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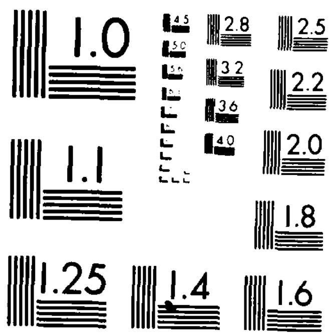

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TIME SERIES PREDICTION OF

HURRICANE LANDFALL

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TIME SERIES PREDICTION OF
HURRICANE LANDFALL

by

THOMAS FRANK CURRY, M.S., B.S.

DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

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May 1986

A C K N O W L E D G E M E N T S

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T. F. C.

The University of Texas at Austin

May, 1986

TIME SERIES PREDICTION OF

HURRICANE LANDFALL

Publication No. __________

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For many years, land development in the coastal regions of the Gulf of Mexico and the eastern seaboard has continued unabated. As coastal populations increase, it is becoming more and more difficult to evacuate people from hurricane-threatened areas and to secure industrial plants. Greater accuracy is required in predicting hurricane landfall in order to insure timely evacuation.

A significant result of this research is the classification of past storms by time series stationarity category which relates to direction of movement. Also, a psi-weight representation of the forecast is used to develop a bivariate Normal confidence ellipse for the threshold autoregressive model.

It is shown that the landfall of North Atlantic hurricanes and tropical storms can be accurately predicted by modeling the storm track as a bivariate (latitude and longitude) fifth-order autoregressive process. A threshold approach is used to allow model parameters to change as the storm moves to a new region of the ocean. For test cases, operational average 72 hour prediction error is at least three standard deviations below the average error of the official forecasts issued by the National Hurricane Center.

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CHAPTER I

INTRODUCTION

"On Saturday, September 8, 1900 a major hurricane struck Galveston, Texas, resulting in the death of approximately 6000 persons" (Carter, 1983). Despite the fact that mariner warnings had been posted the day before, the residents of Galveston were not prepared for the hurricane.

Since that time, land development in the coastal regions of the Gulf of Mexico has continued unabated. Today the population of the Houston-Galveston metropolitan area is over two million people and property is worth billions of dollars. As coastal populations increase, it is becoming more and more difficult to evacuate people from hurricane-threatened areas. A recent study indicates that it would take 26 hours to evacuate Galveston Island and that the evacuation order must come 36 to 38 hours before anticipated hurricane landfall (Carter, 1983).

When hurricane Alicia came ashore at Galveston on August 17-18, 1983, the city was better prepared. Many people had evacuated, yet Alicia still killed 17 people and injured 3000 (Barron and Allred, 1983). The evacuation order for hurricane Alicia came at 10:00 AM on August 17, approximately 14 hours before landfall. As the storm passed, inland freeways were still choked with cars. It was amazing that the loss of life was not greater.

Hurricane scientists are well aware of the evacuation problem and of the need for more accurate landfall prediction. A recent newspaper article stated, "With such enormous growth in the coastal lands, and no real improvement in hurricane forecasting, scientists have concluded they can no longer predict the landfall of the big storms in time to guarantee that everyone can get out alive" (Calonius, 1983).

Another problem that should be addressed here is the cost of securing a threatened area. Neumann (1975) estimates 1975 protection cost at 25 million dollars for a typical 300 mile stretch of Gulf of Mexico coastline. He states that every 10 nautical mile increase in forecast error increases the cost by 5 million dollars, and every 10 nautical mile decrease reduces cost by 2.75 million dollars per storm. Costs have increased since then. At an annual inflation rate of 8 percent, the corresponding total cost would now be 54 million dollars.

The crux of the problem rests with the inadequacy of present forecasting procedures. Currently the National

Hurricane Center (NHC) needs approximately 2 hours and 45 minutes to develop a 72 hour forecast. The major reason for this seemingly lengthy lead time is that the NHC computer is not colocated with the forecasters. The average NHC 72 hour forecast error is 435 nautical miles (Carter, 1983). This large error sometimes results in an inability of the population at large to "suspend disbelief" with regard to that forecast. Thus, city managers are left with difficult assessments. Should they order a costly evacuation based on the forecast, or should they wait and hope the storm misses their city?

Through the use of past hurricane tracks, Dr. William G. Lesso has developed a Markov model that runs in five seconds on a microcomputer but is usually less accurate then the NHC official forecasts (Freeze, 1983). Further refinement of his initial research has lead to a time series model that runs in less than ten seconds on a microcomputer and has an average 72 hour forecast error of 312 nautical miles.

The time series model is developed in this study. The computer models currently used to forecast hurricanes are discussed in Chapter 2. Recent developments in nonlinear time series modeling are outlined in Chapter 3. Chapter 4 includes a discussion of, stationarity as it relates to hurricane tracking, formulation of the time series model, and estimation of the parameters. The chapter also contains several sections on bivariate estimation and the development of a forecast confidence interval that can be applied to the threshold model. Finally, the accuracy of the model is assessed by applying it to historical hurricane tracks from 1945 through 1985.

CHAPTER 2

PREDICTION OF TROPICAL CYCLONE MOTION

The major United States agency involved in hurricane movement forecasting today is the National Hurricane Center (NHC) in Coral Gables, Florida. The NHC is definitely the world leader in hurricane forecasting, and the state-of-the-art is represented by the seven models that the NHC uses to analyze storm movement. In this chapter the forecast methodologies used by the NHC, adaptations of NHC models used by other agencies world-wide, and one of the models developed at The University of Texas at Austin by Lesso and Freeze are discussed.

When an active hurricane is being tracked, the NHC issues forecasts at least every 6 hours and predicts storm movement for lead times up to 72 hours. These forecasts are based partially on seven computer models: NHC-67, SANBAR, HURRAN, CLIPER, NHC-72, NHC-73, and MFM. The official forecast is made by a highly skilled and experienced hurricane forecaster. The forecaster combines the output of the seven models with data from prognostic charts of hemispheric circulation, examines the influence of these large scale features on the motion of the storm, and using his best judgement issues a forecast (Carter, 1983; Freeze, 1983). The seven models are discussed in the following paragraphs.

NHC-67 (Miller et al., 1968) is classified as a statistical-synoptic model. The term synoptic refers to the use of weather data covering a large area at a particular time. NHC-67 is based on the steering principle that a tropical cyclone moves in proportion to the vertically integrated flow around the vortex. Specifically, the smoothed 500, 700, and 1000 millibar (mb) height fields are used in conjunction with the thickness of the 500-700 millibar (mb) and 700-1000 mb pressure bands. The measurements are used to grid the pressure differences across the cyclone. The readings are then combined with previous 1000 mb, 700 mb, and 500 mb readings and used to modify an initial forecast based on climatology and persistence (the degree of steadiness of movement). A stepwise regression is performed to select pressure levels that are significantly correlated with future zonal (east/west) and meridonal (north/south) components of motion (Neumann and Pelissier, 1981). The result is a steering vector that predicts storm movement and velocity. The aforementioned pressure levels are chosen because the air flows into the storm at the 1000 mb (surface) level, the pressure gradient is approximately balanced at the 500 mb level, and the air flows out of the top of the system at about the 200 mb level. (This inverted "bathtub vortex" then spins counterclockwise due to the Coriolis effect.) Thus, NHC-67 uses time dependent pressure gradients that reflect the speed of development and strength of the storm. This model is comparatively accurate for forecast times of 24 hours or less.

The SANBAR model (Sanders and Burpee, 1968), has been in use at NHC since 1970. It is a barotropic model that predicts storm tracks by following minimum stream function and maximum vorticity centers in the belief that conservation of momentum is the primary physical mechanism that determines the motion of the storm (Neumann and Pelissier, 1981). The general assertion is that the storm is steered by the large scale current in which it is embedded. The method uses winds averaged with respect to mass from the surface to the 100 mb level (approximately 55,000 feet), analyzes the wind circulation in terms of stream functions, and predicts displacement of the vorticity maximum value (the eye). It provides good results for prediction periods longer than 36 hours and works best in the tropics (Barney, 1983). SANBAR requires computer facilities which can rapidly process large amounts of data.

HURRAN (HURRicane ANalog) is an analog model based on the historical observation that hurricane tracks tend to be repetitive (Hope and Neumann, 1970). Families of storms are identified by location, motion, and time of year. Selected tracks are moved to a common origin (the current position of the existing storm) and combined with persistence to produce a forecast with assumed bivariate normal error. The centers of the confidence ellipses are used to identify the most likely track. HURRAN is a good predictor of westward movement, but fails accurately to model recurvature (the usual northward turn a hurricane makes when exiting the middle latitudes) (Freeze, 1983). The Navy has adapted the analog techniques of HURRAN to other tropical cyclone basins by using a weighting scheme that gives the most weight to the most similiar storms. Adaptations of HURRAN are also used by India, Australia, Nationalist China, and the Peoples' Republic of China (Hope and Neumann, 1977).

CLIPER (CLImatology and PERsistence) is a regression model that was developed to overcome the problems encountered by HURRAN when no similar storm tracks exist (Neumann, 1968). Based on location, motion, and forecast period, CLIPER mathematically recreates past storm tracks, and applies the same predictor equations to the current storm. Thus it has the advantage of always being able to provide a forecast, even under unusual weather conditions, but it is not reliable in predicting northward movement. Interestingly, it consistently outperforms HURRAN. Consequently, CLIPER is frequently used as a benchmark for comparing the accuracy of more sophisticated models (Neumann and Pelissier, 1981). The term "CLIPER-class model" is now used to refer to a wide range of models that employ climatology and persistence, such as the model developed in this study.

NHC-72 (Neumann et al., 1972) generates two independent sets of forecasts. One forecast uses the 1000, 700, and 500 mb pressure data, without persistence, and the other uses the CLIPER equations. The final equations are developed through regression techniques used to combine the two forecasts. In general the model is a combination of the NHC-67 and CLIPER methodologies. NHC-72 provides a better model of northward movement, but is weak at low latitudes and in the westernmost parts of the Caribbean and is not reliable after recurvature (Neumann and Pelissier, 1981).

NHC-73 (Neumann and Lawrence, 1975) is a combination of the NHC-72 and CLIPER systems. It is similar to NHC-72 in that it combines regression equations based on synoptic data with those of the CLIPER model. However, it also selects predictors from the U. S.

National Meteorological Center's 500 mb Primitive Equation Model geopotential height prognosis. NHC-73 is one of the most accurate models at the NHC, especially for long period forecasts.

The Moveable Fine Mesh (MFM) grid model is considered to be one of the most sophisticated and complex dynamic models in use at NHC (Hope and Neumann, 1977). The grid follows the hurricanes as they move. It is generally the same as other primitive equation models, but has finer resolution and covers less area. It is a ten layer model with a horizontal grid mesh length of 60 kilometers and covers an area of 9 million square kilometers (Neumann and Pelissier, 1981). This model is also used for precipitation prediction. Its results compare favorably with the other models.

The National Hurricane Center is definitely the world leader in prediction models. Hope states that the analog, regression, and synoptic models developed by other meteorological centers are generally adaptations of the NHC models (Hope and Neumann, 1977). There are two exceptions: (1) the Indian Meteorological Service has developed its own models using analog techniques; and (2) the Royal Observatory in Hong Kong has developed an empirical model that combines climatology and persistence with equal weighting. The latter model combines the last twelve hours of movement with historical directions and speeds computed for each 2.5 degree latitude-longitude square. It is quite accurate where there is a high frequency of occurrence, but less reliable above 25 degrees north latitude due to recurvature. The techniques used to develop this model are similar those which were employed in this study.

Another approach has been pursued in one of several hurricane movement models developed by Lesso and Freeze. The model (Model A) is based on a Markov process that uses historical hurricane tracks to predict future tracks (Freeze, 1983). In its simplest form a Markov process represents the probabilities of movement along a line. At each discrete time increment there is a probability P of an object moving in one direction and probability 1-P of moving in the opposite direction. At each future time increment the expected position of the object can be calculated. The analogy extends to two, three, or more dimensions. The distinguishing feature of the process is one of being in a discernable state which can easily be represented by the state variables. Movement to the future state depends only on the current state. With respect to hurricanes, the state is the position of the storm at a particular six hour position report. Future movement is hypothesized to depend only on current position and not on the path the hurricane took to get to that position.

In developing their model, Lesso and Freeze analyzed hurricanes that occurred in the Gulf of Mexico and the north Atlantic Ocean. In Model A they divided the region into latitude bands five degrees in width. Based on the most frequent past movements in each band they discovered, via least squares regression, that the next change in longitude and latitude was described by the following equations:

$$DX = -1.24969 + .001499\,(PO(IP,1))^2$$

$$DY = -.045835 + .022926\,(PO(IP,1))\ .$$

PO(IP,1) was the current latitude of the storm. DX was the forecast change in longitude, and DY was the forecast change in latitude. These quantities and the cumulative forecast error were added to the last position to obtain the forecast position. Standard errors of the coefficients and the forecasts were not discussed. Freeze (1983) considered only the average forecast error.

In order to improve the forecasts, Lesso and Freeze regressed the mean error for each forecast (6 hr., 12 hr., ... , 72 hr.) against the forecast period and discovered they were linearly related. Standard deviations of the estimates were not discussed. Specifically the correction for latitude is given by

$$Y = -.35 + .29X$$

and for longitude

$$Y = -.32 + .36X$$

where X represents the forecast period (X=1 is the 6 hr. forecast, X=2 is the 12 hr. forecast, etc.) and Y is the correction. Seeking an even better correction scheme they found, by trial and error, that the latitude correction should be multiplied by a factor of three.

Error analysis of five historical hurricane tracks shows that Model A is slightly less accurate than the offical forecasts issued by the National Hurricane Center. The comparative data for hurricanes Frederic (August, 1979), Dennis (August, 1981), Allen (July, 1980), Floyd (September, 1981) and Gert (September, 1981) are presented in Table 2.1 on the following page.

TABLE 2.1

FORECAST MEAN ERROR DISTANCE

Hurricane Name	Forecast Error (Nautical Miles) 24 HR	48 HR	72 HR	Forecast Source
Frederic	91	188	286	Model A
	69	143	218	NHC
Dennis	99	261	406	Model A
	92	207	364	NHC
Allen	100	213	325	Model A
	173	353	589	NHC
Floyd	115	272	558	Model A
	93	234	408	NHC
Gert	133	372	859	Model A
	136	236	455	NHC
Average	108	261	487	Model A
	113	235	407	NHC

Sources: Freeze (1983), Hebert et al.,(1980),
Staff, NHC (1982), Taylor et al., (1981)

CHAPTER 3

NONLINEAR TIME SERIES

Many processes occurring in nature, and in a variety of engineering fields, exhibit behavior that can not be adequately represented by a linear time series. This has resulted in significant interest in developing nonlinear time series models (Haggan et al., 1984). Priestly (1980) discusses a general class of nonlinear time series models called "state dependent models" (SDM). In the SDM approach, current values of the coefficients depend on previous values of the time series. When considering the location prediction of a moving target one would like the flexibility of allowing the model parameters to vary over location. Thus, it would be desirable to utilize a model which exhibits properties of the SDM models.

In this chapter univariate and bivariate cases of the state dependent model are considered. The univariate Box-Jenkins Autoregressive Moving Average (ARMA) model is given as a basis for comparison. Then the general SDM approach, the bilinear model, the exponential autoregressive model, and the threshold autoregressive (AR) model for both the linear and nonlinear cases are presented.

Estimation of SDM parameters is also discussed. In the bivariate section the "open-loop threshold autoregressive system," which is very similar to the approach used in this research, is discussed (Tong and Lim, 1980).

The Linear ARMA Model

The linear ARMA (k,ℓ) model is given by

$$X_t + \phi_1 X_{t-1} + \cdots + \phi_k X_{t-k} = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_\ell \varepsilon_{t-\ell}$$

where ε_t is a sequence of zero-mean random error terms and $\theta_1, \ldots, \theta_\ell, \phi_1, \ldots, \phi_k$ are constants. The objective is to predict X_t based on previous observations and random inputs.

The General SDM

The SDM is an extension of a linear ARMA time series model to the case where a process $\{X_t\}$ can be represented by a nonlinear model whose behavior may be approximated locally by a linear ARMA time series model (Haggan et al., 1984). (The term "locally" implies small departures of the model from its current state.) This leads to the general model

$$X_t + \phi_1(x_{t-1})X_{t-1} + \phi_2(x_{t-1})X_{t-2} + \cdots + \phi_k(x_{t-1})X_{t-k}$$
$$= \mu(x_{t-1}) + \varepsilon_t + \theta_1(x_{t-1})\varepsilon_{t-1} + \cdots + \theta_\ell(x_{t-1})\varepsilon_{t-\ell} \ . \quad (3.1)$$

That is, the coefficients depend on the state vector x_t of the process in the previous time period.

There are benefits and disadvantages to using nonlinear SDM models to describe hurricane movement. The advantage is that parameter values can change as the storm moves. This should result in forecasts with less error than forecasts produced by linear models. Unfortunately, the increased accuracy comes at a price. Identification procedures for SDM models are not well established. To circumvent this problem, researchers typically fit "all orders" of a particular model and choose the model that fits the "best" according to some predetermined measure. Tong and Lim (1980) propose a procedure that is outlined later in the chapter. In addition, computation times are sometimes large, and there can be convergence problems when estimating the parameter values.

The hurricane model used in this research is a piecewise linearization of the hurricane movement process. Parameter values are allowed to change only when the storm crosses a "threshold" and enters a new region of the North Atlantic. Thus, most of the usual time series identification, estimation, and forecasting procedures still apply within each region. The segmenting of the North Atlantic ocean into several regions increases the computation time and requires the availability of large numbers of hurricane position reports.

The Bilinear Model

In the linear model, μ , $\{\phi_u\}$, and $\{\theta_u\}$ are fixed. In the bilinear model only μ and $\{\phi_u\}$ are fixed and

$$\theta_u(x_{t-1}) = b_u + \sum_{j=1}^{p} d_{uj}X_{t-j}$$

where b_u and $\{d_{uj}\}$ are scalars. Then (3.1) becomes

$$X_t + \sum_{j=1}^{p} a_j X_{t-j} = \sum_{j=0}^{r} c_j \epsilon_{t-j}$$

$$+ \sum_{k=1}^{m} \sum_{i=1}^{n} b_{ki} X_{t-k} \epsilon_{t-i}$$

which Rao (1981) denotes as BL(p,r,m,n).

Bilinear models arise naturally in economic theory where model components are the products of variables. For example, cost is the product of quantity and price, and return from an investment is the amount invested multiplied by the interest rate. Use of the bilinear model for forecasting can result in complicated expressions due to the dependency structure of the term containing the product (Granger and Anderson, 1978).

Rao (1981) discusses the identification and estimation of parameter values for the bilinear model. Identification is performed by fitting all lags up to an arbitrary upper limit, and then selecting the lag combinations (for X and ϵ) that minimize the Akaike Information

Criteria (AIC) (Akaike, 1973). AIC is defined as

$$AIC = N \ln(RSS/N) + 2k$$

where RSS is the residual sum of squares, N is the number of observations and k is the number of independent parameters. Parameter values are then computed using a nonlinear optimization procedure, such as Newton-Raphson, to minimize the RSS. Starting values for the AR portion of the model are obtained by fitting an AR(p) model. The remaining coefficients are initiated at zero. An alternative starting procedure is to take initial estimates from the model of order BL(p,0,p,q-1) or BL(p-1,0,p-1,q) whichever has the smallest RSS.

Lee (1985) used a biliner model to predict sea state processes. He tentatively identified the order of the autoregressive and moving average components by using the usual identification procedures for linear models. The order of the cross product term was identified by selecting the model that minimized the AIC statistic.

The bilinear model was considered for use in modeling hurricane tracks. However, it was discarded when a comparatively simple additive autoregressive threshold model was found to accurately describe storm movement.

Exponential Autoregressive Model

This class of models is formed by assuming $\mu=0$, $\theta_u=0$, for all u and

$$\phi_u(x_{t-1}) = \psi_u + \pi_u \exp\{-\gamma X^2_{t-1}\}$$

(Haggan et al., 1984). Then (3.1) yields

$$X_t+(\psi_1+\pi_1 \exp\{-X^2_{t-1}\})X_{t-1}+\ldots+(\psi_k+\pi_k \exp\{-X^2_{t-1}\})X_{t-k}=\varepsilon_t.$$

Ozaki (1980) has used this model to analyze self-sustained oscillations of springs and nonlinear oscillations in electric circuits. Model identification was not discussed.

Haggan and Ozaki (1981) describe a procedure for estimating the order k, and the coefficients $(\gamma,(\phi_i, \pi_i); i=1,\ldots,k)$. First γ is fixed at particular grid values and X_t is regressed against $\exp\{-\gamma X^2_{t-1}\}X_s$ $(s < t)$ and against previous X_t values, from order k=1 up through arbitrary order m. Then the model that minimizes the AIC for that γ is selected. The AIC statistic can also be compared across γ values to select the "best" model overall.

This model was not used in this research because the small size of the segmented hurricane tracks made it desirable to limit the number of model parameters. The bivariate threshold autoregressive model that was finally developed had one-half the number of parameters as compared to the exponential autoregressive model.

Threshold AR Model

The idea underlying the threshold AR model is the piecewise linearization of a nonlinear model of the state space by introduction of "thresholds." The models are then locally linear (Tong and Lim, 1980). For this model it is assumed that $\theta_u=0$ for all u and

$$\mu(x_{t-1}) = \begin{cases} \mu_1 \text{ if } X_{t-d} \leq c \\ \mu_2 \text{ if } X_{t-d} > c \end{cases}$$

$$\phi_u(x_{t-1}) = \begin{cases} \phi_u^{(1)} \text{ if } X_{t-d} \leq c \\ \phi_u^{(2)} \text{ if } X_{t-d} > c \end{cases} .$$

The resulting model is

$$X_t + \phi_1^{(1)} X_{t-1} + \cdots + \phi_k^{(1)} X_{t-k} = \mu_1 + \varepsilon_t \quad \text{if } X_{t-d} \leq c$$

$$X_t + \phi_1^{(2)} X_{t-1} + \cdots + \phi_k^{(2)} X_{t-k} = \mu_2 + \varepsilon_t \quad \text{if } X_{t-d} > c$$

(Haggan et al., 1984).

Tong and Lim (1980) used this model to study the nonlinear aspects of "jump resonance" and "amplitude-frequency dependency." Jump resonance is the sudden change in output amplitude that occurs at different input frequencies depending on whether the input frequency is increasing or decreasing. Amplitude-frequency dependency is present when an output signal has different frequencies of oscillation for different amplitudes. The estimation procedure used by Tong and Lim is described later in the chapter.

The threshold AR model is nearly the model used in this research. The differences are that the hurricane model is bivariate, and the parameter values depend upon the region in which the storm is located rather than depending on the value of the last observation X_{t-1}.

Nonlinear Threshold AR Model

This model is a modified form of the threshold AR model (Haggan et al., 1984). Let $\mu = 0$, $\theta_u = 0$ for all u and define

$$\phi_u(X_{t-1}) = \begin{cases} \phi_u + \pi_u |X_{t-1}| & \text{if } |X_{t-1}| \leq c \\ \phi_u + \pi_u c & \text{if } |X_{t-1}| > c \end{cases}$$

where c is some constant. Then

$$X_t + (\phi_1 + \pi_1 |X_{t-1}|) X_{t-1} + \cdots + (\phi_k + \pi_k |X_{t-1}|) X_{t-k} = \varepsilon_t \quad \text{if } |X_{t-1}| \leq c$$

$$X_t + (\phi_1 + \pi_1 c) X_{t-1} + \cdots + (\phi_k + \pi_k c) X_{t-k} = \varepsilon_t \quad \text{if } |X_{t-1}| > c$$

This model has the flexibility to change parameters from period to period, based on a state of the previous period.

This type of model provides the framework for the model used in this research. However, parameter values depend on the previous observation X_{t-1} which is not desirable. For the hurricane process it is belived that the climatology of the storms changes slowly. Piecewise linearization enforces this assumption by allowing the forecast parameters to remain constant until the next threshold is crossed.

Estimation of SDM Parameters

Priestly (1980) showed the estimation procedure for the parameters of the SDM model could be based on the extended Kalman filter. Parameters are assumed to be linear functions of the state vector X_t so that for each u,

$$\mu(X_t) = \mu_u^o + X_t'\alpha$$

$$\theta_u(X_t) = \theta_u^o + X_t'\beta_u$$

and

$$\phi_u(X_t) = \phi_u^o + X_t'\gamma_u$$

where μ_u, θ_u, ϕ_u are constants and α, β_u, γ_u are gradient vectors. Updating equations for the parameters are given by

$$\mu(X_{t+1}) = \mu(X_t) + \Delta X'_{t+1}\alpha^{t+1}$$

$$\theta_u(X_{t+1}) = \theta_u(X_t) + \Delta X'_{t+1}\beta_u{}^{t+1}$$

$$\phi_u(X_{t+1}) = \phi_u(X_t) + \Delta X'_{t+1}\gamma_u{}^{t+1}$$

for all u, where $X'_{t+1} = X_{t+1}-X_t$.

The gradients are unknowns that must be estimated. The basic strategy is to allow the gradients to follow a random walk which can be represented in matrix form by

$$B_{t+1} = B_t + V_{t=1}$$

where $B_t = (\alpha^{(t)}, \beta_1^{(t)}, \ldots , \beta_\ell^{(t)}, \gamma_1^{(t)}, \ldots, \gamma_k^{(t)})'$

and V_t is a sequence of independent matrix-valued random variables distributed as multivariate Normal with zero means. For each t, the procedure determines those values of B_t which minimize the discrepancy between the observed value of X_t and its predicted value from the model. This sequential algorithm resembles the procedure used in the Kalman filter algorithm (Haggan et al., 1984).

Open Loop Threshold Autoregressive System

Tong and Lim (1980) developed a more general representation of the threshold autoregressive (TAR) model. One of their models, the open loop threshold autoregressive system (TARSO), is nearly identical to the hurricane forecasting model. There are two major differences between the TARSO model and the model used in this study. First, Tong and Lim require pairwise independence between all white noise sequences. The white noise sequences in the hurricane model are allowed to have a contemporaneous covariance structure. Second, the TARSO model requires some past observation to be in a particular region before switching parameters. In the hurricane model, the parameters change when the forecast crosses into a new region.

Tong and Lim (1980) represent the TARSO model as

$$X_n = a_o^j + \sum_{i=1}^{m} a_i{}^j X_{n-i} + \sum_{i=0}^{m'} b_i{}^j Y_{n-i} + \varepsilon_n{}^j$$

where $\{X_t\}$ is the output series, $\{Y_t\}$ is the input series, and the coefficients a_o, a_i and b_i are dependent on the value of Y_{n-d} (some previous input). ε_n is a white noise sequence with zero mean and finite variance and is independent of Y_n. The values of the nonlinear series are assigned to j nonoverlapping intervals by percentiles of Y (taken on the observed range). When Y_{n-d} crosses a percentile boundary, the coefficients are allowed to change.

Tong and Lim (1980) use an estimation procedure in which the data are divided into two regions, those values falling above a percentile breakpoint t_q, and those falling below. In each region, AR models are fit up to a maximum (arbitrary) order. The models that minimize the AIC in each region are selected. Let the order of the models be k_1 and k_2. Then one can write

$$AIC(t_q) = AIC(k_1) + AIC(k_2) \quad .$$

Next, t_q is allowed to vary over a preselected set of t_q's. The value of t_q that minimizes the AIC for the applicable k_1 and k_2 order models is selected as the threshold value. Then d (for Y_{n-d} above) is varied over a preselected set of integers. The d that minimizes the AIC for the values of t_q, k_1, and k_2 is selected as the appropriate lag.

Tong and Lim also develop the eventual forecast function. Their forecasts take the form of an oscillatory series of constant period. This is due to the fact that, unlike a linear Box-Jenkins model, a stable nonlinear system will continue to oscillate (converge to a limit cycle, which may degenerate to a constant) after termination of input (Tong and Lim, 1980). To analyze forecast error, Tong and Lim delete the last 10 percent of the observations, and then fit a new model. If the fitted model using the complete data set does not differ significantly from the new model, then the original model is adopted as the final model. Confidence intervals of the forecast were not discussed.

CHAPTER 4

MODEL IDENTIFICATION ESTIMATION AND FORECASTING

The procedure used to model the hurricane tracks is outlined in this chapter. Stationarity and the resulting identification of the appropriate autoregressive model are discussed. Univariate and bivariate models are estimated, and models are checked for adequacy and then used to forecast the longitude and latitude series of actual hurricane tracks.

Stationarity

A stochastic process is strictly stationary if its properties are unaffected by a change in origin; that is, if the joint probability distribution of m observations $Z_1, Z_2, \ldots, Z_m$, made at any set of times $t_1, t_2, \ldots, t_m$ is the same as that of a different set of observations $Z_{1+k}, Z_{2+k}, \ldots, Z_{m+k}$, made at times $t_{1+k}, t_{2+k}, \ldots, t_{m+k}$, for any integer k (Box and Jenkins, 1976).

A convenient example of a process that is mean nonstationary is the latitude position series of a hurricane moving due north at constant velocity. Clearly the value of latitude increases over time. Thus, the latitude does not vary about a constant mean. It is non-stationary. However, the longitude is constant and seems to vary about a constant mean. It is apparently stationary. In an actual hurricane position series, nature supplies random shocks that cause the longitude and latitude to vary about their respective means. The stationarity of the latitude and longitude series is important and is discussed in detail in the next section.

In practical applications it is virtually impossible to test for strict stationarity (Granger and Newbold, 1977). Fortunately, under the assumption of a Gaussian process, the conditions known as weak stationarity are equivalent to strict stationarity. A process $\{w_t\}$ is weakly stationary (covariance stationary) if over time it varies about a constant mean, the process variance remains constant, and the $\mathrm{cov}(w_t, w_{t+k})$ is constant for all t.

In order to obtain a consistent and unbiased estimate of the population mean it is necessary that the process be ergodic. Ergodicity is explicitly defined by Hannan (1970). Ergodicity implies that observations of the process sufficiently far apart in time are uncorrelated, so that when averaging a growing series through time, new information is continually added. Then

$$w_m = (1/m) \sum_{t=1}^{m} w_t \qquad (4.1)$$

can be used to estimate the mean of the process. Similarly, the estimators of $cov(w_t, w_{t+k})$ will also be consistent. An estimator $\hat{\theta}$ is a consistent estimator of θ if, as the sample size increases, it approaches θ in probability. That is if

$$\lim_{n\to\infty} P[\ |\hat{\theta} - \theta| \leq \varepsilon] = 1\ , \qquad \text{for any } \varepsilon > 0\ .$$

The $cov(w_t, w_{t+k})$, the autocovariance at lag k, is defined by

$$\gamma_k = cov\ [w_t, w_{t+k}] = E\ [(w_t - \mu)(w_{t+k} - \mu)] \qquad (4.2)$$

where $\mu = E\ [w_t]$. Given N observations, μ is estimated by

$$\bar{w} = (1/N) \sum_{t=1}^{N} w_t\ . \qquad (4.3)$$

The autocorrelation at lag k is

$$\rho_k = \frac{\gamma_k}{\gamma_o} = \frac{E\ [(w_t - \mu)(w_{t+k} - \mu)]}{E\ [(w_t - \mu)(w_t - \mu)]}$$

$$= E\ [(w_t - \mu)(w_{t-k} - \mu)]/\sigma_w^2\ . \qquad .4)$$

ρ_k is estimated by $r_k = c_k/c_o$ where

$$c_k = \frac{1}{N} \sum_{t=1}^{N-k} (w_t - \bar{w})(w_{t+k} - \bar{w})\ . \qquad (4.5)$$

The c_k are the estimates of γ_k. In order to compare the c_k values, σ_w must remain constant over time. Also c_o, the estimator of the process variance, σ_w^2, must be constant so that the values of r_k may be compared at various lags.

In modeling hurricanes, the approach used in this study involved dividing the North Atlantic region into rectangular grids within which the individual hurricanes moved in a similar "pattern" (i.e. with constant velocity or acceleration during a period of p observations for the AR(p) model). That is, the tracks were divided into approximately homogeneous segments. In order to analyze various orders of AR models, it was necessary to develop a procedure to determine if a given hurricane track was covariance stationary. Within each grid, if the latitude and longitude series (possibly differenced) are individually covariance stationary then, under the assumption that the bivariate process is Normal, the bivariate process is stationary.

Theorem 4.1:

Definitions

Let Y_t and X_t be weakly stationary univariate series with zero mean.

N is the number of observations.

p is the order of the AR model.

ρ_k is the cross correlation $E[Y_t X_{t-k}]/(\sigma_1 \sigma_2)$.

$\phi_{11,i}$ is the autoregressive coefficient of X_{t-i} used to predict X_t.

$\phi_{12,i}$ is the autoregressive coefficient of Y_{t-i} used to predict X_t.

$\phi_{21,i}$ is the autoregressive coefficient of X_{t-i} used to predict Y_t.

$\phi_{22,i}$ is the autoregressive coefficient of Y_{t-i} used to predict Y_t.

ε_1 and ε_2 are the white noise series for X and Y,

$$\varepsilon_1 \sim N(0,\sigma_1^2)$$

$$\varepsilon_2 \sim N(0,\sigma_2^2)$$

and,

$$\text{cov}(\varepsilon_{1t}\ \varepsilon_{2s}) = \begin{cases} 0 & t \neq s \\ \rho\sigma_1\ \sigma_2 & t = s \end{cases} .$$

Then the general bivariate AR(p) model is

$$X_t = \sum_{i=i}^{p} [\ \phi_{11,i}X_{t-i} + \phi_{12,i}Y_{t-i}] + \varepsilon_{1t}$$

$$Y_t = \sum_{i=1}^{p} [\ \phi_{21,i}X_{t-i} + \phi_{22,i}Y_{t-i}] + \varepsilon_{2t} .$$

The covariance matrix of the bivariate process is

$$E[(X,Y)'(X,Y)] = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_1\sigma_2\rho & \sigma_2^2 \end{bmatrix}$$

where X and Y are column vectors of length N.

To prove that the bivariate process is weakly stationary, it must be shown that the means and the covariance matrix of X and Y are constant over time.

Proof:

Clearly the means are constant. It is given that σ_1^2 and σ_2^2 are constant over time. Thus, the proof reduces to showing that ρ_k (the cross correlation at lag k) is constant over time, or $E[X_t Y_{t-k}] = c$, for all t, where $p+k < t < N$. The use of lag k or lag -k is arbitrary. For stationarity, both ρ_k and ρ_{-k} must be constant over time (but not necessarily equal). By definition of stationarity, $E[X_t X_{t-k}] = a_1$, a constant. Then,

$$\begin{aligned} E[X_t X_{t-k}] = {} & \phi_{11,1}E[X_{t-1}X_{t-k}]+\phi_{11,2}E[X_{t-2}X_{t-k}]+ \cdots \\ & +\phi_{11,p}E[X_{t-p}X_{t-k}] \\ & +\phi_{12,1}E[Y_{t-1}X_{t-k}]+\phi_{12,2}E[Y_{t-2}X_{t-k}]+ \cdots \\ & +\phi_{12,p}E[Y_{t-p}X_{t-k}]+E[\varepsilon_{1t}X_{t-k}] = a_1 \end{aligned} \quad (4.6)$$

where $E[\varepsilon_{1t}X_{t-k}] = 0$. Also,

$$\begin{aligned} E[Y_t X_{t-k}] = {} & \phi_{21,1}E[X_{t-1}X_{t-k}]+\phi_{21,2}E[X_{t-2}X_{t-k}]+ \cdots \\ & +\phi_{21,p}E[X_{t-p}X_{t-k}] \\ & +\phi_{22,1}E[Y_{t-1}X_{t-k}]+\phi_{22,2}E[Y_{t-2}X_{t-k}]+ \cdots \\ & +\phi_{22,p}E[Y_{t-p}X_{t-k}]+E[\varepsilon_{2t}X_{t-k}] \end{aligned} \quad (4.7)$$

where $E[\varepsilon_{2t}X_{t-k}] = 0$.

The individual expected values in (4.6) and (4.7) are equal. Only the coefficients, which are known scalars, differ in the two equations. From (4.6) it is clear that

$$\phi_{12,1}E[Y_{t-1}X_{t-k}]+\phi_{12,2}E[Y_{t-2}X_{t-k}]+\cdots$$
$$+\phi_{12,p}E[Y_{t-p}X_{t-k}]$$

must be constant over t. Thus, in (4.7), the quantity

$$\phi_{22,1}E[Y_{t-1}X_{t-k}]+\phi_{22,2}E[Y_{t-2}X_{t-k}]+\cdots$$
$$+\phi_{22,p}E[Y_{t-p}X_{t-k}]$$

must also equal some other constant for all t.

Therefore, $E[Y_tX_{t-k}]$ is constant over t.

It is easy to determine if a univariate process is stationary. For the AR(p) process let

$$w_t = \phi_1 w_{t-1} + \cdots + \phi_p w_{t-p} + a_t \qquad a_t \sim N(0,\sigma_a{}^2) \quad .$$

An equivalent representation using the backshift operator B is given by

$$(1 - \phi_1 B - \cdots - \phi_p B^p)\ w_t = a_t \quad .$$

The effect of B is to shift w_t back one time interim. i.e. $Bw_t = w_{t-1}$. Now, consider the AR(1) process where

$$(1 - \phi_1 B)\ w_t = a_t$$

then

$$w_t = a_t/(1 - \phi_1 B)$$
$$= a_t\ (1+\phi_1 B+\phi_1{}^2 B^2+ \ldots)$$
$$= a_t + \phi_1 a_{t-1} + \phi_1{}^2 a_{t-2} + \ldots$$

and the variance of w_t is given by

$$Var[w_t] = Var[a_t]\ (1+\phi_1{}^2+\phi_1{}^4 + \ldots) \quad .$$

Then

$$\mathrm{Var}[w_t]\ (1-\phi_1{}^2) = \mathrm{Var}[a_t] \quad . \tag{4.8}$$

Clearly if these variances are to be finite, the geometric series in ϕ must converge so $\phi_1{}^2 < 1$.

Equivalently, for a stationary process of any order, the roots of the $\phi(B)$ polynomial lie outside the unit circle. For the AR(1) model this polynomial is given by

$$\phi(B) = (1-\phi_1 B) \quad .$$

Regarding B as a variable, if $\phi(B)$ is set equal to zero and solved for B, for stationarity the roots of the equation $B=1/\phi_1$ must be greater than one. In general, for the AR(p) process, the roots of the $\phi(B)$ polynomial

$$\phi(B) = (1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p)$$

must lie outside the unit circle (Box and Jenkins, 1976).

Hurricane Stationarity

Suppose the last n position reports of a hurricane (or tropical storm) are defined by the ordered pairs (LA_{t-1}, LO_{t-1}) (LA_{t-2}, LO_{t-2}) ... (LA_{t-n}, LO_{t-n}). The set of position reports are two time series, latitude and longitude. It is desired to forecast (LA_t, LO_t) as a linear combination of the n previous position reports.

The general model is

$$LA_t = a_{11,1}LA_{t-1}+a_{11,2}LA_{t-2}+ \cdots +a_{11,n}LA_{t-n}$$
$$+a_{12,1}LO_{t-1}+a_{12,2}LO_{t-2}+ \cdots +a_{12,n}LO_{t-n}+c_1$$
$$LO_t = a_{21,1}LA_{t-1}+a_{21,2}LA_{t-2}+ \cdots +a_{21,n}LA_{t-n}$$
$$+a_{22,1}LO_{t-1}+a_{22,2}LO_{t-2}+ \cdots +a_{22,n}LO_{t-n}+c_2$$

In order to develop a model of the latitude series LA_{t-1}, LA_{t-2}, ... , LA_{t-n} and longitude series LO_{t-1}, LO_{t-2}, ..., LO_{t-n}, it is necessary that the series be weakly stationary. Hurricane series representing six hour position reports are too short for nonconstant variance to be detected, but often are nonstationary in their means.

While it would seem that a hurricane which is continually in motion could never be considered to be stationary, this is not typically the case. If a storm is moving due west (W) or east (E), the latitude series remains constant. In this case the hurricane is latitude-position stationary, i.e. the time series LA_{t-1}, LA_{t-2}, ... , LA_{t-n} varies about a constant mean. A storm moving due north (N) or south (S), is longitude-position stationary.

If the storm is moving northwest (NW), northeast (NE), southwest (SW), or southeast (SE), it is neither latitude-position stationary nor longitude-position stationary. When this occurs, stationarity can be induced by differencing (calculating the change per unit time interval) the latitude and longitude series. If the new series (which now represent velocities) vary about a constant mean, the hurricane is said to be latitude-velocity and/or longitude-velocity stationary. It describes a storm moving (say NW) at constant velocity and is used to predict the next velocity, i.e. the next change in position. This is the basis of the model developed by Lesso and Freeze, although their model uses the previous latitude to predict velocity and is based on all past storm tracks (Freeze, 1983). If only the previous velocity were used to predict the next change in position, the model would be first-order autoregressive (AR1), or equivalently, a Markov random walk in velocity.

If the hurricane is accelerating (say in latitude), the latitude series must be differenced twice to induce stationarity. The process of determining whether the latitude and longitude series are stationary in position, velocity, or acceleration, results in nine possible classifications (categories) for a particular track. These categories have a useful physical interpretation related to the direction of motion (Table 4.1).

TABLE 4.1

HURRICANE STATIONARITY CATEGORIES

LATITUDE	LONGITUDE POSITION	LONGITUDE VELOCITY	LONGITUDE ACCELERATION
POSITION	1 Standing Still	2 Moving West (East)	3 Accelerating West (East)
VELOCITY	4 Moving North (South)	5 Moving NE,SE,SW,NW	6 Recurving N,S to E,W
ACCELERATION	7 Accelerating North (South)	8 Recurving W,E to N,S	9 Accelerating NE,SE,SW,NW

Hurricane Model Identification

Once stationarity is confirmed, it is necessary to determine the order of the autoregressive process. The general p^{th} order bivariate (latitude, longitude) model is given by

$$LA_t = \sum_{i=1}^{p} [\phi_{11,i} LA_{t-i} + \phi_{12,i} LO_{t-i}] + a_{1t}$$

$$LO_t = \sum_{i=1}^{p} [\phi_{21,i} LA_{t-i} + \phi_{22,i} LO_{t-i}] + a_{2t} \quad .$$

Tiao and Box (1981) developed a useful identification procedure for such multivariate models. The process involves fitting AR models of successively higher order and examining the cross-correlation and partial auto-regression matrices after each fit. Let $\Gamma(\ell)$ be the lag ℓ cross-covariance matrix. Then for the bivariate process

$$\Gamma(\ell) = \{\gamma_{i,j}(\ell)\}, \qquad \ell = 0, \pm 1, \pm 2, \ldots \quad i,j = 1, 2$$

and $\rho(\ell) = \{\rho_{ij}(\ell)\}$ is the corresponding cross-correlation matrix. For a stationary multivariate AR(p) process, the auto and cross-correlations decay slowly to zero as the lag increases. The p^{th} partial autoregression matrix P(p) is the matrix of autoregressive coefficients. i.e. at lag p

$$P(p) = \hat{\Phi}_p$$

where

$$\hat{\Phi}_p = \{\phi_{ij,p}\} \qquad i,j = 1,2$$

For a stationary AR(p) process, the partial autoregression matrix is zero beyond lag p,and the estimates $\hat{\Phi}_1,\ldots,\hat{\Phi}_p$ are asymptotically jointly Normally distributed. The significance of the parameters may be tested. (The test results can be represented by filling the partial autoregression matrix with a '+' sign if the coefficient is more than two standard deviations above zero, or a '-' sign if it is more than two standard deviations below zero.) Their procedure is implemented in the time series package by Statistical Computing Associates (SCA; Liu et al., 1983).

Unfortunately, due to the segmenting of hurricane tracks and the resultant missing values, SCA could not be used. Instead Statistical Package for the Social Sciences (SPSS; Nie, et al, 1975) was employed. The Box and Tiao approach was adapted to the partial F statistic computed by SPSS. The lagged data were sequentially regressed and the plus and minus signs were assigned based on a 90% confidence interval for the regression coefficients.

In the first analysis, all velocity lags of latitude and longitude were considered up to lag 5 (30 hours prior to the current velocity observation). As the lag increased, a gradual decrease in the auto and cross-correlations was noted. Significant coefficients occurred at lags 1, 4, and 5. This implied that the process could be autoregressive with a "cyclic" component at lag 4, representing a 24 hour lag. This component could physically reflect the diurnal effect of the sun (the slowing of the storm at night).

In the second set of regressions lags 1, 4, and 5 were considered. In general, the lag 5 parameter was weak, so it was dropped from the model. This led to the general model with velocity coefficients evaluated at lags 1 and 4.

Based on the analysis by Lesso and Freeze, it was believed that the model coefficients should be allowed to change as the storm moved. To capture this change, more than 300 historical North Atlantic hurricane tracks (from the National Oceanic and Atmospheric Administration magnetic tape described in Appendix F) were segmented by latitude bands eight degrees in width with a three degree overlap. Larger band widths masked the diurnal effect, and smaller band widths resulted in too few observations per track. Segmenting the longitude into similar bands increased forecast error primarily due to the reduction of latitude observations. Models were then fit to each region. The models are discussed in Chapter 5 and are listed in Appendix A.

Data Manipulation

For each individual grid, the physical manipulation of the data that produced the model coefficients required five major steps. In the first step, the raw data file of hurricane tracks was condensed. Storms that occurred before 1945 were deleted due to concerns about the accuracy of the observations. Then portions of storm tracks outside the current grid of interest were deleted. Finally, the subtropical storms (maximum wind less than 45 knots) were eliminated due to their weak persistence. In the second step, the lag one coefficient for storm tracks in the grid was calculated and used to determine the stationarity category of the storm. Next, based on the stationarity category, the latitude and longitude series were differenced appropriately, and the lagged data matrices were constructed. In the fourth step, the least squares regression coefficients were calculated. Finally, in step five, the coefficients were used to forecast the storms. The results of these forecasts were compared with the "best track" data. This produced the empirical forecast errors used in the examples presented later in this chapter. A detailed discussion of the data files and the required data manipulation is in Appendix F.

UNIVARIATE ESTIMATION

When stationarity is achieved for a series, and the order of the model is tentatively identified, the model parameters must be estimated. The next several sections include detailed discussions of the joint density of the time series observations and derivation of the maximum likelihood point and interval estimators for the univariate process.

Joint Density of the Observations

Let $w_t = \phi_1 w_{t-1} + \phi_2 w_{t-2} + \cdots + \phi_p w_{t-p} + a_t$

where

$$a_t \sim N(0,\sigma_a^2) \ , \qquad w_t \sim N(0,\sigma_w^2) \ .$$

a_t represents a random shock (input) to the system at time t, w_t is the observed value of the series at time t. Let $w_n' = (w_1, w_2, \ldots, w_n)$. Suppose the w_i are independent Normally distributed random variables with zero mean and constant variance. Then the probability density function (pdf) of w_i is

$$f(w_i = w \mid \phi, \sigma_w) = (2\pi\sigma_w^2)^{-1/2} \exp\{-w^2/2\sigma_w^2\} \quad .$$

Because the observations are independent and identically distributed (iid), the joint pdf is given by the product of the marginal pdf's

$$f(w_1, w_2, \ldots, w_n \mid \phi, \sigma_w) = (2\pi\sigma_w^2)^{-n/2} \exp\{(-1/2) \sum_{i=1}^{n} (w_i^2/\sigma_w^2)\}.$$

Let Σ represent the diagonal n x n covariance matrix. Then

$$\Sigma = \begin{bmatrix} \sigma_w^2 & & & \\ & \sigma_w^2 & & \\ & & \ddots & \\ & & & \sigma_w^2 \end{bmatrix}$$

because the w_i are iid. Then

$$f(w_1, w_2, \ldots, w_n \mid \phi, \sigma_w) = (2\pi)^{-n/2} |\Sigma|^{-1/2} \exp\{(-1/2)\, w_n' \Sigma^{-1} w_n\} \qquad (4.9)$$

If the w_i are correlated, the functional form of the pdf is the same, but the off-diagonal elements of Σ are nonzero (Morrison, 1976).

Box and Jenkins (1976) and Hannan (1970) use the notation

$$f(w_n|\phi,\sigma_a)=(2\pi\sigma_a^2)^{-n/2}|M_n^p|^{1/2}\exp\{(-1/2\sigma_a^2)w_n'M_n^p w_n\} \quad (4.10)$$

where $M_n^p = \Sigma^{-1}\sigma_a^2$, w_n is a vector of n observations, and p is the order of the AR model. Substituting for M_n^p in equation (4.10) yields equation (4.9). Consequently, the exact likelihood is

$$f(w_n|\phi,\sigma_a) = L(\phi,\sigma_a|w_n) =$$

$$= (2\pi\sigma_a^2)^{-n/2}|M_p^p|^{1/2}\exp\{-(1/2\sigma_a^2)w_n'\ M_n^p\ w_n\} \quad (4.11)$$

where $|M_n^p|=|M_p^p|$, because the w_t are uncorrelated with observations more than p time periods in the past. Thus, all the covariances beyond row and column p in M_n^p are zero, and with σ_a^2 factored out, the variance elements (from p+1 through n) are one.

Maximum Likelihood Estimators

The method of maximum likelihood can be used to derive estimators for the parameters and the variance of the noise series. Using the likelihood function (4.11), the log likelihood function is

$$\ell(\phi,\sigma_a|w_n)=(-n/2)\ln 2\pi-(n/2)\ln\sigma_a^2+(1/2)\ln|M_p^p|-1/(2\sigma_a^2)w_n'M_n^p w_n \quad (4.12).$$

Let $S(\phi)= w_n'\ M_n^p\ w_n$,

then $\partial \ell / \partial \sigma_a = (-n/2)(2\sigma_a/\sigma_a^2) - (1/2)(-2)(\sigma_a^{-3})\ S(\phi)$

$$= (-n/\sigma_a) + S(\phi)/\sigma_a^3$$

equals zero at the maximum, which implies

$$\hat{\sigma}_a^2 = S(\phi)/n \quad .$$

The derivation of the estimator for the vector $\underset{\sim}{\phi}$ is somewhat more tedious because M_p^p and M_n^p are functions of $\underset{\sim}{\phi}$.

For example, for an AR(1) process with

$$w_n' = \{w_1, \ldots, w_n\}$$

from (4.12) $\ell(\phi, \sigma_a | w_n) =$

$$(-n/2)\ln 2\pi - (n/2)\ln \sigma_a^2 + (1/2)\ln(1-\phi_1^2) - (1/2\sigma_a^2)\underbrace{\{(1-\phi_1^2)w_1^2 + \sum_{t=2}^{n}(w_t - \phi_1 w_{t-1})^2\}}_{a} \quad (4.13)$$

Expanding a in (4.13) yields,

$$\{w_1^2 - \phi_1^2 w_1^2 + w_2^2 - 2\phi_{w1} w_2 + \phi_1^2 w_1^2 + \sum_{t=3}^{n} (w_t - \phi_1 w_{t-1})^2\}$$

$$= \{w_1^2 + w_2^2 - 2\phi_1 w_2 w_1 + \sum_{t=3}^{n} (w_t - \phi_1 w_{t-1})^2\}$$

$$= \{-2\phi_1 w_1 w_2 + \sum_{t=2}^{n-1} \phi_1^2 w_t^2 - 2\phi_1 \sum_{t=2}^{n} w_t w_{t-1}\} \quad .$$

Then

$$\partial\ell/\partial\phi_1 = (1/2)[(-2\phi_1)/(1-\phi_1^2)] - (1/2\sigma_a^2)\{-2w_1 w_2 + 2\phi_1 \sum_{t=2}^{n-1} w_t^2 - \sum_{t=2}^{n} w_t w_{t-1}\}$$

$$= [\sigma_w^2(1-\phi_1^2)(-\phi_1)/(1-\phi_1^2)] - \{-2w_1 w_2 + 2\phi_1 \sum_{t=2}^{n-1} w_t^2 - \sum_{t=2}^{n} w_t w_{t-1}\}$$

$$= -\gamma_1 + \sum_{t=2}^{n} w_t w_{t-1} - \phi_1 \sum_{t=2}^{n-1} w_t^2 .$$

Setting this equal to zero and solving for ϕ_1, yields

$$\hat{\phi}_1 = \left(\sum_{t=2}^{n} w_t w_{t-1} - \gamma_1\right) / \left(\sum_{t=2}^{n-1} w_t^2\right) . \qquad (4.14)$$

Clearly, 4.14 is asymptotically equivalent to the ordinary least squares estimate of ϕ_1 given by

$$\hat{\phi}_1 = \left(\sum_{t=2}^{n} w_t w_{t-1}\right) / \left(\sum_{t=2}^{n} w_t^2\right) .$$

When n is large enough, Box and Jenkins, and the commercially available software, prefer to ignore γ_1 and simply use the ordinary least squares estimate. A convenient iterative least squares algorithm will be presented later in the paper.

In the general AR(p) model, the ϕ's can be estimated by conventional ordinary least squares or by solving the Yule-Walker equations. The Yule-Walker estimates are calculated by solving the system of equations

$$\begin{aligned} r_1 &= \hat{\phi}_1 + \hat{\phi}_2 r_1 + \cdots + \hat{\phi}_p r_{p-1} \\ r_2 &= \hat{\phi}_1 r_1 + \hat{\phi}_2 + \cdots + \hat{\phi}_p r_{p-2} \\ &\vdots \\ r_p &= \hat{\phi}_1 r_{p-1} + \hat{\phi}_2 r_{p-2} + \cdots + \hat{\phi}_p \end{aligned}$$

where $r_k = c_k / c_o$ and

$$c_k = (1/n) \sum_{t=1}^{n-k} w_t w_{t+k} .$$

In matrix form let

$$r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_p \end{bmatrix}, \quad R = \begin{bmatrix} 1 & r_1 & \cdots & r_{p-1} \\ r_1 & 1 & \cdots & r_{p-2} \\ \vdots & & & \vdots \\ r_{p-1} & r_{p-2} & \cdots & 1 \end{bmatrix}$$

then

$$\underset{\sim}{r} = \underset{\sim}{R}\hat{\underset{\sim}{\phi}} \quad \text{so} \quad \hat{\underset{\sim}{\phi}} = \underset{\sim}{R}^{-1}\underset{\sim}{r} \ .$$

Variance of the Maximum Likelihood Estimators

Derivation of the variance of the maximum likelihood estimators is reviewed in this section. Then, an iterative procedure that can be used to calculate estimates of the autoregressive parameters is discussed, and those estimates are compared to least squares estimates. An actual hurricane track is then used to illustrate the variances of parameter estimates computed using both the least squares and maximum likelihood criteria approaches.

The variance of the parameter estimates for the autoregressive process is

$$V[\hat{\phi}] = I^{-1}(\phi)$$

where $I(\phi)$, the information matrix, is

$$I[\phi] = E[U'U]\ \sigma_a^{-2} \ ,$$

and U is an nxp matrix containing n+p observations lagged up to p periods (Box and Jenkins, 1976). In least squares estimation of ϕ, $u_{t,j}$ is defined as $u_{t,j} = -\partial a_t/\partial \phi_j$ given $\phi_{j,o}$ (Granger and Newbold, 1977). Thus, $u_{t,j}$ represents the change of the error of the estimate of the t^{th} value in the time series with respect to changes in the value of the j^{th} parameter. The gradient $(u_{t,j})$ can be approximated using a first order Taylor series expansion (Fig. 4.1) as

$$\begin{aligned} u_{t,j} &\cong -(a_{t,o} - a_t)/(\phi_{j,o} - \phi_j) \\ &= (a_t - a_{t,o})/(\phi_{j,o} - \phi_j) \quad . \end{aligned}$$

Solving for a_t

$$a_t = -(\phi_j - \phi_{j,o})u_{t,j} + a_{t,o} \qquad (4.16)$$

where $\phi_{j,o}$, and $a_{t,o}$ are initial guesses of the parameter and error values respectively.

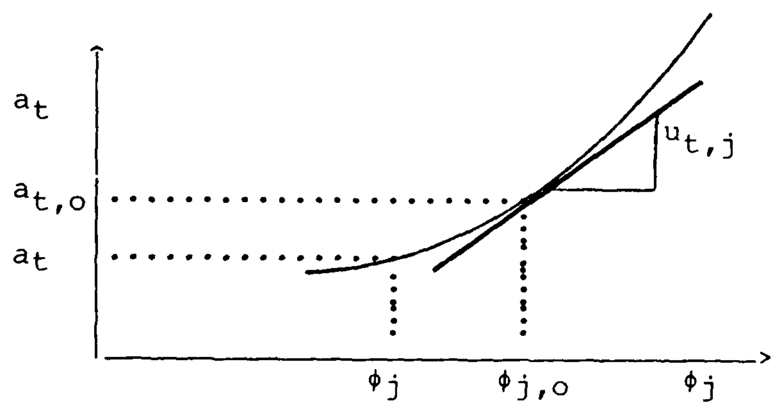

Figure 4.1 The Gradient $u_{t,j}$

The one dimensional case is shown in Fig. 4.1. In general, for the AR(p) model, solving for a_t yields

$$a_t \cong a_{t,o} - \sum_{j=1}^{p} (\phi_j - \phi_{j,o})\, u_{t-j} \quad . \tag{4.17}$$

This expression is the first order Taylor series expansion commonly used in nonlinear gradient search algorithms (Granger and Newbold, 1977). If a_t is a function of $\underset{\sim}{\phi}$, then

$$\underset{\sim}{u}_t = -\partial a_t / \partial \underset{\sim}{\phi} \quad .$$

The minimum error with respect to the parameters is determined along the direction of the negative gradient. For moderate and large samples, when an ellipsoidal quadratic approximation to the likelihood function is appropriate, the global minimum occurs where the sum of squares in the exponent of the likelihood function is minimized. Solving (4.17) for $a_{t,o}$ yields

$$a_{t,o} \cong \sum_{j=1}^{p} (\phi_j - \phi_{j,o})\, u_{t-j} + a_t$$

in the form of a linear regression model. The $(\phi_j - \phi_{j,o})$ are estimated as

$$\phi_j - \phi_{j,o} = (U'U)^{-1}\, U'\, a_o$$

where

$$U=\begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1p} \\ u_{21} & u_{22} & \cdots & u_{2p} \\ \vdots & \vdots & & \vdots \\ u_{n1} & u_{n2} & \cdots & u_{np} \end{bmatrix}, \quad a_{t,o}=\begin{bmatrix} a_{1,o} \\ a_{2,o} \\ \vdots \\ a_{n,o} \end{bmatrix}, \quad a_{t,o} \sim N(0,\sigma_a^2) .$$

At the final iteration the well known result of linear least squares theory yields

$$V[\hat{\phi}] = S^2(U'U)^{-1} \text{ where } S^2 = (n-p)^{-1} \sum_{t=1}^{n} a_t^2 . \qquad (4.18)$$

The covariance matrix of the estimates is also given by

$$V[\hat{\phi}] \cong I^{-1}(\phi) = \{E[U'U]\sigma_a^2\}^{-1} = (n\Gamma_p)^{-1}\sigma_a^2 = M_p^p/n$$

(Box and Jenkins, 1976). $E(U'U)^{-1}$ can be shown to be equal to $(n\Gamma_p)^{-1}$. For an AR(p) model

$$w_t = \sum_{j=1}^{p} \phi_j w_{t-j} + a_t$$

so

$$a_t = w_t - \sum_{j=1}^{p} \phi_j w_{t-j}$$

therefore

$$\partial a_t/\partial \phi_j = u_{t,j} = -w_{t-j}$$

and

$$E[u_{t,j}^2] = E[w_{t-j}^2] = \gamma_o .$$

Also

$$E[w_{t-k} w_{t-j}] = \gamma_{k-j} .$$

Then E[U'U] contains all cross product terms and

$$E[U'U] = n\sigma_a^2 \begin{bmatrix} \gamma_o & \gamma_1 & \cdots & \gamma_{p-1} \\ \gamma_1 & \gamma_o & \cdots & \gamma_{p-2} \\ \vdots & & & \vdots \\ \gamma_{p-1} & & \cdots & \gamma_o \end{bmatrix} = n\Gamma_p$$

when n is moderate or large. Thus, the approximate covariance matrix of the parameter estimates may be obtained from a transformation of the covariance matrix of the data.

A Univariate Example

Consider the following velocities (in degrees of latitude per six hour interval) of an example hurricane in 1960 (.8,1.0,1.1,.7,.1,-.6,-.3,.6,.7). A short series was chosen to better illustrate the differences between estimation procedures. An AR(1) model was appropriate for this series.

Let $w_t = \{.8,1.0,1.1,.7,.1,-.6,-.3,.6,.7\}$.
Then there are eight pairs of observations that can be used to calculate ϕ_1. They are (.8,1.0)(1.0,1.1)(1.1,.7)(.7,.1)(.1,-.6)(-.6,-.3)(-.3,.6)(.6,.7). Let {x} denote the series of first elements in each ordered pair. Let {y} denote the series of second elements in each ordered pair.

The sample mean of {x} is .425 . The sample mean of {y} is .4125 . Adjusting {x} and {y} to zero mean yields

X' = {.5875,.6875,.2875,-.3125,-1.0125,-.7125,.1875,.2875}

Y' = {.375 ,.575 ,.675 , .275 ,- .325 ,-1.025,-.725,.175}.

The maximum likelihood point estimate of ϕ_1 is

$$\hat{\Phi}_1 = [\sum_{t=2}^{n} w_t w_{t-1}/(n-1)]/[\sum_{t=2}^{n-1} w_t^2/(n-2)] \quad (4.19)$$

$$= [1.6975/8]/[2.5744/7] = .5769 \quad .$$

The least squares point estimate is

$$\hat{\Phi}_1 = [\sum_{t=2}^{9} w_t w_{t-1}]/[\sum_{t=2}^{9} w_t^2] = 1.6975/2.7150 = .6252 \quad . \quad (4.20)$$

Now in the usual least squares regression estimator

$$\hat{\Phi}_1 = (X'X)^{-1}X'Y \quad (4.21)$$

where

$$X'X = \sum_{m=1}^{8} X_m^2 = \sum_{t=1}^{8} (w_t - .425)^2 \quad .$$

The question of whether to use the sum of the w_t^2 from index 2 to 9, or 1 to 8, to represent X'X is academic, because as the number of observations increases, the difference becomes negligible. In fact, the SCA time series computer package uses the Gauss-Marquardt search algorithm which converges in three iterations to the least squares estimate .6252 (Liu et al., 1983).

The estimated standard deviation of the maximum likelihood estimate of ϕ_1, for a first order model is

$$\text{std dev } [\hat{\phi}_1] = [M_p{}^P/9] = [(1-\phi_1{}^2)/9]^{1/2} = \begin{cases} .272 & \hat{\phi}_1 = .5769 \\ .260 & \hat{\phi}_1 = .6252 \end{cases}$$

From least squares the estimate is

$$\text{std dev } [\hat{\phi}_1] = [S/(X'X)]^{1/2} = \{[\sum_{t=2}^{9} a_t{}^2/(8-1)]/[\sum_{t=2}^{9} w_t{}^2]$$

$$= \{[1.699/(8-1)]/[4.16]\}^{1/2} = .242 \quad .$$

SCA calculates the standard deviation as

$$\text{std dev } [\hat{\phi}_1] = \{[1.699/(8)]/[4.16]\}^{1/2} = .226$$

which uses the biased *maximum likelihood estimator for S.*

The smallest hurricane model has 178 observations and most parameter standard deviations are approximately .05. Even with that small standard deviation, the estimates of ϕ_1 are not statistically significantly different. Since the SCA estimate matches the result of SPSS to four significant digits (all that SCA reports), it seems appropriate to use the least squares point estimator in equation (4.21). In addition, because of the way the hurricane tracks are segmented by region, there is missing data which SCA can not handle. Thus, the least squares estimates from SPSS are used in all regions.

Univariate Forecasting

The time series models are to be used to forecast hurricane positions. It is usually a simple matter to calculate a point forecast. The more difficult task, especially in the multivariate case, is to calculate approximate probability limits surrounding the point forecast. As a prelude to the bivariate section, this section contains the development of the univariate point and interval forecasts.

Suppose an AR(1) model has a nonzero mean. Then

$$w_t - \mu = \phi_1 (w_{t-1} - \mu) + a_t$$
$$w_t = \phi_1 w_{t-1} - \phi_1 \mu + \mu + a_t$$
$$w_t = \phi_1 w_{t-1} + \mu(1-\phi_1) + a_t \quad .$$

Note that $-\phi_1\mu+\mu$ can be thought of as $\overline{Y}-\beta_1\overline{X} = \beta_o$ (the intercept) in a simple linear regression.

For the AR(2) model,

$$w_t-\mu = \phi_1(w_{t-1}-\mu) + \phi_2(w_{t-2}-\mu) + a_t$$
$$w_t-\mu = \phi_1 w_{t-1} - \phi_1\mu + \phi_2 w_{t-2} - \phi_2\mu + a_t$$
$$w_t = \phi_1 w_{t-1} + \phi_2 w_{t-2} + a_t + \mu(1-\phi_1-\phi_2)$$

In general for the AR(p) process,

$$w_t = \phi_1 w_{t-1} + \cdots + \phi_p w_{t-p} + \mu(1-\phi_1-\phi_2-\cdots-\phi_p) + a_t \quad .$$

Consider the one step ahead forecast of the AR(1) process with nonzero mean.

$$w_t(1) = \phi_1(w_t-\mu) + \mu = \phi_1 w_t - \phi_1\mu + \mu \quad .$$

Then the two step ahead forecast is

$$\begin{aligned} w_t(2) &= \phi_1(w_t(1)-\mu) + \mu \\ &= \phi_1\{\phi_1 w_t - \phi_1\mu\} + \mu \\ &= \phi_1{}^2\{w_t - \mu\} + \mu \quad , \end{aligned}$$

and the ℓ^{th} step ahead forecast is

$$w_t(\ell) = \phi_1{}^{\ell}\{w_t-\mu\} + \mu \quad .$$

For stationarity it has been shown that $|\phi_1| < 1$. Therefore,

$$\lim_{\ell\to\infty} w_t(\ell) = \mu$$

and the forecast converges to the mean of the series. For hurricane forecasting this means that the greater the lead time, the less information one has concerning the storm. Consequently, the average velocity of past storms is the best guess for a predicted path when the lead time is large.

The forecast error for the AR(1) one step ahead forecast is given by

$$e_t(1) = w_{t+1} - \hat{w}_{t+1} = w_{t+1} - w_t(1)$$

$$= \{\phi_1(w_t-\mu)+\mu+a_{t+1}\}-\{\phi_1(w_t-\mu)+\mu\}$$
$$= a_{t+1} \quad .$$

Thus, the one step ahead forecast error is the noise series. The error for the two step ahead forecast is

$$e_t(2) = w_{t+2} - \hat{w}_{t+2} = w_{t+2} - w_t(2)$$
$$= \{\phi_1(w_{t+1}-\mu)+\mu+a_{t+2}\}-\{\phi_1{}^2(w_t-\mu)+\mu\}$$
$$= a_{t+2}+\phi_1[(w_{t+1}-\mu)-\phi_1(w_t-\mu)]$$
$$= a_{t+2}+\phi_1(a_{t+1}) \quad .$$

Similarly, the error in the ℓ^{th} step ahead forecast is

$$e_t(\ell) = a_{t+\ell} + \phi_1(a_{t+\ell-1})+\phi_1{}^2(a_{t+\ell-2})+\cdots+\phi_1{}^{\ell-1}a_{t+1}$$

and, because the a_t are independent and identically distributed (iid), the variance of the ℓ^{th} step ahead forecast is given by

$$V[e_t(\ell)]=V[a_{t+\ell}]+\phi_1{}^2V[a_{t+\ell-1}]+\phi_1{}^4V[a_{t+\ell-2}]+\cdots$$
$$+\phi_1{}^{2\ell-2}V[a_{t+1}]$$
$$= \sigma_a{}^2 \; [(1-\phi_1{}^{2\ell})/(1-\phi_1{}^2)] \quad .$$

For the AR(2) model, the one step ahead forecast error is

$$e_t(1)= w_{t+1} - \hat{w}_{t+1}$$
$$=\{\phi_1(w_t-\mu)+\phi_2(w_{t-1}-\mu)+\mu+a_{t+1}\}-\{\phi_1(w_t-\mu)+\phi_2(w_{t-1}-\mu)+\mu\}$$
$$= a_{t+1}$$

and,

$$
\begin{aligned}
e_t(2) &= w_{t+2} - \hat{w}_{t+2} = w_{t+2} - w_t(2) \\
&= \{\phi_1(w_{t+1}-\mu)+\phi_2(w_t-\mu)+\mu+a_{t+2}\} \\
&\quad -\{\phi_1(w_t(1)-\mu)+\phi_2(w_t-\mu)+\mu\} \\
&= \{\phi_1 w_{t+1}+\phi_2 w_t+a_{t+2}\}-\{\phi_1[\phi_1 \quad +\phi_2 w_{t-1}]+\phi_2 w_t\} \\
&= \phi_1(w_{t+1}-(\phi_1 w_t+\phi_2 w_{t-1})) + a_{t+2} \\
&= a_{t+2} + \phi_1 a_{t+1} \quad .
\end{aligned}
$$

These results are based on the assumption that the ϕ_i are constant parameters and σ_a is known. In practice these quantities are random variables and must be estimated. Consequently, the confidence regions for the forecast error are approximate.

The calculation of $e_t(\ell)$, for AR models is simply an algebraic exercise. However, as ℓ increases the computation becomes laborious. Indeed, there is an easier way to calculate the ℓ^{th} step ahead forecast error for the AR(p) model. It involves the ψ weight representation of the AR process which is defined by expressing w_t in terms of the past values of the noise a_t. The psi-weight form of the AR(p) model is derived from the difference equation form of the AR(p) model

$$w_t = \phi_1 w_{t-1}+\phi_2 w_{t-2}+\cdots+\phi_p w_{t-p}+a_t$$

then

$$
\begin{aligned}
(1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p)w_t &= a_t \\
w_t &= (1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p)^{-1}a_t \\
&= (1-\psi_1 B-\psi_2 B^2- \cdots)a_t
\end{aligned}
$$

$$= a_t - \psi_1 a_{t-1} - \psi_2 a_{t-2} - \cdots$$

For the AR(1) model, the ψ weights are successive powers of ϕ_1.

$$w_t - \mu = \phi_1(w_{t-1} - \mu) + a_t$$
$$(1-\phi_1 B)(w_t - \mu) = a_t$$
$$w_t - \mu = (1-\phi_1 B)^{-1} a_t$$
$$w_t = \mu + a_t + \phi_1 a_{t-1} + \phi_1{}^2 a_{t-2} + \cdots$$

where the a_t are the residuals of the one step ahead forecasts of the previous observations.

For the AR(2) model,

$$w_t - \mu = (1-\phi_1 B - \phi_2 B^2)^{-1} a_t \quad .$$

Expanding $(1-\phi_1 B-\phi_2 B^2)^{-1}$ by a Maclaurin series yields the approximate ψ weights (Abraham and Ledolter, 1983).

$$f(B) = (1-\phi_1 B-\phi_2 B^2)^{-1}$$
$$f'(B) = (\phi_1+2\phi_2 B)(1-\phi_1 B-\phi_2 B^2)^{-2}$$
$$f''(B) = 2[(\phi_1+2\phi_2 B)^2(1-\phi_1 B-\phi_2 B^2)^{-3}+\phi_2(1-\phi_1 B-\phi_2 B^2)^{-2}]$$
$$f'''(B) = 6[(\phi_1+2\phi_2 B)^3(1-\phi_1 B-\phi_2 B^2)^{-4}$$
$$+(\phi_1+2\phi_2 B)(1-\phi_1 B-\phi_2 B^2)^{-3}(2\phi_2)]$$
$$f(0) = 1$$
$$f'(0) = \phi_1$$
$$f''(0) = 2[(\phi_1)^2+\phi_2]$$
$$f'''(0) = 6[(\phi_1)^3+2\phi_1\phi_2]$$

$$w_t - \mu = [1+\phi_1 B+(\phi_1{}^2+\phi_2)B^2+(\phi_1{}^3+2\phi_1\phi_2)B^3+\cdots]a_t$$

$$w_t = \mu+[\psi_o+\psi_1 B+\psi_2 B^2+\psi_3 B^3+\cdots]a_t \quad . \qquad (4.22)$$

Equation (4.22) may be expanded to any desired degree of accuracy and is easily obtained for the low order AR models with which one is usually concerned. For example, consider the AR(4) model with ϕ_2 and ϕ_3 equal to zero. Then

$$\psi_1=\phi_1,\ \psi_2=\phi_1{}^2,\ \psi_3=\phi_1{}^3,\ \psi_4=\phi_1{}^4+\phi_4$$

and, in general,

$$\psi_j = \phi_1\psi_{j-1}+\phi_4\psi_{j-4} \quad .$$

Next, the forecast error for the AR(p) model is derived in terms of the ψ weights. Let the ℓ^{th} step ahead forecast be given by

$$w_t(\ell) = \mu+\xi_o a_t+\xi_1 a_{t-1}+\xi_2 a_{t-2}+\cdots \qquad (4.23)$$

where the ξ_j are to be determined. Then the forecast error (known only after time $t+\ell$) is

$$e_t(\ell) = w_{t+\ell}-w_t(\ell) = (\mu+\psi_o a_{t+\ell}+\psi_1 a_{t+\ell-1}+\psi_2 a_{t+\ell-2}+\cdots)$$
$$-(\mu+\xi_o a_t +\xi_1 a_{t-1} +\xi_2 a_{t-2} +\cdots)$$

and because the a_t are iid, the variance of the ℓ^{th} step ahead forecast is given by

$$E[e_t^2(\ell)] = [a_{t+\ell}+\psi_1 a_{t+\ell-1}+\psi_2 a_{t+\ell-2}+\cdots+\psi_{\ell-1}a_{t+1}$$

$$+(\psi_\ell-\xi_o)a_t+(\psi_{\ell+1}-\xi_1)a_{t-1}+\cdots]^2$$

$$= (1+\psi_1^2+\psi_2^2+\cdots+\psi_{\ell-1}^2)\sigma_a^2 + \sum_{j=0}^{\infty} (\psi_{\ell+j}-\xi_j)^2\sigma_a^2 \; .$$

This variance is minimized when $\psi_{\ell+j} = \xi_j$.

Thus, the minimum mean square error forecast from (4.23) is

$$w_t(\ell) = \mu+\psi_\ell a_t+\psi_{\ell+1}a_{t-1}+\psi_{\ell+2}a_{t-2}+\cdots \; .$$

Therefore,

$$\begin{aligned} e_t(\ell) &= (\mu+a_{t+\ell}+\psi_1 a_{t+\ell-1}+\psi_2 a_{t+\ell-2}+\cdots) \\ &\quad - (\mu+\psi_\ell a_t+\psi_{\ell+1}a_{t-1}+\psi_{\ell+2}a_{t-2}+\cdots) \\ &= (a_{t+\ell}+\psi_1 a_{t+\ell-1}+\cdots+\psi_{\ell-1}a_{t+1}+\psi_\ell a_t+\psi_{\ell+1}a_{t-1}+\cdots) \\ &\quad - (\psi_\ell a_t+\psi_{\ell+1}a_{t-1}+\cdots) \\ &= a_{t+\ell}+\psi_1 a_{t+\ell-1}+\cdots+\psi_{\ell-1}a_{t+1} \; , \end{aligned} \qquad (4.24)$$

and the variance of the ℓ^{th} step ahead forecast error is

$$\begin{aligned} V[e_t(\ell)] &= E[a_{t+\ell}^2]+\psi_1^2 E[a_{t+\ell-1}^2]+\cdots+\psi_{\ell-1}^2 E[a_{t+1}^2] \\ &= \sigma_a^2(1+\psi_1^2+\psi_2^2+\cdots+\psi_{\ell-1}^2) \; . \end{aligned} \qquad (4.25)$$

There are three important conclusions: (1) the forecast error is a linear sum of independent Normally distributed noise terms, and so is Normally distributed with mean zero and variance given by (4.25), (2) the forecast error is a function of unknown future shocks (noise) which must be estimated, (Consequently, forecast errors beyond one period are correlated.), (3) this approach assumes that the ψ_i (and consequently the ϕ_i) are constant and σ_a^2 is known. In practice these quantities are random variables which must be estimated. Thus, the confidence region surrounding the forecast is approximate. Also, if the variances of the ϕ_i are large (as in the univariate example) the confidence interval may be severely understated (Pankratz, 1983).

In order to reduce the variance of the parameter estimates it was necessary to determine a way to combine hurricane tracks so as to maximize the number of observations in each grid. This was accomplished by differencing the individual storm tracks (if required) and then using the pairwise deletion option of SPSS. For example, two tracks were being combined, each having ten observations. Then for the velocity model there were 18 observations available to compute the lag 1 parameter ϕ_1, 16 available to compute ϕ_2, 14 available to compute ϕ_3 and so forth. This procedure increased the number of observations in a particular region and so decreased the variance of the parameter estimates (to approximately .0016 for a parameter value near .7).

Univariate Interval Forecast Example

In this example, for the arbitrary region 10-15 degrees north latitude, and 45-83 degrees west longitude, confidence intervals associated with the 24, 48, and 72 hour latitude forecast are desired. A univariate AR(4) model of the applicable latitude position series (differenced once) yields the model

$$Vel_t = .69558Vel_{t-1} + .10069Vel_{t-4}$$

where $Vel_t = (LA_t - LA_{t-1})$, and LA_t is the latitude (in degrees) at time t. Consequently,

$$LA_t = 1.69558LA_{t-1} - .69558LA_{t-2} + .10069(LA_{t-4} - LA_{t-5}) .$$

The psi weights for this model are computed as shown in the previous section. Here $\psi_o=1$, $\psi_1=1.69558$,

$$\psi_2 = 1.69558\psi_1 + (-.69558)\psi_o = 2.17941$$

$$\psi_3 = \phi_1\psi_2 + \phi_2\psi_1 = 2.51595$$

$$\psi_4 = \phi_1\psi_3 + \phi_2\psi_2 + \phi_4\psi_o = 2.85073$$

$$\psi_5 = \phi_1\psi_4 + \phi_2\psi_3 + \phi_4\psi_1 + \phi_5\psi_o = 3.15364$$

$$\psi_6 = \phi_1\psi_5 + \phi_2\psi_4 + \phi_4\psi_2 + \phi_5\psi_1 = 3.41305$$

$$\psi_7 = \phi_1\psi_6 + \phi_2\psi_5 + \phi_4\psi_3 + \phi_5\psi_2 = 3.62737$$

Similarly,

$\psi_8=3.81016$, $\psi_9=3.96781$, $\psi_{10}=4.10358$, $\psi_{11}=4.21960$.

The one step ahead latitude forecast errors have mean zero and standard deviation 9.1645 nautical miles (n mi). Thus, the variance of the 24 hour forecast is

$$V[e_t(4)] = (9.1645)^2(1+\psi_1^2+\psi_2^2+\psi_3^2) = 84.0(14.9548) = 1256.2$$

and the standard deviation (SD) of $e_t(4)$ is 35.4 n mi. This compares favorably with the empirical standard deviation of 35.9 n mi obtained from the forecast models in Appendix A. For the 48 and 72 hour forecast

$$SD[e_t(8)] = 69.7 \text{ n mi}$$

$$SD[e_t(12)] = 101.5 \text{ n mi}$$

compared to the empirical values of 67.0 and 82.2 n mi respectively. Consequently, the 90% confidence intervals are the point forecasts ± 58.2, 114.6, and 167.0 nautical miles respectively.

BIVARIATE ESTIMATION

Let $\{Z_{1,t}\}$, $\{Z_{2,t}\}$ be two time series, each with n observations. If the series are autoregressive of order p then

$$Z_{1,t} = \phi_{11,1}Z_{1,t-1} + \phi_{11,2}Z_{1,t-2} + \cdots + \phi_{11,p}Z_{1,t-p} + \phi_{12,1}Z_{2,t-1} + \cdots + \phi_{12,p}Z_{2,t-p} + C_1 + a_{1,t}$$

and

$$Z_{2,t} = \phi_{21,1}Z_{1,t-1} + \cdots + \phi_{21,p}Z_{1,t-p} + \phi_{22,1}Z_{2,t-1} + \cdots + \phi_{22,p}Z_{2,t-p} + C_2 + a_{1,t}$$

or, in matrix notation,

$$\begin{bmatrix} Z_{1,t} \\ Z_{2,t} \end{bmatrix} = \begin{bmatrix} \phi_{11,1} & \phi_{12,1} \\ \phi_{21,1} & \phi_{22,1} \end{bmatrix} \begin{bmatrix} Z_{1,t-1} \\ Z_{2,t-1} \end{bmatrix} + \cdots + \begin{bmatrix} \phi_{11,p} & \phi_{12,p} \\ \phi_{21,p} & \phi_{22,p} \end{bmatrix} \begin{bmatrix} Z_{1,t-p} \\ Z_{2,t-p} \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix}$$

or,

$$\underset{\sim}{\phi}(B)\underset{\sim}{Z}_t = \underset{\sim}{C} + \underset{\sim}{a}_t \tag{4.26}$$

where the $\{\underset{\sim}{a}_t\}$ are iid $N(0,\Sigma)$. The $\underset{\sim}{a}_t$ noise series are assumed to have zero covariance except at time t. That is, for two noise series, say $\{a_1\}$ and $\{a_2\}$,

$$E[a_{1,t}, a_{2,s}] = \begin{cases} 0 & t \neq s \\ \Sigma & t = s \end{cases},$$

C is a 2 x 1 vector of constants, and

$$\phi(B) = I - \phi_1 B - \cdots - \phi_p B^p$$

where ϕ_i is a 2 x 2 matrix of coefficients at lag i as shown in (4.26) above. The individual series are assumed to be weakly stationary.

The model in (4.26) can be expressed as a multivariate linear model and $\underset{\sim}{\phi}$ can then be obtained via multivariate least squares (Tiao and Tsay, 1983). Specifically, let

$$Y = \begin{bmatrix} Z'_{p+1} \\ \vdots \\ Z'_{n} \end{bmatrix}, \quad X = \begin{bmatrix} 1 & Z'_{p} & \cdots & Z'_{1} \\ \vdots & \vdots & & \vdots \\ 1 & Z'_{n-1} & \cdots & Z'_{n-p} \end{bmatrix}, \quad \eta = \begin{bmatrix} C' \\ \phi_1' \\ \vdots \\ \phi_p' \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} a'_{p+1} \\ \vdots \\ a_n' \end{bmatrix}$$

Then $Y=X\eta+\varepsilon$ is a multivariate linear model. Least squares estimates can be obtained directly (Hillmer and Tiao, 1979). Although they are biased (Granger and Newbold, 1977), the least squares parameter estimates converge in probability to the true parameter values (Hannan, 1970).

<u>Bivariate Point Forecast</u>

Similar to univariate models, bivariate forecasts are based on the difference-equation form of the model. Specifically, the general form of the AR(5) position model used in all regions is

$$LA_t=\phi_{11,1}LA_{t-1}+\phi_{11,2}LA_{t-2}+\phi_{11,3}LA_{t-3}+\phi_{11,4}LA_{t-4}+\phi_{11,5}LA_{t-5}$$
$$+\phi_{12,1}LO_{t-1}+\phi_{12,2}LO_{t-2}+\phi_{12,3}LO_{t-3}+\phi_{12,4}LO_{t-4}+\phi_{12,5}LO_{t-5}$$
$$+ C_1$$

$$LO_t=\phi_{21,1}LA_{t-1}+\phi_{21,2}LA_{t-2}+\phi_{21,3}LA_{t-3}+\phi_{21,4}LA_{t-4}+\phi_{21,5}LA_{t-5}$$
$$+\phi_{22,1}LO_{t-1}+\phi_{22,2}LO_{t-2}+\phi_{22,3}LO_{t-3}+\phi_{22,4}LO_{t-4}+\phi_{22,5}LO_{t-5}$$
$$+ C_2$$

where, LA_t represents the latitude at time t, LO_t represents the longitude at time t, and C_1 and C_2 are scalar constants.

For the region 25-30 degrees north latitude, 45-100 degrees west longitude (the Gulf of Mexico coastal region), the model is

$$LA_t = 1.777LA_{t-1} - .777LA_{t-2} + .016LA_{t-4} - .016LA_{t-5} - .101LO_{t-1} + .101LO_{t-2} + .083LO_{t-4} - .083LO_{t-5} + .071$$

$$LO_t = .103LA_{t-1} - .103LA_{t-2} - .198LA_{t-4} + .198LA_{t-5} + 1.837LO_{t-1} - .837LO_{t-2} + .032LO_{t-4} - .032LO_{t-5} + .052 \quad .$$

Parameters not significantly different from zero have been included for ease of programming, and model comparison. The one step ahead forecast position (LA_t, LO_t) is based on the use of the five previous (six hour) position reports. To obtain the two step ahead forecast, (LA_t, LO_t) is treated as the last observed position, and the one step ahead forecast (from time t) is computed. Forecasts for lead times up to n steps ahead are computed in a similar manner.

Point Forecast Example

Consider the following five position reports from hurricane Elena (1985).

$$(LA_{t-5}, LO_{t-5}) = (28.7,\ 85.2)$$
$$(LA_{t-4}, LO_{t-4}) = (28.8,\ 84.4)$$
$$(LA_{t-3}, LO_{t-3}) = (28.9,\ 83.9)$$
$$(LA_{t-2}, LO_{t-2}) = (28.8,\ 83.8)$$
$$(LA_{t-1}, LO_{t-1}) = (28.5,\ 84.0)$$

The last position report (time t-1) was for 0500 Central Daylight Time (CDT) on September 1. Actual landfall of Elena was 54 hours later at Gulfport, Mississippi (30.3, 88.8). The sequence of position forecasts from the 25-30N latitude model is (28.3,84.1), (28.1,84.3), (28.0,84.4), (28.0,84.6), (28.1,84.9), (28.2,85.3), (28.3,85.6), (28.4,86.0), (28.6,86.4). These forecasts have an important characteristic. Even though the initial motion vector points to the southwest, the model "turns" the storm to the northwest. That is, given that the hurricane is "wandering" (as Elena was at this point) the forecast of future motion is the average velocity vector of past storms which points to the northwest. The landfall forecast error of the time series model is approximately 150 n mi.

This is more accurate than other forecasts of this storm. Model A (Freeze, 1983), which depends heavily on the initial motion vector, forecasts landfall in 72 hours on the Yucatan peninsula. The error of model A is approximately 480 n mi. At the time of this position report the National Hurricane Center had hurricane warnings posted from Pensacola, Florida to Fort Meyers, Florida and was forecasting landfall near Cedar Key, Florida. This was an error of approximately 250 n mi. Thus, it appears the time series approach is potentially effective.

<u>Interval Forecast of One Variable</u>

If the random shocks $\{a_{1t}\}$, $\{a_{2t}\}$ are Normally distributed (as assumed), and the appropriate model has been estimated with a sufficiently large sample, then the forecasts are Normally distributed (Pankratz, 1983). Consequently, appropriate 90% confidence intervals for the ℓ^{th} step ahead latitude or longitude forecast are

$$LA_t(\ell) \pm 1.645\ SD[e_{1t}(\ell)]$$

$$LO_t(\ell) \pm 1.645\ SD[e_{2t}(\ell)]$$

where $e_{1t}(\ell)$ and $e_{2t}(\ell)$ are the ℓ^{th} step ahead forecast errors for latitude and longitude respectively.

In the bivariate case, the psi weights necessary for computation of $e_{1t}(\ell)$ and $e_{2t}(\ell)$ are computed using the general model

$$LA_t=\phi_{11,1}LA_{t-1}+\phi_{11,2}LA_{t-2}+\phi_{11,3}LA_{t-3}+\phi_{11,4}LA_{t-4}+\phi_{11,5}LA_{t-5}$$
$$+\phi_{12,1}LO_{t-1}+\phi_{12,2}LO_{t-2}+\phi_{12,3}LO_{t-3}+\phi_{12,4}LO_{t-4}+\phi_{12,5}LO_{t-5}$$
$$+ C_1 + a_{1t}$$

$$LO_t=\phi_{21,1}LA_{t-1}+\phi_{21,2}LA_{t-2}+\phi_{21,3}LA_{t-3}+\phi_{21,4}LA_{t-4}+\phi_{21,5}LA_{t-5}$$
$$+\phi_{22,1}LO_{t-1}+\phi_{22,2}LO_{t-2}+\phi_{22,3}LO_{t-3}+\phi_{22,4}LO_{t-4}+\phi_{22,5}LO_{t-5}$$
$$+ C_2 + a_{2t} \quad ,$$

where a_{1t} and a_{2t} represent the noise in the forecast for period t for latitude and longitude respectively. Solving for LA_t and LO_t in terms of the $\phi(B)$ polynomials, and eliminating the constants (which do not contribute to the error), yields

$$LA_t=\frac{(\phi_{12,1}B+\phi_{12,2}B^2+\phi_{12,3}B^3+\phi_{12,4}B^4+\phi_{12,5}B^5)LO_t+a_{1t}}{(1-\phi_{11,1}B-\phi_{11,2}B^2-\phi_{11,3}B^3-\phi_{11,4}B^4-\phi_{11,5}B^5)} \quad (4.27)$$

$$LO_t=\frac{(\phi_{21,1}B+\phi_{21,2}B^2+\phi_{21,3}B^3+\phi_{21,4}B^4+\phi_{21,5}B^5)LA_t+a_{2t}}{(1-\phi_{22,1}B-\phi_{22,2}B^2-\phi_{22,3}B^3-\phi_{22,4}B^4-\phi_{22,5}B^5)} \quad (4.28)$$

Solving these two simultaneous linear equations for LA_t and LO_t in terms of a_{1t} and a_{2t} results in

$$LA_t=\{(\phi_{12,1}B+\cdots+\phi_{12,5}B^5)a_{2t}+(1-\phi_{22,1}B-\cdots-\phi_{22,5}B^5)a_{1t}\}/$$
$$\{(1-\phi_{11,1}B-\cdots-\phi_{11,5}B^5)(1-\phi_{22,1}B-\cdots-\phi_{22,5}B^5)-$$
$$(\phi_{12,1}B+\cdots+\phi_{12,5}B^5)(\phi_{21,1}B+\cdots+\phi_{21,5}B^5)\} \quad (4.29)$$

$$LO_t=\{(\phi_{21,1}B+\cdots+\phi_{21,5}B^5)a_{1t}+(1-\phi_{11,1}B-\cdots-\phi_{11,5}B^5)a_{2t}\}/$$
$$\{(1-\phi_{11,1}B-\cdots-\phi_{11,5}B^5)(1-\phi_{22,1}B-\cdots-\phi_{22,5}B^5)-$$
$$(\phi_{12,1}B+\cdots+\phi_{12,5}B^5)(\phi_{21,1}B+\cdots+\phi_{21,5}B^5)\} \ . \quad (4.30)$$

where the ratios of the above polynomials yield the psi weights. The error of the ℓ^{th} step ahead forecast from time origin t is given by

$$e_{1t}(\ell)=a_{1t}+\psi_{11,1}a_{1t+\ell-1}+\psi_{11,2}a_{1t+\ell-2}+\cdots+\psi_{11,\ell-1}a_{1t+1}$$
$$+\psi_{12,1}a_{2t+\ell-1}+\cdots+\psi_{12,\ell-1}a_{2t+1}$$

$$e_{2t}(\ell)=a_{2t}+\psi_{22,1}a_{2t+\ell-1}+\psi_{22,2}a_{2t+\ell-2}+\cdots+\psi_{22,\ell-1}a_{2t+1}$$
$$+\psi_{21,1}a_{1t+\ell-1}+\cdots+\psi_{21,\ell-1}a_{1t+1} \quad .$$

The $\mathrm{cov}(a_{1t},a_{2s}) = 0,\ t \neq s$, so

$$E[e_{1t}^2(\ell)] = (1+\psi_{11,1}^2+\psi_{11,2}^2+\cdots+\psi_{11,\ell-1}^2)\sigma_{a1}^2$$
$$+(\psi_{12,1}^2+\cdots+\psi_{12,\ell-1}^2)\sigma_{a2}^2$$
$$+2(\psi_{11,1}\psi_{12,1}+\cdots+\psi_{11,\ell-1}\psi_{12,\ell-1})E[a_1a_2] \quad (4.31)$$

$$E[e_{2t}^2(\ell)] = (1+\psi_{22,1}^2+\psi_{22,2}^2+\cdots+\psi_{22,\ell-1}^2)\sigma_{a2}^2$$
$$+(\psi_{21,1}^2+\cdots+\psi_{21,\ell-1}^2)\sigma_{a1}^2$$
$$+2(\psi_{22,1}\psi_{21,1}+\cdots+\psi_{22,\ell-1}\psi_{21,\ell-1})E[a_1a_2] \quad (4.32)$$

$$E[e_{1t}(\ell)e_{2t}(\ell)]=\{(1+\psi_{11,1}\psi_{22,1}+\cdots+\psi_{11,\ell-1}\psi_{22,\ell-1})$$
$$+(\psi_{12,1}\psi_{21,1}+\cdots+\psi_{12,\ell-1}\psi_{21,\ell-1})\}E[a_1a_2]$$
$$+(\psi_{11,1}\psi_{21,1}+\cdots+\psi_{11,\ell-1}\psi_{21,\ell-1})\sigma_{a1}^2$$
$$+(\psi_{12,1}\psi_{22,1}+\cdots+\psi_{12,\ell-1}\psi_{22,\ell-1})\sigma_{a2}^2. \quad (4.33)$$

Interval Forecast Example (Latitude)

For the Gulf coast region discussed in point forecasting, the covariance matrix (Σ) of the one step ahead forecast forecasts is

$$\Sigma = \begin{bmatrix} 212.576 & -28.936 \\ -28.936 & 307.301 \end{bmatrix} .$$

By (4.31) the variance of the eight step ahead forecast for latitude is

$$\begin{aligned} V[e_{1t}(8)] = {} & (1+1.777^2+2.370^2+2.815^2+3.155^2+3.439^2+3.694^2 \\ & +3.932^2)\ (213.16) \\ & +\{0+(-.101)^2+(-.264)^2+(-.460)^2+(-.587)^2+(-.665)^2 \\ & +(-.715)^2+(-.752)^2\}(307.301) \\ & +2(-11.837)(-28.936) = 15927.0 \quad . \end{aligned}$$

The corresponding standard deviation of 126.2 n mi compares favorably with the empirical value of 137.2 n mi.

Interval Forecast of Two Variables

Results of chi-square tests of latitude and longitude forecast errors for category five storms do not reject the hypothesis that the joint distribution of the forecasts is bivariate Normal (Appendix D). Specifically, the distribution is

$$f(LA,LO)=\{(2\pi\sigma_1\sigma_2)(1-\rho^2)^{1/2}\}^{-1}\exp(-1/2)(1/[1-\rho^2])\{([LA-\mu_1]/\sigma_1)^2$$
$$-2\rho([LA-\mu_1]/\sigma_1)([LO-\mu_2]/\sigma_2)+(LO-\mu_2]/\sigma_2)^2\} \quad (4.34)$$

where σ_1 and σ_2 represent the standard deviations of the latitude and longitude forecasts as computed from the psi weights and (μ_1, μ_2) is the point forecast. The exponent of (4.34) can be used to specify the equation of a confidence ellipse in two dimensions when it is set equal to some positive constant (say c) (Morrison, 1976).

Let (x_1,x_2) be any point on the ellipse. Then $(x_1-\mu_1,x_2-\mu_2)$ is the vector from (x_1,x_2) to the center (μ_1,μ_2). The length of the vector is maximized when $(x-\mu)'(x-\mu) = \{(x_1-\mu_1)^2+(x_2-\mu_2)^2\}$ is a maximum, subject to

$$(1-\rho^2)^{-1}[(x_1-\mu_1)/\sigma_1]^2-2\rho[(x_1-\mu_1)/\sigma_1][(x_2-\mu_2)/\sigma_2]$$
$$+[(x_2-\mu_2)/\sigma_2]^2 = c \quad .$$

This optimization problem can be solved by use of the Lagrangian L where

$$L=(x_1-\mu_1)^2+(x_2-\mu_2)^2-\lambda(1-\rho^2)^{-1}\{[(x_1-\mu_1)/\sigma_1]^2$$
$$-2\rho[(x_1-\mu_1)/\sigma_1][(x_2-\mu_2)/\sigma_2]+[(x_2-\mu_2)/\sigma_2]^2-c\} \quad .$$

Taking partial derivatives to find a stationary point,

$$\partial L/\partial x_1=2(x_1-\mu_1)-2\lambda(1-\rho^2)^{-1}[(x_1-\mu_1)/\sigma_1{}^2]$$
$$+2\lambda[(\rho/1-\rho^2)][(x_2-\mu_2)/(\sigma_1\sigma_2)] = 0 \quad ,$$

$$\partial L/\partial x_2 = 2(x_2-\mu_2) - 2\lambda(1-\rho^2)^{-1}[(x_2-\mu_2)/\sigma_2{}^2]$$
$$+2\lambda[(\rho/1-\rho^2)][(x_1-\mu_1)/(\sigma_1\sigma_2)] = 0$$

or, equivalently

$$I(X-\mu) - \lambda\Sigma^{-1}(X-\mu) = 0$$
$$(I-\lambda\Sigma^{-1})(X-\mu) = 0 \quad . \qquad (4.35)$$

Pre-multiplication by Σ yields

$$(\Sigma-\lambda I)(X-\mu) = 0 \quad .$$

Thus, the coordinates specifying the principle axis of the ellipse lie on the eigenvectors of Σ. Pre-multiplying (4.35) by $(X-\mu)'$ yields

$$(X-\mu)'(X-\mu) = \lambda(X-\mu)'\Sigma^{-1}(X-\mu)$$
$$= \lambda c \quad .$$

Then the length of the principal axis is maximized when λ is the largest eigenvalue of Σ. The length of the axis is $2(\lambda_1)^{1/2}c$, where λ_1 is the largest eigenvalue of Σ, and c is the number of standard deviations for the chosen confidence interval. The covariance matrix Σ is the matrix for the ℓ^{th} step ahead forecast, and is computed from equations (4.31), (4.32), and (4.33).

Bivariate Interval Forecast Example

In this example the 90% confidence ellipse is derived for the 48 hour forecast for the Gulf coast region (25-30N, 45-100W). From the previous example, the standard deviation of the latitude forecast is 126.2 n mi. For longitude the eight step ahead forecast is given by (4.30) where

$$LO_t=\{(.103B-.103B^2-.198B^4+.198B^5)a_{1t}$$
$$+(1-1.777B+.777B^2-.016B^4+.016B^5)a_{2t}\}/$$
$$\{(1-1.837B+.837B^2-.032B^4+.032B^5)$$
$$(1-1.777B+.777B^2-.016B^4+.016B^5)$$
$$-(-.101B+.101B^2+.083B^4-.084B^5)$$
$$(.103B-.103B^2-.198B^4+.198B^5)\}$$
$$=\{(.103B-.103B^2-.198B^4+.198B^5)a_{1t}$$
$$+(1-1.777B+.777B^2-.016B^4+.016B^5)a_{2t}\}/$$
$$\{1+3.614B+4.888B^2-2.935B^3+.592B^4+.105B^5-.067B^6$$
$$+.009B^7+.017B^8+.032B^9+.017B^{10}\} \quad .$$

Then the estimated variance of the eight step ahead longitude forecast is

$$V[e_{2t}(8)]=(0+.103^2+.269^2+.469^2+.484^2+.385^2+.219^2+.016^2)\sigma_{a1}{}^2$$
$$+(1^2+1.837^2+2.528^2+3.090^2+3.572^2+4.009^2+4.421^2$$
$$+4.817^2)(\sigma_{a2})^2$$
$$+2(6.593)(-28.936)$$
$$=.728(212.576)+91.893(307.301)-381.55$$
$$=28012.154 \quad .$$

Then the estimated standard deviation of the eight step ahead longitude forecast is

$$SD[e_{2t}(8)] = 167.4 \text{ n mi} \quad .$$

The estimated covariance of the eight step ahead longitude and latitude forecast is computed as

$$
\begin{aligned}
E[e_{1t}(8)\ e_{2t}(8)] = {} & [1+1.777(1.837)+2.371(2.528)+2.815(3.090) \\
& +3.155(3.521)+3.439(4.009)+3.694(4.421) \\
& +3.932(4.817)+(-.101)(.103)+(-.264)(.269) \\
& +(-.460)(.469)+(-.587)(.485)+(-.667)(.382) \\
& +(-.715)(.213)+(-.752)(.016)]\ (-28.936) \\
& +\{1.777(.103)+2.370(.269)+2.814(.469)+ \\
& +3.155(.484)+3.439(.382)+3.694(.213) \\
& +4.144(-.191)\}\ (212.576) \\
& +\{(-.101)(1.837)+(-.264)(2.528) \\
& +(-.469)(3.090)+(-.586)(3.571) \\
& +(-.669)(4.009)+(-.715)(4.420) \\
& +(-.752)(4.817)\}\ (307.301) \\
= {} & -5498.78
\end{aligned}
$$

where -28.936, 212.576, and 307.301 are elements of the empirical one step ahead forecast covariance matrix evaluated previously. Then the estimated correlation of the eight step ahead forecast is

$$
\begin{aligned}
\rho &= -5498.78/[(167.4)(126.2)] \\
&= -.260
\end{aligned}
$$

where 167.4 and 126.2 are the estimated standard deviations for longitude and latitude.

The estimated covariance matrix is

$$
\Sigma = \begin{bmatrix} (167.4)^2 & (-.260)(167.4)(126.2) \\ (-.260)(167.4)(126.2) & (126.2)^2 \end{bmatrix}
$$

$$
= \begin{bmatrix} 28022.80 & -5492.72 \\ -5492.72 & 15926.44 \end{bmatrix} .
$$

The estimated eigenvalues (λ_1, λ_2) are the solutions of the quadratic equation

$$(28022.80-\lambda)(15926.44\lambda)-5492.72^2 = 0 \quad .$$

Then,

$$\lambda^2 - 43949.24\lambda + 416133470 = 0 \quad ,$$

which has roots

$$\lambda_1 = 13804.52$$

$$\lambda_2 = 30144.72 \quad .$$

The corresponding eigenvectors are <.360,.938>, and <-.938,.360>. The second eigenvector orients the major axis of the confidence ellipse and has length

$$2(30144.72)^{1/2}(1.645)=571.2 \text{ n mi} \quad .$$

The minor axis has length 386.6 n mi., lies on a magnetic heading of $\tan^{-1}(.360/.938) = 21.0$ degrees, and is orthogonal to the major axis. The ellipse is centered at the point forecast.

In general, the equation of the 90% confidence ellipse for any forecast is given by

$$(x_1/\sigma_1)^2-2\rho(x_1/\sigma_1)(x_2/\sigma_2)+(x_2/\sigma_2)^2=1.645(1-\rho^2)$$

where ρ is the correlation between longitude and latitude forecasts and σ_1 and σ_2 are the standard deviations of the longitude and latitude forecasts respectively.

Thus, for the 48 hour forecast for the Gulf coast region, the equation of the 90% confidence ellipse is

$$(x_1/167.4)^2-2(-.260)(x_1/167.4)(x_2/126.2)-(x_2/126.2)^2$$
$$= 1.645(1-(-.260)^2)$$

or

$$(x_1{}^2/28802.80)+(.520x_1x_2/21125.88)+(x_2{}^2/15926.44)=1.534 .$$

It should be noted that this confidence ellipse is based on the one step ahead forecast for all storms that have passed through the region, so it is fairly robust. However, locations for storms that exhibit rapid changes in velocity may fall outside the confidence ellipse because the model coefficients are based on velocity-stationary storms.

Threshold Point Forecast and Example

Forecast models have been developed for each 5 degree band of latitude from 10 through 45 degrees north latitude (Appendix A). When the forecast latitude LA_t enters a new latitude band, the model associated with that latitude band should be used.

For example, position reports from hurricane Henri (September, 1985) were (21.0,64.0) (21.4,65.0) (22.2,66.7) (22.3,67.8) (23.0,68.7). Using the model for 20-25 degrees north latitude yielded the following forecasts: (23.7,69.6) (24.4,70.5) (25.1,71.5) . At the third forecast, Henri was predicted to enter a new region, and the model parameters were changed. The four step ahead forecast was made using the new parameters for 25-30 north latitude. That forecast was (25.6,72.3).

Threshold Interval Forecast of One Variable

The time series model developed in the next two sections of the paper is similar to the TARSO model (Tong and Lim, 1980) discussed in Chapter Three. There are two major differences between the TARSO model and the hurricane model: (1) all observations on which the forecast is based are not required to lie in the same region; (2) the noise series of the hurricane model are allowed to co-vary at the same lag, while Tong and Lim assume independence. In addition, Tong and Lim do not discuss the distribution of the forecast.

Definitions

$t-(k+1)$ is the time index of the last observed position

$\{\psi_i\}$ $i=0,\ldots,k$ is the set of old grid (R_j) psi weights

j is the index of the region

$\{\xi_i\}$ $i=0,\ldots,\ell-k-1$ is the set of new grid (R_{j+1}) psi weights

k is the number of forecasts in R_j

ℓ is the number of steps ahead from the last observation

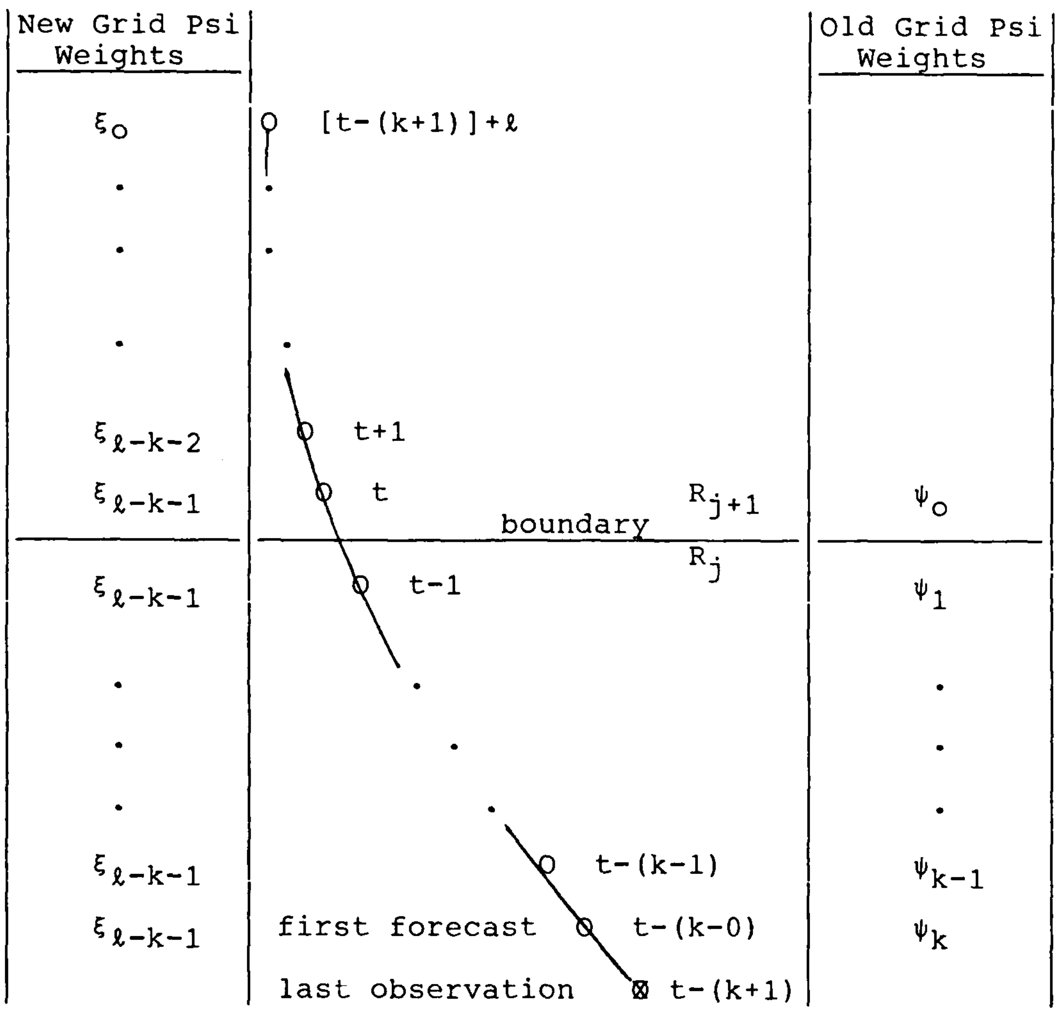

Figure 4.2 Sample Hurricane Track

$e_{t-(k+1)}(\ell,k)$ is the ℓ^{th} step ahead forecast error from time origin t-(k+1)

$\{a_t\}$ is the noise series from R_j $\qquad a_t \sim N(0,\sigma_a^2)$

$\{\eta_t\}$ is the noise series from R_{j+1} $\qquad \eta_t \sim N(0,\sigma_\eta^2)$

$$\mathrm{cov}(a_t,\eta_s) = \begin{cases} 0 & t \neq s \\ \rho\sigma_a\sigma_\eta & t = s \end{cases}$$

ρ is a correlation coefficient

LA_{t+3} is observed latitude at time t+3

$LA_{t-1}(\ell)$ is the ℓ^{th} step ahead forecast; time origin t-1

LA_t is the the first forecast value in R_{j+1}

p is the order of the autoregressive model

$\{\phi_i\}$ is the set of autoregressive parameters for R_j

$\{\Phi_i\}$ is the set of autoregressive parameters for R_{j+1}

Consider, for example, the AR(1) process where the true model is

$$LA_t = \phi_1 LA_{t-1} + a_t \quad .$$

From time origin $t-\ell$, the ℓ^{th} step ahead forecast for LA_t is

$$LA_{t-\ell}(\ell) = \phi_1 \, LA_{t-\ell}(\ell-1) \quad .$$

That is, the ℓ^{th} forecast depends on the $\ell-1^{st}$ forecast. Similarly,

$$LA_{t-\ell}(\ell-1) = \phi_1 \, LA_{t-\ell}(\ell-2)$$
$$\vdots$$
$$LA_{t-\ell}(1) = \phi_1 \, LA_{t-\ell}$$

where $LA_{t-\ell}$ is the last observed latitude. The the ℓ^{th} step ahead forecast may be represented in terms of the last observed latitude as

$$LA_{t-\ell}(\ell) = \phi_1^{\ell} LA_{t-\ell} \quad .$$

Let ℓ=k+1. Then

$$LA_{t-\ell} = LA_{t-(k+1)} \quad ,$$

the time origin in Figure 4.2. For the true model, looking forward from t-(k+1) (i.e. t-ℓ)

$$LA_{t-\ell+1} = \phi_1 LA_{t-\ell} + a_{t-\ell+1}$$

$$LA_{t-\ell+2} = \phi_1 LA_{t-\ell-1} + a_{t-\ell+2}$$

$$= \phi_1(\phi_1 LA_{t-\ell} + \phi_1 a_{t-\ell+1}) + a_{t-\ell+2}$$

$$LA_{t-\ell+3} = \phi_1 LA_{t-\ell+2} + a_{t-\ell+3}$$

$$= \phi_1\phi_1\phi_1 LA_{t-\ell} + \phi_1\phi_1 a_{t-\ell+1} + \phi_1 a_{t-\ell+2} + a_{t-\ell+3}$$

$$\vdots$$

$$LA_t = \phi_1^{\ell} LA_{t-\ell} + \phi_1^{\ell-1} a_{t-(\ell-1)} + \cdots + \phi_1 a_{t-1} + a_t \quad .$$

Then the ℓ^{th} step ahead forecast error is

$$LA_t - LA_{t-\ell}(\ell) = \phi_1^{\ell-1} a_{t-(\ell-1)} + \cdots + \phi_1 a_{t-1} + a_t$$

$$= \psi_{\ell-1} a_{t-(\ell-1)} + \cdots + \psi_1 a_{t-1} + \psi_o a_t \quad .$$

The order of the AR model does not appear in the expression of the error in terms of the psi weights. Indeed, as was shown in univariate forecasting, the psi weights are obtained by inverting the $\phi(B)$ polynomial which generates an infinite series for every order of AR model. Thus, if the forecast errors are represented in terms of the psi weights, the analysis of the forecast error applies to all orders of AR models, so it is functionally sufficient to consider the AR(1) process in developing the approach.

Referring to Figure 4.2, consider the AR(1) forecast of LA_{t+1}. The forecast is made using a "new" set of parameters from R_{j+1}. From time origin t, for the true model

$$LA_{t+1} = \Phi_1 LA_t + \eta_{t+1} \quad .$$

The one step ahead forecast of LA_{t+1} from time origin t is

$$LA_t(1) = \Phi_1 LA_t \quad .$$

For the time origin t-1 the true model is

$$LA_t = \phi_1 LA_{t-1} + a_t$$

and the one step ahead forecast from time origin t-1 is

$$LA_{t-1}(1) = \phi_1 LA_{t-1}$$

and the forecast error is

$$e_{t-1}(1,0) = LA_t - LA_{t-1}(1) = a_t \quad .$$

For two steps ahead,

$$\begin{aligned} LA_{t+1} &= \Phi_1 LA_t + \eta_{t+1} \\ &= \Phi_1 \phi_1 LA_{t-1} + \Phi_1 a_t + \eta_{t+1} \\ LA_{t-1}(2) &= \Phi_1 LA_{t-1}(1) \\ &= \Phi_1 \phi_1 LA_{t-1} \\ e_{t-1}(2,0) &= \Phi_1 a_t + \eta_{t+1} \quad . \end{aligned}$$

Similarly, for three steps ahead (time origin still t-1)

$$LA_{t+2} = \Phi_1 LA_{t+1} + \eta_{t+2}$$

$$= \Phi_1 \Phi_1 \phi_1 LA_{t+1} + \Phi_1 \Phi_1 a_t + \Phi_1 \eta_{t+1} + \eta_{t+2}$$

$$LA_{t-1}(3) = \Phi_1 LA_{t-1}(2)$$

$$= \Phi_1 \Phi_1 \phi_1 LA_{t-1}$$

$$e_{t-1}(3,0) = \Phi_1 \Phi_1 a_t + \Phi_1 \eta_{t+1} + \eta_{t+2} \quad .$$

In general, for ℓ steps ahead,

$$e_{t-1}(\ell,0) = \xi_{\ell-1} a_t + \xi_{\ell-2} \eta_{t+1} + \cdots + \xi_o \eta_{t-1+\ell} \quad .$$

For time origin t-2, and for one step ahead

$$LA_{t-1} = \phi_1 LA_{t-2} + a_{t-1}$$

$$LA_{t-2}(1) = \phi_1 LA_{t-2}$$

$$e_{t-2}(1,1) = a_{t-1} \quad .$$

For two steps ahead,

$$LA_t = \phi_1 LA_{t-1} + a_t$$

$$= \phi_1 \phi_1 LA_{t-2} + \phi_1 a_{t-1} + a_t$$

$$LA_{t-2}(2) = \phi_1 LA_{t-2}(1)$$

$$= \phi_1 \phi_1 LA_{t-2}$$

$$e_{t-2}(2,1) = \phi_1 a_{t-1} + a_t \quad .$$

For three steps ahead the new model must be used. Then

$$LA_{t+1} = \Phi_1 LA_t + \eta_{t+1}$$

$$= \Phi_1 \phi_1 \phi_1 LA_{t-2} + \Phi_1 \phi_1 a_{t-1} + \Phi_1 + \eta_{t+1}$$

$$LA_{t-2}(3) = \Phi_1 LA_{t-2}(2)$$

$$= \Phi_1 \phi_1 \phi_1 LA_{t-2}$$

$$e_{t-2}(3,1) = \Phi_1 \phi_1 a_{t-1} + \Phi_1 a_t + \eta_{t+1} \quad .$$

And finally, for four steps ahead,

$$LA_{t+2} = \Phi_1 LA_{t+1} + \eta_{t+2}$$

$$= \Phi_1\Phi_1\phi_1\phi_1 LA_{t-2} + \Phi_1\Phi_1\phi_1 a_{t-1} + \Phi_1\Phi_1 a_t + \Phi_1\eta_{t+1} + \eta_{t+2}$$

$$LA_{t-2}(4) = \Phi_1 LA_{t-2}(3)$$

$$= \Phi_1\Phi_1\phi_1\phi_1 LA_{t-2}$$

$$e_{t-2}(4,1) = \Phi_1\Phi_1\phi_1 a_{t-1} + \Phi_1\Phi_1 a_t + \Phi_1\eta_{t+1} + \eta_{t+2} \quad .$$

Three important general conclusions are evident.

(1) The ξ weight coefficients of the a's are the same within a particular forecast.

(2) The ψ weight coefficients of the a's are the same for all steps ahead. In other words, the coefficients of the a's depend on k, the number of <u>forecasts</u> that fall in R_j.

(3) The ξ weight coefficients of the η's are the usual psi weights for a "within" grid forecast.

Thus, by a two term induction, (ℓ and k), if the forecast origin is t-(k+1) and the first forecast in R_{j+1} is LA_t, then

$$e_{t-(k+1)}(\ell,k) = \xi_{\ell-k-1}[\psi_k a_{t-k} + \psi_{k-1} a_{t-k+1} + \cdots + \psi_o a_t] + \xi_{\ell-k-2}\eta_{t+1} + \xi_{\ell-k-3}\eta_{t+2} + \cdots + \xi_o \eta_{t-(k+1)+\ell} \quad . \quad (4.36)$$

Equation (4.36) gives the ℓ^{th} step ahead forecast error for the univariate threshold model when k forecasts fall in R_j, $\ell \geq k+1$. $a_i \sim N(0,\sigma_a^2)$ and $\eta_i \sim N(o,\sigma_\eta^2)$. So, when n is sufficiently large, $e_{t-(k+1)}(\ell,k)$ is Normally distributed. The mean of the distribution is

$$E[e_{t-(k+1)}(\ell,k)]=\xi_{\ell-k-1}\{\psi_k E[a_{t-k}]+\psi_{k-1}E[a_{t+k+1}]+\cdots$$
$$+\psi_o E[a_t]\} + \xi_{\ell-k-2}E[n_{t+1}]+\cdots+\xi_o E[n_{t-(k+1)+\ell}]$$
$$= 0 \quad .$$

Within a grid, the noise series does not co-vary with the lagged values of the same series. Also, it is assumed that different noise series are uncorrelated at different time indexes. Therefore, since each of the noise terms in (4.36) has a different time index, there is no covariance between terms. Consequently,

$$E[e_{t-(k+1)}{}^2(\ell,k)]=\xi^2{}_{\ell-k-1}(\psi_k{}^2+\psi_{k-1}{}^2+\cdots\psi_o{}^2)\sigma_a{}^2$$
$$+(\xi_{\ell-k-2}{}^2+\xi_{\ell-k-3}{}^2+\cdots+\xi_o{}^2)\sigma\eta^2 \qquad (4.37)$$

is the variance of the error distribution.

Equation (4.36) is easily generalized for multiple threshold crossings. At each threshold, all previous psi weights are multiplied by the psi weight of the new model. The error variance in (4.37) is merely increased to include the psi weights of the new grid.

Threshold Interval Forecast Example (Latitude)

The following position series is from hurricane Frederic (1979): {(12.0,45.1) (12.5,47.0) (12.9,48.7) (13.3,50.4) (13.8,52.3) (14.3,54.1) (14.9,55.5) (15.5,57.2) (16.3,58.8) (16.7,59.8) (17.1,60.8)

(17.5,61.8) (17.8,62.8) (18.0,63.8) (18.1,64.8)
(18.1,65.8) (18.1,66.8)} .

Suppose 13.8 is the last observed latitude. Then forecasting with a lead time of 72 hours via the appropriate model yields the following forecasts:

{(14.2,54.1) (14.6,55.9) (15.1,57.6) (15.4,59.1)
(15.8,60.6) (16.2,62.1) (16.6,63.5) (17.1,64.8)
(17.5,66.1) (18.0,67.4) (18.4,68.6) (18.8,69.8)} .

For latitude, the 48 hour forecast of 17.1 has estimated variance

$$V[e_5(8,2)] = \xi_5^2[\psi_2^2+\psi_1^2+\psi_o^2]\sigma_a^2+(\xi_4^2+\cdots+\xi_o^2)\sigma_\eta^2 \quad .$$

For the grids 10-15N and 15-20N the variance estimates are

$$\hat{\sigma}_a^2 = (8.7)^2 = 75.7 \ , \ \hat{\sigma}_\eta^2 = (11.2)^2 = 125.4 \quad .$$

From the univariate forecasting confidence interval example $\hat{\psi}_o=1$, $\hat{\psi}_1=1.696$, $\hat{\psi}_2=2.179$. For the new model

$$LA_t=1.766LA_{t-1}-.776LA_{t-2}-.106LA_{t-4}+.106LA_{t-5}$$
$$-.010LO_{t-1}+.010LO_{t-2}+.073LO_{t-4}-.073LO_{t-5}.$$

For this example ignore the longitude coefficients. This yields

$$\xi_o=1, \ \xi_1=1.766, \ \xi_2=1.766(\xi_1)+(-.766)(\xi_o)=2.353.$$

Similarly,

$$\hat{\xi}_3=2.813,\ \hat{\xi}_4=3.059,\ \text{and}\ \hat{\xi}_5=3.166 \quad .$$

Then the estimated eight step ahead error variance and standard deviation are

$$V[e_5(8,2)]=10.023(8.624)75.7+(26.926)125.4=9919.904$$

$$SD[e_5(8,2)]=99.6 \text{ n mi} \quad .$$

<u>Threshold Interval Forecast of Two Variables</u>

Definitions

$t-(k+1)$ is the time index of the last observation

$\begin{bmatrix} \psi_{11,i} & \psi_{12,i} \\ \psi_{21,i} & \psi_{22,i} \end{bmatrix}$ $i=0,1,\ldots,k$ is the i^{th} set of psi weights for the old region R_j

Subscript (11,i) is latitude predicting latitude at lag i
Subscript (12,i) is longitude predicting latitude
Subscript (21,i) is latitude predicting longitude
Subscript (22,i) is longitude predicting longitude

j is the index of the region

$\begin{bmatrix} \xi_{11,i} & \xi_{12,i} \\ \xi_{21,i} & \xi_{22,i} \end{bmatrix}$ $i=0,1,\ldots,\ell-k-1$ is the i^{th} set of psi weights for the new region R_{j+1}

k is the number of forecasts in R_j

ℓ is the number of steps ahead from the last observation

$e_{1,t-(k+1)}(\ell,k)$ is the ℓ^{th} step ahead forecast error for latitude from time origin $t-(k+1)$

$e_{2,t-(k+1)}(\ell,k)$ is the ℓ^{th} step ahead forecast error for longitude from time origin $t-(k+1)$

a_{1t} $t=0,1,\ldots,k$ is the latitude noise series from R_j

a_{2t} $t=0,1,\ldots,k$ is the longitude noise series from R_j

$a_{it} \sim N(0,\sigma_{ai}^2)$ $i=1,2$

η_{1t} $t=k+1,\ldots,n$ is the latitude noise series from R_{j+1}

η_{2t} $t=k+1,\ldots,n$ is the longitude noise series from R_{j+1}

$\eta_{it} \sim N(0,\sigma_{\eta i}^2)$ $i=1,2$

$$\text{cov}(a_{it},\eta_{js}) = \begin{cases} 0 & t \neq s \\ \rho\sigma_{ai}\sigma_{\eta j} & t = s \end{cases}$$

$\begin{bmatrix} LA_{t+3} \\ LO_{t+3} \end{bmatrix}$ is observed latitude and longitude at time t+3

$\begin{bmatrix} LA_{t-1}(\ell) \\ LO_{t-1}(\ell) \end{bmatrix}$ is the ℓ^{th} step ahead forecast from time t-1

$\begin{bmatrix} LA_t \\ LO_t \end{bmatrix}$ is the first forecast to fall in R_{j+1}

$\begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$ are the lag 1 AR coefficients for R_j

$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{21} \end{bmatrix}$ are the lag 1 AR coefficients for R_{j+1}

Consider the AR(1) bivariate process where

$$LA_t = \phi_{11}LA_{t-1} + \phi_{12}LO_{t-1} + a_{1t}$$
$$LO_t = \phi_{21}LA_{t-1} + \phi_{22}LO_{t-1} + a_{2t} \quad .$$

This may be written in matrix form such that

$$\begin{bmatrix} LA_t \\ LO_t \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} LA_{t-1} \\ LO_{t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} .$$

Referring to figure 4.2, consider the bivariate forecast of (LA_{t+1}, LO_{t+1}). The forecast is made using a "new" set of parameters from R_{j+1}. From time origin t, for the true model

$$\begin{bmatrix} LA_{t+1} \\ LO_{t+1} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} LA_t \\ LO_t \end{bmatrix} + \begin{bmatrix} \eta_{1t+1} \\ \eta_{2t+1} \end{bmatrix} .$$

The one step ahead forecast is

$$\begin{bmatrix} LA_t(1) \\ LO_t(1) \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} LA_t \\ LO_t \end{bmatrix} .$$

For a time origin of t-1, for the true model

$$\begin{bmatrix} LA_t \\ LO_t \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} LA_{t-1} \\ LO_{t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} ,$$

the one step ahead forecast is

$$\begin{bmatrix} LA_{t-1}(1) \\ LO_{t-1}(1) \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} LA_{t-1} \\ LO_{t-1} \end{bmatrix} ,$$

and the forecast error is

$$\begin{bmatrix} e_{1t-1}(1,0) \\ e_{2t-1}(1,0) \end{bmatrix} = \begin{bmatrix} LA_t \\ LO_t \end{bmatrix} - \begin{bmatrix} LA_{t-1}(1) \\ LO_{t-1}(1) \end{bmatrix} = \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} .$$

For two steps ahead,

$$\begin{bmatrix} LA_{t+1} \\ LO_{t+1} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} LA_t \\ LO_t \end{bmatrix} + \begin{bmatrix} \eta_{1t+1} \\ \eta_{2t+1} \end{bmatrix}$$

AD-A170 742 TIME SERIES PREDICTION OF HURRICANE LANDFALL(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH T F CURRY MAY 86 AFIT/CI/NR-86-75D 2/2

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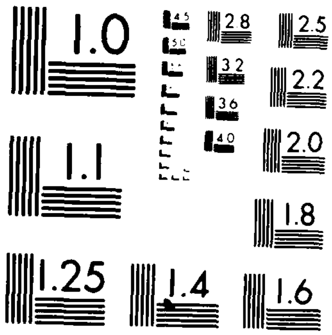

MICROCOPY RESOLUTION TEST CHART

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$$= \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \left\{ \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} LA_{t-1} \\ LO_{t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} \right\} + \begin{bmatrix} \eta_{1t+1} \\ \eta_{2t+1} \end{bmatrix} .$$

Then the forecast error is

$$\begin{bmatrix} e_{1t-1}(2,0) \\ e_{2t-1}(2,0) \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} + \begin{bmatrix} \eta_{1t+1} \\ \eta_{2t+1} \end{bmatrix} .$$

For time origin t-2, for one step ahead

$$\begin{bmatrix} LA_{t-1} \\ LO_{t-1} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} LA_{t-2} \\ LO_{t-2} \end{bmatrix} + \begin{bmatrix} a_{1t-1} \\ a_{2t-1} \end{bmatrix}$$

and the forecast error is

$$\begin{bmatrix} e_{1t-2}(1,1) \\ e_{2t-2}(1,1) \end{bmatrix} = \begin{bmatrix} a_{1t-1} \\ a_{2t-1} \end{bmatrix} .$$

For two steps ahead (time origin still t-2)

$$\begin{bmatrix} LA_{t} \\ LO_{t} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} LA_{t-1} \\ LO_{t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix}$$

$$= \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} LA_{t-2} \\ LO_{t-2} \end{bmatrix} + \begin{bmatrix} a_{1t-1} \\ a_{2t-2} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix}$$

and the forecast error is

$$\begin{bmatrix} e_{1t-2}(2,1) \\ e_{2t-2}(2,1) \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} a_{1t-1} \\ a_{2t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} .$$

For three steps ahead, the new model should be used. Then

$$\begin{bmatrix} LA_{t+1} \\ LO_{t+1} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} LA_{t} \\ LO_{t} \end{bmatrix} + \begin{bmatrix} \eta_{1t+1} \\ \eta_{2t+1} \end{bmatrix}$$

$$= \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} LA_{t-2} \\ LO_{t-2} \end{bmatrix}$$

$$+ \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} a_{1t-1} \\ a_{2t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix}$$

$$+ \begin{bmatrix} \eta_{1t+1} \\ \eta_{2t+1} \end{bmatrix} .$$

These results are the multivariate analogy identical to the univariate case, except they are in matrix form. Thus, the conclusions are similar. If the forecast origin is t-(k+1) and the first forecast in R_{j+1} is (LA_t, LO_t), then the ℓ^{th} step ahead forecast error is given by

$$\begin{bmatrix} e_{1t-(k+1)}(\ell,k) \\ e_{2t-(k+1)}(\ell,k) \end{bmatrix} = \begin{bmatrix} \xi_{11,\ell-k-1} & \xi_{12,\ell-k-1} \\ \xi_{21,\ell-k-1} & \xi_{12,\ell-k-1} \end{bmatrix}$$

$$\cdot \left\{ \begin{bmatrix} \psi_{11,k} & \psi_{12,k} \\ \psi_{21,k} & \psi_{22,k} \end{bmatrix} \begin{bmatrix} a_{1t-k} \\ a_{2t-k} \end{bmatrix} + \cdots + \begin{bmatrix} \psi_{11,o} & \psi_{12,o} \\ \psi_{21,o} & \psi_{22,o} \end{bmatrix} \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} \right\}$$

$$+ \begin{bmatrix} \xi_{11,\ell-k-2} & \xi_{12,\ell-k-2} \\ \xi_{21,\ell-k-2} & \xi_{22,\ell-k-2} \end{bmatrix} \begin{bmatrix} \eta_{1t+1} \\ \eta_{2t+1} \end{bmatrix} + \cdots$$

$$+ \begin{bmatrix} \xi_{11,o} & \xi_{12,o} \\ \xi_{21,o} & \xi_{22,o} \end{bmatrix} \begin{bmatrix} \eta_{1t-(k+1)+\ell} \\ \eta_{2t-(k+1)+\ell} \end{bmatrix} . \qquad (4.38)$$

The psi weights for (4.38) are the values calculated from (4.29) and (4.30). As in the univariate case, the individual forecast errors are a linear combination of independent Normally distributed noise terms, so they are Normally distributed, and the distribution of the bivariate threshold forecast is bivariate Normal. The covariance matrix for the ℓ^{th} step ahead forecast can now be calculated. From equation (4.38),

$$\begin{aligned} e_{1t-(k+1)}(\ell,k) = & (\xi_{11,\ell-k-1}\psi_{11,k}+\xi_{12,\ell-k-1}\psi_{21,k})a_{1t-k} \\ & +(\xi_{11,\ell-k-1}\psi_{12,k}+\xi_{12,\ell-k-1}\psi_{22,k})a_{2t-k} \\ & + \cdots \\ & +(\xi_{11,\ell-k-1}\psi_{11,o}+\xi_{12,\ell-k-1}\psi_{21,o})a_{1t} \\ & +(\xi_{11,\ell-k-1}\psi_{12,o}+\xi_{12,\ell-k-1}\psi_{22,o})a_{2t} \\ & +(\xi_{11,\ell-k-2}\eta_{1t+1}+\xi_{12,\ell-k-2}\eta_{2t+1})+\cdots \\ & +(\xi_{11,o}\eta_{1t-(k+1)+\ell}+\xi_{12,o}\eta_{2t-(k+1)+\ell}) \quad . \end{aligned}$$

For notational convenience, this can be rewritten as

$$\begin{aligned} & e_{1t-(k+1)}(\ell,k)=c_{1t-k}a_{1t-k}+c_{2t-k}a_{2t-k}+\cdots+c_{1t}a_{1t}+c_{2t}a_{2t} \\ & +c_{1t+1}\eta_{1t+1}+c_{2t+1}\eta_{2t+1}+\cdots+c_{1t-(k+1)+\ell}\eta_{1t-(k+1)+\ell} \\ & +c_{2t-(k+1)+\ell}\eta_{2t-(k+1)+\ell} \quad . \end{aligned}$$

Then,

$$\begin{aligned} & E[e_{1t-(k+1)}{}^2(\ell,k)]=(c_{1t-k}{}^2+\cdots+c_{1t}{}^2)\sigma_{a1}{}^2 \\ & +(c_{2t-k}{}^2+\cdots+c_{2t}{}^2)\sigma_{a2}{}^2 \\ & +2(c_{1t-k}c_{2t-k}+\cdots+c_{1t}c_{2t})\mathrm{cov}(a_{1t}a_{2t}) \\ & +(c_{1t+1}{}^2+\cdots+c_{1t-(k+1)+\ell}{}^2)\sigma_{\eta 1}{}^2 \\ & +(c_{2t+1}{}^2+\cdots+c_{2t-(k+1)+\ell}{}^2)\sigma_{\eta 2}{}^2 \\ & +2(c_{1t+1}c_{2t+1}+\cdots+c_{1t-(k+1)+\ell}c_{2t-(k+1)+\ell})\mathrm{cov}(\eta_{1t}\eta_{2t}) . \quad (4.39) \end{aligned}$$

Similarly,

$$e_{2t-(k+1)}(\ell,k)=(\xi_{21,\ell-k-1}\xi_{11,k}+\xi_{22,\ell-k-1}\xi_{21,k})a_{1t-k}$$
$$+(\xi_{21,\ell-k-1}\xi_{12,k}+\xi_{22,\ell-k-1}\xi_{22,k})a_{2t-k}$$
$$+\cdots$$
$$+(\xi_{21,\ell-k-1}\xi_{11,o}+\xi_{22,\ell-k-1}\xi_{21,o})a_{1t}$$
$$+(\xi_{21,\ell-k-1}\xi_{12,o}+\xi_{22,\ell-k-1}\xi_{22,o})a_{2t}$$
$$+(\xi_{21,\ell-k-1}\xi_{1t+1}+\xi_{22,\ell-k-2}\xi_{2t+1})+\cdots$$
$$+(\xi_{21,o}\eta_{1t-(k+1)+\ell}+\xi_{22,o}\eta_{2t-(k+1)+\ell}),$$

and this may be rewritten as

$$E[e_{2t-(k+1)}{}^2(\ell,k)]=(d_{1t-k}{}^2+\cdots+d_{1t}{}^2)\sigma_{a1}{}^2$$
$$+(d_{2t-k}{}^2+\cdots+d_{2t}{}^2)\ {}_{a2}{}^2\quad\sigma$$
$$+2(d_{1t-k}d_{2t-k}+\cdots+d_{1t}d_{2t})\mathrm{cov}[a_{1t}a_{2t}]$$
$$+(d_{1t+1}{}^2+\cdots+d_{1t-(k+1)+\ell}{}^2)\sigma_{\eta 1}{}^2$$
$$+(d_{2t+1}{}^2+\cdots+d_{2t-(k+1)+\ell}{}^2)\sigma_{\eta 2}{}^2$$
$$+2(d_{1t+1}d_{2t+1}+\cdots$$
$$+d_{1t-(k+1)+\ell}d_{2t-(k+1)+\ell})\mathrm{cov}[\eta_{1t}\eta_{2t}]; \qquad (4.40)$$

so,

$$E[e_{1t-(k+1)}(\ell,k)e_{2t-(k+1)}(\ell,k)]$$
$$=(c_{1t-k}d_{1t-k}+\cdots+c_{at}d_{1t})\sigma_{a1}{}^2$$
$$+\{(c_{1t-k}d_{2t-k}+\cdots+c_{1t}d_{2t})$$
$$+(c_{2t-k}d_{1t-k}+\cdots+c_{2t}d_{1t})\}\mathrm{cov}[a_{1t}a_{2t}]$$
$$+(c_{2t-k}d_{2t-k}+\cdots+c_{2t}d_{2t})\sigma_{a2}{}^2$$
$$+(c_{1t+1}d_{1t+1}+\cdots+c_{1t-(k+1)+\ell}d_{1t-(k+1)+\ell})\sigma_{\eta 1}{}^2$$

$$+[(c_{1t+1}d_{2t+1}+\cdots+c_{1t-(k+1)+\ell}d_{2t-(k+1)+\ell})$$

$$+(c_{2t+1}d_{1t+1}+\cdots+c_{2t-(k+1)+\ell}d_{1t-(k+1)+\ell})]\mathrm{cov}[\eta_1\eta_2]$$

$$+(c_{2t+1}d_{2t+1}+\cdots+c_{2t-(k+1)+\ell}d_{2t-(k+1)+\ell})\sigma_{\eta 2}^2 \quad . (4.41)$$

Although these calculations are tedious, it may be noted that equations (4.39) through (4.41) are the Kronecker product of the error terms defined by (4.38) where the Kronecker product is

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \otimes \begin{bmatrix} e_1 & e_2 \end{bmatrix} = \begin{bmatrix} e_1^2 & e_1e_2 \\ e_1e_2 & e_2^2 \end{bmatrix} . \quad (4.42)$$

The expected value of (4.42) is the covariance matrix of the forecast of interest.

The case of multiple threshold crossings is easily handled. The first psi weight matrix of the new grid is used to multiply all previous psi weight matrices. Thus, the constants in the 2 by 2 matrices of equation (4.38) change, and an additional sum is needed to account for the new grid. For example, in the case of two threshold crossings (the most that would normally be expected in 72 hours) where there are k_1, k_2, and k_3 forecasts in the first, second, and third grid respectively ($k_1+k_2+k_3=\ell$), ψ, ξ, and β are 2 by 2 psi weight matrices, and a, η, and b represent the applicable noise terms, a notationally simplified version of (4.38) is given by

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} \beta_{k_3-1} \end{bmatrix} \begin{bmatrix} \xi_{k_2} \end{bmatrix} \left\{ \begin{bmatrix} \psi_{k_1} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \cdots + \begin{bmatrix} \psi_o \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \right\}$$
$$+ \begin{bmatrix} \beta_{k_3-1} \end{bmatrix} \left\{ \begin{bmatrix} \xi_{k_2-1} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \cdots + \begin{bmatrix} \xi_o \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \right\}$$
$$+ \begin{bmatrix} \beta_{k_3-2} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \cdots + \begin{bmatrix} \beta_o \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} .$$

Equations (4.38) - (4.41) can be used to determine the confidence ellipse around a forecast that crosses a grid boundary. The equations are the theoretical contribution of this research.

Bivariate Threshold Interval Forecast Example

The same hurricane track is used in this example (hurricane Frederic 1979). It should be noted that the correlation and standard deviations are based on all storms rather than only category five storms. This increases the standard deviations used in the confidence ellipse by approximately 40 percent due to the inclusion of accelerating storms. The increase affords additional protection when forecasting storms that are not category five.

Details of the following calculations used to compute the 90% confidence ellipse are in Appendix C. Psi weight matrices and covariance matrices for the one step ahead forecast for each grid are in Appendix B. For this example, Frederic is forecast from origin (13.8,52.3) which is the fifth report of the series (time origin 5). Two forecasts fall in the old region from 10-15N (k=2). The following procedure is used to define the 90% confidence ellipse centered at the eight step ahead forecast:

(1) Compute the point forecast using the models in Appendix A. Here the point forecast is (17.1,64.8). Note the number of forecasts that fall in the old region (k=2).

(2) Write the models in $\phi(B)$ polynomial form (see Appendix C).

(3) Compute the psi weight matrices for each region the storm crosses up through the eleventh term. Eleven matrices (12 including ψ_0) are needed to compute the variance of the 12 step ahead (72 hour) forecast.

(4) Let Σ_a be the covariance matrix of the one step ahead forecast errors for the old region. Σ_η is the covariance matrix of the one step ahead forecast errors for the new region. Form the Kronecker product of (4.38) as shown in (4.39) through (4.41) and Appendix C. This yields Σ_8, the estimated covariance matrix of the eight step ahead forecast.

(5) Compute the eigenvalues λ_1, and λ_2 (λ_2 is the largest) and eigenvectors for Σ_8. Compute the

estimated coefficient of correlation (ρ) for Σ_8.

(6) Then the approximate 90% confidence ellipse for this example is

$$(x_1/168.6)^2 - 2(.076)(x_1/168.6)(x_2/110.9) + (x_2/110.9)^2 = 1.645(1-(.076)^2)$$

where the minor axis is oriented on a magnetic heading of 4.99 degrees and is centered at the point forecast.

CHAPTER 5

FORECASTING RESULTS

The results of applying the models in Appendix A to historical hurricane tracks are presented in this chapter. The analysis that led to the use of velocity-stationary (category 5) models is discussed. Next, the actual empirical models are presented and compared with NHC official forecasts, and forecasts for storms from 1945 through 1983. Then the results for 1985 storms are discussed.

Predicting the Stationarity Category

Transition matrices were investigated as a means of forecasting the correct future stationarity category. A window of seven previous six hour observations from each track was used to develop the matrices. Segments of longer length missed short term accelerations, and segments of shorter length resulted in large variances in the parameter estimates. At each new position, the stationarity category was determined. For example, at the seventh position report, the stationarity category was based on positions one through seven. At the eighth, the stationarity category was based on positions two through eight, etc.. In this manner a storm category (direction of movement) was associated with the terminal point in each "window". Numerical results of this analysis are presented in Appendix E.

A transition matrix was then constructed based on a previous category. For example, regardless of which category the storm was in at position seven, it was most frequently in category five at later positions. This implied that any acceleration of the storm was short lived. Consequently, for six hour data a good guess of the future category of any hurricane would be latitude-velocity, longitude-velocity stationary.

The Forecast Models

Forecasts were made using the category five model in each latitude band. The model is bivariate AR(4) in velocity where latitude and longitude are functions of each other at lags one and four. The model uses five previous positions (differenced once to obtain four velocities), and thus adapts itself to the changing motion of the storm.

There are definite relationships between the model structure and known meteorological phenomena associated with hurricanes. Specifically, segmenting by latitude band relates to large scale climatology differences observed as the distance from the equator increases. The lag one parameter captures persistence, and the lag four parameter models the diurnal effect of the sun (the slowing of the storm at night).

Probably the most significant result is that the forecast parameters are based only on velocity-stationary hurricanes. It is not valid to use accelerating storms in a velocity model. Admitting such storms to the data base, when forecasting the next change in position, biases the parameter estimates and increases the variance of the forecast error.

The forecast models for the five degree latitude bands from 10 degrees north latitude to 45 degrees north latitude are listed in Appendix A. Three digits after the decimal are reported (even when insignificant) so that the form of the models remains the same in each band. A parameter standard deviation is reported for the lag one parameter value even though it was derived directly from the lag two value (by adding the constant one). The lag one parameter value increases as the storm moves north and remains fairly constant above 25 degrees north latitude. Also, latitude seems to be a better predictor of longitude as opposed to predicting via the reverse relationship. In general the lag four relationships are weak, but they are significant for some regions.

To use the models, the most recent position report becomes (LA_{t-1}, LO_{t-1}), and the forecast (LA_t, LO_t) is calculated for a six hour leadtime. Twelve to 72 hour forecasts are then obtained by treating (LA_t, LO_t) as the last true position and then forecasting another 6 hour increment. When LA_t moves into a new latitude band the model associated with that latitude band should be used.

Great Circle Distance

A popular measure of forecast accuracy, and the measure used in this dissertation, is the great circle distance between the actual and the forecast position of the eye of the hurricane (Pike, 1985). Let (LAF,LOF) be the latitude and longitude of the forecast position, and let (LA,LO) be the actual position. Then the great circle distance in nautical miles between the two points is

$$60 \cos^{-1}[\sin(LAF)\sin(LA)+\cos(LAF)\cos(LA)\cos(LO-LOF)] \quad .$$

Forecast Results

Overall forecast errors for all storms from 1945 through 1983 in each latitude band were computed using great circle distance (Table 5.1). Sample standard deviations (n mi) and sizes are reported below each mean error. Similarly to Neumann and Pelissier (1981), the average error was computed for the southern (10-25N) and the northern (25-45N) region and then summarized (Table 5.2). Results are compared with the NHC average errors. The

TABLE 5.1

TIME SERIES BEST TRACK MEAN FORECAST ERRORS (N MI(KM))

Region	Forecast interval (hours) 24	48	72
10-15N 45-83W	57.9(107.3) (35.1,190)	131.9(244.4) (75.7,103)	204.8(379.5) (118.1,50)
15-20N 45-87W	70.6(130.8) (47.6,721)	154.6(286.5) (109.4,403)	253.0(468.8) (160.0,221)
20-25N 45-100W	76.6(142.0) (49.9,956)	165.6(306.9) (96.4,519)	260.1(482.0) (135.2,279)
25-30N 45-100W	96.2(178.3) (66.6,889)	203.2(376.6) (131.6,448)	293.9(544.7) (163.7,223)
30-35N 45-80W	107.7(199.6) (67.2,598)	242.4(449.2) (138.0,365)	358.6(664.5) (188.9,230)
35-40N 45-76W	121.0(224.2) (74.3,344)	267.5(495.7) (132.8,176)	391.8(726.0) (199.5,79)
40-45N 45-70W	138.7(257.0) (79.7,106)	269.8(500.0) (178.1,47)	206.3(382.3) (203.9,21)

time series storm tracks are not the same as those used by Neumann and Pelissier because the time series sample is larger. However, time series analysis of 1973-1979 storms in the southern region (a subset of the 1945-1983 data base) produced deviations of less than 10 n mi from the reported average errors for the entire sample (Table 5.2). Thus, comparisons should be fairly accurate. Assuming equivalent NHC standard deviations and sample sizes, the time series (TS) errors (Table 5.2) are significantly less (95% confidence) than the NHC errors at 48 and 72 hours.

It should be noted that TS results are based on best track data instead of operational position reports. Best tracks are constructed in a careful post-storm analysis that combines position data from all available sources. Some subjective smoothing is then employed to plot the best track. Neumann observed that the average error for real time position reports is approximately

TABLE 5.2

COMPARISON OF MEAN FORECAST ERRORS [N MI(KM)]

Group	Source	Forecast interval (hours) 24	48	72
1. Entire sample	NHC	110.1(204)	244.0(452)	362.0(671)
	TS	109.8(203)	214.6(398)	312.0(578)
2. Northern region	NHC	131.3(243)	304.4(564)	421.0(780)
	TS	126.5(234)	251.0(465)	351.5(651)
3. Southern region	NHC	84.5(157)	179.2(332)	317.3(588)
	TS	92.4(171)	177.9(330)	272.2(504)

20 n mi (Barney, 1983). In order to approximate true error, the time series errors listed in Table 5.2 include an additional 20 n mi error over the actual value obtained using the model.

To better compare the procedures, the five storms in Table 2.1 were forecast using operational position reports. The average forecast errors were 94.8(176), 163.1(302), and 227.4(421) n mi (km) as compared to 77.3(143), 179.4(332), and 317.3(588) for the NHC. The differences are significant at the 95% confidence level at 24 and 72 hours. This analysis also confimed that 20 n mi is an appropriate amount to add to the best track forecasts to account for operational position error.

In 1985 there were eleven hurricanes and tropical storms in the North Atlantic region. The National Weather Service office in Austin issued position reports for eight of them. Two of the eight storms were at sea for less than five position reports and could not be forecast by the time series model. The remaining 6 storms yielded 55 24 hour forecasts, 31 48 hour forecasts, and 10 72 hour forecasts. The average forecast errors for the time series model were 118, 273, and 241 n mi respectively, indicating that the storms were not category five. Indeed, looping and accelerating storms caused the NHC to alert wide areas of the Gulf coast where four of the storms came ashore. Forecast accuracies for the NHC will not be available until late in 1986.

It is evident that the time series approach is valuable in reducing forecast error especially for the long range forecast. The fact that it rivals the official forecast accuracy of the National Hurricane Center is significant. This statement is made in consideration of the fact that the time series model has used only latitude and longitude as predictor variables, while the NHC has available a plethora of predictive tools. Incorporation of exogenous variables into the time series model, and various extensions discussed in Chapter 6, can only be expected to improve the forecast accuracies.

CHAPTER 6

SUMMARY AND CONCLUSIONS

Both the theoretical and application oriented results of Chapters 4 and 5 are summarized in this chapter, as they relate to hurricane forecasting and threshold autoregressive time series models. Topics for future research are also discussed.

Hurricane Modeling Results

The original objectives of this research were: (1) to predict hurricane movement based on time series of observations of latitude and londitude taken at six hour intervals, (2) to determine the confidence intervals surrounding the forecasts, and (3) to use a model whose coefficients could change as the storm moved. The models were to be categorized by time of year and region of the ocean in which the storm was located.

The first problem encountered in the research involved construction of the data files. Segmenting the hurricane tracks by region resulted in several short bivariate time series, each containing approximately 10-14 observations. These tracks could not be directly concatenated, because the last observation of one storm would have been used to predict the first observation of the next storm which was intuitively incorrect. The lagged data needed to be "storm unique." The commercially available time series computer programs were unable to lag the data from the segmented tracks in the required manner. A FORTRAN program was written to lag the data six time periods, and SPSS was used to compute regression coefficients.

The initial forecasting model (in which the position forecasts were based on past positions) was nonstationary. The latitude and longitude series were differenced and the coefficients recomputed. Two findings were immediately apparent. The level of differencing of the latitude and/or longitude series required to satisfy stationarity resulted in nine possible stationarity categories for the storm. These categories were related to direction of movement (Table 4.1). In addition, parameters significantly different from zero occurred at lags one and four. It was believed that these parameters related to known meteorological phenomena associated with hurricanes. Specifically, the lag one parameter captured persistence, and the lag four parameter modeled the diurnal effect of the sun (the slowing of the storm at night).

For storms that were velocity-stationary (the latitude and longitude series were differenced once) the last change in storm position was the best predictor of the next change in storm position regardless of location or time of year. Segmenting data by month increased forecast accuracy for some months, but resulted in an overall decrease in average forecast accuracy. Time of year was discarded as a predictor variable. By trial and error it was determined that segmenting the North Atlantic by latitude bands eight degrees in width with a three degree overlap resulted in the smallest forecast errors. This led to a "threshold" approach in which parameter values were allowed to change when the storm crossed into a new region. There were nine models for each region depending on the stationarity category of the storm. Forecast errors based on the velocity-stationary models were small.

In order to increase forecast accuracy, empirical transition matrices were computed in a Markov chain approach to modeling the transition of storms between categories (Appendix E). It was determined that future storm categories were usually category five (velocity-stationary) regardless of the current category. This implied that storm accelerations were short lived. Thus, a velocity model, especially one based on the last change in position, was expected to accurately forecast all categories of storms, at least for 6-hour position report data.

For the time series model the average error for the 24 hour forecast is 110 n mi. The official forecasts of the NHC have the same average error. At a 72 hour lead time, the average error of the time series model is 312 n mi as compared to 362 n mi for the NHC. The accurate long range forecast is probably due to two important characteristics of the time series model. For a stationary model, the "importance" of the last observed velocity decays exponentially as the forecast lead time increases. Thus, if the last velocity observation is inaccurate, its effect diminishes rapidly. This rapid decay causes the long range forecasts to be based primarily on the mean of the latitude and longitude series, which is the average motion vector of past storms in the region. This suggests that, on the average, in particular regions of the ocean, hurricanes move in the same direction.

Future Hurricane Modeling

In this study, the velocity-stationary model is used to predict movement of all hurricanes. This is a very "broad brush" approach. Although category five occurs most frequently, there are eight other categories into which a storm can be classified. The typical storm spends only 30-40 percent of its life as a category five storm. The rest of the time the storm is accelerating.

The key to more accurate movement prediction is the ability to forecast these accelerations. This is an extremely difficult problem whose solution has always eluded hurricane scientists. Predictors such as surrounding pressure fields, maximum wind velocity, upper level steering winds, and central pressure seem to be useful for some storms, but not for others. It is possible that certain of these exogenous variables are good leading indicators only for storms in a particular category.

It is also possible to make better use of the category transition matrices. There are nine models for each grid. Another forecasting approach would be to examine the percentage of storms that (say, beginning in category five) are in each of the categories 48 hours later. 48 hour forecasts could then be calculated by combining forecasts from each of the nine models, weighted by the applicable percentage of storms that historically transitioned to that category.

Satellite photographs taken at 30 minute intervals are available. These photographs could be processed to yield position reports. The use of more frequent observations might increase the significance of the parameter estimates corresponding to the 24 hour lag and/or introduce a moving average parameter.

In this study the data were only lagged up to six periods. Coefficients should be computed at longer lags to determine if there are other significant predictor variables. Caution must be exercised when lagging the data depending on the size of the grid containing the storm track. In the case of latitude bands five degrees in width, storm tracks with many observations are usually associated with storms moving in a westerly direction. Thus, the variables beyond lag 10 (or so) are not from a representative sample of hurricanes.

Theoretical Results

The bivariate threshold autoregressive model used in this research represents a piecewise linearization of the hurricane movement process. It is slightly different from the TARSO model developed by Tong and Lim (1980) because allows a contemporaneous covariance structure between latitude and longitude.

In order to compute forecast confidence intervals, it was necessary to determine the bivariate distribution of the forecast errors. A psi-weight representation of the univariate and bivariate threshold autoregressive process was used to show that the distribution of the forcast errors was bivariate Normal. This allowed the point forecast to be bounded with an approximate confidence ellipse.

In addition, it was discovered that when a forecast sequence contained a threshold crossing, the confidence interval calculation required that the psi-weight associated with the first forecast in the new region be multiplied by all the previous psi-weights. This resulted in an increase in the size of the confidence region that accounted for the change in model parameter values as the threshold was crossed. This adjustment was shown to be necessary in the univariate and bivariate cases.

Future Theoretical Development

The useful theoretical extensions are based on the possible improvements that might be made to the forecasting model. If it is possible to forecast the hurricanes by using a weighted combination of models, the confidence interval calculations will have to be changed accordingly. If more frequent observations introduce a moving average component, it would be a theoretical extension to derive the confidence interval for the threshold ARMA model. This extension is probably most worthwhile, and might also be easily accomplished using a psi-weight representation of the forecast. Finally, a prediction interval that accounts for the variation of the model coefficients could be constructed. This is a difficult problem if the psi-weight representation is used because model coefficients are the product of several Normally distributed random variables.

Overall Evaluation

The model developed during this study is important because it groups the storms based on the stationarity category. The nine categories are a direct link between the theoretical requirement of stationarity and the physical movement of the storm. This relationship means that it is possible to describe statistically the physical process that causes a hurricane or tropical storm to stand still, move at constant velocity, or accelerate. It is believed that application of known meteorological predictor variables to the different categories would result in the ability to forecast the stationarity category. This, in turn, would result in further significant decreases in forecast error.

A P P E N D I X A

FORECAST MODELS

LA_{t-1} is the most recent latitude report. LO_{t-1} is the most recent longitude report. Standard deviations of parameter estimates are in parentheses. When LA_t enters a new band, the appropriate model from that band is utilized.

|10-15N| |45-83W|

$$LA_t = \underset{(.07)}{1.696}LA_{t-1} - \underset{(.07)}{.696}LA_{t-2} + \underset{(.07)}{.101}(LA_{t-4} - LA_{t-5}) + \underset{(.04)}{.048}$$

$$LO_t = \underset{(.11)}{.233}(LA_{t-1} - LA_{t-2}) - \underset{(.12)}{.075}(LA_{t-4} - LA_{t-5}) + \underset{(.07)}{1.607}LO_{t-1} - \underset{(.07)}{.607}LO_{t-2} + \underset{(.07)}{.251}(LO_{t-4} - LO_{t-5}) + \underset{(.07)}{.127}$$

|15-20N| |45-87W|

$$LA_t = \underset{(.03)}{1.766}LA_{t-1} - \underset{(.03)}{.766}LA_{t-2} - \underset{(.04)}{.106}(LA_{t-4} - LA_{t-5}) - \underset{(.02)}{.010}(LO_{t-1} - LO_{t-2}) + \underset{(.02)}{.014}(LO_{t-4} - LO_{t-5}) + \underset{(.02)}{.053}$$

$$LO_t = \underset{(.05)}{.012}(LA_{t-1} - LA_{t-2}) - \underset{(.06)}{.050}(LA_{t-4} - LA_{t-5}) + \underset{(.03)}{1.775}LO_{t-1} - \underset{(.03)}{.775}LO_{t-2} + \underset{(.03)}{.088}(LO_{t-4} - LO_{t-5}) + \underset{(.04)}{.139}$$

|20-25N | |45-100W|

$$LA_t = \underset{(.03)}{1.849}LA_{t-1} - \underset{(.03)}{.849}LA_{t-2} + \underset{(.03)}{.013}(LA_{t-4} - LA_{t-5}) - \underset{(.03)}{.064}(LO_{t-1} - LO_{t-2}) + \underset{(.03)}{.073}(LO_{t-4} - LO_{t-5}) + \underset{(.02)}{.054}$$

$$LO_t = \underset{(.04)}{.026}(LA_{t-1} - LA_{t-2}) - \underset{(.04)}{.052}(LA_{t-4} - LA_{t-5}) + \underset{(.03)}{1.779}LO_{t-1} - \underset{(.03)}{.779}LO_{t-2} + \underset{(.03)}{.121}(LO_{t-4} - LO_{t-5}) + \underset{(.03)}{.067}$$

|25-30N | |45-100W|

$$LA_t = \underset{(.04)}{1.777}LA_{t-1} - \underset{(.04)}{.777}LA_{t-2} + \underset{(.04)}{.016}(LA_{t-4} - LA_{t-5}) - \underset{(.03)}{.101}(LO_{t-1} - LO_{t-2}) + \underset{(.03)}{.083}(LO_{t-4} - LO_{t-5}) + \underset{(.03)}{.071}$$

$$LO_t = \underset{(.05)}{.103}(LA_{t-1} - LA_{t-2}) - \underset{(.05)}{.198}(LA_{t-4} - LA_{t-5}) + \underset{(.04)}{1.837}LO_{t-1} - \underset{(.04)}{.837}LO_{t-2} + \underset{(.04)}{.032}(LO_{t-4} - LO_{t-5}) + \underset{(.03)}{.052}$$

| 30-35N | 45-80W |

$$LA_t = \underset{(.06)}{1.746}LA_{t-1} - \underset{(.06)}{.746}LA_{t-2} - \underset{(.06)}{.015}(LA_{t-4} - LA_{t-5})$$
$$- \underset{(.05)}{.049}(LO_{t-1} - LO_{t-2}) - \underset{(.05)}{.011}(LO_{t-4} - LO_{t-5}) + \underset{(.04)}{.072}$$

$$LO_t = \underset{(.07)}{.050}(LA_{t-1} - LA_{t-2}) - \underset{(.07)}{.030}(LA_{t-4} - LA_{t-5})$$
$$+ \underset{(.06)}{1.841}LO_{t-1} - \underset{(.06)}{.841}LO_{t-2} + \underset{(.06)}{.068}(LO_{t-4} - LO_{t-5}) - \underset{(.05)}{.044}$$

| 35-40N | 45-76W |

$$LA_t = \underset{(.06)}{1.735}LA_{t-1} - \underset{(.06)}{.735}LA_{t-2} - \underset{(.06)}{.094}(LA_{t-4} - LA_{t-5})$$
$$- \underset{(.04)}{.075}(LO_{t-1} - LO_{t-2}) + \underset{(.04)}{.023}(LO_{t-4} - LO_{t-5}) + \underset{(.05)}{.087}$$

$$LO_t = \underset{(.08)}{.030}(LA_{t-1} - LA_{t-2}) - \underset{(.06)}{.005}(LA_{t-4} - LA_{t-5})$$
$$+ \underset{(.05)}{1.881}LO_{t-1} - \underset{(.05)}{.881}LO_{t-2} - \underset{(.08)}{.006}(LO_{t-4} - LO_{t-5}) - \underset{(.06)}{.109}$$

| 40-45N | 45-70W |

$$LA_t = \underset{(.10)}{1.742}LA_{t-1} - \underset{(.10)}{.742}LA_{t-2} + \underset{(.10)}{.013}(LA_{t-4} - LA_{t-5})$$
$$- \underset{(.08)}{.067}(LO_{t-1} - LO_{t-2}) - \underset{(.08)}{.003}(LO_{t-4} - LO_{t-5}) - \underset{(.09)}{.086}$$

$$LO_t = -\underset{(.12)}{.145}(LA_{t-1} - LA_{t-2}) + \underset{(.13)}{.125}(LA_{t-4} - LA_{t-5})$$
$$+ \underset{(.10)}{1.831}LO_{t-1} - \underset{(.10)}{.831}LO_{t-2} - \underset{(.12)}{.042}(LO_{t-4} - LO_{t-5}) - \underset{(.14)}{.205}$$

A P P E N D I X B

PSI WEIGHT AND COVARIANCE MATRICES

The psi-weight matrices are listed from ψ_0 through ψ_{11}.

|10-15N|
|45-83W|

1.00000	0.00000	1.69589	-.00041	2.18006	-.00094
0.00000	1.00000	.23301	1.60668	.53652	1.97465
2.51686	-.00147	2.85208	-.00222	3.15546	-.00313
.83347	2.19776	1.01723	2.58356	1.21311	2.96590
3.41529	-.00408	3.62993	-.00500	3.81298	-.00593
1.44244	3.29566	1.69130	3.54926	1.91324	3.79963
3.97084	-.00689	4.10678	-.00786	4.22289	-.00879
2.11691	4.04808	2.31526	4.28038	2.5136	4.48470

The psi-weights are followed by the covariance matrix of the one step ahead forecast error computed using all storms in the region.

75.6900	2.4420
2.4420	201.3561

|15-20N|
|45-87W|

1.00000	0.00000	1.76569	-.00960	2.35186	-.02439
0.00000	1.00000	.01206	1.77455	.03064	2.37436
2.80050	-.04147	3.24963	-.04534	3.67499	-.04304
.05209	2.83877	.02382	3.28640	-.03011	3.70181
4.06349	-.03864	4.40934	-.03434	4.72193	-.02853
-.09460	4.07719	-.16054	4.40979	-.23253	4.70710
5.00624	-.02102	5.26492	-.01224	5.49947	-.00282
-.31066	4.97395	-.39298	5.21358	-.47683	5.42840

125.2161	-2.9775
-2.9775	257.6025

|20-25N|
|45-100W|

1.00000	0.00000	1.84867	-.06432	2.56723	-.16898
0.00000	1.00000	.02610	1.77858	.06857	2.38309
3.17432	-.29669	3.69960	-.36173	4.15842	-.39159
.12039	2.85101	.12420	3.33268	.09955	3.80332
4.56213	-.40421	4.91924	-.41083	5.23258	-.40822
.05982	4.24724	.01387	4.65601	-.03964	5.04549
5.50628	-.39626	5.74493	-.37624	5.95312	-.35090
-.10014	5.38938	-.16604	5.71947	-.23538	6.02665

143.2809	6.1945
6.1945	179.5600

|25-30N|
|45-100W|

1.00000	0.00000	1.77698	-.10098	2.37029	-.26399
0.00000	1.00000	.10287	1.83734	.26894	2.52809
2.81451	-.46040	3.15533	-.58649	3.43948	-.66503
.46902	3.08971	.48432	3.57154	.38166	4.00860
3.69387	-.71528	3.93228	-.75231	4.14408	-.78303
.21277	4.42070	.01594	4.81731	-.19133	5.18586
4.32554	-.80898	4.47784	-.82946	4.60518	-.84333
-.40259	5.52073	-.61655	5.82149	-.83347	6.09115

212.5764	-28.9362
-28.9362	307.3009

|30-35N|
|45-80W|

1.00000	0.00000	1.74583	-.04904	2.29962	-.12684
0.00000	1.00000	.05037	1.84062	.13028	2.54479
2.70874	-.21940	2.99394	-.32865	3.19124	-.44627
.22535	3.13281	.29555	3.69010	.34972	4.21140
3.32638	-.56639	3.41767	-.68491	3.47875	-.79966
.39382	4.69368	.43171	5.13563	.46427	5.54218
3.51908	-.90931	3.54521	-1.01312	3.56167	-1.11073
.49240	5.91698	.51696	6.26278	.53871	6.58173

270.6025 -63.7864
-63.7864 390.0625

|35-40N|
|45-76W|

1.00000	0.00000	1.73508	-.07490	2.27314	-.19593
0.00000	1.00000	.03048	1.88079	.07973	2.65430
2.66497	-.34283	2.85457	-.47828	2.92116	-.59419
.13951	3.33191	.19889	3.91796	.25294	4.42487
2.91666	-.68797	2.87445	-.76019	2.82412	-.81537
.29945	4.86357	.33787	5.24361	.36905	5.57316
2.77979	-.85792	2.74682	-.89159	2.72590	-.91930
.39429	5.85916	.41490	6.10749	.43203	6.32317

336.3556 -11.8191
-11.8191 428.9041

|40-45N|
|45-70W|

1.00000	0.00000	1.74183	-.06665	2.30179	-.17149
0.00000	1.00000	-.14469	1.83112	-.37288	2.53152
2.73235	-.29594	3.08236	-.43071	3.36253	-.56523
-.64245	3.12881	-.80439	3.60144	-.89091	3.97068
3.58379	-.69279	3.75626	-.80950	3.88867	-.91328
-.92390	4.25466	-.91826	4.46862	-.88804	4.62674
3.98864	-1.00347	4.06270	-1.08035	4.11637	-1.14476
-.84346	4.74095	-.79187	4.82111	-.73839	4.87533

367.8724 -68.7230
-68.7230 538.7041

APPENDIX C

BIVARIATE THRESHOLD CONFIDENCE ELLIPSE EXAMPLE
(Hurricane Frederic, 1979)

This example illustrates use of the forecast model parameters (Appendix A) and the psi weights and covariance matrices (Appendix B) to compute a confidence ellipse for a forecast that crosses a threshold boundary. The positions of interest for hurricane Frederic are

{ (12.0,45.1) (12.5,47.0) (12.9,48.7) (13.3,50.4)
(13.8,52.3) (13.3,54.1) (14.9,55.5) (15.5,57.2)
(16.3,58.8) (16.7,59.8) (17.1,60.8) (17.5,61.8)
(17.8,62.8) (18.0,63.8) (18.1,64.8) (18.1,65.8)
(18.1,66.8) } .

From time origin 5 (13.8,52.3), the models in Appendix A yield forecasts for lead times of 6 through 48 hours:

{ (14.2,54.1) (14.6,55.9) (15.1,57.6) (15.4,59.1)
(15.8,60.6) (16.2,62.1) (16.6,63.5) (17.1,64.8) }.

The 90% confidence ellipse for the 48 hour forecast can be readily constructed. From the model for 10-15N,

$$LA_t = 1.696LA_{t-1} - .696LA_{t-2} + .101(LA_{t-4} - LA_{t-5}) + .048$$

$$LO_t = .233(LA_{t-1} - LA_{t-2}) - .075(LA_{t-4} - LA_{t-5})$$
$$+1.607LO_{t-1} - .607LO_{t-2} + .251(LO_{t-4} - LO_{t-5}) + .127$$

and for 15-20N,

$$LA_t = 1.766LA_{t-1} - .766LA_{t-2} - .106(LA_{t-4} - LA_{t-5})$$
$$-.010(LO_{t-1} - LO_{t-2}) + .014(LO_{t-4} - LO_{t-5}) + .053$$

$$LO_t = .012(LA_{t-1}-LA_{t-2})-.050(LA_{t-4}-LA_{t-5})$$
$$+1.775LO_{t-1}-.775LO_{t-2}+.088(LO_{t-4}-LO_{t-5})+.139$$

These models can be specified in terms of the noise series a_{1t}, a_{2t}, η_{1t}, and η_{2t} which represent the latitude and longitude shocks in the region 10-15N, and the latitude and longitude shocks in the region 15-20N respectively. First represent the models in terms of the $\phi(B)$ polynomials. The polynomial for 10-15N is

$$LA_t = (1.696B-.696B^2+.101B^4-.101B^5)LA_t + 0.0LO_t + a_{1t} \quad (C1)$$

$$LO_t = (.233B-.233B^2-.075B^4+.075B^5)LA_t$$
$$+(1.607B-.607B^2+.251B^4-.251B^5)LO_t + a_{2t} \quad (C2)$$

and for 15-20N,

$$LA_t = (1.766B-.766B^2+.106B^4-.106B^5)LA_t$$
$$+(-.010B+.010B^2+.014B^4-.014B^5)LO_t + \eta_{1t} \quad (C3)$$

$$LO_t = (.012B-.012B^2-.050B^4+.050B^5)LA_t$$
$$+(1.775B-.775B^2+.088B^4-.088B^5)LO_t + \eta_{2t} \quad (C4)$$

Next solve (C3) and (C4) for LA_t and LO_t in terms of the noise to yield, for 15-20N

$$LA_t = a/b$$

where $a = \{(1-1.775B+.775B^2-.088B^4+.088B^5)a_{1t}$
$+(-.010B+.010B^2+.014B^4-.014B^5)a_{2t}\}$

and $b = \{(1-1.766B+.766B^2-.106B^4+.106B^5)$
$(1-1.775B+.775B^2-.088B^4+.088B^5)$
$-(-.010B+.010B^2+.014B^4-.014B^5)$
$(.012B-.01B^2-.050B^4+.050B^5)\}$. (C5)

$$LO_t = c/b$$

where $c = (.012B-.012B^2-.050B^4+.050B^5)a_{1t}$

$$+(1-1.766B+.766B^2-.106B^4+.106B^5)a_{2t} \quad . \qquad (C6)$$

Similarly, for 10-15N,

$$LA_t = d/e$$

where $d = \{ (1-1.607B+.607B^2-.251B^4+.251B^5)a_{1t} + 0.0a_{2t} \}$

and $e = \{(1-1.696B+.696B^2-.101B^4+.101B^5)$

$$(1-1.607B+.607B^2-.251B^4+.251B^5)$$

$$-(0)(.233B-.233B^2-.075B^4+.075B^5)\} \quad . \qquad (C7)$$

$$LO_t = f/e$$

where $f = (.233B-.233B^2-.075B^4+.075B^5)a_{1t}$

$$+(1-1.696B+.696B^2-.101B^4+.101B^5)a_{2t} \quad . \qquad (C8)$$

The ratios in (C7) and (C8) are the psi weight matrices for the region 10-15N. These 2 by 2 matrices are denoted ψ_i, where i denotes the lag from the forecast position. Similarly, the ratios in (C5) and (C6) yield the psi weights for the region 15-20N. These are also 2 by 2 matrices are denoted ψ_i. Then, following the development in the section on Threshold Interval Forecast of Two Variables,

$$\psi_0=\begin{bmatrix}1.000 & 0.000\\ 0.000 & 1.000\end{bmatrix} \quad \psi_1=\begin{bmatrix}1.696 & 0.000\\ .233 & 1.607\end{bmatrix} \quad \psi_2=\begin{bmatrix}2.180 & 0.000\\ .537 & 1.975\end{bmatrix}$$

$$\xi_0=\begin{bmatrix}1.000 & 0.000\\ 0.000 & 1.000\end{bmatrix} \quad \xi_1=\begin{bmatrix}1.766 & -.010\\ .012 & 1.775\end{bmatrix} \quad \xi_2=\begin{bmatrix}2.352 & -.024\\ .031 & 2.374\end{bmatrix}$$

$$\xi_3=\begin{bmatrix}2.801 & -.042\\ .052 & 2.839\end{bmatrix} \quad \xi_4=\begin{bmatrix}3.250 & -.045\\ .024 & 3.286\end{bmatrix} \quad \xi_5=\begin{bmatrix}3.675 & -.043\\ -.030 & 3.702\end{bmatrix} \; .$$

The covariance matrix of the one step ahead forecast errors for 10-15N is

$$\Sigma_a = \begin{bmatrix} 75.69 & -2.44 \\ -2.44 & 201.36 \end{bmatrix} .$$

For 15-20N,

$$\Sigma_\eta = \begin{bmatrix} 125.22 & -2.98 \\ -2.98 & 257.60 \end{bmatrix} .$$

From (4.39), (4.40), and (4.41),

$$\begin{bmatrix} c_{11} & c_{21} \\ d_{11} & d_{21} \end{bmatrix} = \xi_5 \psi_o = \begin{bmatrix} 3.675 & -.043 \\ -.030 & 3.702 \end{bmatrix} \tag{C9}$$

$$\begin{bmatrix} c_{12} & c_{22} \\ d_{12} & d_{22} \end{bmatrix} = \xi_5 \psi_1 = \begin{bmatrix} 6.222 & -.071 \\ .812 & 5.948 \end{bmatrix} \tag{C10}$$

$$\begin{bmatrix} c_{13} & c_{23} \\ d_{13} & d_{23} \end{bmatrix} = \xi_5 \psi_2 = \begin{bmatrix} 7.989 & -.088 \\ 1.921 & 7.310 \end{bmatrix} \tag{C11}$$

Summing the squares of the elements of the matrices in (C9), (C10), and (C11) yields,

$$\begin{bmatrix} c_{11}^2+c_{12}^2+c_{13}^2 & c_{21}^2+c_{22}^2+c_{23}^2 \\ d_{11}^2+d_{12}^2+d_{13}^2 & d_{21}^2+d_{22}^2+d_{23}^2 \end{bmatrix} = \begin{bmatrix} 116.043 & .0147 \\ 4.350 & 102.520 \end{bmatrix}$$

These are the multipliers for σ_{a1}^2, and σ_{a2}^2 in equation (4.39) and (4.40). The multipliers for the $\text{cov}(a_1 a_2)$ in (4.39) are given by

$$c_{11}c_{21}+c_{12}c_{22}+c_{13}c_{23} = -1.306$$

and for $\text{cov}(a_1 a_2)$ in (4.40),

$$d_{11}d_{21}+d_{12}d_{22}+d_{13}d_{23} = 18.761 .$$

For Σ_η, the multipliers are the ξ weights. Specifically, for (4.39), the sum

$$c_{14}^2 + \cdots + c_{18}^2 = (3.250)^2 + (2.801)^2 + (2.353)^2 + (1.766)^2 + (1)^2$$
$$= 28.082$$

is the coefficient of $\sigma_{\eta 1}^2$. The sum

$$c_{24}^2 + \cdots + c_{28}^2 = (-.045)^2 + (-.042)^2 + (-.025)^2 + (-.012)^2 + (0)^2$$
$$= .00456$$

is the coefficient of $\sigma_{\eta 2}^2$ in (4.39), and,

$$c_{14}c_{24} + \cdots + c_{18}c_{28} = -.340$$

is the coefficient of $cov(\eta_1\eta_2)$ in (4.39).

For (4.40), $\sigma_{\eta 1}^2$ is multiplied by

$$d_{14}^2 + \cdots + d_{18}^2 = (.024)^2 + (.052)^2 + (.031)^2 + (.012)^2 + (0)^2$$
$$= .00439$$

and $\sigma_{\eta 2}^2$ is multiplied by

$$d_{24}^2 + \cdots + d_{28}^2 = 28.644 \quad .$$

Then $cov(\eta_1\eta_2)$ is multiplied by

$$d_{14}d_{24} + \cdots + d_{18}d_{28} = .321 \quad .$$

The coefficients of σ_{a1}^2, $cov(a_1a_2)$, σ_{a2}^2, $\sigma_{\eta 1}^2$, $cov(\eta_1\eta_2)$, and $\sigma_{\eta 2}^2$ in equation (4.41) are 20.289, 108.787, -1.228, .318, 28.348, and -.344 . So, from (4.39) the variance of the 8 step ahead latitude forecast from time origin 5 with 2 forecasts falling in the old region is

$$V[e_{15}(8,2)] = (116.043)\sigma_{a1}^2 + (.0147)\sigma_{a2}^2 + 2(-1.306)\mathrm{cov}(a_1a_2)$$
$$+ (28.082)\sigma_{\eta 1}^2 + (.00456)\sigma_{\eta 2}^2 + 2(-.340)\mathrm{cov}(\eta_1\eta_2)$$
$$= (116.21)(75.69) + (.0147)(201.36) + 2(-1.306)(-2.44)$$
$$+ (28.082)(125.22) + (.00456)(257.60) + 2(-.340)(-2.98)$$
$$= 12295.427 \quad .$$

Thus, the standard deviation of the 8 step ahead forecast of latitude is

$$SD[e_{15}(8,2)] = (12295.427)^{1/2} = 110.9 \text{ n mi} \quad .$$

This compares well with the empirical standard deviation of 102.8 n mi.

From (4.40) the variance of the eight step ahead forecast of longitude is

$$V[e_{25}(8,2)] = 4.350(75.69) + 102.520(201.36) + 2(18.761)(-2.44)$$
$$+ .00439(125.22) + 28.644(257.60) + 2(.321)(-2.98)$$
$$= 28439.717$$

and

$$[e_{25}(8,2)] = 168.6 \text{ n mi}$$

which compares favorably with the empirical value of 149.7 n mi. From (4.41),

$$E[e_{15}(8,2)e_{25}(8,2)] = 20.289(75.69) + 108.787(-2.44)$$
$$+ (-1.228)(201.36) + .318(125.22)$$
$$+ 28.348(-2.98) + (-3.44)(257.60)$$
$$= 1421.0$$

Consequently, the covariance matrix for the eight step ahead forecast is

$$\Sigma_8 = \begin{bmatrix} 12324.9 & 1421.0 \\ 1421.0 & 28258.5 \end{bmatrix} .$$

The confidence ellipse can be specified by using the procedure described in Interval Forecast of Two Variables. The eigenvalues for Σ_8 are $\lambda_1 = 12171.30$, and $\lambda_2 = 28563.84$, and the eigenvectors are

$$<.087,.996> \quad <.996,-.087> \quad .$$

The major axis has length

$$2(\lambda_2)^{.5}(1.645) = 556.0 \text{ n mi.}$$

The minor axis has length

$$2(\lambda_1)^{.5}(1.645) = 377.5 \text{ n mi.}$$

The minor axis *lies on a magnetic heading of*

$$360 + \tan^{-1}(.087/.996) = 4.99 \quad \text{degrees}$$

and is centered at the point forecast (17.1, 64.8) . Finally, the equation of the 90% confidence ellipse is given by

$$(x_1/168.6)^2 - 2(.076)(x_1/168.6)(x_2/110.9) + (x_2/110.9)^2$$
$$= 1.645[1-(.076)^2]$$

where x_1 represents the east-west direction and x_2 represents the north-south direction and

$$\rho = .076 = [(1421.0)(12295.4)^{-.5}(28439.7)^{-.5}] \quad .$$

APPENDIX D

ANALYSIS OF RESIDUALS

Chi-square contingency table tests (99% confidence) were used to evaluate the independence of the latitude and longitude one step ahead forecasts. The tests rejected the null hypothesis of independence which was acceptable because a dependency structure was allowed. However, chi-square goodness of fit tests also rejected the null hypothesis that the forecasts were Normally distributed which was not acceptable.

The region posessing the worst chi-square value (20-25N) was examined in detail. Further goodness of fit tests did not reject the double exponential distribution for both marginals. This indicated the category five forecast distributions had heavier tails than expected. Outliers were examined case by case. It was discovered that a few storms that were classified as category five had short non-category five track segments. That is, the storms accelerated during the time they were supposed to be moving at constant velocity. The segments containing the accelerations were removed and the category five model was then fit to the new data. New coefficients were not significantly different from old coefficients (Table D1). Mean forecast accuracy was not significantly

TABLE D1

OLD VS NEW COEFFICIENT MATRICES

		Old coefficients		New Coefficients	
		LA	LO	LA	LO
Lag 1	LA	.84	-.06	.81	-.08
	LO	.03	.78	.05	.76
Lag 4	LA	.01	.07	.04	.08
	LO	-.05	.12	-.09	.12

different. Deleting the track segments removed approximately 4% of the forecasts from each tail of the forecast error distributions. All the deleted forecasts were beyond three standard deviations from the mean.

A second set of chi-square tests did not reject the null hypothesis of Normally distributed errors. In addition, the contingency table tests did not reject the null hypothesis of independence.

Outliers beyond three standard deviations (at most 4% from each tail) were deleted from the one step ahead forecast errors in other regions. Chi-square tests did not reject independence or Normality.

Sequential regression analysis of lagged residuals showed no significant auto or cross-correlations at lags one through four.

To afford additional protection when forecasting accelerating storms, the empirical one step ahead forecast error covariance matrices based on all storms (Appendix B) are used in confidence ellipse computations.

APPENDIX E

TRANSITION MATRICES

A representative analysis of the four step ahead transition matrix for the region 25-30N is presented in this appendix. The heuristic procedure used to develop the matrix involved moving a window of seven previous six hour position reports along each hurricane track. Segments of longer length resulted in large estimated parameter variances, while shorter segments missed short term accelerations. At each position report a stationarity category was determined. For example, at the seventh position report, the stationarity category was based on positions one through seven. At the eighth position report the stationarity category was based on positions two through eight, etc.. In this manner, a stationarity category was associated with the terminal point in each "window." The transition matrix was then constructed based on those categories.

Some difficulty was encountered in determining the storm categories primarily due to the small window. With seven observations, the standard deviations of the parameter estimates were approximately .1 . If the storm was in category nine, the series had to be differenced twice for stationarity. This resulted in five observations being available to compute the lag one parameter.

To sort the storms into the various categories, the lag one parameter was calculated for each latitude and longitude series. The raw position series was analyzed first. By trial and error it was determined that coefficient values below .8 were good indicators of mean stationarity. If the lag one parameter was greater than .8 (two standard deviations below 1.0), the series was differenced and the lag one parameter was calculated for the velocity model. If the parameter was still greater than .8, the series was differenced again and the parameter was calculated for the acceleration model. Usually, at some point in the procedure, the series were found to be stationary. In a very few cases (less than 1%) the series were always nonstationary. Rather than difference the series a third time, these cases were discarded.

Given that the storm is in a particular category at time t, the category at time t+1, t+2, ... , can be determined. By recording the results for all storm tracks it is possible, given the current category, to determine the probability the storm will transition to some other category at a future time. The result of the analysis is a transition matrix (Table E.1). For example, given that the storm is presently in category 2, there is a 38.8% probability that it will be in category 5 in 24 hours. For this example the category five state is the most frequent future state regardless of the current category.

Forecasts were computed by using different combinations of the nine possible models depending on past categories (or probable future categories) without any reduction in forecast error. Further study is required to determine the acceptability of analyses that make use of the transition matrices.

TABLE E.1

24 Hour Transition Matrix for he region 25-30N

Present Category	Future Category 1	2	3	4	5	6	7	8	9
1	.120	.160	.000	.200	.200	.080	.040	.080	.120
2	.060	.149	.000	.194	.388	.030	.030	.134	.015
3	.000	.143	.000	.000	.714	.000	.143	.000	.000
4	.014	.137	.027	.137	.466	.096	.041	.055	.027
5	.057	.134	.000	.159	.459	.096	.045	.032	.019
6	.029	.171	.000	.114	.543	.000	.029	.086	.029
7	.100	.200	.000	.100	.400	.100	.000	.000	.100
8	.035	.035	.000	.172	.483	.069	.035	.138	.035
9	.071	.143	.000	.214	.357	.143	.000	.000	.071

APPENDIX F

REQUIRED DATA MANIPULATION

The data files and the five major steps required to calculate the model coefficients for each latitude band are described in the following pages. The five steps are: data reduction, determination of the stationarity category, construction of the data matrix, coefficient computation, and analysis of empirical forecast error.

"Best track" storm data were provided via magnetic tape by the National Environmental Satellite Data and Information Service (an agency of the National Oceanic and Atmospheric Administration), Asheville, North Carolina. The data set contained position reports at 6 hour intervals for 815 North Atlantic storms (including hurricanes, tropical storms, and subtropical storms) dating from 1886 through 1983. The tape format had 80 characters per record. There were three types of card images: the Title Card (Table F.1), the Storm Classification Card (Table F.2), and the Storm Data Cards (Table F.3) (Jarvinen, Neumann, and Davis, 1984).

In each region, the first step was to condense the data file by eliminating data that was not required. Based on recommendations from the analysts at the National Hurricane Center, all storms occurring before 1945 were

TABLE F.1

TITLE CARD - FORMAT AND CONTENTS

Computer Card Columns	Contents
1 - 5	Card sequence number
7 - 8	Month
10 - 11	Day (first day of storm)
13 - 16	Year
20 - 21	Value of M (M=number of days the storm existed)
23 - 24	Storm number for the year
25 - 30	Blank
31 - 34	Cumulative Storm Number
36 - 47	Storm Name
48 - 52	Blank
53	(1=hit coastline, 0=did not)
54 - 58	Blank
59	Saffir/Simpson Hurricane Scale number
60 - 79	Blank
80	Last storm of the year if = L

TABLE F.2

CLASSIFICATION CARD - FORMAT AND CONTENTS

Computer Card Columns	Contents
1 - 5	Card sequence number
7 - 8	Maximum status of the storm during its life

TABLE F.3

STORM DATA CARD - FORMAT AND CONTENTS

Latitude and longitude are rounded to the nearest tenth. Wind speed is rounded to the nearest five knots. Pressure is rounded to the nearest millibar. Storm types are: '*'-tropical storm or hurricane, 'D'- tropical disturbance, 'S'-subtropical storm, 'W'-tropical wave, and 'E'-extratropical storm.

Computer Card Columns	Contents
1 - 5	Card sequence number
7 - 8	Month
10 - 11	Day
12	Storm type at 0000Z
13 - 15	Latitude at 0000Z
16 - 19	Longitude at 0000Z
20	Blank
21 - 23	Wind speed at 0000Z
24	Blank
25 - 28	Central pressure at 0000Z
29	Storm type at 0600Z
30 - 32	Latitude at 0600Z
33 - 36	Longitude at 0600Z
37	Blank
38 - 40	Wind speed at 0600Z
41	Blank
42 - 45	Central pressure at 0600Z
46	Storm type at 1200Z
47 - 49	Latitude at 1200Z
50 - 53	Longitude at 1200Z
54	Blank
55 - 57	Wind speed at 1200Z
58	Blank
59 - 62	Central pressure at 1200Z
63	Storm type at 1800Z
64 - 66	Latitude at 1800Z
67 - 70	Longitude at 1800Z
71	Blank
72 - 74	Wind speed at 1800Z
75	Blank
76 - 79	Central pressure at 1800Z

deleted due to concerns about the accuracy of the observations. All storms that attained less than tropical storm status (12 of them) were deleted because of their weak persistence. Next, all storm tracks outside the region of interest were deleted. Finally, because the focus of the research was to accurately predict hurricane landfall based on the past track, position reports following landfall on the continental United States were eliminated as were the central pressures and wind velocities. This left 362 storm tracks containing a total of over 10,000 position reports. The final condensed data file contained the cumulative storm number, the year, month, and day of the first position report, the storm name, the number of position reports, and the latitude and longitude coordinates of each position report (Table F.4).

In the second step, the following procedure was used to determine the stationarity category of the storm. For latitude,

(1) Adjust the latitude series to zero mean.

(2) Lag the data one period.

(3) Use least squares regression to calculate the lag one <u>position</u> forecast coefficient.

(4) If the coefficient is less than .8 (the storm is latitude position-stationary) go to (12).

(5) Difference the series the first time.

(6) Use least squares regression to calculate the lag one <u>velocity</u> forecast coefficient.

(7) If the coefficient is less than .8 (the storm is latitude velocity-stationary) go to (12).

(8) Difference the series a second time.

(9) Use least squares regression to calculate the lag one acceleration forecast coefficient.

(10) If the coefficient is less than .8 (the storm is latitude acceleration-stationary) go to (12).

(11) Discard the storm. Go to (14).

(12) Perform steps (1)-(11) for longitude.

(13) Classify the storm according to Table 4.1.

(14) Read the next storm track.

Various stationarity "cutoffs" were examined from ϕ=.6 through ϕ=1.2. The value of ϕ=.8 was selected because it resulted in the minimum forecast errors and because, for individual storms, the standard deviation of the lag one coefficient was approximately .1. Thus, coefficient values below .8 were good indicators of mean stationarity. Once the stationarity category was determined, it was appended to the storm name. A two storm sample of the resulting data file is shown in Table F.4.

The data matrices were then constructed. There are nine data matrices for each grid, one for each of the storm categories in Table 4.1. For example, the category 1 and category 5 data matrices for the storms in Table F.4 are shown in Tables F.5 and F.6.

TABLE F.4

CONDENSED DATA FILE AND SAMPLE RECORDS

Title Card Columns	Contents
1 - 2	Blank
3 - 6	Cumulative sequence number
7	Blank
8 - 11	Year
12	Blank
13 - 14	Month
15	Blank
16 - 17	Day of first position report
18	Blank
19 - 33	Name
34	Blank
35 - 36	Number of position reports
37	Blank
38 - 39	Maximum storm type attained
40 - 41	Blank
42	Stationarity category

Position Card Columns	
1	Blank
2 - 5	Latitude
6	Blank
7 - 10	Longitude

Sample Records For Grid 25-30N

```
  690 1970 07 31 CELIA           08 TS 5
 25.3 89.6
 25.8 90.8
 26.2 92.0
 26.6 93.5
 27.0 94.9
 27.5 96.3
 28.1 97.8
 28.6 99.3
  694 1970 09 12 FELICE          08 HR 5
 25.3 84.0
 25.8 85.2
 26.5 86.5
 27.2 88.4
 28.0 90.2
 28.8 92.2
 29.4 94.1
 29.9 95.5
```

TABLE F.5

CATEGORY 1 REGRESSION DATA MATRIX
(For records in Table F.4)

This is position data for Celia and Felice adjusted to zero mean. This example is mean nonstationary. Missing data are denoted by an 'X'. Actual data matrices were lagged up to six periods. This example is lagged four periods for clarity of presentation. The dependent variables are at lag0. The predictor variables are at lag1 through lag4. This is only an example of the category 1 format. Celia and Felice are category 5 storms.

Latitude					Longitude					
lag0	lag1	lag2	lag3	lag4	lag0	lag1	lag2	lag3	lag4	
-1.6	X	X	X	X	-4.7	X	X	X	X	C
-1.1	-1.6	X	X	X	-3.5	-4.7	X	X	X	E
-0.7	-1.1	-1.6	X	X	-2.3	-3.5	-4.7	X	X	L
-0.3	-0.7	-1.1	-1.6	X	-0.8	-2.3	-3.5	-4.7	X	I
0.1	-0.3	-0.7	-1.1	-1.6	0.6	-0.8	-2.3	-3.5	-4.7	A
0.6	0.1	-0.3	-0.7	-1.1	2.0	0.6	-0.8	-2.3	-3.5	
1.2	0.6	0.1	-0.3	-0.7	3.5	2.0	0.6	-0.8	-2.3	
1.7	1.2	0.6	0.1	-0.3	5.0	3.5	2.0	0.6	-0.8	
-2.3	X	X	X	X	-5.5	X	X	X	X	F
-1.8	-2.3	X	X	X	-4.3	-5.5	X	X	X	E
-1.1	-1.8	-2.3	X	X	-3.0	-4.3	-5.5	X	X	L
-0.4	-1.1	-1.8	-2.3	X	-1.1	-3.0	-4.3	-5.5	X	I
0.4	-0.4	-1.1	-1.8	-2.3	0.7	-1.1	-3.0	-4.3	-5.5	C
1.2	0.4	-0.4	-1.1	-1.8	2.7	0.7	-1.1	-3.0	-4.3	E
1.8	1.2	0.4	-0.4	-1.1	4.6	2.7	0.7	-1.1	-3.0	
2.3	1.8	1.2	0.4	-0.4	6.0	4.6	2.7	0.7	-1.1	

TABLE F.6

CATEGORY 5 REGRESSION DATA MATRIX
(For records in Table F.4)

This is velocity data for Celia and Felice. This example is mean stationary. Missing data are denoted by an 'X'. Actual data matrices were lagged up to five periods. This example is lagged four periods for clarity of presentation. The dependent variables are at lag0. The predictor variables are at lag1 through lag4. These are the actual data matrices used for Celia and Felice.

Latitude					Longitude					
lag0	lag1	lag2	lag3	lag4	lag0	lag1	lag2	lag3	lag4	
-0.5	X	X	X	X	-1.2	X	X	X	X	C
-0.4	-0.5	X	X	X	-1.2	-1.2	X	X	X	E
-0.4	-0.4	-0.5	X	X	-1.5	-1.2	-1.2	X	X	L
-0.4	-0.4	-0.4	-0.5	X	-1.4	-1.5	-1.2	-1.2	X	I
-0.5	-0.4	-0.4	-0.4	-0.5	-1.4	-1.4	-1.5	-1.2	-1.2	A
-0.6	-0.5	-0.4	-0.4	-0.4	-1.5	-1.4	-1.4	-1.5	-1.2	
-0.5	-0.6	-0.5	-0.4	-0.4	-1.5	-1.5	-1.4	-1.4	-1.5	
-0.5	X	X	X	X	-1.2	X	X	X	X	F
-0.7	-0.5	X	X	X	-1.3	-1.2	X	X	X	E
-0.7	-0.7	-0.5	X	X	-1.9	-1.3	-1.2	X	X	L
-0.8	-0.7	-0.7	-0.5	X	-1.8	-1.9	-1.3	-1.2	X	I
-0.8	-0.8	-0.7	-0.7	-0.5	-2.0	-1.8	-1.9	-1.3	-1.2	C
-0.6	-0.8	-0.8	-0.7	-0.7	-1.9	-2.0	-1.8	-1.3	-1.3	E
-0.5	-0.6	-0.8	-0.8	-0.7	-1.4	-1.9	-2.0	-1.8	-1.3	

In this research only the forecast models for the category five storms were developed. A portion of the category five data matrix for 25-30N latitude is shown in Table F.6. The data in Table F.6 represent velocities in units of degrees of displacement per six hour interval.

In the fourth step, the lag zero latitude and longitude columns were treated as dependent variables and a least squares regression was performed using SPSS. The resulting coefficients are in Appendix A.

Finally, in step five, the models were used to forecast the storms, and the forecast error was analyzed. Initially there was concern over the fact that the hurricanes being forecast were the same ones used to develop the model coefficients. To analyze the effect of this "unfair" advantage, in a sample region, the storms were deleted (one at a time) from the data base, and model coefficients were computed and used to forecast only the excluded storm. There was no significant change in forecast accuracy. For example, for the 48 hour forecast, the error changed by 2 n mi.

It was concluded that the large number of observations in each region tended to diminish the contribution of individual storms. This allowed computation of model coefficients in approximately 90 seconds (per region) on a CDC Dual Cyber 170/750. Had it been necessary to delete individual storms prior to computing forecast coefficients, it is estimated that run times would have increased by a factor of 1000. A separate run would have been required for each storm in each region. Most storms had track segments in more than one region.

Once the model coefficients are computed, forecasting can be accomplished rapidly. Complete forecasts and error analysis for up to 72 hours in the future requires approximately 17 seconds of CDC processor time per region. This includes the analysis of approximately 7700 forecasts per region. For individual storms, forecasts with lead times as large as 72 hours in the future (without error analysis) can be accomplished on an Apple IIe micro-computer in approximately four seconds.

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