

MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS 1963-A

①

AFIT/GSO/ENS/85D-5

AD-A170 591

STATISTICAL MODELS FOR PREDICTING  
THE CHANGE IN MEAN MOTION OF A  
SATELLITE OVER TIME INCLUDING  
THE EFFECTS OF SOLAR FLUX

THESIS

James M. Burns, B.S.  
Captain, USAF  
AFIT/GSO/ENS/85D-5

DTIC FILE COPY

DTIC  
ELECTE  
AUG 5 1986  
B

Approved for public release; distribution unlimited

## **DISCLAIMER NOTICE**

**THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.**

AFIT/GSO/ENS/85D-5

STATISTICAL MODELS FOR PREDICTING THE CHANGE IN  
MEAN MOTION OF A SATELLITE OVER TIME  
INCLUDING THE EFFECTS OF SOLAR FLUX

THESIS

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology  
Air University  
In Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science in Space Operations

James M. Burns, B.S.

Captain, USAF

December, 1985

Approved for public release; distribution unlimited

## PREFACE

The purpose of this study was to fill a need of the Space Operations Directorate of the North American Aerospace Defense Command (NORAD). One of the missions of the Space Operations Directorate is to maintain positional data on all man made objects in space. Occasionally some of these objects cannot be located. To find a lost object, it is necessary to estimate where the object could be from historical positional data on the satellite. This is simple to do over a short time but the error increases rapidly over longer times because of changes in the satellite's period due to orbital perturbations. For near earth satellites one of the most important perturbations is atmospheric drag which is influenced by solar activity. A method to predict this change in period (or mean motion) from initial data was needed. Required characteristics for accomplishing this are simplicity and computational efficiency since many objects must be updated at any time.

The method developed in this thesis is simple and rapid. Tests were performed on actual data to verify the model.

I would like to thank all those who helped in the production of this thesis. First of all, I would like to thank my advisor, LtC Charles Ebeling, for his assistance and patience on this project. I would also like to thank Lt Terry Sparks of NORAD for his help in collecting the data needed to carry out this thesis. I have special thanks for Mary Browning and Pam McCarthy of the library for their help in locating some very obscure references.

James M. Burns

TABLE of CONTENTS

|  | Page  |
|--|-------|
| Preface . . . . .  | ii    |
| List of Figures . . . . .  | iv    |
| List of Tables . . . . .   | v     |
| Notation . . . . .   | vi    |
| Abstract . . . . .   | vii   |
| I. Introduction . . . . .  | 1.1   |
| Background . . . . .   | 1.1   |
| Research Objective . . . . .   | 1.2   |
| Scope . . . . .  | 1.3   |
| II. Theory . . . . .   | 2.1   |
| Orbital Parameters . . . . .   | 2.1   |
| Orbital Mechanics . . . . .  | 2.5   |
| Current Models . . . . .   | 2.6   |
| Measures of Effectiveness . . . . .  | 2.9   |
| III. Methodology . . . . .   | 3.1   |
| Data . . . . .   | 3.1   |
| Model Development . . . . .  | 3.2   |
| IV. Results . . . . .  | 4.1   |
| Solar Flux . . . . .   | 4.1   |
| BMDP Analysis . . . . .  | 4.2   |
| Post BMDP Analysis . . . . .   | 4.7   |
| V. Recommendations . . . . .   | 5.1   |
| Appendix A: Data File Used for BMDP Analysis . . . . .                           | A.1   |
| Appendix B: NORAD Elements for Satellites<br>15363, 14476, 13043, 7840 . . . . . | B.1   |
| Bibliography . . . . .   | BIB.1 |
| Vita . . . . .   | V.1   |

LIST OF FIGURES

| FIGURE   | PAGE |
|--|------|
| 2.1 NORAD 2-card element sets . . . . .  | 2.4  |
| 3.1 Typical Mean Motion for Days 84300 through 85084 . . . . .   | 3.3  |
| 3.2 Solar Flux for Days 84300 through 85084 . . . . .  | 3.8  |
| 3.3 Daily Change in Mean Motion for Days 84300 through 85084 . . . . .   | 3.9  |
| 4.1 Actual and Calculated Values of $n$ for Satellite 7840 using Unaveraged $\dot{n}$ and $\ddot{n}$ . . . . . | 4.16 |
| 4.2 Actual and Calculated Values of $n$ for Satellite 7840 using Averaged $\dot{n}$ and $\ddot{n}$ . . . . .   | 4.17 |

RE: Appendix B  
 Best available pages per Ms. Melonie  
 Dahmer, AFIT/EN

|               |                                     |
|---------------|-------------------------------------|
| Accession For |                                     |
| NTIS          | <input checked="" type="checkbox"/> |
| DTIC          | <input type="checkbox"/>            |
| Unavail       | <input type="checkbox"/>            |
| Just          | <input type="checkbox"/>            |
| By            |                                     |
| Dist          |                                     |
| Acct          |                                     |
| Dist          |                                     |
| A-1           | 23 TDC                              |



LIST OF TABLES

| TABLE   | PAGE |
|---|------|
| 4.1 BMDP Results for Model n1 . . . .   | 4.3  |
| 4.2 BMDP Results for Model n2 . . . .   | 4.4  |
| 4.3 BMDP Results for Model n3 . . . .   | 4.5  |
| 4.4 BMDP Results for Model n4 . . . .   | 4.6  |
| 4.5 Value of $\dot{n}$ , $\ddot{n}$ , and $\dot{N}/2$ for each Satellite. . . . | 4.7  |
| 4.6 Mean Motion Values for Model n0 . . . .                                     | 4.8  |
| 4.7 Mean Motion Values for Model n1 . . . .                                     | 4.9  |
| 4.8 Mean Motion Values for Model n2 . . . .                                     | 4.10 |
| 4.9 Mean Motion Values for Model n3 . . . .                                     | 4.11 |
| 4.10 Mean Motion Values for Model n4 . . . .                                    | 4.12 |
| 4.11 Mean Motion Values for Model n5 . . . .                                    | 4.13 |
| 4.12 100 Day Error Absolute Values for all<br>Satellites and Models . . . .     | 4.14 |
| 4.13 Average 100 Day Error Values for each Model . . . .                        | 4.14 |
| 4.14 Average 100 Day Error Values for Acceptable Models. . . .                  | 4.15 |
| 4.15 Average Values of $\dot{n}$ and $\ddot{n}$ for each Satellite . . . .      | 4.18 |
| 4.16 Mean Motion Values for Model n1 . . . .                                    | 4.18 |
| 4.17 Mean Motion Values for Model n2 . . . .                                    | 4.19 |
| 4.18 Mean Motion Values for Model n3 . . . .                                    | 4.20 |
| 4.19 Mean Motion Values for Model n5 . . . .                                    | 4.21 |
| 4.20 100 Day Error Absolute Values for all Satellites<br>and Models . . . .     | 4.21 |
| 4.21 Average 100 Day Error Values for Averages $\dot{n}$ and $\ddot{n}$ . . . . | 4.22 |
| 4.22 Sample Values of Mean Motion . . . .                                       | 4.23 |

## NOTATION

|                  |  |
|------------------|--|
| $n$              | Mean motion                                      |
| $\dot{n}$        | First time derivative of mean motion             |
| $\ddot{n}$       | Second time derivative of mean motion            |
| $\dot{N}/2$      | Differance between Brouwer and Kozai mean motion |
| $P$              | Period of the orbit                              |
| $\dot{P}$        | Change in period per day                         |
| $t$              | Time   |
| $dt$             | Change in time                                   |
| $SF$             | Solar flux F10.7 value                           |
| $S$              | Change in SF at time $t$                         |
| $F$              | Change in SF at time $t-dt$                      |
| $\bar{D}$        | Drag force                                       |
| $C$              | Drag coefficient                                 |
| $P$              | Atmospheric density                              |
| $A$              | Satellite's cross sectional area                 |
| $v$              | Relative satelllite velocity                     |
| $\ddot{\bar{r}}$ | Acceleration due to drag                         |
| $\dot{\bar{r}}$  | Total satelllite velocity                        |
| $m$              | Satellite mass                                   |

ABSTRACT

This investigation derived a simple model to determine the change in mean motion over time when the actual values are unknown. A method was developed to include effects of solar flux by calculating an average value of  $n$  over 30 days. The model requires a knowledge of the mean motion for about 30 days before the time of interest to calculate this average.

The analysis was done using BMDP on a CDC Cyber 6000 computer using element set data from actual satellites.

This model does not attempt absolute accuracy, but is intended to be a method to quickly approximate a new mean motion when real values are not available. A limitation of this model is the amount of historical data and analyst judgement which are required.

STATISTICAL MODELS FOR PREDICTING THE CHANGE IN  
MEAN MOTION OF A SATELLITE OVER TIME  
INCLUDING THE EFFECTS OF SOLAR FLUX

I. Introduction

BACKGROUND

Satellites in near earth orbit (those at an altitude of under 1000 km) show a loss of orbital period known as decay. This decay is caused by the drag on a satellite due to the upper atmosphere. It would be a simple matter to account for this decay if the drag were constant. Drag, however, is not constant. It is a function of atmospheric density. This density is, in turn, a function of altitude and solar flux (changes in solar flux cause changes in the atmospheric density at all altitudes). It can be concluded, therefore, that the decay of a satellite is a function of its altitude and of the solar flux.

There are several ways to calculate and predict this decay. One method requires an accurate model of the upper atmosphere. The model must include both solar flux and altitude dependence. One such atmospheric model is the Jacchia model. Using this model and the known parameters of the satellite, it is possible, through astrodynamics, to calculate a very accurate near term orbit for the satellite. This method is very time consuming and requires up-to-the-minute knowledge of the satellite's position. It also decreases in accuracy as the position is predicted further into the future (1).

A second method is to calculate the rate of change in mean motion

(the number of orbital revolutions per day, known as  $n$ ) of a satellite. This is called NDOT. If NDOT is known, then the mean motion of the satellite and its position (or "element set") can be predicted for any time. This prediction is known as "propagation of the element set." NDOT is found by a least squares fit of actual data to two different astrodynamic models (known as the SGP model and the GP4 model). This produces a fairly accurate model of the satellite's orbit in far less time than a complete model such as the Jacchia requires. It is also less accurate than the Jacchia model for the same time, but it loses accuracy less quickly than the Jacchia. Over longer times the new  $n$  produced from the simple model is more accurate than that from an exact model. That is, the model is valid for several days instead of hours. Both of the models are in use at the North American Aerospace Defense Command (NORAD).

While other models have been tried, none of them have outperformed these two. In tests done by NORAD, the models which provide some improvement in accuracy do so at the expense of greatly increased computer run times (2-3).

#### RESEARCH OBJECTIVE

A method that is accurate over long delta times and requires no knowledge of the satellite's current position is needed for predicting  $n$ . It should not require an increase in computer run times or size, above that of the current methods. It is not a replacement for any of the methods currently in use. It is to be considered as an additional method for an area where existing methods are weakest.

Research questions include:

- 1) Is there a statistical relation between  $n$  and solar flux?
- 2) What is the best statistical relation between  $n$  and  $\dot{n}$ ?
- 3) What is the best statistical relation between  $n$  and  $\ddot{n}$ ?
- 4) Is there a statistical relation between  $n$  and other orbital elements?

### SCOPE

There are several types of possible errors in satellites orbits that must be allowed for. These errors may be divided into three main types. They are altitude errors, plane errors, and in-track errors. Altitude errors are errors in the satellites orbital altitude. Plane errors are errors in the satellites orbital plane and can include errors in inclination and errors in right ascension of node. In-track errors are time bias errors between the time a satellite should be at some place and the time it actually arrives there. Altitude errors are normally measured in units of length. Plane errors are measured in units of angle. In-track errors are measured in terms of time. These errors may be combined for a satellite and given in terms of absolute distance error, but that is not normally done in routine cases. This thesis will concentrate on in-track errors only.

This thesis will develop and test some methods of predicting mean motion of an orbit. Chapter II will begin this development by providing some background and basic theory of orbital decay. It will define the terms that will be used in this thesis, and will explore some of the current models and methods used for satellite decay. Chapter III will then develop the models that will be tested in this thesis. Chapter IV will

study the test results of these models and compare them to each other and a current model. Chapter V will then present the conclusions of this study. Extensive use will be made of tables and figures to illustrate the study that was done.

## II. THEORY

### ORBITAL PARAMETERS

A satellite's orbit is described by several parameters. There are several different sets of parameters used for this. These sets include the set of position and velocity vectors, the Keplerian element set, and the F&G series (3). The parameters used in this thesis are those used by NORAD, which are almost the same as those in the Keplerian element set. There are some added parameters which will be included in the following discussion.

The parameters used in the NORAD element set (elset) are:

**Epoch time:** As used by NORAD in a general perturbations element set the epoch time of a satellite is taken as the time of passage through the equatorial plane on the last ascending pass of the satellite.

**Inclination:** Inclination is defined as the angle made between the satellite's orbital plane and the equatorial plane in the direction of satellite movement at the ascending node.

**Ascending node:** The ascending node is the point of passage of the satellite's orbit through the equatorial plane in a south to north direction. It is measured in degrees right ascension for the first point of Aries.

**Eccentricity:** Eccentricity is a measure of the flatness of an orbit since the orbit is in the shape of an ellipse.

**Argument of perigee:** The argument of perigee is the angular distance between the ascending node and the perigee of the orbit in the direction of satellite motion.



Mean anomaly: Mean anomaly is a measure of the angular distance between the argument of perigee and the satellite's position in the orbit, in the direction of satellite motion, at the epoch time. In the NORAD general perturbations element set the mean anomaly and argument of perigee must add to 360 degrees.

Mean motion ( $n$ ): Mean motion is a representation of a satellite's orbital period. It is not, however, expressed in the time to complete one revolution, but in terms of revolutions per day. Its relation to period is given by  $n=1440/P$ , where  $P$  is in minutes.

$\dot{N}/2$  ( $\dot{N}/2$ ):  $\dot{N}/2$  is a term given with the NORAD elset as the difference between a Brouwer and a Kozai mean motion (1), where Kozai used a fourth order general perturbations model and Brouwer used a simplified model of a satellite's orbit. It estimates orbital decay rate.

BSTAR: BSTAR is a synthetic drag term for use in the equations of motion. It is a comparison of the orbit of a satellite to some standard reference satellite. It does not account for size or shape differences or solar flux. Its prime use is to indicate which satellites have been more affected by changes in solar flux. There is little other practical use for this term.

Figure 2.1 shows some typical NORAD 2-card elsets.

Three other parameters are used in this thesis, though neither is in a normal elset. One of them is  $\dot{P}$ , which is the rate of change of orbital period per day. It is provided by the Naval Space Surveillance Center (NAVSPASUR). Another parameter used in this thesis is  $\dot{n}$ . For the purpose of this thesis  $\dot{n}$  will be estimated as the difference between  $n$  at two different times, divided by the difference in time

such that:

$$\dot{n}(t_0) = \frac{n(t_1) - n(t_0)}{t_1 - t_0} \quad (2.1)$$

The final parameter is  $\ddot{n}$ , which, in this thesis, is the difference in  $\dot{n}$  at two times divided by the delta time so:

$$\ddot{n}(t_0) = \frac{\dot{n}(t_1) - \dot{n}(t_0)}{t_1 - t_0} \quad (2.2)$$

The parameters  $N/2$ ,  $\dot{n}$ ,  $\ddot{n}$  will be used in this thesis to estimate new values of  $n$ .

01-APP-85

12:01:00

UNCLASSIFIED

```
1 14R241 81 11 " 85064.6501045H .00008641 00000-0 15285-3 0 02870  
2 14R24 27.427H 239.0351 0562080 69.7509 296.7249 14.5824008516707A
```

Figure 2.1 NORAD 2-card element set

The Format of the 2-card elset is:

Line 1: line number, satellite number, international designator,  
epoch time,  $\dot{N}/2$ ,  $\ddot{N}/6$ , Bstar, elset number.

Line 2: line number, satellite number, inclination, right ascension,  
eccentricity, argument of perigee, mean anomaly, mean motion.

## ORBITAL MECHANICS

A satellite in near earth orbit is affected by the upper reaches of the atmosphere. The satellite has a velocity of about 7 km/s relative to the upper atmosphere. As in the movement of any body through a medium, there will be a retarding force on the body opposite to the direction of motion. This retarding force is called drag and is given by:

$$\vec{D} = - \frac{C_p A v^2 \vec{v}}{2v} \quad (2.3)$$

where  $\vec{D}$  = drag force

$C$  = drag coefficient

$p$  = atmospheric density

$A$  = satellite cross sectional area

$\vec{v}$  = relative vehicle velocity (3-21)

This will give an acceleration of the satellite such that:

$$\ddot{\vec{r}} = - \frac{C A p v \dot{\vec{r}}}{2m} \quad (2.4)$$

where  $\ddot{\vec{r}}$  = acceleration due to drag

$m$  = mass of the satellite

$\dot{\vec{r}}$  = total satellite velocity (4-423)

These equations can be combined with the standard equations of motion and integrated to find the satellites orbit. The largest obstruction to this is finding the correct value(s) of  $p$  to use in the model. Both altitude and solar flux affect  $p$  and must be accounted for.

## CURRENT MODELS

Satellite decay is covered by several models. These can be either estimating type models such as that developed in this thesis, or atmospheric density models which compute values of  $p$  for use in the equations of motion. Each type of model has advantages and disadvantages. If accuracy is required and values of solar flux are known, then the atmospheric density models are best. If solar flux values are unknown or long time estimates are needed, estimating models are more suitable. The exact choice of model must be made in view of the product desired.

Of the atmospheric density models, one of the most useful to date is the Jacchia atmospheric model. There are several versions of the Jacchia atmosphere model. One of them, the J65 model produces the density ( $p$ ) values through a table "look up" method. It uses the values of the solar flux to find density values in a table. It relates values of the solar flux at a wavelength of 10.7 cm to density given in a table compiled by Nicolet (2-B14). It should be noted that the 10.7 cm flux ( $F_{10.7}$ ) does not heat the upper atmosphere. The upper atmosphere is heated by the extreme ultra violet (EUV) radiation which cannot be observed at the earth's surface, whereas the  $F_{10.7}$  flux can be measured (2-B16). The J70 model uses both an average value of  $F_{10.7}$  taken at a time of  $t - 400$  days and the changing daily value for a time lag of one day such that:

$$T = 383 + 3.3\bar{F} + 1.8(F - \bar{F}) \quad (2.5)$$

where  $\bar{F}$  = average value of F10.7 flux at t-400 days

F = previous days value of F10.7 flux

T = temperature (2-B17)

The J70 model can then calculate the density through suitable manipulation of thermodynamic gas laws.

Other models produce the p values in different ways. For example, the DENSEL model uses a power function in altitude and theoretical values of F10.7 (2-B14,B16) to find p. There is also the exponential atmosphere:

$$p = p_0 \exp[-c(h - h_0)] \quad (2.6)$$

where  $p_0$  = density at height  $h_0$

c = a constant (2-B13)

The exponential atmosphere is similar to the model used by Desmond King-Hele to determine satellite lifetimes, where King-Hele uses:

$$p = p_0 \exp[-(y - y_0)/H] \quad (2.7)$$

where  $p_0$  = density at  $y_0$

H = scale height (5-182)

and then:

$$z = ae/H \quad (2.8)$$

where a = semi major axis

e = orbital eccentricity (5-182)

so that the satellite lifetime L may be written as:

$$L = \frac{3en}{4\dot{n}} \left[ \frac{1+7e+5e^2}{6} + \frac{1}{16z} \left( \frac{1+11e+3e^2}{12} + \frac{3}{4z} + \frac{3}{4z^2} \right) + 0(e, 0.5/z) \right] \quad (5-182) \quad (2.9)$$

Any of the models that calculate density are accurate for the time calculated, but are valid for only a short time before the orbit changes beyond prediction limits. This is due to uncertainty of future solar flux, and, therefore, atmospheric density values. Another problem with this type of model is the calculation time required, on the order of 31 seconds to produce the densities (2-3), and minutes to hours to integrate the equations of motion for each satellite. This quickly grows beyond acceptable limits when dealing with many satellites.

A method for predicting long term changes is in use by Naval Space Surveillance (NAVSPASUR). This method is based on calculating values of PDOT. PDOT is the change in period per day of a satellites orbit. It is not highly accurate, but offers two advantages:

- 1) It gives a general indication of a satellites period over very long time spans.
- 2) It is very quick, requiring less than a minute on a hand-held calculator. The equation used by this method is:

$$P(t) = P(0) + \dot{P}t \quad (2.10)$$

This method is used by NAVSPASUR and NORAD to determine if a lost satellite may have decayed. Because of the inaccuracies, the satellite is not counted as decayed unless the period has been below 87.5 minutes (the standard cutoff for an orbit) for over a month, and there are no possible unknown satellites between its old position and possible current positions. In practice the use of this method is more an art

than a science. It is only used by the most qualified analysts because a great deal of judgment on the part of the analyst is required.

A third method, used mostly by NORAD, is based upon NDOT. NDOT is the difference between a Brouwer and a Kozai mean motion for the same time (1). Brouwer and Kozai are two methods of calculating mean motion used in the NORAD system. The method is very similar to the PDOT except that the change is calculated for mean motion instead of period. In application the equation used is:

$$n(t) = n(0) + 2(\dot{N}/2)dt \quad (2.11)$$

Like the PDOT, the NDOT requires experience to use and produces similar results.

The primary model developed in this thesis is based upon the NDOT model. It is employed in much the same manner as described above but includes an allowance for solar flux and error boundaries on the accuracy of the calculated value of  $n$ . This provides a measure of the uncertainty in predicting a new value for  $n$ . Solar flux is addressed by calculating an average  $\dot{N}/2$  from several points out of a 30 day interval since solar flux varies over about a 27 day interval (6-189).

#### MEASURES OF EFFECTIVENESS

The accuracy limits are determined to allow a reasonable prediction at 100 days and still have a recoverable satellite. From experience this was set at about 0.1 min/rev which gives an error limit of 0.01595 rev/day (for a 95 minute orbit). This is an error range of 1.58 revs at 100 days.



### III. METHODOLOGY

#### DATA

The data used in this thesis was provided by NORAD/HDS. The data consisted of 50 days of element sets on a group of 5 random satellites chosen from a set of 70 satellites with mean motions between 16 and 14 rev/day. The satellites were chosen at random and were not given individual identifiers to eliminate the possibility of selecting data with some bias. The following data on each of the 5 satellites was collected: time, delta time,  $\dot{n}(t)$ ,  $\dot{n}(t-dt)$ ,  $n(t)$ ,  $n(t=0)$ .

In addition to these data, the values of F10.7 solar flux for the epoch date, the change in F10.7 flux from the previous day and the change in F10.7 flux on the previous day were added. This information was placed in a file on the Cyber computer. Appendix A contains a listing of this file. The data are in the format:

$t, dt, \dot{n}(t), \dot{n}(t-dt), n(t), n(0), S, F, SF.$

The values of  $\dot{n}$  were calculated by equation 2.1 and placed in the file along with the data transferred from the NORAD elsets.

The data in this file were then used in the program BMDP to fit the models to the data.

## MODEL DEVELOPMENT

A model for  $n$  may be developed in several ways. One of the models used at NORAD involves a Taylor series expansion of  $n$  to give:

$$n(t) = n(0) + \dot{n}(t - t_0) + \frac{1}{2}\ddot{n}(t - t_0)^2 + \dots \quad (3.1)$$

where NORAD defined  $\dot{n}$  as  $2(\dot{N}/2)$  and  $\ddot{n}$  as  $3(\ddot{N}/6)$  so that:

$$n(t) = n(0) + 2(\dot{N}/2) (t - t_0) + 3(\ddot{N}/6) (t - t_0)^2 + \dots \quad (3.2)$$

and  $t - t_0$  is the time of interest minus the initial time. The second and higher order terms are of small magnitude compared to  $n$  for the normal values of  $(t - t_0)$  encountered in element propagation. Therefore the second and higher order terms are set to zero. In normal usage the equation used for  $n$  is:

$$n(t) = n(0) + 2(\dot{N}/2) (t - t_0) \quad (3.3)$$

That this formula is appropriate can be seen from figure 3.1 which is a plot of mean motion versus time. This figure shows that the value of  $n$  at any one time depends on the value of  $n$  at some previous time plus the sum of all the changes in  $n$  over the time difference. The sum of all changes can also be considered as an integral over all changes. Under the assumption that all changes over time greater than second order are very small and can be considered zero, the second order change,  $\ddot{n}$ , can be considered a constant. That is:

$$\ddot{n} = a \quad (3.4)$$

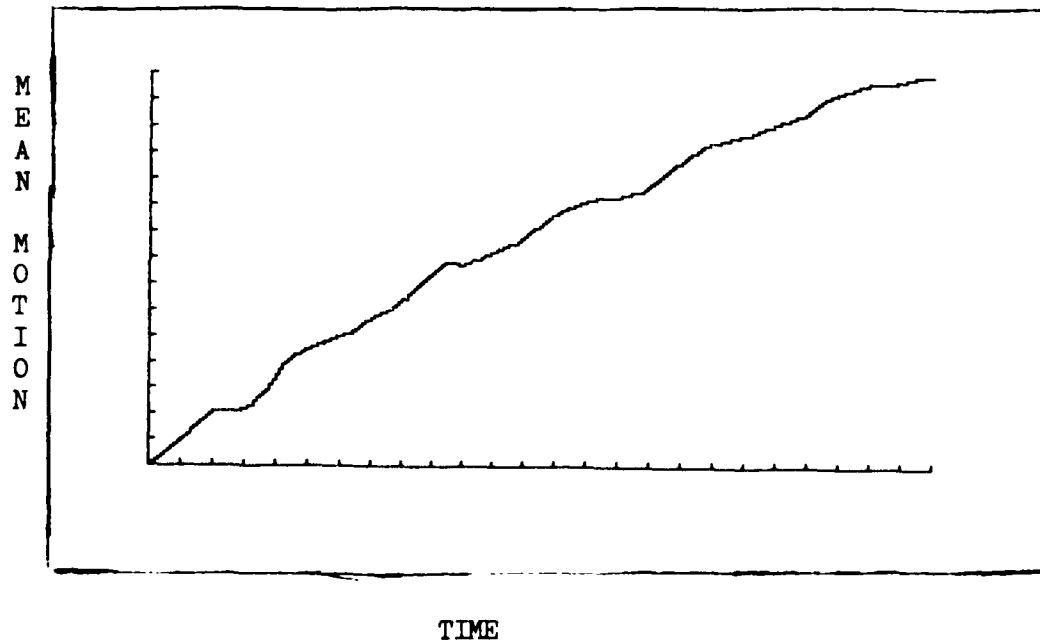


Figure 3.1 Typical Mean Motion for days 84300 through 85084

And since:

$$\ddot{n} = \frac{d^2 n}{dt^2} = \frac{d\dot{n}}{dt} \quad (3.5)$$

This gives:

$$\frac{d\dot{n}}{dt} = a \quad (3.6)$$

Which may be written as:

$$d\dot{n} = a dt \quad (3.7)$$

Equation 3.7 may be integrated:

$$\int d\dot{n} = a \int dt \quad (3.8)$$

$$\dot{n} - \dot{n}_0 = a(t - t_0) \quad (3.9)$$

$$\dot{n} = \dot{n}_0 + a(t - t_0) \quad (3.10)$$

And since:

$$\dot{n} = \frac{dn}{dt} \quad (3.11)$$

$$\frac{dn}{dt} = \dot{n}_0 + a(t - t_0) \quad (3.12)$$

$$dn = \left[ \dot{n}_0 + a(t - t_0) \right] dt \quad (3.13)$$

Integrating equation 3.13 gives:

$$\int dn = \int [\dot{n}_0 + a(t - t_0)] dt \quad (3.14)$$

$$n - n_0 = \dot{n}_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \quad (3.15)$$

$$n = n_0 + \dot{n}_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \quad (3.16)$$

Substituting in equation 3.4 lead to an equation of the form:

$$n(t) = n(0) + \dot{n}(t - t_0) + \frac{1}{2}\ddot{n}(t - t_0)^2 \quad (3.17)$$

This equation is basically the same as that given by a Taylor series, but is arrived at from physical considerations.

From these come two of the models that will be studied in this thesis. Equation 3.17 is the baseline n0 model and is used in BMDP in modified form. Equation 3.3 is the standard NORAD model (herein called model n5).

Model n0 appears in this thesis in three basic forms. It appears in its original version as given by equation 3.17. There are also two modified versions which are used in BMDP. They include model n1 which included regression coefficients in each term and has the intercept defined at  $B_1 n(0)$  such that:

$$n1: \quad n(t) = B_1 n(0) + B_2 \dot{n}(t - t_0) + 0.5B_3 \ddot{n}(t - t_0)^2 \quad (3.18)$$

where the B's are the regression coefficients from BMDP. Next, under the assumption that the regression coefficients contain the physical

constants and the solar flux dependence, and that long term average solar effects are contained in the intercept since over time flux approaches a constant gives:

$$n_2: \quad n(t) = B_0 + B_1 n(0) + B_2 \dot{n}(t - t_0) + B_3 \ddot{n}(t - t_0)^2 \quad (3.19)$$

Next, assuming there is no  $(t - t_0)^2$  dependence, that is, the satellites "forgets" earlier changes such that there is no second order time dependence, gives model n3:

$$n_3: \quad n(t) = B_0 + B_1 n(0) + B_2 \dot{n}(t - t_0) + B_3 \ddot{n}(t - t_0) \quad (3.20)$$

The final model comes from assuming a simple linear relation between the terms. Model n4 is:

$$n_4: \quad n(t) = B_0 + B_1(t - t_0) + B_2 \dot{n} + B_3 n(0) + B_4 F \quad (3.21)$$

Note that only model n4 contains solar flux explicitly, where it is given as the change in the flux for time t-1. Solar flux changes are contained implicitly within the regression coefficients and the  $\dot{n}$  and  $\ddot{n}$  terms in models n1, n2, and n3. The reason for this can be seen by comparison of figures 3.1 and 3.2, where figure 3.2 is a plot of solar flux for the same time frame as figure 3.1. It is obvious from the graph that the daily solar flux value has little long term effect on individual values of n.

Likewise, a comparison of figure 3.2 and figure 3.3 (which is a graph of the daily change in n, i.e.  $\dot{n}$ ) shows a much higher correlation between  $\dot{n}$  and flux. That this is indeed the case will be shown by the data analysis carried out in chapter four.

None of the models contains orbital elements other than those related to mean motion. Eccentricity is not included because it is near zero for the orbits being studied. While it is known that atmospheric density is not constant around an orbit (density changes with latitude and darkness), it is assumed in this thesis that effects relating to inclination and other elements are very small for time spans on the order of 100 days. Their inclusion in a model is an area for additional study.

Below is given a complete list of the models that will be used in the analysis by BMDP, where  $t$  is now defined as  $t - t_0$ .

$$n0: n = n_0 + \dot{n}t + 0.5\ddot{n}t^2 \quad (3.17)$$

$$n1: n = B_1 n_0 + B_2 \dot{n}t + 0.5B_3 \ddot{n}t^2 \quad (3.18)$$

$$n2: n = B_0 + B_1 n_0 + B_2 \dot{n}t + B_3 \ddot{n}t^2 \quad (3.19)$$

$$n3: n = B_0 + B_1 n + B_2 \dot{n}t + B_3 \ddot{n}t \quad (3.20)$$

$$n4: n = B_0 + B_1 t + B_2 \dot{n} + B_3 n_0 + B_4 F \quad (3.21)$$

$$n5: n = n + 2(\dot{N}/2)t \quad (3.3)$$

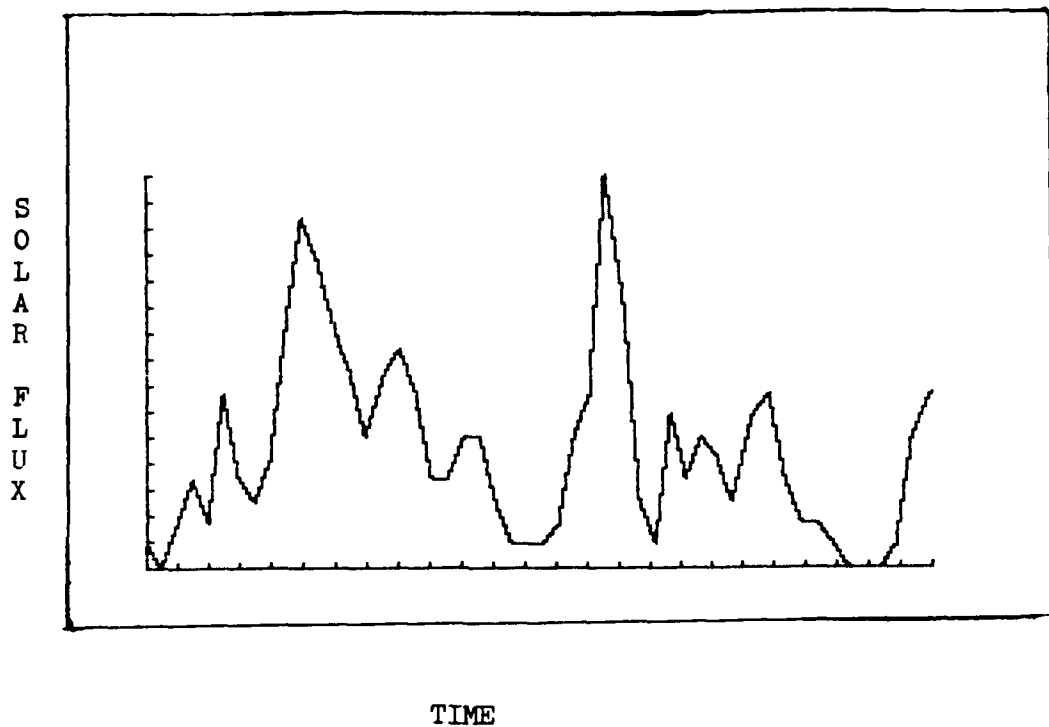


Figure 3.2 Solar Flux for days 84300 through 85084



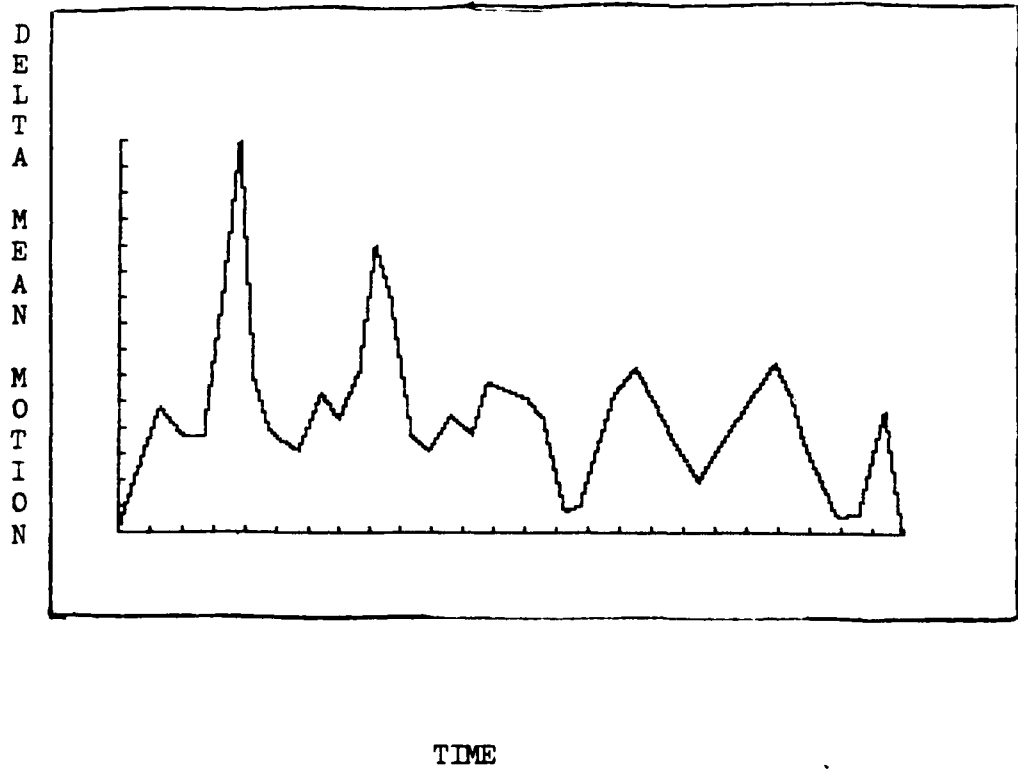


Figure 3.3 Daily Change in Mean Motion for days 84300 through 85084

## IV. RESULTS

### SOLAR FLUX

In chapter three it was stated that solar flux was not a major explicit factor in predicting values of  $n$ . Some justification, on the basis of graphical analysis, was given. Further empirical justification will be given here.

In the BMDP analysis of model n4 it is shown that there is only a 0.0122 correlation between  $F$  and  $n$  (see Table 4.4), while the regression coefficient is of the order of  $10E-4$ . This shows that flux has little effect on  $n$  directly. A separate analysis shows that there is a -0.1021 correlation between  $F$  and  $\dot{n}$ .

Also, if flux is considered as a cumulative average and the daily change from that average, it is seen that the average rapidly approaches a constant plus or minus the daily change. If this constant is included in the regression equations, it must be in the value of the intercept constant. The daily change from a constant will be in the daily values of the second and higher order terms of the  $n$  equation. That is, since for a nearly circular orbit there is no altitude dependence for drag, the rate of decay will contain a constant value for the average density at that altitude. This constant is the first order term,  $\dot{n}$ . The rate of change of  $\dot{n}$ ,  $\ddot{n}$  will be constant for a constant change in solar flux. Since flux is not constant on a daily basis, there will be a change in  $\ddot{n}$ , given by the third and higher order terms, which are nearly zero and are defined as zero such that  $\ddot{n}$  is a constant of small magnitude. Therefore the statement that the solar flux dependence is

implicit in n and the regression coefficients is supported.

### BMDP ANALYSIS

The models included in the BMDP analysis were:

$$n1: n = B_1 n_o + B_2 \dot{n}t + 0.5 B_3 \ddot{n}t^2 \quad (3.18)$$

$$n2: n = B_o + B_1 n_o + B_2 \dot{n}t + B_3 \ddot{n}t^2 \quad (3.19)$$

$$n3: n = B_o + B_1 n_o + B_2 \dot{n}t + B_3 \ddot{n}t \quad (3.20)$$

$$n4: n = B_o + B_1 t + B_2 \dot{n} + B_3 n_o + B_4 F \quad (3.21)$$

BMDP was run against the data file in Appendix A using each of these models independently. The output from BMDP produced the results shown in tables 4.1 through 4.4.

TABLE 4.1

BMDP RESULTS FOR MODEL N1

---

Model            n1:  $n = B_1 n_o + B_2 \dot{n}t + 0.5 B_3 \ddot{n}t^2$             (3.18)

---

Multiple R=1     $R^2=1$     standard error=0.0118

$B_1=1$              $B_2=1.06457$              $B_3=0.01970$

$n = n_o + 1.06457 \dot{n}t + (0.0197/2) \ddot{n}t^2$

---

|            | Sum of Squares | DF | Mean Square | Ratio        | P   |
|------------|----------------|----|-------------|--------------|-----|
| Regression | 22260.2993     | 3  | 7420.0998   | 53696658.325 | 0.0 |
| Residual   | 0.0130         | 94 | 0.0001      |              |     |

---

| Correlation with n | $n_o$  | $\dot{n}$ | $\ddot{n}$ | $\dot{n}t$ | $\ddot{n}t^2$ |
|--------------------|--------|-----------|------------|------------|---------------|
|                    | 0.9962 | 0.8274    | 0.1138     | 0.7135     | 0.1118        |

---

TABLE 4.2

BMDP RESULTS FOR MODEL N2

Model            n2:  $n = B_0 + B_1 n_0 + B_2 \dot{n}t + B_3 \ddot{n}t^2$             (3.19)

Multiple R=0.9992       $R^2=0.9983$       standard error=0.0115

$B_0=-0.20596$        $B_1=1.01384$        $B_2=0.98394$        $B_3=0.00935$

$n=-0.20596+1.01384n_0+0.98394\dot{n}t+0.00935\ddot{n}t^2$

|            | Sum of Squares | DF | Mean Square | Ratio     | P     |
|------------|----------------|----|-------------|-----------|-------|
| Regression | 7.3171         | 3  | 2.4390      | 18347.738 | 0.000 |
| Residual   | 0.0124         | 93 | 0.0001      |           |       |

| Correlation | $n_0$  | $\dot{n}$ | $\ddot{n}$ | $\dot{n}t$ | $\ddot{n}t^2$ |
|-------------|--------|-----------|------------|------------|---------------|
| with n      | 0.9962 | 0.8274    | 0.1138     | 0.7135     | 0.1118        |

TABLE 4.3

BMDP RESULTS FOR MODEL N3

---

Model            n3:  $n = B_0 + B_1 n_0 + B_2 \dot{n}t + B_3 \ddot{n}t$             (3.20)

---

Multiple R=0.9992       $R^2=0.9983$       standard error=0.0114

$B_0=-0.18935$        $B_1=1.01273$        $B_2=0.99835$        $B_3=0.43515$

$n=-0.18935+1.01273n_0+0.99835\dot{n}t+0.43515\ddot{n}t$

---

|            | Sum of Squares | DF | Mean Square | Ratio     | P    |
|------------|----------------|----|-------------|-----------|------|
| Regression | 7.3174         | 3  | 2.4391      | 18659.405 | 0.00 |
| Residual   | 0.0122         | 93 | 0.0001      |           |      |

---

| Correlation with n | $n_0$  | $\dot{n}$ | $\ddot{n}$ | $\dot{n}t$ | $\ddot{n}t$ |
|--------------------|--------|-----------|------------|------------|-------------|
|                    | 0.9962 | 0.8274    | 0.1138     | 0.7135     | 0.1089      |

---

TABLE 4.4

BMDP RESULTS FOR MODEL N4

---

Model      n4:  $n = B_0 + B_1 t + B_2 \dot{n} + B_3 n_0 + E_4 F$       (3.21)

---

Multiple      R=0.9981       $R^2=0.9962$       standard error=0.0174

$B_0=-0.67573$        $B_1=0.00090517$        $B_2=17.45671$        $B_3=1.04379$

$B_4=-0.000097511$

$n=-0.67573+(9.0517E-4)t+17.45671\dot{n}+1.04379n_0+(-9.7511E-5)F$

---

|            | Sum of Squares | DF | Mean Square | Ratio    | P     |
|------------|----------------|----|-------------|----------|-------|
| Regression | 7.7315         | 4  | 1.9329      | 6366.816 | 0.000 |
| Residual   | 0.0294         | 97 | 0.0003      |          |       |

---

|                    | t       | $\dot{n}$ | $n_0$  | F      |
|--------------------|---------|-----------|--------|--------|
| Correlation with n | -0.0023 | 0.8160    | 0.9963 | 0.0122 |

---

## POST BMDP ANALYSIS

The equations produced by BMDP, along with  $n_0$  and  $n_5$ , were used to generate predicted  $n$  values for satellites 15363, 14476, 13043, and 7840. The predictions were done for time spans of 10, 30, 50, and 100 days (as close to these values as possible given the NORAD data). The calculated value of  $n$  was subtracted from the actual value of  $n$  at each time to give a value for the error in  $n$  such that a negative error indicates that the calculated value was greater than the actual value at time  $t$ . Tables 4.5 through 4.11 contain the sample data for these four satellites for each model. The values on  $\dot{n}$  and  $\ddot{n}$  were calculated by equations 2.1 and 2.2, and are shown in Table 4.5.

TABLE 4.5  
VALUES OF  $\dot{n}$ ,  $\ddot{n}$ , AND  $\dot{N}/2$  FOR EACH SATELLITE

| SATELLITE | $\dot{n}$  | $\ddot{n}$ | $\dot{N}/2$ |
|-----------|------------|------------|-------------|
| 15363     | 0.00061318 | 0.00007924 | 0.00033037  |
| 14476     | 0.00002221 | 0.00000051 | 0.00001155  |
| 13043     | 0.00049423 | 0.00001846 | 0.00027705  |
| 7840      | 0.00002121 | 0.00000222 | 0.00001401  |



TABLE 4.6

## MEAN MOTION VALUES FOR MODEL NO

| SATELLITE | t<br>days | ACTUAL n<br>rev/day | CALCULATED n<br>rev/day | ERROR<br>rev/day |
|-----------|-----------|---------------------|-------------------------|------------------|
| 15363     | 0         | 14.61592219         | -----                   | -----            |
|           | 10        | 14.62055640         | 14.6260108              | -0.00545440      |
|           | 30        | 14.62850624         | 14.6699376              | -0.04143136      |
|           | 50        | 14.63611823         | 14.7455305              | -0.10941227      |
|           | 100       | 14.65988312         | 15.0730518              | -0.41316868      |
| 14476     | 0         | 14.47940319         | -----                   | -----            |
|           | 10        | 14.47959223         | 14.4796510              | -0.00005877      |
|           | 31        | 14.48005842         | 14.4803389              | -0.00028048      |
|           | 52        | 14.48061500         | 14.4812537              | -0.00063870      |
|           | 102       | 14.48162641         | 14.4843450              | -0.00271859      |
| 13043     | 0         | 15.66287477         | -----                   | -----            |
|           | 10        | 15.66745489         | 15.6687299              | -0.00128501      |
|           | 30        | 15.67900804         | 15.6860070              | -0.00699896      |
|           | 50        | 15.68914747         | 15.7106564              | -0.02150893      |
|           | 100       | 15.71437096         | 15.8045781              | -0.09020714      |
| 7840      | 0         | 14.56440052         | -----                   | -----            |
|           | 10        | 14.56459216         | 14.5647237              | -0.00013154      |
|           | 29        | 14.56491839         | 14.5659499              | -0.00103151      |
|           | 52        | 14.56555106         | 14.5685076              | -0.00295654      |
|           | 103       | 14.56649157         | 14.5783721              | -0.01188053      |

TABLE 4.7

## MEAN MOTION VALUES FOR MODEL N1

| SATELLITE | t<br>days | ACTUAL n<br>rev/day | CALCULATED n<br>rev/day | ERROR<br>rev/day |
|-----------|-----------|---------------------|-------------------------|------------------|
| 15363     | 0         | 14.61592219         | -----                   | -----            |
|           | 10        | 14.62055640         | 14.6225264              | -0.00197000      |
|           | 30        | 14.62850624         | 14.6362026              | -0.00769626      |
|           | 50        | 14.63611823         | 14.6505026              | -0.05286367      |
|           | 100       | 14.65988312         | 14.6889819              | -0.02909878      |
| 14476     | 0         | 14.47940319         | -----                   | -----            |
|           | 10        | 14.47959223         | 14.4796401              | -0.00004787      |
|           | 31        | 14.48005842         | 14.4801410              | -0.00008258      |
|           | 52        | 14.48061500         | 14.4806463              | -0.00003130      |
|           | 102       | 14.48162641         | 14.4818675              | -0.00024109      |
| 13043     | 0         | 15.66287477         | -----                   | -----            |
|           | 10        | 15.66745489         | 15.6681544              | -0.00069951      |
|           | 30        | 15.67900804         | 15.6788227              | 0.00018534       |
|           | 50        | 15.68914747         | 15.6896365              | -0.00048903      |
|           | 100       | 15.71437096         | 15.7173072              | -0.00293624      |
| 7840      | 0         | 14.56440052         | -----                   | -----            |
|           | 10        | 14.56459216         | 14.5646204              | -0.00002824      |
|           | 29        | 14.56491839         | 14.5650397              | -0.00012131      |
|           | 52        | 14.56555106         | 14.5655565              | -0.00000551      |
|           | 103       | 14.56649157         | 14.5668036              | -0.00031203      |

TABLE 4.8

## MEAN MOTION VALUES FOR MODEL N2

| SATELLITE | t<br>days | ACTUAL n<br>rev/day | CALCULATED n<br>rev/day | ERROR<br>rev/day |
|-----------|-----------|---------------------|-------------------------|------------------|
| 15363     | 0         | 14.61592219         | -----                   | -----            |
|           | 10        | 14.62055640         | 14.6183525              | 0.00220390       |
|           | 30        | 14.62850624         | 14.6310084              | -0.00250216      |
|           | 50        | 14.63611823         | 14.6442566              | -0.00813837      |
|           | 100       | 14.65988312         | 14.6799675              | -0.01979188      |
| 14476     | 0         | 14.47940319         | -----                   | -----            |
|           | 10        | 14.47959223         | 14.4745202              | 0.00553513       |
|           | 31        | 14.48005842         | 14.4745202              | 0.00553822       |
|           | 52        | 14.48061500         | 14.4749875              | 0.00562750       |
|           | 102       | 14.48162641         | 14.4751171              | 0.00550931       |
| 13043     | 0         | 15.66287477         | -----                   | -----            |
|           | 10        | 15.66745489         | 15.6785692              | -0.01111431      |
|           | 30        | 15.67900804         | 15.6884331              | -0.00942506      |
|           | 50        | 15.68914747         | 15.6984351              | -0.00928763      |
|           | 100       | 15.71437096         | 15.7240441              | -0.00967314      |
| 7840      | 0         | 14.56440052         | -----                   | -----            |
|           | 10        | 14.56459216         | 14.5602155              | 0.00437666       |
|           | 29        | 14.56491839         | 14.5606034              | 0.00431499       |
|           | 52        | 14.56555106         | 14.5610904              | 0.00446066       |
|           | 103       | 14.56649157         | 14.5622384              | 0.00425317       |

TABLE 4.9

## MEAN MOTION VALUES FOR MODEL N3

| SATELLITE | t<br>days | ACTUAL n<br>rev/day | CALCULATED n<br>rev/day | ERROR<br>rev/day |
|-----------|-----------|---------------------|-------------------------|------------------|
| 15363     | 0         | 14.61592219         | -----                   | -----            |
|           | 10        | 14.62055640         | 14.6190976              | 0.00145880       |
|           | 30        | 14.62850624         | 14.6320271              | -0.00352086      |
|           | 50        | 14.63611823         | 14.6449565              | -0.00883827      |
|           | 100       | 14.65988312         | 14.6772802              | -0.01739708      |
| 14476     | 0         | 14.47940319         | -----                   | -----            |
|           | 10        | 14.47959223         | 14.4746000              | 0.00492230       |
|           | 31        | 14.48005842         | 14.4750703              | 0.00498812       |
|           | 52        | 14.48061500         | 14.4755406              | 0.00050744       |
|           | 102       | 14.48162641         | 14.4766604              | 0.00496601       |
| 13043     | 0         | 15.66287477         | -----                   | -----            |
|           | 10        | 15.66745489         | 15.6779276              | -0.01047271      |
|           | 30        | 15.67900804         | 15.6879566              | -0.00894796      |
|           | 50        | 15.68914747         | 15.6979856              | -0.00883813      |
|           | 100       | 15.71437096         | 15.7230580              | -0.00868704      |
| 7840      | 0         | 14.56440052         | -----                   | -----            |
|           | 10        | 14.56459216         | 14.5606685              | 0.00392366       |
|           | 29        | 14.56491839         | 14.5610645              | 0.00385389       |
|           | 52        | 14.56555106         | 14.5615438              | 0.00400726       |
|           | 103       | 14.56649157         | 14.5620670              | 0.00442457       |

TABLE 4.10

## MEAN MOTION VALUES FOR MODEL N4

| SATELLITE | t<br>days | ACTUAL n<br>rev/day | CALCULATED n<br>rev/day | ERROR<br>rev/day |
|-----------|-----------|---------------------|-------------------------|------------------|
| 15363     | 0         | 14.61592219         | -----                   | -----            |
|           | 10        | 14.62055640         | 14.5997121              | 0.02084430       |
|           | 30        | 14.62850624         | 14.6182269              | 0.01027934       |
|           | 50        | 14.63611823         | 14.6361425              | -0.00002427      |
|           | 100       | 14.65988312         | 14.6812678              | -0.05138468      |
| 14476     | 0         | 14.47940319         | -----                   | -----            |
|           | 10        | 14.47959223         | 14.4470907              | 0.03250153       |
|           | 31        | 14.48005842         | 14.4662946              | 0.01376382       |
|           | 52        | 14.48061500         | 14.4851002              | -0.00448520      |
|           | 102       | 14.48162641         | 14.5303213              | -0.04869489      |
| 13043     | 0         | 15.66287477         | -----                   | -----            |
|           | 10        | 15.66745489         | 15.6907502              | -0.03004711      |
|           | 30        | 15.67900804         | 15.7087211              | -0.02971306      |
|           | 50        | 15.68914747         | 15.7269894              | -0.03784193      |
|           | 100       | 15.71437096         | 15.7722919              | -0.05792094      |
| 7840      | 0         | 14.56440052         | -----                   | -----            |
|           | 10        | 14.56459216         | 14.5358578              | 0.02873436       |
|           | 29        | 14.56491839         | 14.5529297              | 0.01198869       |
|           | 52        | 14.5655106          | 14.5740243              | -0.00847324      |
|           | 103       | 14.56649157         | 14.6201866              | -0.05369503      |

TABLE 4.11

## MEAN MOTION VALUES FOR MODEL N5

| SATELLITE | t<br>days | ACTUAL n<br>rev/day | CALCULATED n<br>rev/day | ERROR<br>rev/day |
|-----------|-----------|---------------------|-------------------------|------------------|
| 15363     | 0         | 14.61592219         | -----                   | -----            |
|           | 10        | 14.62055640         | 14.6225296              | -0.00198320      |
|           | 30        | 14.62850624         | 14.6357444              | -0.00723816      |
|           | 50        | 14.63611823         | 14.6489592              | -0.01284097      |
|           | 100       | 14.65988312         | 14.6819962              | -0.02211308      |
| 14476     | 0         | 14.47940319         | -----                   | -----            |
|           | 10        | 14.47959223         | 14.4796342              | -0.00004197      |
|           | 31        | 14.48005842         | 14.4801193              | -0.00006088      |
|           | 52        | 14.48061500         | 14.4806044              | 0.00001060       |
|           | 102       | 14.48162641         | 14.4817594              | -0.00013299      |
| 13043     | 0         | 15.66287477         | -----                   | -----            |
|           | 10        | 15.66745489         | 15.6684158              | -0.00096091      |
|           | 30        | 15.67900804         | 15.6794978              | -0.00048976      |
|           | 50        | 15.68914747         | 15.6905798              | -0.00143233      |
|           | 100       | 15.71437096         | 15.7182848              | -0.00391384      |
| 7840      | 0         | 14.56440052         | -----                   | -----            |
|           | 10        | 14.56459216         | 14.5646807              | -0.00021484      |
|           | 29        | 14.56491839         | 14.5652131              | -0.00029471      |
|           | 52        | 14.56555106         | 14.5658576              | -0.00030654      |
|           | 103       | 14.56649157         | 14.5672866              | -0.00079503      |

Since the criteria for quality of a model was set at 0.01595 rev/day at 100 days, the absolute value of the error at 100 days (as near to 100 days as the data allowed) was averaged over the four satellites for each model. This average error was used to measure the model quality and compare the models to each other. Table 4.12 compares the 100 day error values of all satellites and all models.

TABLE 4.12

## 100 DAY ERROR ABSOLUTE VALUES FOR ALL SATELLITES AND MODELS

| MODEL/SATELLITE | 15363      | 14476      | 13043      | 7840       |
|-----------------|------------|------------|------------|------------|
| n0:             | 0.41316868 | 0.00271859 | 0.09020714 | 0.01188053 |
| n1:             | 0.02909878 | 0.00024109 | 0.00293624 | 0.00031203 |
| n2:             | 0.01979188 | 0.00550931 | 0.00967314 | 0.00425317 |
| n3:             | 0.01739708 | 0.00496601 | 0.00868704 | 0.00442457 |
| n4:             | 0.05138468 | 0.04869489 | 0.05792094 | 0.05369503 |
| n5:             | 0.02211308 | 0.00013299 | 0.00391384 | 0.00079503 |

Table 4.13 shows the average of the 100 day error values for each model in order from smallest to largest.

TABLE 4.13

## AVERAGE 100 DAY ERROR VALUES FOR EACH MODEL

| MODEL | ERROR rev/day |
|-------|---------------|
| n5    | 0.0067387     |
| n1    | 0.0081470     |
| n3    | 0.0088687     |
| n2    | 0.0098068     |
| n4    | 0.0529239     |
| n0    | 0.1295098     |

Analysis of the data for each model indicates the following:

- 1) Models n0 and n4 have a 100 day error greater than 0.01595 rev/day and are removed from consideration.
- 2) Models n1, n2, n3, and n5 all have acceptable 100 day error values as shown in table 4.14.

TABLE 4.14

## AVERAGE 100 DAY ERROR VALUES FOR ACCEPTABLE MODELS

| MODEL | ERROR rev/day |
|-------|---------------|
| n5    | 0.0067387     |
| n1    | 0.0081470     |
| n3    | 0.0088687     |
| n2    | 0.0098068     |

3) Model n5 is the best in terms of 100 day error. It also produces very accurate values of  $n$  at times other than 100 days.

4) Models n2 and n3 are very consistent in the magnitude of the error for any satellite as shown in Tables 4.8 and 4.9. This is shown in figures 4.1 and 4.2 which are plots of actual and calculated values on  $n$ .

In studying the data it was found that the greatest error, for example for satellite 15363, occurred when  $\dot{n}$  was unstable. To eliminate this,  $\dot{N}/2$  for satellite 15363 was averaged over a 30 day span using 10 day intervals. This decreased the 100 day error for that satellite by a factor of seven in the n5 model. The 30 day time span is indicated by the about 27 day length of the solar cycle. Since this first test improved the results a general test was done using equations 2.1 and 2.2 for  $\dot{n}$  and  $\ddot{n}$ . The times were chosen to be at the beginning, middle, and end of the 30 day period. Averaging over this time seemed to eliminate some of the randomness in the flux dependent values. The results are shown in tables 4.15 through 4.19.



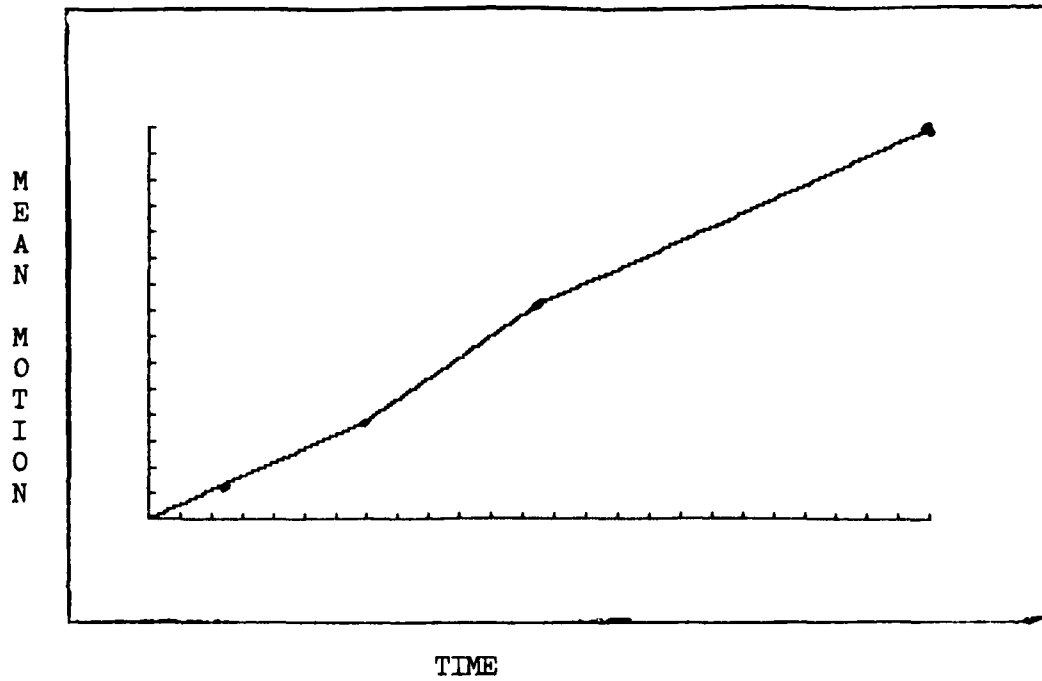
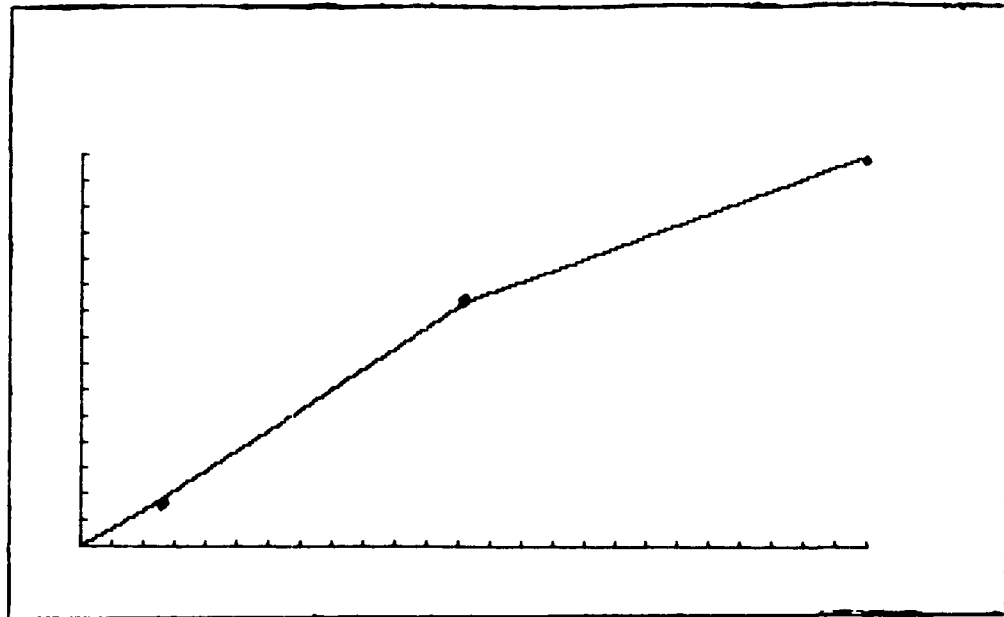


Figure 4.1 Actual and Calculated values of  $n$  for Satellite 7840 using Unaveraged  $\dot{n}$  and  $\ddot{n}$

M  
E  
A  
N  
  
M  
O  
T  
I  
O  
N



T  
I  
M  
E

Figure 4.2 Actual and Calculated values of  $n$  for Satellite 7840 using Averaged  $\dot{n}$  and  $\ddot{n}$

TABLE 4.15

AVERAGE VALUES OF  $\dot{n}$  AND  $\ddot{n}$  FOR EACH SATELLITE

| SATELLITE | $\dot{n}$  | $\ddot{n}$ | $\ddot{N}/2$ |
|-----------|------------|------------|--------------|
| 15363     | 0.00044071 | 0.00000295 | 0.00023550   |
| 14476     | 0.00001892 | 0.00000029 | 0.00001156   |
| 13043     | 0.00049658 | 0.00000552 | 0.00026520   |
| 7840      | 0.00001943 | 0.00000022 | 0.00001057   |

TABLE 4.16

MEAN MOTION VALUES FOR MODEL N1

| SATELLITE | t<br>days | ACTUAL n<br>rev/day | CALCULATED n<br>rev/day | ERROR<br>rev/day |
|-----------|-----------|---------------------|-------------------------|------------------|
| 15363     | 0         | 14.62860624         | -----                   | -----            |
|           | 10        | 14.63270865         | 14.63320080             | -0.00049215      |
|           | 50        | 14.64873275         | 14.65203730             | -0.00330445      |
|           | 100       | 14.67174206         | 14.67571370             | -0.00397164      |
| 14476     | 0         | 14.48005842         | -----                   | -----            |
|           | 9         | 14.48028419         | 14.48023990             | 0.00004429       |
|           | 51        | 14.48126626         | 14.48109410             | 0.00017316       |
|           | 101       | 14.48210420         | 14.48212170             | -0.00001750      |
| 13043     | 0         | 15.67900804         | -----                   | -----            |
|           | 11        | 15.68473145         | 15.68482970             | -0.00009825      |
|           | 50        | 15.70452036         | 15.70557610             | -0.00105574      |
|           | 100       | 15.73148592         | 15.73241580             | -0.00092988      |
| 7840      | 0         | 14.56491839         | -----                   | -----            |
|           | 11        | 14.56516626         | 14.56514620             | 0.00002006       |
|           | 52        | 14.56622862         | 14.56599960             | -0.00022902      |
|           | 105       | 14.56703165         | 14.56711360             | -0.00008195      |

TABLE 4.17

## MEAN MOTION VALUES FOR MODEL N2

| SATELLITE | t<br>days | ACTUAL n<br>rev/day | CALCULATED n<br>rev/day | ERROR<br>rev/day |
|-----------|-----------|---------------------|-------------------------|------------------|
| 15363     | 0         | 14.62860624         | -----                   | -----            |
|           | 10        | 14.63270865         | 14.62934390             | 0.00336475       |
|           | 50        | 14.64873275         | 14.64675540             | 0.00197735       |
|           | 100       | 14.67174206         | 14.66864400             | 0.00309806       |
| 14476     | 0         | 14.48005842         | -----                   | -----            |
|           | 9         | 14.48028419         | 14.47467020             | 0.00561399       |
|           | 51        | 14.48126626         | 14.47545890             | 0.00580736       |
|           | 101       | 14.48210420         | 14.47641010             | 0.00569410       |
| 13043     | 0         | 15.67900804         | -----                   | -----            |
|           | 11        | 15.68473145         | 15.69542640             | -0.01069495      |
|           | 50        | 15.70452036         | 15.71460470             | -0.01008434      |
|           | 100       | 15.73148592         | 15.73942180             | -0.00793588      |
| 7840      | 0         | 14.56491839         | -----                   | -----            |
|           | 11        | 14.56516626         | 14.56074740             | 0.00441886       |
|           | 52        | 14.56622862         | 14.56153640             | 0.00469222       |
|           | 105       | 14.56703165         | 14.56256640             | 0.00446525       |

TABLE 4.18

## MEAN MOTION VALUES FOR MODEL N3

| SATELLITE | t<br>days | ACTUAL n<br>rev/day | CALCULATED n<br>rev/day | ERROR<br>rev/day |
|-----------|-----------|---------------------|-------------------------|------------------|
| 15363     | 0         | 14.62860624         | -----                   | -----            |
|           | 10        | 14.63270865         | 14.62978980             | 0.00291885       |
|           | 50        | 14.64873275         | 14.64744050             | 0.00129225       |
|           | 100       | 14.67174206         | 14.66950400             | 0.00223806       |
| 14476     | 0         | 14.48005842         | -----                   | -----            |
|           | 9         | 14.48028419         | 14.47521070             | 0.00507349       |
|           | 51        | 14.48126626         | 14.47600930             | 0.00525696       |
|           | 101       | 14.48210420         | 14.47696010             | 0.00514410       |
| 13043     | 0         | 15.67900804         | -----                   | -----            |
|           | 11        | 15.68473145         | 15.69473160             | -0.01000015      |
|           | 50        | 15.70452036         | 15.71415990             | -0.00963954      |
|           | 100       | 15.73148592         | 15.73906790             | -0.00758198      |
| 7840      | 0         | 14.56491839         | -----                   | -----            |
|           | 11        | 14.56516626         | 14.56119420             | 0.00397206       |
|           | 52        | 14.56622862         | 14.56199330             | 0.00423532       |
|           | 105       | 14.56703165         | 14.56302630             | 0.00400535       |

TABLE 4.19

## MEAN MOTION VALUES FOR MODEL N5

| SATELLITE | t<br>days | ACTUAL n<br>rev/day | CALCULATED n<br>rev/day | ERROR<br>rev/day |
|-----------|-----------|---------------------|-------------------------|------------------|
| 15363     | 0         | 14.62860624         | -----                   | -----            |
|           | 10        | 14.63270865         | 14.63221620             | 0.00049245       |
|           | 50        | 14.64873275         | 14.65205620             | -0.00332345      |
|           | 100       | 14.67174206         | 14.67560620             | -0.00386414      |
| 14476     | 0         | 14.48005842         | -----                   | -----            |
|           | 9         | 14.48028419         | 14.48026650             | 0.00001769       |
|           | 51        | 14.48126626         | 14.48123750             | 0.00002876       |
|           | 101       | 14.48210420         | 14.48239350             | -0.00028930      |
| 13043     | 0         | 15.67900804         | -----                   | -----            |
|           | 11        | 15.68473145         | 15.68484240             | -0.00011095      |
|           | 50        | 15.70452036         | 15.70552800             | -0.00100764      |
|           | 100       | 15.73148592         | 15.73204800             | -0.00056208      |
| 7840      | 0         | 14.56491839         | -----                   | -----            |
|           | 11        | 14.56516626         | 14.56515090             | 0.00001536       |
|           | 52        | 14.56622862         | 14.56601770             | 0.00021092       |
|           | 105       | 14.56703165         | 14.56713810             | -0.00010645      |

TABLE 4.20

## 100 DAY ERROR ABSOLUTE VALUES FOR ALL SATELLITES AND MODELS

| MODEL/SATELLITE | 15363      | 14476     | 13043      | 7840       |
|-----------------|------------|-----------|------------|------------|
| n1:             | 0.00049215 | 0.0000175 | 0.00092988 | 0.00008195 |
| n2:             | 0.00309806 | 0.0056941 | 0.00793588 | 0.00446525 |
| n3:             | 0.00223806 | 0.0051441 | 0.00758198 | 0.00400535 |
| n5:             | 0.00386414 | 0.0002893 | 0.00056208 | 0.00010645 |

TABLE 4.21

AVERAGE 100 DAY ERROR VALUES FOR AVERAGED  $\dot{n}$  AND  $\ddot{n}$ 

| MODEL | ERROR rev/day |
|-------|---------------|
| n5    | 0.0012055     |
| n3    | 0.0047424     |
| n1    | 0.0050009     |
| n2    | 0.0052980     |

These values are about a factor of two (at least) improvement over the values produced by unaveraged  $n$  values. The NORAD model (n5) shows the most improvement and is still the most accurate. Its greatest problem is the instability of the error for a single satellite over time as shown in Table 4.19. In contrast, model n3, which has a small average error, shows great consistency in the magnitude of the error as shown in Table 4.18. This means that if a value of  $n$  is known at some time around the start time, the size of the error to be expected from the model can be determined for that satellite. This could be of great help in locating lost objects. For example, if an elset is available for the time at which the satellite went lost, or start time, and two to three elsets are available for ten to thirty days prior to the start time,  $\dot{n}$  and  $\ddot{n}$  may be calculated using the elset prior to the start time. Then a calculated value of  $n$  may be found and compared to the actual value of  $n$  at the start time to find a limit on the magnitude of the error. Next, at some future time, the value of  $n$  and a plus or minus range can be found. This will give the analyst general bounds within which to look for the satellite. For example satellite 519 has the following values of  $n$  at the times shown.

TABLE 4.22

## SAMPLE VALUES OF MEAN MOTION

| DATE  | n rev/day  |
|-------|------------|
| 84312 | 15.0054437 |
| 84324 | 15.0059095 |
| 84331 | 15.0062630 |
| 84338 | 15.0063419 |
| 85072 | 15.0091569 |

Using equations 2.1 and 2.2 for times 84312, 84324, 84331 gives:

$$\dot{n} = 0.00003857 \quad \ddot{n} = 0.00000911$$

Predicting from 84331 to 84338 using model n3 gives:  $n = 15.00824$  with an error of  $-0.0018981$ . Therefore the approximate error in the future is plus/minus  $0.002$ . The actual error at 85072 is  $-0.0026$ . The plus/minus error gives the analyst a range within which to look for the satellite.

An attempt was made to calculate  $\dot{n}$  from a simple statistical model:

$$\dot{n}(t) = B_0 + B_1 \dot{n}(t-1) + B_2 t + B_2 n_0 + B_4 F \quad (4.1)$$

This model would be used in place of equation 2.1 to find  $\dot{n}$ . BMDP was unable to produce an accurate result. The residuals from the model were of the same order of magnitude as the value being calculated. This results in no gain over conventional methods.



## V. CONCLUSIONS AND RECOMMENDATIONS

The  $n_5$  model has been shown to be the most accurate when used with 30 day average values of  $\dot{N}/2$ . The  $n_3$  model is a close second to the  $n_5$  model. Since the  $n_3$  model is stable, if some data for  $n$  near the the start time is available, limits on the error in  $n$  at some future time can be set. Because of this, the  $n_3$  model is better for the satellite orbit types studied here, that is, low eccentricity orbits with  $14 \leq n \leq 16$ .

Further study is recommended in four main areas. The first area for further study is the possible use of  $\dot{N}/2$  to find a value for  $\dot{n}$  and the use of these values in model  $n_3$ . This would determine if the greater accuracy available in the  $n_5$  model can be combined with the consistency of the  $n_3$  model to give more accurate in-track areas in which to search for lost objects.

A second item for further study is the extension of the model to times greater than 100 days for prediction.

The third subject for study is the inclusion of mean motions outside the range included in this thesis and for more eccentric satellite orbits to be included.

The final study should determine if other orbital elements need to be included in the model.

APPENDIX A

DATA FILE USED FOR BMDP

| t  | dt | $\dot{n}(t)$ | $\dot{n}(t - dt)$ | $n(t)$     | $n(\emptyset)$ | S  | F  | SF |
|----|----|--------------|-------------------|------------|----------------|----|----|----|
| 2  | 0  | 0.0000401    | 0.0000000         | 14.9814358 | 14.9813442     | -1 | 0  | 70 |
| 3  | 1  | 0.0000286    | 0.0000401         | 14.9814758 | 14.9813442     | 4  | -1 | 70 |
| 6  | 3  | 0.0000505    | 0.0000286         | 14.9815617 | 14.9813442     | -1 | 4  | 71 |
| 9  | 3  | 0.0000558    | 0.0000505         | 14.9817133 | 14.9813442     | 0  | -1 | 73 |
| 18 | 9  | 0.0000668    | 0.0000558         | 14.9822159 | 14.9813442     | 1  | 0  | 74 |
| 22 | 4  | 0.0000849    | 0.0000668         | 14.9825630 | 14.9813442     | 0  | 0  | 72 |
| 24 | 2  | 0.0000629    | 0.0000849         | 14.9827328 | 14.9813442     | 6  | 0  | 73 |
| 27 | 3  | 0.0000458    | 0.0000629         | 14.9829216 | 14.9813442     | 1  | 6  | 80 |
| 28 | 1  | 0.0000525    | 0.0000458         | 14.9829674 | 14.9813442     | 2  | 1  | 80 |
| 30 | 2  | 0.0000615    | 0.0000525         | 14.9830723 | 14.9813442     | 0  | 2  | 83 |
| 34 | 4  | 0.0000690    | 0.0000615         | 14.9833183 | 14.9813442     | 0  | 0  | 83 |
| 37 | 3  | 0.0000854    | 0.0000690         | 14.9835253 | 14.9813442     | 1  | 0  | 79 |
| 39 | 2  | 0.0000825    | 0.0000854         | 14.9836960 | 14.9813442     | -2 | 1  | 80 |
| 41 | 2  | 0.0001192    | 0.0000825         | 14.9838610 | 14.9813442     | 0  | -2 | 76 |
| 42 | 1  | 0.0000324    | 0.0001192         | 14.9839802 | 14.9813442     | 1  | 0  | 75 |
| 44 | 2  | 0.0000792    | 0.0000324         | 14.9840450 | 14.9813442     | 1  | 1  | 76 |
| 45 | 1  | 0.0000563    | 0.0000792         | 14.9841242 | 14.9813442     | 3  | 1  | 77 |
| 47 | 2  | 0.0000896    | 0.0000563         | 14.9842367 | 14.9813442     | -1 | 3  | 81 |
| 49 | 2  | 0.0000656    | 0.0000896         | 14.9844160 | 14.9813442     | 0  | -1 | 79 |
| 6  | 0  | 0.0000150    | 0.0000000         | 14.8682432 | 14.8681669     | -1 | 0  | 71 |
| 16 | 10 | 0.0000208    | 0.0000150         | 14.8683929 | 14.8681669     | -1 | -1 | 77 |
| 22 | 6  | 0.0000334    | 0.0000208         | 14.8685179 | 14.8681669     | 0  | -1 | 72 |
| 23 | 1  | 0.0000203    | 0.0000334         | 14.8685513 | 14.8681669     | 1  | 0  | 73 |
| 30 | 7  | 0.0000149    | 0.0000203         | 14.8686934 | 14.8681669     | 0  | 1  | 83 |
| 38 | 8  | 0.0000222    | 0.0000149         | 14.8688126 | 14.8681669     | -2 | 0  | 79 |
| 42 | 4  | 0.0000101    | 0.0000222         | 14.8689013 | 14.8681669     | 1  | -2 | 75 |
| 47 | 5  | 0.0000105    | 0.0000101         | 14.8689518 | 14.8681669     | -1 | 1  | 81 |
| 48 | 1  | 0.0000146    | 0.0000105         | 14.8689623 | 14.8681669     | -2 | -1 | 80 |
| 2  | 0  | 0.0000141    | 0.0000000         | 14.6311455 | 14.6310959     | -1 | 0  | 70 |
| 6  | 4  | 0.0000207    | 0.0000141         | 14.6312017 | 14.6310959     | -1 | -1 | 71 |
| 9  | 3  | 0.0000211    | 0.0000207         | 14.6312637 | 14.6310959     | 0  | -1 | 73 |
| 15 | 6  | 0.0000223    | 0.0000211         | 14.6313906 | 14.6310959     | -2 | 0  | 74 |
| 25 | 10 | 0.0000181    | 0.0000223         | 14.6316137 | 14.6310959     | 0  | -2 | 74 |
| 29 | 4  | 0.0000134    | 0.0000181         | 14.6316862 | 14.6310959     | 2  | 0  | 81 |
| 32 | 3  | 0.0000105    | 0.0000134         | 14.6317263 | 14.6310959     | -3 | 2  | 85 |

|    |   |           |           |            |            |    |    |    |
|----|---|-----------|-----------|------------|------------|----|----|----|
| 35 | 3 | 0.0000160 | 0.0000105 | 14.6317577 | 14.6310959 | 0  | -3 | 75 |
| 39 | 4 | 0.0000141 | 0.0000160 | 14.6318216 | 14.6310959 | -2 | 0  | 80 |
| 44 | 5 | 0.0000111 | 0.0000141 | 14.6318922 | 14.6310959 | 1  | -2 | 76 |
| 8  | 0 | 0.0000312 | 0.0000000 | 15.0053215 | 15.0049667 | 0  | 0  | 70 |
| 12 | 4 | 0.0000238 | 0.0000312 | 15.0054464 | 15.0049667 | 1  | 0  | 72 |
| 14 | 2 | 0.0000300 | 0.0000238 | 15.0054941 | 15.0049667 | 3  | 1  | 72 |
| 16 | 2 | 0.0000243 | 0.0000300 | 15.0055542 | 15.0049667 | -1 | 3  | 77 |
| 20 | 4 | 0.0000606 | 0.0000243 | 15.0056515 | 15.0049667 | -3 | -1 | 74 |
| 22 | 2 | 0.0000391 | 0.0000606 | 15.0057726 | 15.0049667 | 0  | -3 | 72 |
| 23 | 1 | 0.0000982 | 0.0000391 | 15.0058117 | 15.0049667 | 1  | 0  | 73 |
| 24 | 1 | 0.0000458 | 0.0000982 | 15.0059891 | 15.0049667 | 6  | 6  | 73 |
| 26 | 2 | 0.0000367 | 0.0000458 | 15.0060806 | 15.0049667 | 0  | 6  | 80 |
| 28 | 2 | 0.0000591 | 0.0000367 | 15.0061541 | 15.0049667 | 2  | 0  | 80 |
| 29 | 1 | 0.0000248 | 0.0000591 | 15.0062132 | 15.0049667 | 2  | 2  | 81 |
| 31 | 2 | 0.0000264 | 0.0000248 | 15.0062628 | 15.0049667 | 1  | 2  | 85 |
| 34 | 3 | 0.0000200 | 0.0000264 | 15.0063419 | 15.0049667 | 0  | 1  | 83 |
| 36 | 2 | 0.0000267 | 0.0000200 | 15.0063820 | 15.0049667 | 0  | 0  | 79 |
| 38 | 2 | 0.0000341 | 0.0000267 | 15.0064354 | 15.0049667 | -2 | 0  | 79 |
| 42 | 4 | 0.0000277 | 0.0000341 | 15.0065718 | 15.0049667 | 1  | -2 | 75 |
| 46 | 4 | 0.0000404 | 0.0000277 | 15.0066824 | 15.0049667 | -1 | 1  | 78 |
| 49 | 3 | 0.0000715 | 0.0000404 | 15.0068035 | 15.0049667 | 0  | -1 | 79 |
| 50 | 1 | 0.0000467 | 0.0000715 | 15.0068750 | 15.0049667 | 0  | 0  | 77 |
| 1  | 0 | 0.0005770 | 0.0000467 | 15.3779602 | 15.3755236 | 0  | 0  | 70 |
| 2  | 1 | 0.0007029 | 0.0005770 | 15.3789206 | 15.3755236 | -1 | -1 | 70 |
| 3  | 1 | 0.0012178 | 0.0007029 | 15.3796234 | 15.3755236 | 4  | -1 | 70 |
| 4  | 1 | 0.0012465 | 0.0012178 | 15.3808413 | 15.3755236 | -2 | 4  | 69 |
| 5  | 1 | 0.0005617 | 0.0012465 | 15.3831081 | 15.3755236 | 0  | 0  | 73 |
| 6  | 1 | 0.0007572 | 0.0005617 | 15.3840485 | 15.3755236 | -1 | -1 | 71 |
| 7  | 1 | 0.0010633 | 0.0007572 | 15.3855209 | 15.3755236 | 3  | 3  | 71 |
| 8  | 1 | 0.0010376 | 0.0010633 | 15.3875751 | 15.3755236 | 0  | 0  | 70 |
| 9  | 1 | 0.0016823 | 0.0010376 | 15.3899431 | 15.3755236 | 0  | 0  | 73 |
| 10 | 1 | 0.0014992 | 0.0016823 | 15.3922539 | 15.3755236 | -1 | -1 | 73 |
| 11 | 1 | 0.0017462 | 0.0014992 | 15.3937531 | 15.3755236 | -1 | -1 | 73 |
| 12 | 1 | 0.0015879 | 0.0017462 | 15.3957367 | 15.3755236 | 1  | 1  | 72 |
| 13 | 1 | 0.0014381 | 0.0015879 | 15.3973246 | 15.3755236 | 2  | 1  | 71 |
| 14 | 1 | 0.0017490 | 0.0014381 | 15.3987627 | 15.3755236 | 3  | 2  | 72 |
| 15 | 1 | 0.0017366 | 0.0017490 | 15.4005117 | 15.3755236 | -2 | 3  | 74 |
| 16 | 1 | 0.0016155 | 0.0017366 | 15.4022484 | 15.3755236 | -1 | -2 | 77 |
| 17 | 1 | 0.0017204 | 0.0016155 | 15.4046593 | 15.3755236 | -1 | -1 | 75 |

|    |   |           |           |            |            |    |    |    |
|----|---|-----------|-----------|------------|------------|----|----|----|
| 18 | 1 | 0.0013862 | 0.0017204 | 15.4063797 | 15.3755236 | 1  | -1 | 74 |
| 20 | 2 | 0.0026684 | 0.0013862 | 15.4091520 | 15.3755236 | -3 | 1  | 74 |
| 21 | 1 | 0.0032047 | 0.0026684 | 15.4118204 | 15.3755236 | 1  | -3 | 75 |
| 24 | 3 | 0.0019274 | 0.0032047 | 15.4214344 | 15.3755236 | 6  | 1  | 73 |
| 25 | 1 | 0.0006914 | 0.0019274 | 15.4243202 | 15.3755236 | 0  | 0  | 74 |
| 26 | 1 | 0.0007496 | 0.0006914 | 15.4255304 | 15.3755236 | 0  | 0  | 80 |
| 27 | 1 | 0.0011072 | 0.0007496 | 15.4262800 | 15.3755236 | 1  | 0  | 80 |
| 28 | 1 | 0.0018005 | 0.0011072 | 15.4283867 | 15.3755236 | 2  | 2  | 80 |
| 29 | 1 | 0.0020504 | 0.0018005 | 15.4307737 | 15.3755236 | 2  | 2  | 81 |
| 30 | 1 | 0.0022745 | 0.0020504 | 15.4328241 | 15.3755236 | 0  | 2  | 83 |
| 31 | 1 | 0.0017424 | 0.0022745 | 15.4350986 | 15.3755236 | 1  | 0  | 85 |
| 32 | 1 | 0.0013084 | 0.0017424 | 15.4372549 | 15.3755236 | -3 | -3 | 85 |
| 33 | 1 | 0.0025711 | 0.0013084 | 15.4393463 | 15.3755236 | -4 | -4 | 86 |
| 34 | 1 | 0.0018959 | 0.0025711 | 15.4419174 | 15.3755236 | 0  | -4 | 83 |
| 35 | 1 | 0.0021210 | 0.0018959 | 15.4438133 | 15.3755236 | 0  | 0  | 79 |
| 36 | 1 | 0.0021133 | 0.0021210 | 15.4476538 | 15.3755236 | 0  | 0  | 79 |
| 38 | 2 | 0.0016947 | 0.0021133 | 15.4548283 | 15.3755236 | -2 | -2 | 79 |
| 40 | 2 | 0.0012426 | 0.0016947 | 15.4597807 | 15.3755236 | -1 | -1 | 78 |
| 41 | 1 | 0.0012321 | 0.0012426 | 15.4616613 | 15.3755236 | 0  | 0  | 76 |
| 42 | 1 | 0.0024109 | 0.0012321 | 15.4628935 | 15.3755236 | 1  | 0  | 75 |
| 43 | 1 | 0.0017233 | 0.0024109 | 15.4653044 | 15.3755236 | 1  | 1  | 75 |
| 44 | 1 | 0.0010376 | 0.0017233 | 15.4681396 | 15.3755236 | 1  | 1  | 76 |
| 45 | 1 | 0.0005436 | 0.0010376 | 15.4779492 | 15.3755236 | 3  | 3  | 77 |
| 46 | 1 | 0.0025892 | 0.0005436 | 15.4714928 | 15.3755236 | -1 | 3  | 78 |
| 47 | 1 | 0.0013638 | 0.0025892 | 15.4756136 | 15.3755236 | -1 | -1 | 81 |
| 48 | 1 | 0.0022907 | 0.0013638 | 15.4769773 | 15.3755236 | -2 | -1 | 80 |
| 49 | 1 | 0.0012608 | 0.0022907 | 15.4815140 | 15.3755236 | 0  | 0  | 79 |
| 50 | 1 | 0.0035906 | 0.0012608 | 15.4827747 | 15.3755236 | 0  | 0  | 77 |





|          |       |   |       |        |          |          |            |       |
|----------|-------|---|-------|--------|----------|----------|------------|-------|
| 2 153601 | 84110 | H | 85015 | 050110 | 147.2445 | 214.2721 | 14.6571003 | 00945 |
| 2 153602 | 84110 | H | 85016 | 050110 | 147.2446 | 214.2721 | 14.6571017 | 14350 |
| 1 153603 | 84110 | H | 85017 | 050110 | 147.2447 | 214.2721 | 14.6571031 | 00951 |
| 2 153604 | 84110 | H | 85018 | 050110 | 147.2448 | 214.2721 | 14.6571045 | 14466 |
| 2 153605 | 84110 | H | 85019 | 050110 | 147.2449 | 214.2721 | 14.6571059 | 00967 |
| 2 153606 | 84110 | H | 85020 | 050110 | 147.2450 | 214.2721 | 14.6571073 | 14843 |
| 1 153607 | 84110 | H | 85021 | 050110 | 147.2451 | 214.2721 | 14.6571087 | 00973 |
| 2 153608 | 84110 | H | 85022 | 050110 | 147.2452 | 214.2721 | 14.6571101 | 15087 |
| 1 153609 | 84110 | H | 85023 | 050110 | 147.2453 | 214.2721 | 14.6571115 | 00989 |
| 2 153610 | 84110 | H | 85024 | 050110 | 147.2454 | 214.2721 | 14.6571129 | 15272 |
| 1 153611 | 84110 | H | 85025 | 050110 | 147.2455 | 214.2721 | 14.6571143 | 01000 |
| 2 153612 | 84110 | H | 85026 | 050110 | 147.2456 | 214.2721 | 14.6571157 | 15286 |
| 1 153613 | 84110 | H | 85027 | 050110 | 147.2457 | 214.2721 | 14.6571171 | 01010 |
| 2 153614 | 84110 | H | 85028 | 050110 | 147.2458 | 214.2721 | 14.6571185 | 15376 |
| 1 153615 | 84110 | H | 85029 | 050110 | 147.2459 | 214.2721 | 14.6571199 | 01025 |
| 2 153616 | 84110 | H | 85030 | 050110 | 147.2460 | 214.2721 | 14.6571213 | 15491 |
| 1 153617 | 84110 | H | 85031 | 050110 | 147.2461 | 214.2721 | 14.6571227 | 01037 |
| 2 153618 | 84110 | H | 85032 | 050110 | 147.2462 | 214.2721 | 14.6571241 | 15654 |
| 1 153619 | 84110 | H | 85033 | 050110 | 147.2463 | 214.2721 | 14.6571255 | 01047 |
| 2 153620 | 84110 | H | 85034 | 050110 | 147.2464 | 214.2721 | 14.6571269 | 15713 |
| 1 153621 | 84110 | H | 85035 | 050110 | 147.2465 | 214.2721 | 14.6571283 | 01067 |
| 2 153622 | 84110 | H | 85036 | 050110 | 147.2466 | 214.2721 | 14.6571297 | 15799 |
| 1 153623 | 84110 | H | 85037 | 050110 | 147.2467 | 214.2721 | 14.6571311 | 01074 |
| 2 153624 | 84110 | H | 85038 | 050110 | 147.2468 | 214.2721 | 14.6571325 | 15887 |
| 1 153625 | 84110 | H | 85039 | 050110 | 147.2469 | 214.2721 | 14.6571339 | 01087 |
| 2 153626 | 84110 | H | 85040 | 050110 | 147.2470 | 214.2721 | 14.6571353 | 16013 |
| 1 153627 | 84110 | H | 85041 | 050110 | 147.2471 | 214.2721 | 14.6571367 | 01093 |
| 2 153628 | 84110 | H | 85042 | 050110 | 147.2472 | 214.2721 | 14.6571381 | 16097 |
| 1 153629 | 84110 | H | 85043 | 050110 | 147.2473 | 214.2721 | 14.6571395 | 01103 |
| 2 153630 | 84110 | H | 85044 | 050110 | 147.2474 | 214.2721 | 14.6571409 | 16227 |
| 1 153631 | 84110 | H | 85045 | 050110 | 147.2475 | 214.2721 | 14.6571423 | 01119 |
| 2 153632 | 84110 | H | 85046 | 050110 | 147.2476 | 214.2721 | 14.6571437 | 16257 |
| 1 153633 | 84110 | H | 85047 | 050110 | 147.2477 | 214.2721 | 14.6571451 | 01127 |
| 2 153634 | 84110 | H | 85048 | 050110 | 147.2478 | 214.2721 | 14.6571465 | 16305 |
| 1 153635 | 84110 | H | 85049 | 050110 | 147.2479 | 214.2721 | 14.6571479 | 01133 |
| 2 153636 | 84110 | H | 85050 | 050110 | 147.2480 | 214.2721 | 14.6571493 | 16389 |
| 1 153637 | 84110 | H | 85051 | 050110 | 147.2481 | 214.2721 | 14.6571507 | 01141 |
| 2 153638 | 84110 | H | 85052 | 050110 | 147.2482 | 214.2721 | 14.6571521 | 16395 |
| 1 153639 | 84110 | H | 85053 | 050110 | 147.2483 | 214.2721 | 14.6571535 | 01154 |
| 2 153640 | 84110 | H | 85054 | 050110 | 147.2484 | 214.2721 | 14.6571549 | 16445 |
| 1 153641 | 84110 | H | 85055 | 050110 | 147.2485 | 214.2721 | 14.6571563 | 01160 |
| 2 153642 | 84110 | H | 85056 | 050110 | 147.2486 | 214.2721 | 14.6571577 | 16478 |
| 1 153643 | 84110 | H | 85057 | 050110 | 147.2487 | 214.2721 | 14.6571591 | 01183 |
| 2 153644 | 84110 | H | 85058 | 050110 | 147.2488 | 214.2721 | 14.6571605 | 16865 |
| 1 153645 | 84110 | H | 85059 | 050110 | 147.2489 | 214.2721 | 14.6571619 | 01193 |
| 2 153646 | 84110 | H | 85060 | 050110 | 147.2490 | 214.2721 | 14.6571633 | 17027 |
| 1 153647 | 84110 | H | 85061 | 050110 | 147.2491 | 214.2721 | 14.6571647 | 01207 |
| 2 153648 | 84110 | H | 85062 | 050110 | 147.2492 | 214.2721 | 14.6571661 | 17177 |
| 1 153649 | 84110 | H | 85063 | 050110 | 147.2493 | 214.2721 | 14.6571675 | 01215 |
| 2 153650 | 84110 | H | 85064 | 050110 | 147.2494 | 214.2721 | 14.6571689 | 17319 |
| 1 153651 | 84110 | H | 85065 | 050110 | 147.2495 | 214.2721 | 14.6571703 | 01229 |
| 2 153652 | 84110 | H | 85066 | 050110 | 147.2496 | 214.2721 | 14.6571717 | 17410 |
| 1 153653 | 84110 | H | 85067 | 050110 | 147.2497 | 214.2721 | 14.6571731 | 01247 |
| 2 153654 | 84110 | H | 85068 | 050110 | 147.2498 | 214.2721 | 14.6571745 | 17470 |
| 1 153655 | 84110 | H | 85069 | 050110 | 147.2499 | 214.2721 | 14.6571759 | 01261 |
| 2 153656 | 84110 | H | 85070 | 050110 | 147.2500 | 214.2721 | 14.6571773 | 17557 |
| 1 153657 | 84110 | H | 85071 | 050110 | 147.2501 | 214.2721 | 14.6571787 | 01257 |
| 2 153658 | 84110 | H | 85072 | 050110 | 147.2502 | 214.2721 | 14.6571801 | 17713 |
| 1 153659 | 84110 | H | 85073 | 050110 | 147.2503 | 214.2721 | 14.6571815 | 01261 |
| 2 153660 | 84110 | H | 85074 | 050110 | 147.2504 | 214.2721 | 14.6571829 | 17810 |
| 1 153661 | 84110 | H | 85075 | 050110 | 147.2505 | 214.2721 | 14.6571843 | 01277 |
| 2 153662 | 84110 | H | 85076 | 050110 | 147.2506 | 214.2721 | 14.6571857 | 17961 |
| 1 153663 | 84110 | H | 85077 | 050110 | 147.2507 | 214.2721 | 14.6571871 | 01287 |
| 2 153664 | 84110 | H | 85078 | 050110 | 147.2508 | 214.2721 | 14.6571885 | 18149 |
| 1 153665 | 84110 | H | 85079 | 050110 | 147.2509 | 214.2721 | 14.6571899 | 01295 |
| 2 153666 | 84110 | H | 85080 | 050110 | 147.2510 | 214.2721 | 14.6571913 | 18283 |
| 1 153667 | 84110 | H | 85081 | 050110 | 147.2511 | 214.2721 | 14.6571927 | 01306 |
| 2 153668 | 84110 | H | 85082 | 050110 | 147.2512 | 214.2721 | 14.6571941 | 18353 |











1 13430 82 7 P 85016 26322411 100025482 00000-0 21490-3 0 00870  
2 13430 74.0390 117.3591 0009450 241.7764 115.2459 15.7045203816692  
1 13430 82 7 H 85015 2700704 100024511 00000-0 19460-3 0 00880  
2 13430 74.0389 104.3325 0009406 240.8804 115.3361 15.70512191167195  
1 13430 82 7 H 85014 2412378 100022983 00000-0 18660-3 0 00899  
2 13430 74.0383 102.0297 0009400 241.3673 119.0493 15.70554374167159  
1 13430 82 7 H 85014 23371620 100022923 00000-0 18560-3 0 00905  
2 13430 74.0390 98.4957 0009433 240.4906 124.2264 15.70627692167600  
1 13430 82 7 H 85014 2380435 100022774 00000-0 17940-3 0 00912  
2 13430 74.0392 95.2465 0009433 240.8678 125.9480 15.70695335167821  
1 13430 82 7 H 85014 2130221 100021131 00000-0 16250-3 0 00921  
2 13430 74.0391 93.4547 0009424 242.4610 127.5781 15.70702627167914  
1 13430 82 7 H 85014 2024934 100020296 00000-0 15362-3 0 00930  
2 13430 74.0399 91.6537 0009431 243.8450 129.4031 15.7072476168070  
1 13430 82 7 H 85014 1879335 100020259 00000-0 14520-3 0 00943  
2 13430 74.0394 89.4927 0009494 242.9859 133.0670 15.70792760168029  
1 13430 82 7 H 85014 1821353 100020431 00000-0 14011-3 0 00957  
2 13430 74.0403 86.4834 0009452 242.7012 132.1437 15.70817711680990  
1 13430 82 7 H 85023 1676355 100020448 00000-0 13360-3 0 00969  
2 13430 74.0403 84.1747 0009456 245.5815 134.4673 15.7084941168457  
1 13430 82 7 H 85024 1261344 100020683 00000-0 12360-3 0 00972  
2 13430 74.0402 84.8825 0009492 245.7952 134.6543 15.7099432168545

1 13430 82 7 H 85024 1079785 100020620 00000-0 11540-3 0 00981  
2 13430 74.0401 84.0167 0009407 245.7733 135.9793 15.70930604168601  
1 13430 82 7 H 85027 9331703 100020572 00000-0 10470-3 0 00991  
2 13430 74.0390 77.3975 0009423 245.0543 131.0829 15.71077540169064  
1 13430 82 7 H 85027 8507160 100020491 00000-0 16770-3 0 01007  
2 13430 74.0370 76.6527 0009434 245.6945 144.3074 15.71074796169113  
1 13430 82 7 H 85028 7328426 100020913 00000-0 16620-3 0 01010  
2 13430 74.0379 75.7987 0009435 244.7562 145.8066 15.71089936169174  
1 13430 82 7 H 85029 7251613 100020387 00000-0 18450-3 0 01022  
2 13430 74.0380 73.4924 0009434 242.4573 147.6108 15.71147833169334  
1 13430 82 7 H 85030 7073645 100020206 00000-0 13150-3 0 01043  
2 13430 74.0394 71.3117 0009460 242.4898 147.6257 15.71272098169425  
1 13430 82 7 H 85030 7007376 100020797 00000-0 22730-3 0 01047  
2 13430 74.0362 71.3293 0009471 209.7743 147.2999 15.71235496169481  
1 13430 82 7 H 85031 6878745 100020485 00000-0 31110-3 0 01061  
2 13430 74.0399 69.1819 0009407 210.7576 149.3155 15.71336819169635  
1 13430 82 7 H 85031 6479448 100020346 00000-0 30200-3 0 01073  
2 13430 74.0404 68.3031 0009424 212.5745 147.4641 15.71342627169694  
1 13430 82 7 H 85032 6311104 100020321 00000-0 26310-3 0 01087  
2 13430 74.0400 67.1501 0009475 216.1574 143.8959 15.71374335169773  
1 13430 82 7 H 85032 6211717 100020015 00000-0 24660-3 0 01090  
2 13430 74.0390 66.7106 0009408 215.1876 144.8942 15.71371071169805  
1 13430 82 7 H 85032 1421516 100020689 00000-0 21860-3 0 01113  
2 13430 74.0393 64.5554 0007770 215.0574 144.8254 15.7147080169046





## BIBLIOGRAPHY

1. Major, Paul. Personal interviews. Directorate of Astrodynamics, NORAD, Colorado Springs, CO. 1983 through 1985.
2. Space Command USAF. An Analysis of the Use of Empirical Atmospheric Density Models in Orbital Mechanics. Spacetrack Report No 4. Colorado Springs, Co. Space Command USAF, February 1983.
3. Air Training Command. Space System Analyst Course. Colorado Springs, Co. 1 October 1980.
4. Bate, Roger R. et al. Fundamentals of Astrodynamics. New York: Dover Publications Incorporated, 1971.
5. King-Hele, D. G. "Methods for Predicting Satellite Orbital Lifetime," British Interplanetary Society Journal, 31: 181-196 (May 1978).
6. Fominov, A. M. "Correlation Between the Solar Activity Indices and the Use of these Indices in the Study of the Motion of Earth Satellites," Proceedings of the Symposium On Dynamics of Satellites. 189-193. Prague, Czechoslovakia, May 20-24, 1969.

## VITA

Captain James M. Burns was born on 6 November 1958 in Evansville, Indiana. He graduated from high school in Lockhart, South Carolina, in 1977 and attended Clemson University from which he received the degree of Bachelor of Science in Physics in May 1981. He received a commission in the USAF through the ROTC program and came on active duty in November 1981. He served as a crew orbital analyst and then as an orbital analyst leader until February 1983 when he became the lost satellite and breakup specialist in the Space Operations Directorate at the North American Aerospace Defense Command Cheyenne Mountain Complex in Colorado Springs, Colorado. One of his primary jobs was to predict new orbital elements for lost objects from outdated data and the determination of possible orbital decay rates on these objects. He remained there until his entry in the School of Engineering, Air Force Institute of Technology, in June 1984.

Permanent address: Route 5 Box 240

Union, South Carolina 29379



REPORT DOCUMENTATION PAGE

|   |  |  |  |
|---|--|--|--|
| 1a. REPORT SECURITY CLASSIFICATION<br><b>Unclassified</b>   |  | 1b. RESTRICTIVE MARKINGS<br><b>N/A</b>   |  |
| 2a. SECURITY CLASSIFICATION AUTHORITY   |  | 3. DISTRIBUTION/AVAILABILITY OF REPORT<br><b>Approved for public release<br/>distribution unlimited</b>  |  |
| 2b. DECLASSIFICATION/DOWNGRADING SCHEDULE   |  |  |  |
| 4. PERFORMING ORGANIZATION REPORT NUMBER(S)<br><b>AFIT/GSO/ENS/85D-5</b>  |  | 5. MONITORING ORGANIZATION REPORT NUMBER(S)  |  |
| 6a. NAME OF PERFORMING ORGANIZATION<br><b>School of Engineering<br/>Air Force Institute of Tech.</b>  | 6b. OFFICE SYMBOL<br><i>(If applicable)</i><br><b>AFIT/ENS</b> | 7a. NAME OF MONITORING ORGANIZATION  |  |
| 6c. ADDRESS (City, State and ZIP Code)<br><b>Air Force Institute of Technology<br/>Wright-Patterson AFB OH 45433</b>  |  | 7b. ADDRESS (City, State and ZIP Code)   |  |
| 8a. NAME OF FUNDING/SPONSORING ORGANIZATION   | 8b. OFFICE SYMBOL<br><i>(If applicable)</i>                    | 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER  |  |
| 8c. ADDRESS (City, State and ZIP Code)  |  | 10. SOURCE OF FUNDING NOS.   |  |
|   |  | PROGRAM ELEMENT NO.  | PROJECT NO.  |
|   |  | TASK NO.   | WORK UNIT NO.  |
| 11. TITLE (Include Security Classification)<br><b>See Box 19</b>  |  |  |  |
| 12. PERSONAL AUTHOR(S)<br><b>James M. Burns BS Capt USAF</b>  |  |  |  |
| 13a. TYPE OF REPORT<br><b>MS Thesis</b>   | 13b. TIME COVERED<br>FROM _____ TO _____                       | 14. DATE OF REPORT (Yr., Mo., Day)<br><b>1985 December</b>   | 15. PAGE COUNT<br><b>76</b>  |
| 16. SUPPLEMENTARY NOTATION  |  |  |  |
| 17. COSATI CODES  |  | 18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)  |  |
| FIELD   | GROUP  | SUB. GR.   |  |
| 22  | 03   |  | <b>Space Craft-Artificial Satellites<br/>Orbits-Earth Orbits<br/>Orbital Decay</b> |
| 19. ABSTRACT (Continue on reverse if necessary and identify by block number)  |  |  |  |
| <p>Title: <b>Statistical Models For Predicting the Change in Mean Motion of a Satellite Over Time Including the Effects of Solar Flux.</b></p> <p>Thesis Chairman: <b>Charles Ebeling, LTC, USAF</b> (Unclassified)</p> |  |  |  |
|   |  | <p><i>Approved for public release - EAW 278 180-5</i><br/> <i>John E. Wolaver 13 Feb 86</i><br/>                 Dean for Research and Professional Development<br/>                 Air Force Institute of Technology (AFIT)<br/>                 Wright-Patterson AFB OH 45433</p> |  |
| 20. DISTRIBUTION/AVAILABILITY OF ABSTRACT<br><b>UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS <input type="checkbox"/></b>                                |  | 21. ABSTRACT SECURITY CLASSIFICATION<br><b>Unclassified</b>  |  |
| 22a. NAME OF RESPONSIBLE INDIVIDUAL<br><b>Charles Ebeling, LTC, USAF</b>  |  | 22b. TELEPHONE NUMBER<br><i>(Include Area Code)</i>  | 22c. OFFICE SYMBOL<br><b>AFIT/ENS</b>  |

(19)

This investigation derived a simple model to determine the change in mean motion over time when the actual values are unknown. A method was developed to include effects of solar flux by calculating an average value of  $\dot{n}$  over about 30 days. The model requires a knowledge of the mean motion for about 30 days before the time of interest to calculate this average.

The analysis was done using BMDP on a CDC Cyber 6000 Computer using element set data from actual satellites.

This model does not attempt absolute accuracy, but is intended to be a method to quickly approximate a new mean motion when real values are not available. A limitation of this model is the amount of historical data and analyst judgement which are required.

END

DITIC

9 - 86