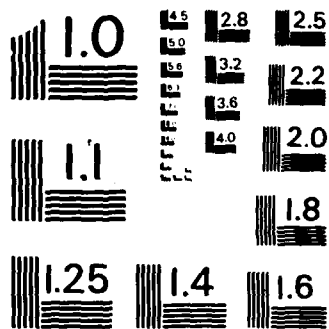MICROCOPY RESOLUTION TEST CHART

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THEORETICAL STUDIES ON DETERMINATIONS, PREDICTIONS AND ACCURACIES OF GEODETIC PARAMETERS AND GRAVIMETRIC QUANTITIES

Urho A. Uotila

The Ohio State University
Research Foundation
Columbus, Ohio 43212

Final Report
10 November 1981 – 9 April 1986

May 1986

The views and conclusions contained in this document are those of the authors, and should not be interpreted as representing the official policies, either expressed or implied, of the United States Government.

This technical report has been reviewed and is approved for publication.

CHRISTOPHER JEKELI
Contract Manager

THOMAS P. ROONEY, Chief
Geodesy & Gravity Branch

FOR THE COMMANDER

DONALD H. ECKHARDT, Director
Earth Sciences Division

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
This report is a final report of the project. It briefly summarizes the research described in detail in twelve scientific reports. The research described includes theories of nutation and polar motion, spherical spline interpolation, local geoid determination in mountain regions, mass point models, prediction of gravity disturbance vector at high altitudes, isostatic earth model and application of the finite element method to the geodetic boundary value problem.

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FOREWORD

This report was prepared by Urho A. Uotila, Professor, Department of Geodetic Science and Surveying, The Ohio State University, under Air Force Contract No. F19628-82-K-0017, and the Ohio State University Research Foundation Projects No. 714255, 714256 and 716187. This report is the final report of the contract that started November 10, 1981 and ended April 9, 1986. This project has been administered by the Air Force Geophysics Laboratory, Air Force Systems Command, Hanscom AFB, Massachusetts, with Dr. Christopher Jekeli, Contract Manager.

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1. Introduction

This final report covers the period of Contract No. F19628-82-K-0017 from November 10, 1981 through April 9, 1986. The research conducted under this contract has been carried out under the general theme of the project which was Theoretical Studies on Determination, Prediction and Accuracies of Geodetic Parameters and Gravimetric Quantities. The results of the research have been reported in 10 scientific reports published and two additional reports which are being typed at this time. One of the reports currently being typed will possibly be submitted to the Graduate School as a Ph.D dissertation.

Other scientific activies are given in Section 3 and a list of personnel associated with this project is given in Section 4. The complete list of reports is given in Section 5 and papers and presentations in Section 6.

The progress and other activities under the project have been reported in seventeen quarterly progress reports. The best sources of information about the research carried out under the project are the individual reports. A brief discussion of various reports is given in the following section.

2. Summaries of the Reports

Scientific Report No. 1 (by Willi Freeden) introduces spherical spline functions by use of Green's (surface) functions with respect to the Beltrami operator on the sphere. The method of interpolation by spherical splines is formulated as variational problem on minimizing a (sobolev) "energy" norm under interpolatory constraints. The process is constructed so as to have the so-called permanence property, i.e. the transition from the interpolation spline with respect to N data to the interpolating spline with respect to N + 1 data necessitates merely the addition of one more term, all the terms obtained formerly remaining unchanged. The algorithm is numerically stable and very economical as regards the number of operations.

Scientific Report No. 2 (by Helmut Moritz) is the third and last of a series of reports on the theoretical description of polar motion and nutation. In order to have a complete picture, the earlier work is reviewed here to give a

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proper background. Moritz's earlier report, ("Concepts in Geodetic Reference Frames", 1979, OSU DOGS 294, AFGL-TR-80-0052) provides a basis of the ensuing research by reviewing conceptual aspects and problems arising in a precise definition of reference frames for the purpose of geodesy and geodynamics. This includes physical and conventional aspects, as well as relativistic effects. In view of the nonrigidity of the earth, a terrestrial system should be "at rest on the average" with respect to the earth, for which various definitions are possible: "geographical axes", which represent an average over globally distributed fundamental observations; "Tisserand axes", for which the equations of dynamics assume a particularly simple form; and several other possibilities, among them, most obvious but practically least suitable, "principal axes of inertia". Various such definitions are intercompared. Practically the best choice seems to be a conventional terrestrial system, close to but more precisely defined then the present BIH system, and at the same time close to a mean Tisserand frame for the mantle (and also to a mean system of principal axes of inertia; the concept of average or mean is especially important here since the instantaneous system of principal axes performs, about this mean system, daily oscillations of an amplitude around 60 meters!).

Further progress was possible only through a detailed theoretical study of the earth's rotation, since earth-fixed and inertial reference systems are related through precession, nutation, and polar motion.

Moritz's report "Theories of Nutation and Polar Motion I" (1980, OSU DOGS 309, AFGL-TR-80-0363), as a first step, treats various simplified earth models: the rigid earth (still important as a reference), the purely elastic earth, and the Poincaré model consisting of a rigid mantle and a homogeneous liquid core. Since the liquid core exerts a rather strong resonance effect on earth rotation, especially on nutation, the Poincaré model represents a step in the right direction, but it is still necessary to take into account the nonrigidity of the mantle.

After the pioneering work of Jeffreys (1949, 1957), a breakthrough was achieved by Molodensky in 1960. Enormous subsequent work was based on his approach, culminating in the extremely accurate values for nutation, etc., by Wahr (1979), which were adopted by both the IAU and IUGG. Therefore, such models, featuring an elastic mantle and liquid core (possibly with a solid inner core) have been called Molodensky models.

Moritz in his report "Theories of Nutation and Polar Motion II" (1981, OSU DOGSS 318, AFGL-TR-81-0288) reviews and interrelates the methods of Jeffreys, Molodensky, Sasao et al., and Wahr. This was not an easy enterprise since the fundamental papers by Jeffreys, Jeffreys and Vicente, and Molodensky are very obscurely written and the work of these authors is unrelated to the other.

A new approach to the rotation of an earth model of Molodensky type was presented in Scientific Report No. 2, "Theories of Nutation and Polar Motion III" (1982, OSU DOGSS 342, AFGL-TR-82-0387). The well-known equations of Sasao, Okubo and Saito (1980), which represent probably the simplest formulation of Molodensky's problem but were found by these authors in a very complicated and obscure way, were derived in this scientific report No. 2 for the first time in a logically simple way by using a little-known variational principle of classical dynamics, due to Poincaré (1901). This provides considerable insight into the mathematical and physical structure of the problem. Expressions for nutation and polar motion for the Molodensky model were also derived.

Precession, lunisolar nutation, and _forced_ (lunisolar) polar motion can now be modeled very adequately in this way, to about 0.001", corresponding to 3 cm for pole position on the earth's surface.

What remains a problem is _free_ polar motion. It now seems that 80% of the irregularities of free polar motion may be explained by the exchange of angular momentum between the atmosphere and the solid earth, that is, by the seasonally varying global atmospheric circulation. This is an important area of contemporary research and is theoretically already rather well understood, as well as tidal effects. Other geophysical effects on free polar motion (earthquakes, electromagnetic core-mantle coupling, etc.) are still very poorly understood, so that direct observation, especially by Doppler, laser and VLBI, will be indispensable, at least during the next decades.

Scientific Report No. 3 (by Willi Freeden) transcribes the standard procedure of expanding an external gravitational potential by trial functions (e.g., mass points, multipoles, spherical harmonics) from the classical spherical case to non-spherical models.

The external gravitational potential of the earth is shown to be expandable into a series being convergent outside and on any closed surface surrounding

the earth's surface in outer space. In addition, for each compact subset of the outer space with positive distance to the earth's surface, the convergence is uniform.

A numerical method of computing the external gravitational potential by truncated series expansions concludes the paper.

Scientific Report No. 4 (by Hans Sünkel) is his second report discussing mass point models. Among the various methods for the representation of the earth's anomalous gravity field, the point mass model representation is particularly attractive both because of its conceptual simplicity and probably because of its closest "neighborhood" to geological reality among all proposed methods. The generation of a point mass model, however, is not only non-unique, but also not quite simple. In his former report, "Point Mass Models and the Anomalous Gravitational Field" (1981, OSU DOGSS 328, AFGL-TR-82-0084), Sünkel demonstrated the reasons for a multi-level mass point model employing known statistical properties of the earth's gravity field. In the present contribution the conceptual mathematical problems have been solved and a conceivable algorithm for the actual generation of a mass point model presented. Frequency domain methods have been frequently used throughout this report.

The evaluation of the kernel which relates point masses and mean gravity disturbances requires integration on the sphere over a limited area, a time-consuming process. In order to speed up calculation, a fast approximation has to be designed and the approximation error to be estimated. Using Peano's theorem on the sphere and the method of Sard, "best" approximations have been derived for various kernel approximation functions and the corresponding approximation errors have been estimated.

The relation between the depth of the point mass level and a set of mean gravity data at zero level has been derived from the principle of minimum deviation of two operators: the smoothing operator (which transforms point data into mean values) and the operator which turns point masses at a certain level into its gravitation.

The algorithm for the mass point model generation given in the report is entirely based on fast Fourier transform methods. Due to the use of mean values of various kind, a recursive procedure had to be designed.

Scientific Report No. 5 (by Helmut Moritz) reviews and interrelates various methods for the local or regional determination of the geoid or of height anomalies according to Molodensky from deflection of the vertical -- the astrogeodetic method -- or by a combination of vertical deflections with gravity anomalies by least-squares collocation. Extensive consideration is given to the geometrical and physical structure of the problem, especially to reduction for the curvature of the plumb line and to topographic-isostatic reduction of vertical deflections in the classical sense, which is a reduction to sea level, and in the modern sense, where the point remains on the earth's surface. The application of collocation, refined by a higher-degree reference field and by topographic-isostatic reduction, to the present problem is discussed.

This method is currently being applied to the determination of a precise detailed geoid in the alpine region of Austria.

Scientific Report No. 6 (by Hans Sünkel) discusses splines and their equivalence to collocation. Least squares collocation and spline functions have been used for the purpose of processing homogeneous as well as heterogeneous data for more than one decade yielding comparable results. Therefore, a relation between these two methods could be anticipated. Since essential features of mathematical methods can be studied best if regular data patterns are investigated, a one-dimensional data distribution with equal data spacing along the real line has been investigated. Very simple relations between all members of the infinite family of spline functions have been found. It could be demonstrated that spline solutions correspond essentially to collocation solutions; moreover, the elementary properties of the frequently used Bjerhammer/Shepard interpolation could be traced back to the properties of splines of infinite degree, strange effects of that kind of interpolation could be explained in a simple way. Of great practical relevance is a rule of thumb which related the prediction error, the data sampling rate, and the correlation length of the covariance function, which has been derived purely theoretically and meanwhile proved numerically.

Based on the experiences from the study of one-dimensional data distributions along the real line, the spline-collocation concept has been extended to the two-dimensional case for applications in the plane and its associated half-space as well as on the sphere and its outer space. A great

deal of effort has been put into a systematical treatment of all spline functions, which has been made possible by employing the concept of Green's functions corresponding to differential operators of various degree and complexity. Frequency domain methods have been used extensively. The very answer of that approximation work is that spline and collocation solutions are essentially equivalent.

Scientific Report No. 7 (by Hans Sünkel) is a feasibility study for the prediction of the components of the gravity disturbance vector at high altitudes on a multitude of data distribution patterns. Basically two techniques have been investigated and compared: the integral solution and the collocation approach; differences between the solutions were within a 10% range. Considering the large amount of computer time required for a collocation estimate, the subsequent investigations were confined to the integral approach only. Only two significant error sources and their impact on the error budget of the estimated gravity vector components have been dealt with: the representation error implied by the use of mean anomalies and the estimation error due to data noise.

Under these assumptions, a prediction error of 1 mgal for the vertical component and about 2.1 mgal for the horizontal components of the gravity disturbance vector can be achieved for a typical elevation of 50,000 feet. Suggestions have been given as how to reduce the prediction error in particular for the horizontal components.

These feasibility studies made a strong dependence of the prediction error particularly of the vertical component of the gravity disturbance vector, on the rms error of the available data obvious. Therefore, all known effects which contribute to this mean anomaly error budget had to be investigated in detail. Emphasis has been put on the impact of the data sampling rate, both in terms of gravity anomaly sampling and of digital terrain information sampling, and on the particular way of processing this kind of information.

Scientific Report No. 8 (by Willi Freeden) reports a development of a technique for generating spherical harmonics by exact computation (in integer mode). This technique circumvents any source of rounding errors.

Essential results of the theory of spherical harmonics are recapitulated by intrinsic properties of the space of homogeneous harmonic polynomials. Exact

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computation of (maximal) linearly independent and orthonormal systems of spherical harmonics is explained using exclusively integer operations. The numerical efficiency is discussed.

The development of exterior gravitational potential in a series of outer (spherical) harmonics is investigated. Some numerical examples are given for solving exterior Dirichlet's boundary-value problems by use of outer (spherical) harmonic expansions for not-necessarily spherical boundaries.

Scientific Report No. 9 (by Dingbo Chao and Edward Baker) discusses how the circular errors of a gravity meter could be effectively calibrated in a laboratory. An optimization method is used in this study. Minimization of the trace of the variance-covariance matrix of adjusted parameters is adopted as the criterion for the optimization. The mathematical analysis of the trace is made in the case of one wavelength in order to find the best distribution of observations, as well as the worst. For several wavelengths, a number of simulative computations are carried out for finding the most effective distribution of observations and the best weights, as well as the worst. A set of numerical solutions for the equations over a certain range of observations is obtained.

Based on the simulative studies, the concepts of phase distribution and effectiveness of observations in the periodic error calibration are presented and so a design for the most effective distribution of observations is introduced. For the calibration of periodic errors with several wavelengths, it is preferable to select two weights that can be mutually compensated in fitting them with all involved periods. some possible compensative weights for an LCR "G" gravity meter with periodic errors of 1206, 1206/17, 1206/34, 134/17, 134/34 and 1 counter units (c.u.) are presented. An attempt is made to answer how many observations should be made for determining the periodic screw errors with reasonable accuracy.

Scientific Report No. 10 (by Hans Sünkel) examines the concept of the conventional Airy/Heiskanen isostatic model from scratch, based on the harmonic analysis of the topographic-isostatic potential. First and higher order approximations for those coefficients are discussed and rule of thumb formulas are given.

The estimated frequency transfer function between the power spectrum of

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the observed gravitational field and the power spectrum implied by the isostatic model strongly suggests a smoothing of the compensation surface according to Vening Meinesz with a smoothing operator of Gaussian bell-shaped type, and a depth of compensation of about 24 km. A proof of equivalence of using a standard Airy/Heiskanen model with a larger compensation depth and a corresponding Poisson smoothed Vening Meinesz model at a smaller depth has been given for the case of linear approximation, yielding an entirely new interpretation of recently discussed isostatic models.

An iterative least-squares process has been designed which provided parameter estimates of that isostatic model in best possible agreement with the observed gravitational potential of the earth. Based on these parameters a set of harmonic coefficients of the topographic-isostatic potential, complete up to degree and order 180, has been computed. Several maps of topography-isostasy implied geoidal heights are presented for comparison and illustration purposes.

Two scientific reports are to be sent to AFGL for review and possible approval:

1) One by Edward Baker is an application of the finite element method. The report begins with introductory materials to place the work in perspective. A brief literature review and overview of the general theory of finite element methods is included here.

Theoretical aspects of the application of the finite element method to the geodetic boundary problem follows the introduction. Alternative approaches to the application are possible: Several of these are shown to lead to the same solution. It is easily shown also that the reference surface may be spherical or ellipsoidal. The choice of basis functions is covered. The recommendations of Meissl ("The Use of Finite Elements in Physical Geodesy", OSU DOGS Report No. 313, AFGL-TR-81-0114, 1981) are followed here. To add some new detail, the algebraic structure of the basis functions and associated linear spaces are treated. This also puts some theory in place for error analysis. connections between spherical harmonic representations of the geopotential and finite element representation enable a relationship to be derived between domain partitioning and the relative error analysis.

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The report also discusses the treatment of numerical issues. Details of implementing the theory beyond those given by Meissl are presented. Two global solutions are made. The first is the highest resolution solution possible in the core of the OSU mainframe, with small height extension. The second solution is of lower resolution, but greater height extent. Computational statistics (normal equation formation, solution, etc.) are presented. Regional and local solutions are discussed, but not numerically computed.

Comparative assessments of the operational use of the two models are given. Computational speed and storage requirements of the gravity disturbance vector are the focal point of the attention. Comparison is made against the spherical harmonic expansion of Rapp of similar resolution.

2) The second scientific report to be submitted is written by Bela Szabo. It is a review paper, which starts with discussions of geopotential models such as GEM, OSU, SAO and GRIM and some special models. Following these discussions the utilization of spherical harmonic models in geodesy is reviewed. The description of the global gravity field by spherical harmonics includes most of the gravity models published since the late 1970's. The observation programs and techniques for improvement of the gravity field models are discussed including those which are utilized currently and those which are in the planning stage for future use. The present techniques and potential improvements for adjustment and data combination for gravity modeling are summarized. Regarding the local and regional gravity field estimation the following topics are discussed:

(a) the spectral properties of various data types

(b) estimation of gravimetric quantities in local and regional areas.

Reviews also include the prediction of gravity disturbance at high altitudes and airborne gradiometry with accuracy and feasibility studies. The summary of the report includes recommended actions and studies for improvements in each area reviewed.

3. Other Activities

Dr. Ivan Mueller with two Graduate Associates, Priovolos and Tsaoussi, participated in the activities of Project MERIT. The Project MERIT ended on September 30, 1984 and data received was analyzed. The results were presented at the MERIT workshop held July 31 - August 2, 1985 in Columbus, Ohio. Dr. Mueller was convenor of the workshop, which was the conference of the MERIT-COTES Joint working Groups of International Astronomical Union and International Association of Geodesy. Two volumes of the "Proceedings of the International Conference on Earth Rotation and the Terrestrial Reference Frame" were published by the Department of Geodetic Science and Surveying.

4. Personnel

The *following persons served as senior research Associates*: Drs. Helmut Moritz, Hans Sünkel and Willi Freeden. Bela Szabo served as a consultant. The following Graduate Research Associates were employed under this contract: Edward Baker, George Priovolos and Lucia Tsaoussi. Mr. Dingbo Chao participated in the research related to the project but was not appointed in the project in any capacity. Roberta Canegali and Karen Wasielewski provided secretarial support. Urho A. Uotila was the principal investigator and project supervisor for OSU Research Foundation Projects No. 714255 and 714256 and Ivan I. Mueller was principal investigator for Project No. 716187.

5. The Scientific Reports Produced Under the Contract

Scientific
Report No.

1   Freeden, Willi, On the Permanence Property in Spherical Spline Interpolation, DOGSS Report No. 341, AFGL-TR-82-0386, 88 pages, November 1982. ADA126263.

2   Moritz, Helmut, Theories of Nutation and Polar Motion III, DOGSS Report No. 342, AFGL-TR-82-0387, 116 pages, December 1982. ADA126289.

3   Freeden, Willi, Least Square Approximation by Linear Combinations of (Multi)Poles, DOGSS Report No. 344, AFGL-TR-83-0017, 86 pages, April 1983. ADA123688.

4   Sünkel, Hans, The Generation of a Mass Point Model from Surface Gravity Data, DOGSS Report No. 353, AFGL-TR-83-0318, 34 pages, December 1983. ADA142327.

5   Moritz, Helmut, Local Geoid Determination in Mountain Regions, DOGSS Report No. 352, AFGL-TR-84-0042, 47 pages, December 1983. ADA145799.

6   Sünkel, Hans, Splines: Their Equivalence to Collocation, DOGSS Report No. 357, AFGL-TR-84-0215, 55 pages, August 1984. ADA155250.

7   Sünkel, Hans, On the Reduction of Gravity Data for the Prediction of the Gravity Disturbance Vector at Altitudes, DOGSS Report No. 359, AFGL-TR-85-0084, 50 pages, July 1984. ADA160358.

8   Freeden, Willi, Computation of Spherical Harmonics and Approximation by Spherical Harmonic Expansions, DOGSS Report No. 362, AFGL-TR-85-0063, 133 pages, February 1985. ADA160445.

9   Chao, Dingbo, and Baker, Edward M., A Study of the Optimization Problem for Calibrating a LaCoste and Romberg "G" Gravity Meter to Determine Circular Errors, DOGSS Report No. 364, AFGL-TR-85-0208, 94 pages, September 1985. ADA165218.

10   Sünkel, Hans, An Isostatic Earth Model, DOGSS Report No. 367,
     AFGL-TR-85-0239, 53 pages, September 1985. ADA164608.

6. Papers and Presentations

The research described in the following list was supported fully, or in part, by this contract.

Moritz, Helmut: A Variational Principle for Molodensky's Liquid-core Problem, Bulletin Geodesique, Vol. 56, No. 4, 1982.

Mueller, Ivan I. and Ziging Wei: MERIT Main Campaign: Reference Frame Intercomparisons, Proceedings of the International Conference on Earth Rotation and the Terrestrial Reference Frame, Vol. 2, pp. 667-672. DOGSS, OSU Columbus 1985.

Uotila, Urho A: Gravity Base Station Networks and their Adjustments, Invited paper presented at the 1982 ACSM-ASP Convention, March 14-20, Denver Colorado, printed in Technical Papers of the 42nd Annual Meeting of the American Congress on Surveying and Mapping, pp. 75-84, 1982.

Uotila, Urho A: Report on Activities of Working Group #2 of IGB "World Gravity Standards", presented at the General Meeting of the International Association of Geodesy in Tokyo, May 13, 1982, Bulletin D'Information No. 51, Bureau Gravimetrique International, pp. 32-33, 1982.

Uotila, Urho A: Some Analyses of Absolute and Relative Gravity Measurements in the U.S.A., Bulletin D'Information No. 51, Bureau Gravimetrique International, pp. 69-76, 1982.

The forthcoming book "Earth Rotation: Theory and Determination" by H. Moritz and I.I. Mueller will systematically cover, at the present level of knowledge, all problems raised by Items 4 and 5 (and probably also of Item 11) of the Work Statement. Appropriate credit is given to AFGL.

END

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