TM 86-1 TP

COPY NO. 29

2

Technical Memorandum

DERIVATION OF THE RADAR RANGE EQUATION FOR A PULSE-DOPPLER RADAR WITH RANGE AND VELOCITY GATING AND COHERENT AND NONCOHERENT PULSE INTEGRATION

by

Dr. George Masters

U.S. Naval Test Pilot School

27 June 1986

DTIC
ELECTE
JUL 30 1986
S D
B

DEPARTMENT OF THE NAVY

NAVAL AIR TEST CENTER
PATUXENT RIVER, MARYLAND

DEPARTMENT OF THE NAVY
NAVAL AIR TEST CENTER
PATUXENT RIVER, MARYLAND 20670-5304

TM 86-1 TP
27 June 1986

The application of advanced radar signal processing techniques to the detection of airborne moving targets is of increasing importance in air warfare. Several test programs at NAVAIRTESTCEN, both current and planned, are concerned with the evaluation of radar systems which utilize such advanced techniques. In order to facilitate the analysis of such systems at NAVAIRTESTCEN, this technical memorandum sets forth the manner in which factors accounting for the more common techniques may be incorporated into the radar range equation.

JK Ready
J. K. READY
Commander, Naval Air Test Center

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

ADA170-56

REPORT DOCUMENTATION PAGE	**READ INSTRUCTIONS BEFORE COMPLETING FORM**
1. REPORT NUMBER TM 86-1 TP	2. GOVT ACCESSION NO.
	3. RECIPIENT'S CATALOG NUMBER
4. TITLE *(and Subtitle)* DERIVATION OF THE RADAR RANGE EQUATION FOR A PULSE-DOPPLER RADAR WITH RANGE AND VELOCITY GATING AND COHERENT AND NONCOHERENT PULSE INTEGRATION	5. TYPE OF REPORT & PERIOD COVERED TECHNICAL MEMORANDUM
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) DR. GEORGE MASTERS	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS NAVAL AIR TEST CENTER U.S. NAVAL TEST PILOT SCHOOL PATUXENT RIVER, MARYLAND 20670-5304	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS NAVAL AIR TEST CENTER DEPARTMENT OF THE NAVY PATUXENT RIVER, MARYLAND 20670-5304	12. REPORT DATE 27 JUNE 1986
	13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS *(if different from Controlling Office)*	15. SECURITY CLASS. *(of this report)* UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

RADAR RANGE EQUATION
PULSE-DOPPLER RADAR
RANGE AND VELOCITY GATING
COHERENT AND NONCOHERENT PULSE INTEGRATION

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

The radar range equation is employed in test planning to predict the probability of target detection for a given set of test conditions (target parameters, range, velocity, aspect angle, radar parameters, etc.). This technical memorandum documents the detailed development of the radar range equation including the effects of range gating, velocity gating, and both predetection and postdetection pulse integration.

SUMMARY

The radar range equation is employed in test planning to predict the probability of target detection for a given set of test conditions (target parameters, range, velocity, aspect angle, radar parameters, etc.). This technical memorandum documents the detailed development of the radar range equation including the effects of range gating, velocity gating, and both predetection and postdetection pulse integration.

THE BASIC RADAR RANGE EQUATION

The radar range equation may be written in a form expressing the signal-to-noise ratio expected for a given set of radar and target parameters. In the following development, an expression is derived for the signal-to-noise ratio to be expected when several modern signal processing techniques are employed. The development begins with the range equation for a basic, pulsed radar, and neglects propagation absorption and scattering losses.

The radar equation for a pulsed radar without pulse integration and without range or velocity gating may be written:

$$(S/N)_o = \frac{P_a G_a^2 \lambda^2 \sigma}{(4\pi)^3 d_t L_{so} N_s R_t^4}$$

where:

$(S/N)_o$ = Signal-to-Noise Ratio at Target Detection Filter (n.d.)
P_a = Average Power at Transmitter Output (W)
G_a = Transmit/Receive Antenna Gain (n.d.)
λ = Carrier Wavelength (m)
σ = Target Radar Cross Section (m^2)
d_t = Transmitter Duty Cycle (n.d.) = $\tau_p f_r$
τ_p = Transmitted Pulse Width (sec)
f_r = Pulse Repetition Frequency (Hz)
L_{so} = Total System Losses Except for Those Associated with Gating and Pulse Integration (n.d.)
N_s = Total Internal System Noise (W)
R_t = Range to the Target (m)

The system loss for the basic pulse radar is given by the equation:

$$L_{so} = L_c^2 L_{dx}^2 L_r L_{da}$$

where:

L_{so} = System Loss Factor (n.d.)
L_c = One-Way Cable Loss (n.d.)
L_{dx} = Duplexer Loss (n.d.)
L_r = Receiver Loss (n.d.)
L_{da} = Detection Algorithm Loss (n.d.)

The system noise, for a system dominated by internal noise, is given by the equation:

$$N_s = kT_o B_{if} F_n$$

where:

N_s = System Noise (W)
k = Boltzmann's Constant (W-sec/deg)
T_o = Standard Reference Temperature = 290°K
B_{if} = Intermediate Frequency Bandwidth (Hz) = $1/\tau_p$
F_n = Effective Receiver Noise Figure (n.d.)

PREDETECTION PULSE INTEGRATION

When predetection (coherent) pulse integration is employed, the signal-to-noise ratio is increased, except for processing losses, by a factor equal to the number of pulses coherently integrated, N_{ci}. Thus:

$$(S/N) = (S/N)_o(N_{ci}/L_{ci}) = (S/N)_o(f_rT_s/L_{ci})$$

where:

$(S/N)_o$ = Signal-to-Noise Ratio of Basic Pulse Radar (n.d.)
$(S/N)_o$ = $P_aG_a^2\lambda^2\sigma/(4\pi)^3d_tL_{so}N_sR_t^4$
f_r = Pulse Repetition Frequency (Hz)
T_s = Coherent (Predetection) Integration Sampling (Integration) Time (sec)
L_{ci} = Losses Associated with Coherent Pulse Integration (n.d.)
L_{ci} = L_sL_q (n.d.)
L_s = Sampling Loss (n.d.)
L_q = Quantization Loss (n.d.)
N_{ci} = Number of Pulses Coherently Integrated (n.d.)

POSTDETECTION PULSE INTEGRATION

When postdetection (noncoherent) pulse integration is employed, the signal-to-noise ratio is increased, except for processing losses, (L'_{nci}), by a factor equal to the number of pulses (samples) noncoherently integrated, N_{nci}, times the noncoherent integration efficiency factor, E_{nci}. Thus:

$$(S/N) = (S/N)_o(N_{nci}E_{nci}/L'_{nci}) = (S/N)_o(f_sT_{nci}/L_{nci})$$

or:

$$(S/N) = (S/N)_o(T_{nci}/T_sL_{nci})$$

where:

f_s = Predetection Sampling Frequency (Hz)
T_s = Predetection Sampling Period (sec)
T_{nci} = Noncoherent Integration Time (sec)
L_{nci} = Losses Associated with Noncoherent Integration, Including Efficiency Factor (n.d.)
L_{nci} = $(N_{nci})^{0.2} = (f_sT_{nci})^{0.2}$ (n.d.)
N_{nci} = Number of Samples Noncoherently Integrated (n.d.)

When both predetection (coherent) and postdetection (noncoherent) pulse integration are employed, the radar equation becomes:

$$(S/N) = (S/N)_o(f_rT_{nci}/L_{ci}L_{nci})$$

RANGE GATING

When range gating is employed, two effects change the signal-to-noise ratio: range gate eclipsing and range gate noise reduction. The effect of range gate eclipsing is to reduce the target return signal-to-noise ratio by losing that part of the return signal not within the range gate. The effect of range gate mismatch is to reduce the target return signal-to-noise ratio by increasing the noise received when the gate width is excessively large. These effects can be included in the range equation by defining two effective duty cycles in addition to the transmitter duty cycle (see Skolnik, Radar Handbook, chapter 19). Thus:

$$(S/N) = (S/N)_o(d_r^2/d_g)\ (\text{n.d.})$$

where:

d_r = Variable Received Gated-Pulse Duty Cycle due to Position of Target with Respect to the Range Gate
d_r = τ_r/τ_p (n.d.)
d_g = Receiver Duty Cycle due to Range Gating
d_g = τ_g/T_r (n.d.)
T_r = Pulse Repetition Interval (sec)
τ_g = Range-Gate Interval (Width) (sec)
τ_p = Pulse Width (sec)
τ_r = Received Pulse Width (After Eclipsing) (sec) (This factor is defined in terms of target range in the following material.)

Employing the foregoing expressions for d_r and d_g, the signal-to-noise ratio is given by:

$$(S/N) = (S/N)_o(\tau_r^2 T_r/\tau_p^2 \tau_g) = (S/N)_o(\tau_r/\tau_p)^2(T_r/\tau_g)$$

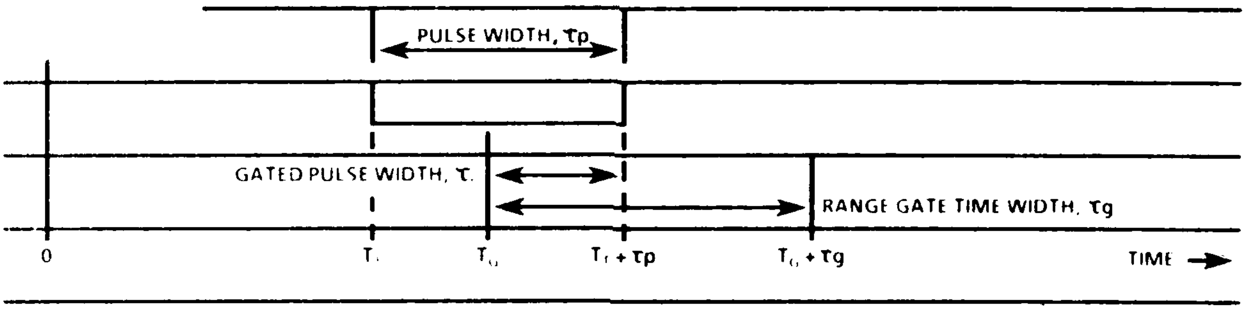
or defining a range gate eclipsing gain, G_{rge}, (≤ 1) and a range gate noise-reduction gain, G_{rgnr}(≥ 1):

$$(S/N) = (S/N)_o\ G_{rge}G_{rgnr}$$

where:

$$G_{rge} = (\tau_r/\tau_p)^2 \text{ and } G_{rgnr} = (T_r/\tau_g)$$

The range gate eclipsing gain, G_{rge}, is a function of the range of the target missile with respect to the range gate, as well as the size of the range gate and the pulse width of the radar. The range gate eclipsing gain is best represented by the following equations in terms of the target and range gate times depicted below.



where:

T_t	Time (of Two-Way Propagation) Corresponding to Range of Target Missile (sec)
T_g	Time Corresponding to Range of Inner Edge of Range Gate (sec)
τ_g	Range Gate Interval (Width) (sec)
τ_p	Transmitted Pulse Width (sec)

In terms of range, T_t and T_g are given by the equations:

$$T_t = 2R_t/C \text{ and } T_g = 2R_g/C$$

where:

R_t	Range to Target (m)
R_g	Range to Inner Edge of Range Gate (m)
C	Velocity of Propagation (m/sec)

For a matched gate ($\tau_g = \tau_p$), the range gate eclipsing gain is given by the following expressions.

For $T_t < T_g - \tau_p$:

$$G_{rge} = 0$$

For $T_g - \tau_p \leq T_t \leq T_g$:

$$G_{rge} = (1 - \frac{T_g - T_t}{\tau_p})^2$$

For $T_t = T_g$:

$$G_{rge} = 1$$

For $T_g < T_t \leq T_g + \tau_g$:

$$G_{rge} = (1 - \frac{T_t - T_g}{\tau_p})^2$$

For $T_t > T_g + \tau_g$:

$$G_{rge} = 0$$

For a stretched gate ($\tau_g > \tau_p$), the range gate eclipsing gain is given by the following expressions.

For $T_t < T_g - \tau_p$:

$$G_{rge} = 0$$

For $T_g - \tau_p \leq T_t < T_g$:

$$G_{rge} = (1 - \frac{T_g - T_t}{\tau_p})^2$$

For $T_g \leq T_t \leq T_g + \tau_g - \tau_p$:

$$G_{rge} = 1$$

For $T_g + \tau_g - \tau_p < T_t \leq T_g + \tau_g$:

$$G_{rge} = (1 - \frac{T_t - (T_g + \tau_g - \tau_p)}{\tau_p})^2$$

For $T_t > T_g + \tau_g$:

$$G_{rge} = 0$$

For a compressed gate ($\tau_g < \tau_p$), the range gate eclipsing gain is given by the following expressions.

For $T_t < T_g - \tau_p$:

$$G_{rge} = 0$$

For $T_g - \tau_p \leq T_t < T_g + \tau_g - \tau_p$:

$$G_{rge} = (1 - \frac{T_g - T_t}{\tau_p})^2$$

For $T_g + \tau_g - \tau_p \leq T_t \leq T_g$:

$$G_{rge} = (\tau_g/\tau_p)^2$$

For $T_g < T_t \leq T_g + \tau_g$:

$$G_{rge} = (1 - \frac{T_t + \tau_p - T_g - \tau_g}{\tau_p})^2$$

For $T_t > T_g + \tau_g$:

$$G_{rge} = 0$$

The range gate noise reduction gain is given by the equation:

$$G_{rgnr} = (T_r/\tau_g) \text{ (n.d.)}$$

VELOCITY GATING

When velocity gating (Doppler clutter filtering) is employed to reduce external noise (clutter), the signal-to-noise ratio is reduced by the Doppler clutter filter loss factor, L_{cf}. That is:

$$(S/N) = (S/N)_o(1/L_{cf})$$

THE COMPLETE RADAR RANGE EQUATION

When range gating, velocity gating, and both coherent and noncoherent pulse integration are employed, the expression for the signal-to-noise ratio becomes:

$$(S/N) = (S/N)_o(f_r T_{nci}/L_{ci} L_{nci} L_{cf})(d_r^2/d_g)$$

or:

$$(S/N) = (S/N)_o(f_r T_{nci}/L_{ci} L_{nci} L_{cf}) G_{rge} G_{rgnr}$$

or:

$$(S/N) = (\frac{P_a G_a^2 \lambda^2 \sigma}{(4\pi)^3 d_t N_s L_{so} R_t^4})(\frac{f_r T_{nci}}{L_{ci} L_{nci} L_{cf}}) G_{rge} G_{rgnr}$$

REDUCTION TO THE BASIC RADAR RANGE EQUATION

When pulse integration and Doppler filtering are not employed:

$f_r T_{nci} = 1$
$L_{ci} = L_{nci} = 1$
$L_{cf} = 1$

When range gating is not employed:

$\tau_r = \tau_p$
$\tau_g = T_r$
$d_r = d_g = 1$
$G_{rge} = 1$
$G_{rgnr} = 1$

The signal-to-noise ratio is then given by:

$$(S/N) = (S/N)_o = P_a G_a{}^2 \lambda^2 \sigma/(4\pi)^3 d_t L_{so} N_s R_t{}^4$$

ALTERNATE FORMS OF THE RADAR RANGE EQUATION

The pulse integration and clutter filter losses can be incorporated into the total system loss factor, L_s, to yield the equation:

$$(S/N) = \frac{P_a G_a{}^2 \lambda^2 \sigma T_{nci} G_{rge} G_{rgnr}}{(4\pi)^3 \tau_p L_s N_s R_t{}^4}$$

where:

$$L_s = L_{so} L_{ci} L_{nci} L_{cf}$$

or:

$$L_s = L_c{}^2 L_{dx}{}^2 L_r L_{da} L_{ci} L_{nci} L_{cf}$$

The radar equation can be solved for the maximum range for detection to yield the equation:

$$R_{tMax} = \left(\frac{P_a G_a{}^2 \lambda^2 \sigma T_{nci} G_{rge} G_{rgnr}}{(4\pi)^3 \tau_p L_s N_s (S/N)_{Min\ Det}}\right)^{1/4}$$

Where:

R_{tMax} = Maximum Range for Detection (m)

$(S/N)_{Min\ Det}$ = Minimum Signal-to-Noise Ratio, at the Detection Filter, Required for Detection (n.d.)

THIS PAGE INTENTIONALLY LEFT BLANK

REFERENCES

1. Barton, D. K., Radar System Analysis, Artech House, Inc., Dedham, MA, 1976.

2. Barton, D. K., and Ward, H. R., Handbook of Radar Measurement, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1969.

3. DiFranco, J. V., and Rubin, W. L., Radar Detection, Artech House, Inc., Dedham, MA, 1980.

4. Masters, G. W., Radar System Test and Evaluation, U.S. Naval Test Pilot School Textbook, U.S. Naval Test Pilot School, Patuxent River, MD, 1981.

5. Nathanson, F. E., Radar Design Principles, McGraw-Hill Book Company, New York, NY, 1969.

6. Skolnik, M. I., Introduction to Radar Systems, McGraw-Hill Book Company, New York, NY, 1962.

7. Skolnik, M. I., Radar Handbook, McGraw-Hill Book Company, New York, NY, 1970.

DISTRIBUTION:

Sanders Associates, Inc., Nashua, NH	(6)
NAVAIRTESTCEN (CT24)	(3)
NAVAIRTESTCEN (AT)	(1)
NAVAIRTESTCEN (RW)	(1)
NAVAIRTESTCEN (SA)	(1)
NAVAIRTESTCEN (SY)	(1)
USNTPS	(10)
DTIC	(2)

Naval Air Test Center, Patuxent River, Maryland 20670-5304

DERIVATION OF THE RADAR RANGE EQUATION FOR A PULSE-DOPPLER RADAR WITH RANGE AND VELOCITY GATING AND COHERENT AND NONCOHERENT PULSE INTEGRATION

27 June 1986 16 pages
TM 86-1 TP

Approved for public release; distribution unlimited.

1. Radar Range Equation
2. Pulse-Doppler Radar
3. Range and Velocity Gating
4. Coherent and Noncoherent Pulse Integration
5. Masters, G., Dr.

UNCLASSIFIED

Naval Air Test Center, Patuxent River, Maryland 20670-5304

DERIVATION OF THE RADAR RANGE EQUATION FOR A PULSE-DOPPLER RADAR WITH RANGE AND VELOCITY GATING AND COHERENT AND NONCOHERENT PULSE INTEGRATION

27 June 1986 16 pages
TM 86-1 TP

Approved for public release; distribution unlimited.

1. Radar Range Equation
2. Pulse-Doppler Radar
3. Range and Velocity Gating
4. Coherent and Noncoherent Pulse Integration
5. Masters, G., Dr.

UNCLASSIFIED

Naval Air Test Center, Patuxent River, Maryland 20670-5304

DERIVATION OF THE RADAR RANGE EQUATION FOR A PULSE-DOPPLER RADAR WITH RANGE AND VELOCITY GATING AND COHERENT AND NONCOHERENT PULSE INTEGRATION

27 June 1986 16 pages
TM 86-1 TP

Approved for public release; distribution unlimited.

1. Radar Range Equation
2. Pulse-Doppler Radar
3. Range and Velocity Gating
4. Coherent and Noncoherent Pulse Integration
5. Masters, G., Dr.

UNCLASSIFIED

Naval Air Test Center, Patuxent River, Maryland 20670-5304

DERIVATION OF THE RADAR RANGE EQUATION FOR A PULSE-DOPPLER RADAR WITH RANGE AND VELOCITY GATING AND COHERENT AND NONCOHERENT PULSE INTEGRATION

27 June 1986 16 pages
TM 86-1 TP

Approved for public release; distribution unlimited.

1. Radar Range Equation
2. Pulse-Doppler Radar
3. Range and Velocity Gating
4. Coherent and Noncoherent Pulse Integration
5. Masters, G., Dr.

UNCLASSIFIED

BEST AVAILABLE COPY

Naval Air Test Center, Patuxent River, Maryland 20670-5304

DERIVATION OF THE RADAR RANGE EQUATION FOR A PULSE-DOPPLER RADAR WITH RANGE AND VELOCITY GATING AND COHERENT AND NONCOHERENT PULSE INTEGRATION

27 June 1986 16 pages
TM 86-1 TP

Approved for public release; distribution unlimited.

1. Radar Range Equation
2. Pulse-Doppler Radar
3. Range and Velocity Gating
4. Coherent and Noncoherent Pulse Integration
5. Masters, G., Dr.

UNCLASSIFIED

Naval Air Test Center, Patuxent River, Maryland 20670-5304

DERIVATION OF THE RADAR RANGE EQUATION FOR A PULSE-DOPPLER RADAR WITH RANGE AND VELOCITY GATING AND COHERENT AND NONCOHERENT PULSE INTEGRATION

27 June 1986 16 pages
TM 86-1 TP

Approved for public release; distribution unlimited.

1. Radar Range Equation
2. Pulse-Doppler Radar
3. Range and Velocity Gating
4. Coherent and Noncoherent Pulse Integration
5. Masters, G., Dr.

UNCLASSIFIED

Naval Air Test Center, Patuxent River, Maryland 20670-5304

DERIVATION OF THE RADAR RANGE EQUATION FOR A PULSE-DOPPLER RADAR WITH RANGE AND VELOCITY GATING AND COHERENT AND NONCOHERENT PULSE INTEGRATION

27 June 1986 16 pages
TM 86-1 TP

Approved for public release; distribution unlimited.

1. Radar Range Equation
2. Pulse-Doppler Radar
3. Range and Velocity Gating
4. Coherent and Noncoherent Pulse Integration
5. Masters, G., Dr.

UNCLASSIFIED

Naval Air Test Center, Patuxent River, Maryland 20670-5304

DERIVATION OF THE RADAR RANGE EQUATION FOR A PULSE-DOPPLER RADAR WITH RANGE AND VELOCITY GATING AND COHERENT AND NONCOHERENT PULSE INTEGRATION

27 June 1986 16 pages
TM 86-1 TP

Approved for public release; distribution unlimited.

1. Radar Range Equation
2. Pulse-Doppler Radar
3. Range and Velocity Gating
4. Coherent and Noncoherent Pulse Integration
5. Masters, G., Dr.

UNCLASSIFIED