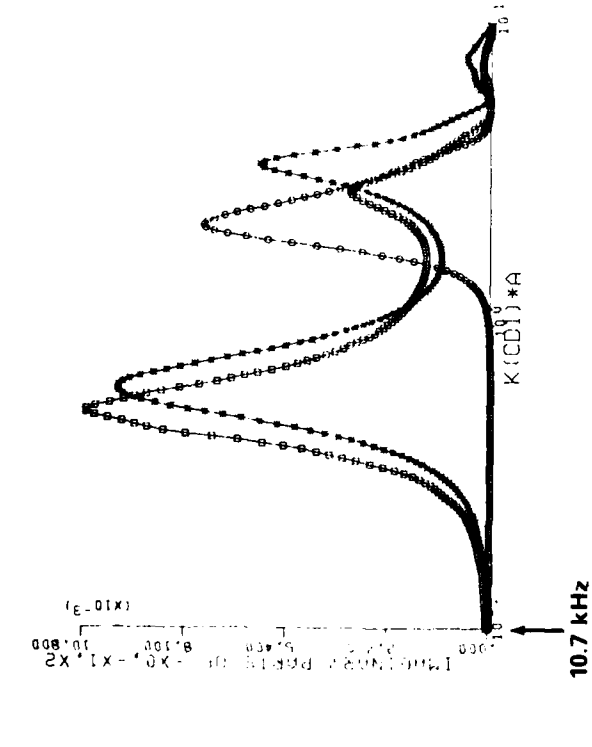


FIGURE 24. HEAVY, LOADED SILICONE RUBBER INCLUSION IN A HEAVY PLASTIC MATERIAL



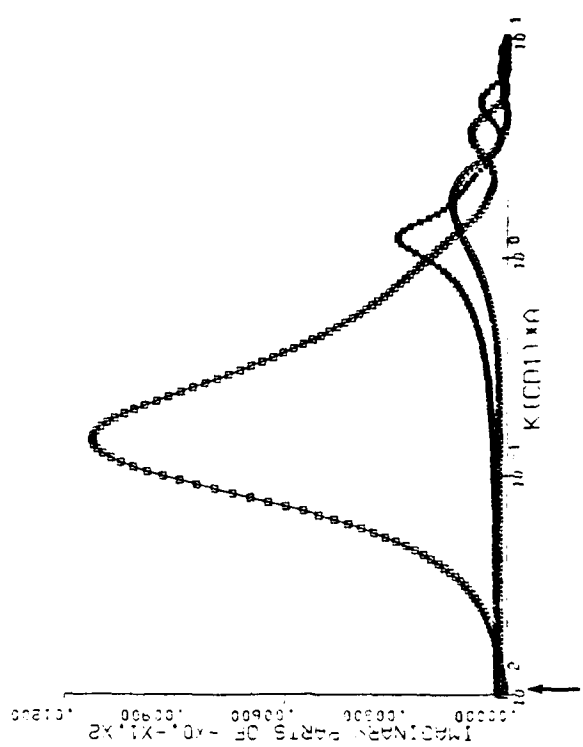
12.1 kHz

FIGURE 26. HEAVY SILICONE RUBBER INCLUSIONS IN PLASTIC MATERIAL



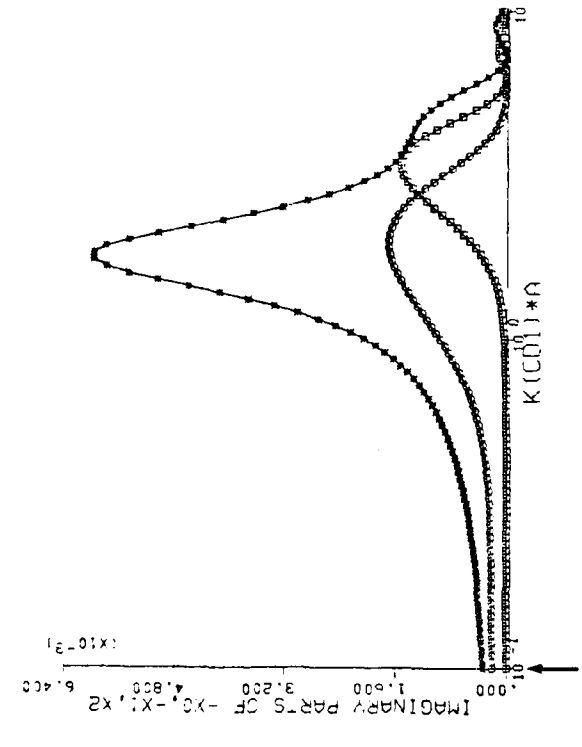
10.7 kHz

FIGURE 25. HEAVY LOADED RUBBER IN A HEAVY PLASTIC MATERIAL



0.803 kHz

FIGURE 28. HEAVY, LOADED PLASTIC IN SILICONE RUBBER



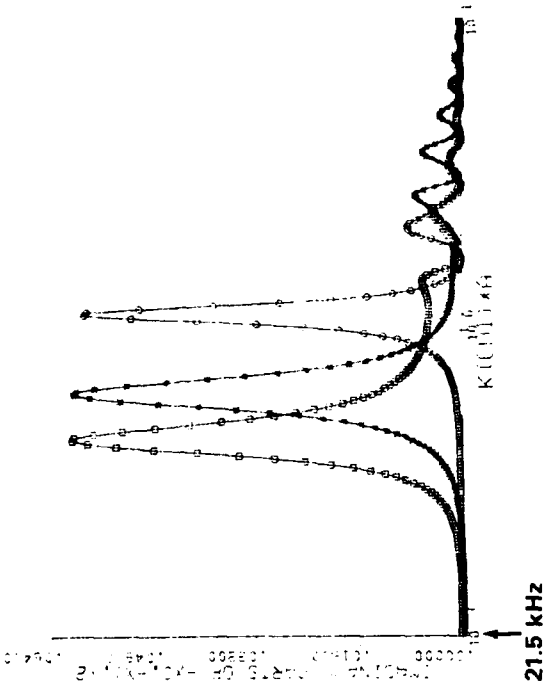
8.03 kHz

FIGURE 27. PLASTIC INCLUSIONS IN SILICONE RUBBER

Viscoelastic Materials in Syntactic Foam

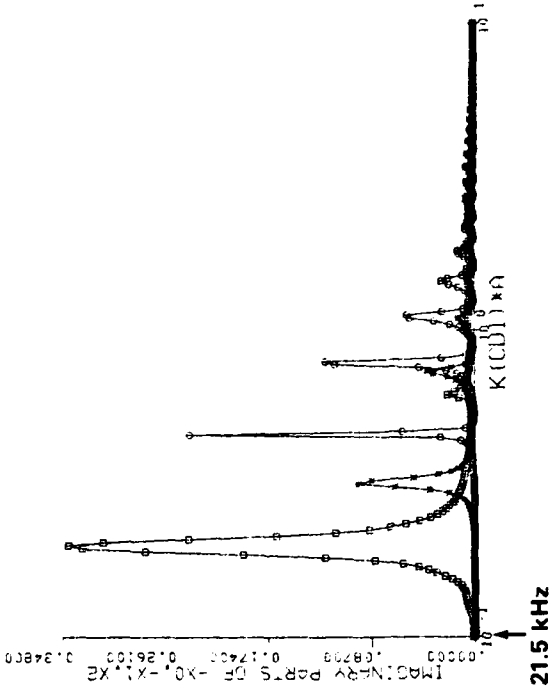
As in the case of elastic foams, resonances are relatively sharp and the dipole is well isolated from the other two. Note also the multiplication factors we have used.

When the inclusions become more highly absorbing, the resonance peaks spread out but the dipole continues to dominate the spectrum (see Fig. 33).



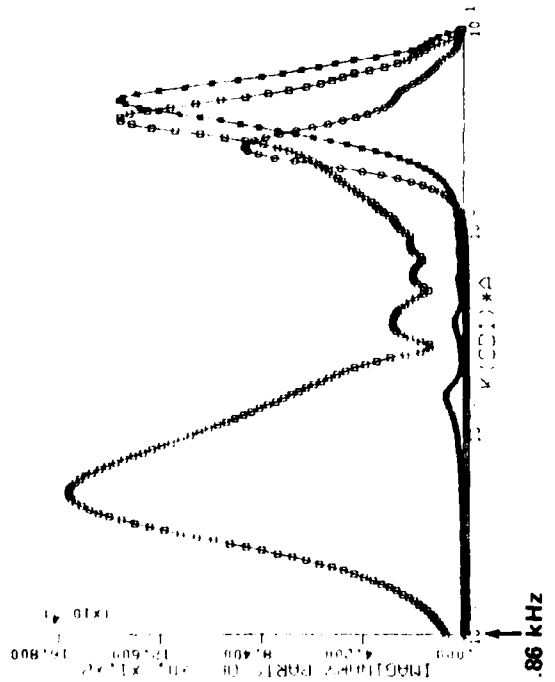
21.5 kHz

FIGURE 30. HEAVY SILICONE RUBBER INCLUSIONS IN SYNTACTIC FOAM



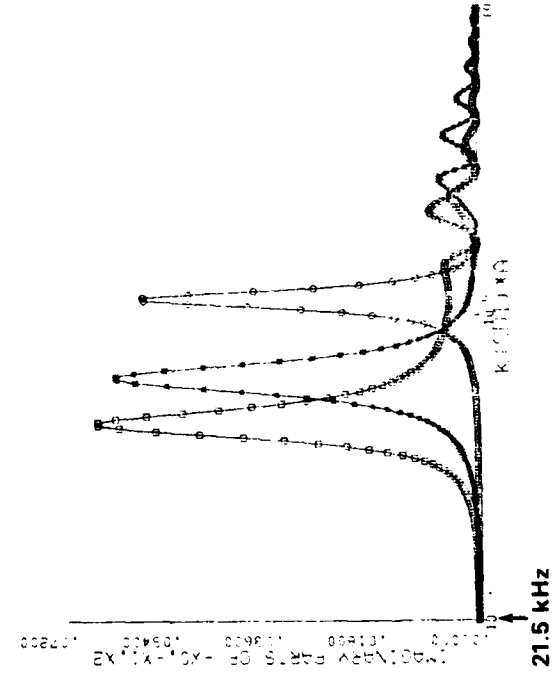
21.5 kHz

FIGURE 32. HEAVY, LOADED SILICONE RUBBER IN SYNTACTIC FOAM



.86 kHz

FIGURE 29. HEAVY SILICONE RUBBER INCLUSIONS IN SILICONE RUBBER



21.5 kHz

FIGURE 31. HEAVY SILICONE RUBBER INCLUSIONS IN SYNTACTIC FOAM

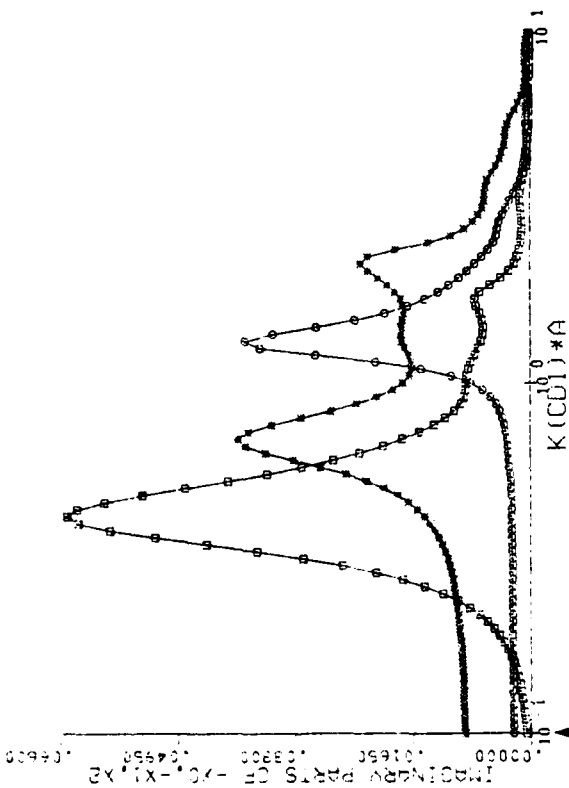


FIGURE 33. HEAVY, LOADED PLASTIC INCLUSIONS IN SYNTACTIC FOAM

CHAPTER 5

CONCLUDING REMARKS

This brings us to the end of our survey. In Table 3 we have summarized the edge frequencies, f_Γ, of the materials in Tables 1 and 2 in monotonic order. Steel in very soft rubber gives the lowest and air in steel the highest f_Γ. Between entries 9 and 10, air in rubber and heavy loaded silicone rubber in a heavy plastic there occurs almost a tripling of f_Γ, from 13.0 to 33.4 kHz. This is the dividing line between highly compressible fluids (air) or moderately heavy to heavy solid materials in relatively soft rubbery media on the one hand, where f_Γ is relatively low, and all other materials of this study, where f_Γ is relatively high.

REFERENCES

1. "Multiple Scattering of Waves," Waterman, P. C. and John Truel, Journal of Mathematical Physics, 2, p. 512, 1961.

2. Akira Ishimaru, Wave Propagation and Scattering in Random Media, Volume 2, Academic Press, 1978.

3. Kerner, E. H., "The Elastic and Thermoelastic Properties of Composite Media," Proc. Physical Society, London B69, 1956, p. 808.

4. Yamakawa, N., "Scattering and Attenuation of Elastic Waves," Geographical Magazine, Tokyo, 31, 1962, p. 63.

5. Waterman, P. C., "Matrix Theory of Elastic Wave Scattering," J. Acoust. Society of America, 60, 1976, p. 567.

6. Yih-Hsing Pao, "Betti's Identity and Transition Matrix for Elastic Waves," J. Acoust. Society of America, 64, 1978, p. 302.

7. Meyer, Erwin; Brendel, Klaus and Tamin, Konrad, "Pulsation Oscillation of Cavities in Rubber," J. Acoust. Society of America, 30, 1958, p. 1116.

8. Gaunaurd, G., Scharnhorst, K. P. and Überall, H., "Giant Monopole Resonances in the Scattering of Waves from Gas-filled Spherical Cavities and Bubbles," J. Acoust. Society of America, 65, 1979, p. 573.

9. Junger, M. C., "Dilatational Waves in an Elastic Solid Containing Lined Gas-filled Spherical Cavities," J. Acoust. Society of America, 69, 1981, p. 1573.

10. Chaban, I. A., "Calculation of the Effective Parameters of Microinhomogeneous Media by the Self Consistent Field Method," Soviet Physics - Acoustics, 11, 1965, p. 81.

11. Gaunaurd, G. C., and Überall, H., "Resonance Theory of the Effective Properties of Perforated Solids," J. Acoust. Society of America, 71, 1982, p. 282.

DISTRIBUTION

	Copies
Commander Naval Sea Systems Command Attn: SEA-55N (C. C. Taylor) Washington, DC 20362	1
Defense Technical Information Center Cameron Station Alexandria, VA 22314	12
Naval Research Laboratory Attn: (J. Jarzynski) Department of the Navy Washington, DC 20375	1
Bolt, Beranek & Newman, Inc. Attn: (E. Kerwin) 50 Moulton Street Cambridge, MA 02138	1
Naval Research Laboratory Underwater Sound Reference Division Attn: (J. Blue) (A. Rudgers) (L. VanBuren) (R. Ting) Department of the Navy Orlando, FL 32806	1 1 1 1
Commander Naval Ocean Systems Center Attn: (S. Speidel) Department of the Navy San Diego, CA 92152	1
Naval Coastal System Center Attn: (L. Flax) Department of the Navy Panama City, FL 32407	1
David Taylor Naval Ship Research & Development Center Attn: (W. Reader) (R. Rippeon) (M. Rummerman) (M. Sevik) Carderock Laboratory Bethesda, MD 20084	1 1 1 1
David Taylor Naval Ship Research & Development Center Attn: (J. Eynck) Annapolis Laboratory Annapolis, MD 21402	1
Naval Underwater System Center Attn: (J. Kalonoski) (R. Radlinski) New London Laboratory Department of the Navy New London, CT 06320	1 1
Office of Naval Research Attn: (R. Ellinthorpe) (D. Bradley) (L. Hargrove) Ballston Centre Tower #1 800 N. Quincy Street Arlington, VA 22217	1 1 1
Applied Research Laboratory Pennsylvania State University Attn: (S. Hayek) (A. Stewart) State College, PA 16804	1 1
Library of Congress Attn: Gift and Exchange Division Washington DC, 20540	4

Internal Distribution:	Copies
E431	9
E432	3
R04	1
R30	2
R31	1
R31 (Madigosky)	1
R31 (Scharnhorst)	5
R31 (Balizer)	1

END

DTIC

9-86