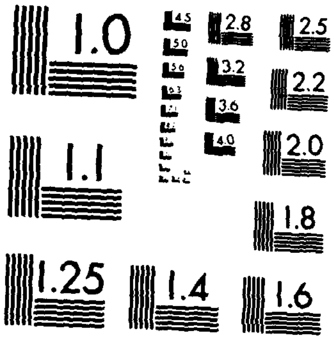
AD-A170 326 HYPOELLIPTICITY OF THE STOCHASTIC PARTIAL DIFFERENTIAL OPERATORS(U) NORTH CAROLINA UNIV AT CHAPEL HILL CENTER FOR STOCHASTIC PROC A S USTUNEL NOV 85 TR-129 1/1

UNCLASSIFIED AFOSR-TR-86-0346 F49620-85-C-0144 F/G 12/1 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AFOSR-TR- 86-0346

AD-A170 326

CENTER FOR STOCHASTIC PROCESSES

Department of Statistics
University of North Carolina
Chapel Hill, North Carolina

HYPOELLIPTICITY OF THE STOCHASTIC PARTIAL

DIFFERENTIAL OPERATORS

by

A.S. Ustunel

DTIC
ELECTE
JUL 23 1986
S D
B

Technical Report No. 129

November ~~1986~~ 1985

DTIC FILE COPY

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER			
AFOSR		F49620 85 C 0144			
8c. ADDRESS (City, State and ZIP Code)		10. SOURCE OF FUNDING NOS.			
Bldg. 410 Bolling AFB, DC		PROGRAM ELEMENT NO. 6.1102F	PROJECT NO. 2304	TASK NO. A5	WORK UNIT NO.
11. TITLE (Include Security Classification) Hypoellipticity of the stochastic partial differential operators"					
12. PERSONAL AUTHOR(S) A.S. Ustunel					
13a. TYPE OF REPORT technical	13b. TIME COVERED FROM 9/85 TO 8/86	14. DATE OF REPORT (Yr., Mo., Day) November 1985	15. PAGE COUNT 25		
16. SUPPLEMENTARY NOTATION					

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	Keywords:
XXXXX	XXXXXXXXX	XXXX	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This work is devoted to the proof of the following result: Let be any $\mathcal{D}'$-valued semimartingale satisfying the following stochastic partial differential equation

$$du_t = (-p + (-1)^{m_i+1}/2\, q_i^2)u_t dt + q_i u_t dW_t^i + dh_t, \quad i=1,\ldots,N,$$

where p, q_i are random partial differential operators with C^∞-coefficients, m_i is the degree of q_i and h is a semimartingale with values in the space of the C^∞-functions. If p satisfies a Garding's inequality on the space of the semimartingales with values in $\mathcal{D}$ and indexed by the uniformly bounded random intervals, then u is a semimartingale with values in the spcae of the C^∞-functions.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION	
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	UNCLASSIFIED	
22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
Peggy Ravitch	919-962-2307	AFOSR/NM

HYPOELLIPTICITY OF THE STOCHASTIC PARTIAL DIFFERENTIAL OPERATORS

by

A.S. Ustunel
Center for Stochastic Processes
Department of Statistics
University of North Carolina
Chapel Hill, NC
and
Centre National d'Etudes des Telecommunications
Issy-les-Moulineaux, France

tract. This work is devoted to the proof of the following result:
be any $\mathcal{D}'$-valued semimartingale satisfying the following stochastic
tial differential equation

$$du_t = (-p + (-1)^{m_i+1}/2\ q_i^2)u_t dt + q_i u_t dW_t^i + dh_t, \quad i = 1,\dots,N,$$

re p,q_i are random partial differential operators with C^∞-coefficients,
is the degree of q_i and h is a semimartinglae with values in the space
the C^∞-functions. If p satisfies a Garding's inequality on the space of
semimartingales with values in $\mathcal{D}$ and indexed by the uniformly bounded
dom intervals, then u is a semimartingale with values in the space of
C^∞-functions.

DTIC
ELECTE
JUL 23 1986
S D
B

earch supported in part by AFOSR Contract No. F49620 85 C 0144.

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DTIC
This technical report has been reviewed and is
approved for public release IAW AFR 190-12.
Distribution is unlimited.
MATTHEW J. KERPER
Chief, Technical Information Division

Introduction

In different branches of science one often encounters the so-called stochastic partial differential equations, e.g., in quantum physics, transport theory, polymer physics, chemistry, signal detection, etc. These equations are then studied in the context of the particular situation from which they originate. In this work we aim to give a start for a systematic treatment of these equations. In fact, we begin with the ideal hypothesis: almost all of the operators are "elliptic" and the equations are driven on one hand with a drift term absolutely continuous with respect to the one dimensional Lebesgue measure and on the other hand, the diffusion term is given by a stochastic integral with respect to a finite dimensional Wiener process. This is typically the case encountered in the filtering of diffusion processes (cf. [2], [5], [10]), except here the drift and diffusion operators are not respectively of the second and first order, they may depend on the whole history, and their coefficients are not necessarily semimartingales. Even at this level there are some interesting problems coming from this fact that the stochastic calculus is essentially a second order calculus. A second difference from the classical literature is the following: in all the works on the subject, it is always assumed that everything is nicely integrable, so that, one can work in some fixed Sobolev space using the hibertian techniques. For the qualitative study of these equations; the difference between integrable and nonintegrable cases is very important since in the latter case one can not handle a stochastic process with values in the space of the distributions as some Sobolev space-valued process. For instance the process

$$\{(I-A)^{W_t}\delta\,;\ t \geq o\}$$

where W is a standard, one dimensional Wiener process and δ is the Dirac's delta function, is a process which visits every Sobolev space of every order. Consequently, it is natural to study these equations in the frame of the distributions-valued stochastic processes and this is one of the essential difference of this work from the literature about the stochastic partial differential equations.

After some preliminary and important results, we begin by a general condition of hypoellipticity for the stochastic partial differential operators on the space of the "integrable" semimartingales with values in the space of the infinitely differentiable functions with compact support and we prove that this condition is sufficient for the hypoellipticity of the corresponding stochastic partial differential equation. By the hypoellipticity of an equation we understand that any distributions-valued solution of the equation is undistinguishable from a semimartingale with values in the space of the infinitely differentiable functions when the latter is injected in the space of the distributions. In the third section we extend the results of the preceding one to the non integrable case by using some recent results on the structure of the trajectories of the nuclear space-valued semimartingales and we end up by giving some examples of the operators satisfying the conditions announced before.

There are some aspects which we want to emphasize: in the equations the drift terms are perturbed by a certain operator derived from the diffusion coefficient. We can erase it by defining a generalized integral of Stratonovitch type for the operators of degree higher than one. If we look at this equation, then it behaves as a deterministic equation (i.e., without the stochastic integral term) as long as we are concerned with its hypoellipticity. The second important observation is the fact that the "purely random" part (i.e., the Stratonovitch integral) of the equation neither helps nor destroys the hypoellipticity of the equation; for the equations of second order this property is illustrated by an example where the operators are with constant coefficients. The same example can be extended to the case where the coefficients are C_b^∞, but, in this case, the flow of diffeomorphisms being nonlinear, the calculations are tedious (cf. [18] for an example).

Accession For
NTIS
Dist
A-1
QUALITY INSPECTED 4

I. Preliminaries and notations

In the following we shall deal only with the stochastic processes with values in the space of the distributions or Sobolev spaces. However, the results of this section are announced in abstract terminology since *they are true for more general spaces than the spaces of the distributions.*

Let (Ω,F,P) be a complete probability space with a right continuous, increasing, completed filtration $(F_t;\ t\in[o,1])$ of F. Suppose that F is a complete, separable nuclear space whose dual F' is also nuclear under its strong topology (denoted by F'_β). Denote by R° the space of the (equivalence classes of) continuous, adapted stochastic processes under the topology defined by the following metric:

$$d(x,o) = E((\sup_{o\leq t\leq 1} |x_t|)\ (\sup_{o\leq t\leq 1} |x_t| + 1)^{-1}) .$$

(R°,d) is a non-locally convex Fréchet space and, if J is a linear, continuous mapping from F into R°, then, there exists an adapted stochastic process $(X_t : t\in[o,1])$ with values in F' having almost surely continuous trajectories in F'_β such that, for any $\phi\in F$, $\langle X,\phi\rangle = (\langle X_t,\phi\rangle;\ t\in[o,1])$ is undistinguishable from $J(\phi)$ (cf. [6], [8]). Moreover, there exists an increasing sequence of absolutely convex, compact subsets $(K_n ;\ n\in\mathbf{N})$ of F' such that $(X_t ;\ t\in[o,1])$ lives almost surely in $\bigcup_{n\in\mathbf{N}} F'[K_n]$, where $F'[K_n]$ denotes the vector subspace of F' spanned by K_n under the norm topology defined by the guage function p_{K_n} of K_n and we can choose them in such a way that each one becomes a separable Hilbert space (cf. [6]). This result implies that such a process is locally bounded: there exists a sequence of stopping times $(T_n\,;\,n\in\mathbb{N})$ increasing to one such that the stopped process $X^n = (X_{t\wedge T_n}\,;\,t\in[o,1])$ lives in a bounded subset of $F'[K_n]$. In fact, it is sufficient to choose T_n as

$$T_n = \inf(t\in[o,1] : X_t \notin nK_n)\ .$$

Since F is nuclear we can suppose the injections $F'[K_n] \hookrightarrow F'[K_{n+1}]$ Hilbert-Schmidt or nuclear (cf. [11]), hence, if X^n is a weak semimartingale in $F'[K_n]$, its image in $F[K_{n+3}]$ becomes a semimartingale (cf. [16], [17]). Let us note that, if F is a countable inductive limit of Fréchet spaces, then, the continuity of the mapping J is equivalent to its sequential continuity (cf. [1]) and this condition is very easy to verify with the help of the closed graph theorem.

Let u be a semimartingale with values in a separable Hilbert space H, having the following decomposition:

$$u_t = u_o + \int_o^t a_s\,ds + \int_o^t b_s^i\ dW_s^i\,,\quad t\in[o,1]\,,$$

where $W_t = (W_t^1,\ldots,W_t^N)$ is a standard Wiener process with values in $\mathbb{R}^N$ and we use the usual summation convention $\cdot$ a and $(b^i\,;\,i=1,\ldots,N)$ are adapted, measurable processes with values in H satisfying

$$\int_o^1 \|a_s\|_H^2\ ds + \sum_i \int_o^1 \|b_s^i\|_H^2\ ds < +\infty \quad \text{a.s.}$$

We shall denote by $S_2(H)$ the space of such semimartingales for which the following norm is finite:

$$|u|_H^2 = E\|u_o\|_H^2 + E\int_o^1 (\|a_s\|_H^2 + \sum_{i=1}^N \|b_s^i\|_H^2)\,ds\ .$$

We also use a weaker topology on $S_2(H)$ defined by the following norm:

$$\|u\|_H^2 = E \int_o^1 \|u_s\|_H^2 \, ds + E\|u_o\|_H^2$$

and the corresponding scalar product is denoted by $(.,.)_H$. The subspace $\{u \in S_2(H) : u_o = o\}$ of $S_2(H)$ will be denoted by $S_2^o(H)$.

If H is a Sobolev space, the corresponding norms and the scalar products will be indicated by replacing H with the index of that Sobolev space. For $u \in S_2(H)$, a and b^i will be denoted respectively by D_+u and $\partial_{W^i}u$, $i=1,\ldots,N$. In fact, for such semimartingales D_+u and $\partial_{W^i}u$ are uniquely defined up to equivalence classes with respect to the measure $dt \times dP$ and D_+u is nothing but the forward derivative of E. Nelson (cf. [9]) and $\partial_{W^i}u$ is the dual operator of the stochastic integration operator and its uniqueness follows from the martingale representation theorem of K. Itô. For $H = \mathbb{R}$, $S_2(\mathbb{R})$ will be written simply as S_2 and for a nuclear space F, $S_2(F)$, and $S_2 \tilde{\otimes} F$ will be the equivalent notations, where, $S_2 \tilde{\otimes} F$ denotes the completed projective tensor product topology when S_2 is equipped with $|\cdot|$-topology. The nuclearity of F implies that $S_2 \tilde{\otimes} F$ is (topologically) isomorphic to the tensor product $S_2 \otimes F$ completed under the topology of bi-equicontinuous convergence, denoted by $S_2 \tilde{\tilde{\otimes}} F$ (cf. [11]), hence any $u \in S_2 \tilde{\tilde{\otimes}} F$ can be represented as an $F[K]$-valued semimartingale, $K \subset F$ being compact, absolutely convex (cf. [21]), hence we can defined D_+u and $\partial_{W^i}u$ as above.

Let us finally indicate that we are not very rigorous about the constants and they are often denoted by the same letters even if they differ from one line to another.

II. A condition for hypoellipticity

Let U be an open domain in $\mathbb{R}^d$ and $p_t(\omega,x,\partial_x)$ be a (random) partial differential operator of constant degree 2m, $m > o$. We suppose that p_t has measurable, adapted, C^∞-coefficients $a_{\alpha,\beta}(t,\omega,x)$ such that, for any $\mu \in \mathbb{N}^d$, $K \subset\subset U$ (i.e., compact subset of U),

$$\sup_{x \in K} \sup_{t \in [o,1]} |D_x^\mu a_{\alpha,\beta}(t,\omega,x)| \leq c_K \quad \text{a.s.},$$

where c_K is a constant depending only on K and μ. We make the following hypothesis about p_t :

(H) There exists $s > o$ such that, for any $K \subset\subset U$, there exists constants $c = c(K,s) > o$ and $\bar{c} = \bar{c}(K,s) > o$ with

$$\|u\|^2_{m+s-1} \leq c(B(u,u) + \bar{c}\, \|u\|^2_o)$$

for any $u \in S_2^o \tilde{\otimes} \mathcal{D}_K(U)$, where $S_2^o \tilde{\otimes} \mathcal{D}_K(U)$ denotes the subspace of $S_2^o \tilde{\otimes} \mathcal{D}(U)$ whose elements have their supports in K and B(u,u) is defined by the linear form

$$B(u,v) = E \int_o^1 ((D_+ + p_s)u_s, v_s)_o ds + 1/2\, E \sum_i \int_o^1 (\partial_{W^i} u_s, \partial_{W^i} u_s)_o ds .$$

Remarks

i) In fact the hypothesis (H) is nothing but the infinite dimensional form of the sharp Garding inequality.

(ii) In the above notations we are not very rigorous, in fact $D_+ u_s$ (also $\partial_{W^i} u_s$) should be understood as $(D_+ u)_s$ (respectively $(\partial_{W^i} u)_s$) but the following we shall continue to use it for the typographical reasons.

In the following, for $\alpha \in \mathbb{R}$, we shall denote by T^{α} the properly supported part of the pseudodifferential operator $(I-\Delta)^{\alpha/2}$ who defines the Sobolev norm of order α (cf. [15]).

Proposition II.1

Let $\alpha \in \mathbb{R}$ and suppose that the hypothesis (H) is satisfied. Then, for any $u \in S_2^0 \tilde{\otimes} \mathcal{D}_K(U)$, one has

$$\|u\|^2_{\alpha+m+s-1} \leq c[B(u,T^{2\alpha}u) + c_1\|u\|^2_{\alpha+m-1}]$$

where c and c_1 are positive constants depending only on α, s and K.

Proof:

By the hypothesis (H), we have

$$\|u\|^2_{\alpha+m+s-1} = \|T^{\alpha}u\|^2_{m+s-1} \leq c(B(T^{\alpha}u,T^{\alpha}u) + \bar{c}\|u\|^2_{\alpha}) .$$

Moreover

$$B(T^{\alpha}u,T^{\alpha}u) = B(u,T^{2\alpha}u) + ([p,T^{\alpha}]u,\ T^{\alpha}u)_o$$

Let S be the following operator:

$$S = T^{\alpha}[p,T^{\alpha}] .$$

Since $p^* = p + 0(2m-1)$, where $0(n)$ denotes and will denote in the following pages a properly supported random pseudodifferential operator of order n whose symbol is adapted, measurable, C^{∞}, having almost surely bounded x-derivatives uniformly in $t \in [o,1]$ on the compact subsets of U, we have

$$\begin{aligned} S^* &= [T^{\alpha},p]T^{\alpha} + 0(2m+2\alpha-2) \\ &= T^{\alpha}[T^{\alpha},p] + [[T^{\alpha},p],\ T^{\alpha}] + 0(2m+2\alpha-2) \\ &= -S + 0(2m+2\alpha-2) \end{aligned}$$

using the well-known results about the commutators (cf. [15], p. 39).

Hence we have the following estimate:

$$([p,T^{\alpha}]u,T^{\alpha}u)_o \leq \bar{c}_1 \|u\|^2_{\alpha+m-1} \quad .$$

//Q.E.D.

Proposition II.2

Suppose that u $S_2^o \tilde{\otimes} \mathcal{D}'(U)$ is a solution of the following stochastic partial differential equation:

$$\text{(II.1)} \qquad du_t = (-p + 1/2(-1)^{m_i+1} q_i^2)u_t \, dt + q_i u_t \, dW_t^i + dh_t$$

where q_i's are the operators as $p = p_t(\omega,x,\partial_x)$, of constant degree $m_i \leq m$ and h is a semimartingale in $S_2 \tilde{\otimes} E(U)$ ($E(U)$ denotes the nuclear Fréchet space of C^{∞} -functions on U). Let ϕ, $\phi_1 \in \mathcal{D}(U)$ with $\phi_1=1$ on the support of ϕ. If, for some $\alpha \in \mathbb{R}$, $\|\phi_1 u\|_{\alpha+m-1} < +\infty$, then one has

$$\|u^{\varepsilon}\|^2_{\alpha+m+s-1} \leq c_{\alpha,m}[\|\phi_1 u\|^2_{\alpha+m-1} + \|\phi_1 u\|_{\alpha} \|\phi_1 \mathcal{D}_+ h\|_{\alpha} + \sum_i \|\phi_1 \partial_{W^i} h\|^2_{\alpha+m}]$$

where $c_{\alpha,m}$ is a positive constant independent of $\varepsilon > o$ and $u^{\varepsilon} = J_{\varepsilon} \phi u$, J_{ε} being a Friedrich's mollifier.

Proof:

By Proposition II.1, we have

$$\|u^{\varepsilon}\|^2_{\alpha+m+s-1} \leq C[((D_+ + p)u^{\varepsilon}, T^{2\alpha}u^{\varepsilon})_o + \frac{1}{2}\sum_i \|\partial_{W^i} u^{\varepsilon}\|^2_{\alpha} + C_1 \|u^{\varepsilon}\|^2_{\alpha+m-1}]$$

Since u is a solution of (I.1), we have

$$D_+ u^{\varepsilon} = D_+ J_{\varepsilon} \phi u = J_{\varepsilon} \phi (-p + \sum_i \frac{(-1)^{m_i+1}}{2} q_i^2)u + J_{\varepsilon} \phi D_+ h \,.$$

Then

$$((D_+ + p)u^\varepsilon,\ T^{2\alpha} u^\varepsilon)_o = (J_\varepsilon \phi(-p + \frac{1}{2}\sum_i (-1)^{m_i+1} q_i^2)u + pJ_\varepsilon \phi u + J_\varepsilon \phi D_+ h,\ T^{2\alpha} u^\varepsilon)_o$$

$$= (-[J_\varepsilon \phi, p]u + \frac{1}{2} J_\varepsilon \phi (\sum_i (-1)^{m_i+1} q_i^2)u +$$

$$+ J_\varepsilon \phi D_+ h,\ T^{2\alpha} u^\varepsilon)$$

$$= (-[J_\varepsilon \phi, p]\phi_1 u + \frac{1}{2} J_\varepsilon \phi (\sum_i (-1)^{m_i+1} q_i^2)\phi_1 u +$$

$$+ J_\varepsilon \phi \phi_1 D_+ h,\ T^{2\alpha} J_\varepsilon \phi \phi_1 u)\ .$$

Letting $v = \phi_1 u$, we have

$$((D_+ + p)u^\varepsilon,\ T^{2\alpha} u^\varepsilon) = (-[J_\varepsilon \phi,\ p]v + \frac{1}{2}\sum_i (-1)^{m_i+1} J_\varepsilon \phi q_i^2 v +$$

$$+ J_\varepsilon \phi \phi_1 D_+ h,\ T^{2\alpha} J_\varepsilon \phi v)\ .$$

Let M be the operator defined by

$$M = \phi J_\varepsilon T^{2\alpha}[J_\varepsilon \phi,\ p]$$

and let us calculate M^*:

$$M^* = [p^*, \phi J_\varepsilon]T^{2\alpha} J_\varepsilon \phi$$

$$= [p, \phi J_\varepsilon]T^{2\alpha} J_\varepsilon \phi + 0(2m+2\alpha-2)$$

since $p^* = p + 0(2m-1)$ (cf. the proof of Proposition II.1 for the notation $0(2m-1)$. Then

$$M^* = [p,\ J_\varepsilon \phi + [\phi,\ J_\varepsilon]]\ T^{2\alpha} J_\varepsilon \phi + 0(2m+2\alpha-2)$$

$$= [p,\ J_\varepsilon \phi]\ T^{2\alpha} J_\varepsilon \phi + 0(2m+2\alpha-2)$$

$$= [p,\ J_\varepsilon \phi]\ T^{2\alpha}(\phi J_\varepsilon + [J_\varepsilon,\ \phi]) + 0(2m+2\alpha-2)$$

$$= [p,\ J_\varepsilon\phi]\ T^{2\alpha}\phi J_\varepsilon + 0(2m+2\alpha-2)$$

$$= [p,\ J_\varepsilon\phi]\ (\phi\ J_\varepsilon\ T^{2\alpha} + [T^{2\alpha},\ \phi J_\varepsilon]) + 0(2m+2\alpha-2)$$

$$= [p,\ J_\varepsilon\phi]\ \phi J_\varepsilon\ T^{2\alpha} + 0(2m+2\alpha-2)$$

$$= \phi\ J_\varepsilon\ T^{2\alpha}[p,\ J_\varepsilon\phi] + [[p,\ J_\varepsilon\phi],\ \phi J_\varepsilon\ T^{2\alpha}] + 0(2m+2\alpha-2)$$

$$= \phi\ J_\varepsilon\ T^{2\alpha}[p,\ J_\varepsilon\phi] + 0(2m+2\alpha-2)$$

$$= -M + 0(2m+2\alpha-2)\ .$$

Using this result and another mollifier and the fact that $\|u\|_{\alpha+m-1}$ is finite we obtain

$$\text{(II.2)}\quad ((D_+ + p)u^\varepsilon,\ T^{2\alpha}\ u^\varepsilon)_o \le c_1\|\phi_1 u\|^2_{m+\alpha-1} + \|\phi_1\ D_+ h\|_\alpha\ \|\phi_1 u\|_\alpha\ +$$

$$+\ 1/2(\sum_i\ (-1)^{m_i+1}\ J_\varepsilon\phi q_i^2\ u,\ T^{2\alpha}\ u^\varepsilon)_o\ .$$

Letting $Q = \phi J_\varepsilon T^{2\alpha}[J_\varepsilon\phi\ ,\ q_i^2]$ and arguing as above, we have

$$Q + Q^* = 0(2m_i + 2\alpha-2)\ ,$$

hence, by the same reasoning, we have

$$\text{(II.3)}\quad ((D_+ + p)u^\varepsilon,\ T^{2\alpha}\ u^\varepsilon)_o \le c_1'\|\phi_1 u\|^2_{m+\alpha-1} + \|\phi_1 D_+ h\|_\alpha\ \|\phi_1 u\|_\alpha\ +$$

$$1/2(\sum_i\ (-1)^{m_i+1}\ q_i^2\ u^\varepsilon,\ T^{2\alpha}u^\varepsilon)_o\ .$$

Let us look at the term $1/2\|\partial_{W^i} u^\varepsilon\|^2_\alpha$:

$$1/2\|\partial_{W^i} u^\varepsilon\|^2_\alpha = 1/2\|T^\alpha\ \partial_{W^i} u^\varepsilon\|^2_o$$

$$= 1/2\|T^\alpha(\partial_W i - q_i)u^\varepsilon\|^2_o + (T^\alpha(\partial_W i - q_i)u^\varepsilon,\ T^\alpha\ q_i u^\varepsilon)_o\ +$$

$$+\ 1/2\|T^\alpha\ q_i u^\varepsilon\|^2_o\ .$$

Adding this term to the right hand side of (II.3) (omitting the summation sign) we see that we have to estimate

$$\text{(II.4)}\quad (-1)^{m_i+1}\, 1/2(q_i^2\, u^\varepsilon,\, T^{2\alpha}\, u^\varepsilon)_o + 1/2\|T^\alpha(\partial_W i - q_i)u^\varepsilon\|_o^2 +$$

$$+ (T^\alpha(\partial_W i - q_i)u^\varepsilon,\, T^\alpha\, q_i u^\varepsilon)_o + 1/2\|T^\alpha\, q_i u^\varepsilon\|_o^2 .$$

We shall do this in several steps:

i) Let us look at the second term of (II.4): we have

$$\partial_{W^i} u^\varepsilon - q_i u^\varepsilon = J_\varepsilon \phi \phi_1 (q_i u + \partial_{W^i} h) - q_i J_\varepsilon \phi \phi_1 u .$$

Letting $v = \phi_1 u$, this sum can be written as

$$(\partial_W i - q_i)u^\varepsilon = [J_\varepsilon \phi,\, q_i]v + J_\varepsilon \phi \phi_1\, \partial_{W^i} h .$$

Therefore

$$\|T^\alpha(\partial_W i - q_i)u^\varepsilon\|_o^2 \leq 2\|T^\alpha[J_\varepsilon \phi,\ q_i]v\|_o^2 + 2\|T^\alpha J_\varepsilon \phi \phi_1\, \partial_{W^i} h\|_o^2$$

$$\leq 2c\|\phi_1 u\|_{m+\alpha-1}^2 + 2c'\|\phi_1\, \partial_{W^i} h\|_\alpha^2 .$$

ii) Now let us look at the third term of (II.4):

$$(T^\alpha(\partial_W i - q_i)J_\varepsilon \phi \phi_1\, u,\, T^\alpha\, q_i\, J_\varepsilon \phi\phi_1\, u)_o =$$

$$= (T^\alpha[J_\varepsilon \phi,\, q_i]v + T^\alpha\, J_\varepsilon \phi \phi_1\, \partial_{W^i} h,\, T^\alpha\, q_i J_\varepsilon \phi v)_o$$

$$\leq (T^\alpha[J_\varepsilon \phi,\ q_i]v,\, T^\alpha\, q_i J_\varepsilon \phi v)_o + \|\phi_1\, \partial_{W^i} h\|_\alpha\, \|\phi_1\, u\|_{m_i+\alpha-1} +$$

$$+ \|\phi_1\, \partial_{W^i} h\|_{2m_i+\alpha} \|\phi_1\, u\|_\alpha .$$

Let N be the operator

$$N = \phi J_\varepsilon\, q_i^*\, T^{2\alpha}[J_\varepsilon \phi,\, q_i] .$$

Then we have, as above,

$$N^* = [q_i^*, J_\varepsilon \phi] T^{2\alpha} q_i J_\varepsilon \phi$$

$$= \phi J_\varepsilon q_i T^{2\alpha}[q_i^*, J_\varepsilon \phi] + 0(2m_i+2\alpha-2)$$

$$= \phi J_\varepsilon q_i^* T^{2\alpha}[q_i, J_\varepsilon \phi] + 0(2m_i+2\alpha-2)$$

$$= -\phi J_\varepsilon q_i^* T^{2\alpha}[J_\varepsilon \phi, q_i] + 0(2m_i+2\alpha-2)$$

$$= -N + 0(2m_i+2\alpha-2)$$

since we have $q_i^* = (-1)^{m_i} q_i + r_i$ where r_i is an operator as q_i but whose degree is less than or equal to $m_i - 1$ and, similarly

$q_i = (-1)^{m_i} q_i^* + s_i$. Consequently, we have the following estimate:

$$(T^\alpha [J_\varepsilon \phi, q_i] v, T^\alpha q_i J_\varepsilon \phi v)_o \leq c\|\phi_1 u\|^2_{m_i+\alpha-1} \leq c'\|\phi_1 u\|^2_{m+\alpha-1}$$

iii) Now let us study the first and the last terms of (II.4):

(II.5) $1/2(T^\alpha q_i u^\varepsilon, T^\alpha q_i u^\varepsilon)_o + (((-1)^{m_i+1})/2)\ (q_i^2 u^\varepsilon, T^{2\alpha} u^\varepsilon)_o$.

We have

$$T^\alpha q_i u^\varepsilon = [T^\varepsilon, q_i] u^\varepsilon + q_i T^\alpha u^\varepsilon,$$

hence

$$\begin{aligned} 1/2\|T^\alpha q_i u^\varepsilon\|_o^2 &= 1/2\|[T^\alpha, q_i] u^\varepsilon\|_o^2 + ([T^\alpha, q_i] u^\varepsilon, q_i T^\alpha u^\varepsilon)_o + \\ &\quad + 1/2\|q_i T^\alpha u^\varepsilon\|_o^2 \\ &\leq c/2\|\phi_1 u\|^2_{\alpha+m_i-1} + ([T^\alpha, q_i] u^\varepsilon, q_i T^\alpha u^\varepsilon)_o + \\ &\quad + 1/2\|q_i T^\alpha u^\varepsilon\|_o^2 . \end{aligned}$$

The second term at the right hand side of this inequality can be written as

$$1/2((L + L^*)u^\varepsilon,\ u^\varepsilon)_o$$

where $L = T^\alpha q_i^*[T^\alpha,\ q_i]$ and one can show as above that

$$L^* = -L + O(2\alpha+2m_i-2)\ .$$

Hence

$$1/2((L + L^*)u^\varepsilon,\ u^\varepsilon)_o \leq c\|\phi_1 u\|^2_{m_i+\alpha-1}$$

and (II.5) can be majorated by

$$\text{(II.6)}\quad c\|\phi_1 u\|_{\alpha+m-1} + 1/2\|q_i T^\alpha u^\varepsilon\|^2_o + (-1/2)^{m_i+1}\ (q_i^2\ u^\varepsilon,\ T^{2\alpha}\ u^\varepsilon)_o\ .$$

The second term of (II.6) can be written as

$$1/2\|q_i T^\alpha u^\varepsilon\|^2_o = 1/2(([q_i^*\ q_i,\ T^\alpha] + T^\alpha\ q_i^*\ q_i)u^\varepsilon,\ T^\alpha\ u^\varepsilon)_o\ .$$

Let H be the operator defined as

$$H = T^\alpha[q_i^*\ q_i,\ T^\alpha]$$

then it is easy to see that

$$H^* = -H + O(2m_i+2\alpha-2)$$

therefore

$$1/2\|q_i\, T^\alpha\ u^\varepsilon\|^2_o \leq c\|\phi_1\, u\|^2_{m_i+\alpha-1} + 1/2(T^\alpha\ q_i^*\ q_i\ u^\varepsilon,\ T^\alpha\ u^\varepsilon)_o$$

and we can see that (II.6) can be majorated by

$$c\|\phi_1 u\|_{m+\alpha-1} + 1/2(q_i^*\ q_i u^\varepsilon,\ T^{2\alpha}\ u^\varepsilon)_o + (-1/2)^{m_i+1}(q_i^2\ u^\varepsilon,\ T^{2\alpha}\ u^\varepsilon)_o\ .$$

Since $q_i^* = (-1)^m\ q_i+r_i$, we have

$$1/2(q_i^*\ q_i\ u^\varepsilon,\ T^{2\alpha}\ u^\varepsilon)_o + (-1/2)^{m_i+1}\ (q_i^2\ u^\varepsilon,\ T^{2\alpha}\ u^\varepsilon)_o = 1/2(r_i q_i\ u^\varepsilon,\ T^{2\alpha}\ u^\varepsilon)_o\ .$$

Let θ_i denote the operator $r_i q_i$. Then θ_i is an operator of degree less than or equal to $2m_i-1$ and θ_i^* can be written as $-\theta_i + t_i$ where t_i is another such operator of degree less than or equal to $2m_i-2$. Let Z be defined as $T^{2\alpha}\theta_i$. Then we have

$$Z^* = \theta_i^* T^{2\alpha} = -\theta_i T^{2\alpha} + 0(2\alpha+2m-2)$$

$$= -Z + 0(2\alpha+2m_i-2) .$$

Consequently (II.6) can be majorated by

$$C\|\phi_1 u\|_{\alpha+m-1}$$

where $C > o$ is independent of $\varepsilon > o$ and adding all the majorations, we see that the proof is completed.

//Q.E.D.

We have the immediate

<u>Theorem II.1</u>

For any u which is a solution of the equation (II.1) *in* $S_2 \tilde{\otimes} \mathcal{D}'(U)$, *there* exists a semimartingale u belonging to $S_2 \otimes E(U)$ such that $u(\phi)$ and $\hat{u}(\phi)$ are undistinguishable for any $\phi \in \mathcal{D}(U)$.

<u>Proof</u>:

We have $u_o = h_o$, so, modifying h, we may suppose that $u_o = o$. Since $u \in S_2 \tilde{\otimes} \mathcal{D}'(U)$, for any $\psi \in \mathcal{D}(U)$, there exists some α $\mathbb{R}$ such that $|u\psi|_\alpha$ (hence $\|u\psi\|_\alpha$) is finite (cf. [16], [24]). Let now (η_k) be a sequence in $\mathcal{D}(U)$ such that

i) supp $\psi \subset$ supp η_k , for any $k \geq 1$,

ii) $\eta_k = 1$ on supp η_{k+1} , for any $k \geq 1$.

Then, for any $k \geq 1$, we have also $\|\eta_k \psi u\|_\alpha < +\infty$, and Proposition II.2 implies that $\|\eta_{k+1}\psi u\|_{s+\alpha} < +\infty$, for any $k \geq 1$, hence $\|\psi u\|_{s+\alpha} < +\infty$, and repeating this argument sufficiently many times we see that $\|\psi u\|_\beta$ is finite for any given $\beta \in \mathbb{R}$. Consequently, ψu defines a linear, continuous mapping u^ψ from $\mathcal{D}'(U)$ into the Hilbert space of the dtx dP-square integrable, measurable, adapted, real-valued processes denoted by L_{22}. Define $\tilde{u}$ as

$$\tilde{u} = \sum_\alpha u^{\psi_\alpha}$$

where (ψ_α) is a locally finite partition of unity of U. Then the Theorem of Banach-Steinhaus implies that $\tilde{u}$ is a continuous mapping from $\mathcal{E}'(U)$ into L_{22}, hence there exists an element of $\mathcal{E}(U) \tilde{\otimes} L_{22}$, denoted again by $\tilde{u}$ corresponding to this (nuclear) mapping. Then, it is sufficient to define $\hat{u}$ by

$$\hat{u}_t = h_t + \int_o^t (-p + ((-1)^{m_i+1}/2)q_i^2)\, \tilde{u}_s\, ds + \int_o^t q_i \tilde{u}_s\, dW_s^i .$$

//Q.E.D.

<u>Remark:</u>

Suppose that we have an equation of the following type:

(II.7) $du_t = -pu_t\, dt + q_i u_t\, dW_t^i + dh_t$.

In order to apply Corollary (II.1) to this equation we write it in the following form:

$$du_t = (-\tilde{p} + ((-1)^{m_i+1}/2)q_i^2)\, u_t\, dt + q_i u_t\, dW_t^i + dh_t ,$$

where

$$\tilde{p} = p + ((-1)^{m_i+1}/2)q_i^2 .$$

III. Some examples and extensions

In the preceding section we have supposed that the solutions of the equation (I.1) were in $S_2 \tilde{\otimes} \mathcal{D}'(U)$, i.e., that D_+u and $\partial_{W^i}u$ were square integrable with respect to $dt \times dP$ on $[o,1] \times \Omega$ for $i=1,\ldots,N$. In fact this is rather a restricted case as one can see by looking at the following equation:

$$\text{(III.1)} \quad du_t = 1/2\, \Delta\, u_t\, dt - \partial_i\, u_t\, dW_t^i + dh_t\,, \quad \partial_i = \partial/\partial x_i\,,$$

where h is a semimartingale with values in $\mathcal{E}(\mathbf{R}^d)$ growing faster than the function $\exp(|x|^2/2)$ at infinity. One can show, using the integration by parts formula (cf. [16], [18], [20]) that the equation (III.1) is hypoelliptic (it has even a unique solution) but u_t is non-integrable for any $t > o$.

In order to solve the *hypoellipticity problem in this case* we shall study with the processes indexed by $\mathbf{R}_+$ instead of $[o,1]$ and modify the hypothesis (H) in the following way:

(H') For any bounded stopping time T, for any $K \subset\subset U$, one has

$$\|u\|^2_{m+s-1,\,T} \leq C(B_T(u,u) + \bar{c}\|u\|^2_{o,T})$$

for any u $S_2^o \tilde{\otimes} \mathcal{D}_K(U)$, where

$$\|u\|^2_{\alpha,T} = E \int_o^T \|u_s\|^2_\alpha \; ds, \quad \alpha \in \mathbf{R}\,,$$

$$B_T(u,v) = E \int_o^T ((D_+ + p)u_s,\, v_s)ds + (1/2)E \int_o^T (\partial_{W^i}u_s,\; \partial_{W^i}v_s)ds$$

and C and $\bar{c}$ are positive constants depending only on K, s and T.

Now we have the following

Theorem III.1

Suppose that u is a semimartingale with values in $\mathcal{D}'(U)$ satisfying the following equation:

$$du_t = (-p + \frac{1}{2}(-1)^{m_i+1} q_i^2)\, u_t\, dt + q_i u_t\, dW_t^i + dh_t$$

and h is a semimartingale with values in $\mathcal{E}(U)$ having the following decomposition:

$$dh_t = D_+ h_t\, dt + \partial_{W^i} h_t\, dW_t^i .$$

If the hypothesis (H') is satisfied, then there exists a semimartingale $\hat{u}$ with values in $\mathcal{E}(U)$ such that $\hat{u}(\phi)$ and $u(\phi)$ are undistinguishable for any $\phi \in \mathcal{D}(U)$.

Proof:

Obviously, it is sufficient to show the above result on $[o,n) \times \Omega$ for any $n \in \mathbb{N}$. Hence we shall suppose that n=1 for the notational simplicity.

Since $\mathcal{D}(U)$ is the countable inductive limit of nuclear Fréchet spaces, the fact that $u(\phi)$ has almost surely continuous trajectories for any $\phi \in \mathcal{D}(U)$ implies the almost sure strong continuity of the trajectories of u as a $\mathcal{D}'(U)$-valued semimartingale (cf. Section I). Moreover there exists a sequence of stopping times (T_n) increasing to one such that the stopped process $u^n = (u_{t \wedge T_n} ; t \;\; [o,1])$ is bounded (in $\mathcal{D}'(U)$). Consequently u^n is an element of $S_2 \check{\otimes} \mathcal{D}'(U)$ on the stochastic interval $[\![o, T_n]\!] = \{(t,\omega);\ t \leq T_n(\omega)\}$. Then, applying Propositions II.1, II.2 and Corollary II.1 (in which we replace the limits of integration with respect to dt by $[o, T_n(\omega)]$) we see that there exists a semimartingale $\hat{u}^n$ with values in $\mathcal{E}(U)$ such that, for any $\phi \in \mathcal{D}(U)$, $\hat{u}^n(\phi)$ and $u^n(\phi)$ are undistinguishable on the stochastic interval $[\![o, T_n]\!]$. Now define $\hat{u}_t$ on $[o,1)$ as

$$\hat{u}_t = \hat{u}^n_t \quad \text{if} \quad t < T_n$$

for some $n \in \mathbb{N}$. Since (T_n) increases to one with probability one, u is well defined on $[o,1) \times \Omega$ and this completes the proof.

///Q.E.D.

Examples

1) The simplest example of the operator p satisfying the hypothesis (H') is any deterministic operator $p(x, \partial_x)$ which is elliptic of order 2m. In fact, for such operators, using Garding's inequality (cf. [22]) we have

$$||u||^2_{m,T} \leq C((pu, u)_{o,T} + ||u||^2_{o,T})$$

for any bounded stopping time T and u $S^o_2 \tilde{\otimes} \mathcal{D}_K(U)$. Using Itô's formula for $||u_t||^2_o$:

$$||u_t||^2_o = 2 \int_o^t (u_s, du_s)_o + \int_o^t ||\partial_{W^i} u_s||^2_o \, ds$$

we obtain

$$||u||^2_{m,T} \leq C(((D_+ + p)u,u)_{o,T} + (1/2) \sum_i ||\partial_{W^i} u||^2_{o,T} + \bar{c}||u||^2_{o,T})$$

for any bounded stopping time T and u $S^o_2 \tilde{\otimes} \mathcal{D}_K(U)$.

2) Let us look at the following equation:

$$du_t = [1/2 \sum_i^m X_i^2 + 1/2 \sum_i^N Z_j^2 + X_o + c]u_t \, dt + Z_j u_t \, dW^j_t + dh_t$$

where X_i, X_o, Z_j are random operators as before but all of them are of the first degree, c is an adapted, measurable process with values in $E(U)$ having almost surely uniformly bounded derivatives on the compact subsets of U and h is an $E(U)$-valued semimartingale having a (unique) decomposition as in Theorem III.1. Then we have

Proposition III.1

Suppose that we have, for $i=1,\dots,d$,

$$\partial_i = \sum_1 a_q C_q$$

where C_q is defined by

$$C_1 = X_{j_1},\ C_q = [X_{j_q}, C_{q-1}];\ j_1,\dots,j_q \in \{1,\dots,m\},$$

a_q is an $E(U)$-valued, adapted, measurable process with almost surely uniformly bounded derivatives on the compact subsets of U. Suppose further that, for an $\varepsilon \in (o,1)$, there exists some $\alpha \in (o,1)$ such that, for any $K \subset U$, bounded stopping time T,

$$\text{(III.2)}\quad \sup_{\|v\|_{\alpha,T} \leq 1,\ v \in S_2^o \tilde{\otimes} \mathcal{D}_K(U)} \Big\| \sum_{q \geq r(\varepsilon)} a_q C_q v \Big\|_{\alpha-1,T} \leq \varepsilon/2d$$

for some $r(\varepsilon) \in \mathbb{N}$, depending on α,ε and K where

$$r(\varepsilon) \leq -\ln \alpha/\ln 2 .$$

Then the hypothesis (H') is satisfied for

$$p = (-1/2) \sum_i X_i^2 - X_o - c .$$

Proof:

Since the proof is similar to those of [2] and [3], we shall underline only the parts of it where the condition (III.2) plays an important role.

Let α,ε and $r(\varepsilon)$ be as in the hypothesis. Then we have

$$\begin{aligned}
\|u\|^2_{\alpha,T} &\leq \|u\|^2_{o,T} + \sum_{i \in d} \|\partial_i u\|^2_{\alpha-1,T} \\
&= \|u\|^2_{o,T} + \sum_{i \leq d} \Big\|\sum_q a^i_q C^i_q u\Big\|^2_{\alpha-1,T} \\
&\leq \|u\|^2_{o,T} + 2 \sum_{i \leq d} \Big(\Big\|\sum_{q \leq r(\varepsilon)} a^i_q C^i_q u\Big\|^2_{\alpha-1,T} + \Big\|\sum_{q > r(\varepsilon)} a^i_q C^i_q u\Big\|^2_{\alpha-1,T}\Big) .
\end{aligned}$$

By the hypothesis, we have

$$(1-\varepsilon)\,\|u\|^2_{\alpha,T} \le \|u\|^2_{o,T} + 2\sum_{i\le d}\|\sum_{q\le r(\varepsilon)} a^i_q C^i_q u\|^2_{\alpha-1,T}$$

for any $u \in S^o_2 \tilde{\otimes} \mathcal{D}_K(U)$. Using the same method as in [2] or [3], one can show that

$$\|C_q u\|^2_{\alpha-1,T} \le C\,\{\sum_{i=1}^{m} \|X_i u\|^2_{2^q\alpha-1,T} + \|u\|^2_{o,T}\},\ q \le r(\varepsilon)\,.$$

By the hypothesis about $r(\varepsilon)$, we have $2^q\alpha \le 1$, hence

$$\text{(III.3)}\quad \|C_q u\|^2_{\alpha-1,T} \le C\,\{\sum_{1}^{m} \|X_i u\|^2_{o,T} + \|u\|^2_{o,T}\}\,.$$

Applying the following energy inequality (cf. [2], [3])

$$\|X_i u\|^2_{\beta,T} \le C[((D_+ + p)u,\ T^{2\beta}u)_{o,T} + (1/2)\sum_i \|\partial_{W^i} u\|^2_{\beta,T} + \bar{c}\|u\|^2_{\beta,T}]$$

to the right hand side of (III.3) with β=o and summing all the terms up to $r(\varepsilon)$, we obtain

$$(1-\varepsilon)\,\|u\|^2_{\alpha,T} \le C_1[((D_+ + p)u,u)_{o,T} + (1/2)\sum_i \|\partial_{W^i} u\|^2_{o,T} + \bar{c}_1\|u\|^2_{o,T}]$$

for any $u \in S^o_2 \tilde{\otimes} \mathcal{D}_K(U)$ and stopping time T.

///Q.E.D.

Remark:

The above case differs from the one treated in [2] since the vector fields may depend on the past and the solution takes its values in the space of the distributions, not in the space of the Radon measures. The hypothesis that we have used above is implied by the restricted Hörmander's condition with uniformly bounded number of Lie brackets on each small neighbourhood. Note that using the stopping technics as above, the results of [2] can be immediately extended to the solutions taking their values in the space of the distributions.

3) In this example we shall study a very simple equation in order to observe the effect of the stochastic integral part of the equation on its hypoellipticity: Suppose we have the following equation

$$\text{(III.4)} \quad du_t = (-p + 1/2\Delta)u_t\,dt - \partial_i u_t\,dW_t^i + dh_t$$

with $N = d$ and p is a partial differential operator with constant coefficients and h is a semimartingale with values in $E(\mathbf{R}^d)$. Let us denote by v_t the following process

$$v_t = u_t * \delta_{-W_t}$$

where δ_{-W_t} is the Dirac's delta function whose support is $W_t(\omega)$. Using the integration by parts formula (cf. [16], [18]) we see that $v = (v_t;\ t \geq o)$ is a semimartingale with values in $\mathcal{D}'(\mathbf{R}^d)$ satisfying the following equation:

$$dv_t = -pv_t\,dt + dk_t$$

where $k = (k_t;\ t \geq o)$ is another $E(\mathbf{R}^d)$-valued semimartingale. Consequently, all the hypoellipticity of the equation (III.4) is coming from the operator p but not from the operator $-1/2\Delta$. In other words, if we write (III.4) in the following form

$$du_t = -pu_t\,dt - \partial_i u_t \circ dW_t^i + dh_t$$

where the stochastic integral with respect to W is taken in the sense of Stratonovitch, then we see that this integral neither contributes to, nor troubles the hypoellipticity of this equation.

REFERENCES

[1] Adasch, N., Ernst, B. and Keim, D. (1978). Topological Vector Spaces. Lecture Notes in Math. Vol. 639. Springer, Berlin-Heidelberg-New York.

[2] Chaleyat-Maurel, M. and Michel, D. (1983). Hypoellipticity theorems and conditional laws. Preprint.

[3] Chaleyat-Maurel, M. (1981). La condition d'hypoellipticité d'Hörmander. Astérisque 84-85, p. 189-202. SMF.

[4] Gilkey, P.B. (1974). The Index Theorem and the Heat Equation. Math. Lecture Series 4. Publish or Perish, Inc.

[5] Krylov, N. and Rozovskii, B. (1980). On the first integrals and Liouville equations for diffusion processes. Proceedings of the 3rd IFIP Working Conference. Lecture Notes in Control and Information Sciences, Vol. 36, p. 145-165. Springer-Verlag. Berlin-Heidelberg-New York.

[6] Martias, C. (1983). Sur les supports des processus à valeurs dans des espaces nucléaires. Preprint.

[7] Métivier, M. and Pellaumail, J. (1980). Stochastic Integration. Academic Press, New York.

[8] Mitoma, I. (1983). On the sample continuity of $\mathcal{S}'$-processes. To appear in J. Math. Soc. Japan.

[9] Nelson, E. (1967). Dynamical Theories of Brownian Motion. Princeton University Press. Princeton, New Jersey.

[10] Pardoux, E. (1982). Equations du filtrage non linéaire de la prédiction et du lissage. Stochastics, Vol. 6, p. 193-231.

[11] Schaefer, H.H. (1971). Topological Vector Spaces. Graduate Texts in Math., Springer. Berlin-Heidelberg-New York.

[12] Schechter, M. (1977). Modern Methods in Partial Differential Equations. McGraw-Hill International Company, New York.

[13] Schwartz, L. (1973). Théorie des Distributions. 3rd Edition. Hermann-Paris.

[14] Taylor, M.E. (1981). Pseudodifferential Operators. Princeton University Press. Princeton, New Jersey.

[15] Treves, F. (1980). Introduction to Pseudodifferential and Fourier Integral Operators, Vol. 1. Plenum Press, New York and London.

[16] Ustunel, A.S. (1982). A characterization of semimartingales on nuclear spaces. Zeit. Wahrsch. verw. Gebiete, 60, p. 21-39.

[17] Ustunel, A.S. (1983). An erratum to "A characterization of semimartingales on nuclear spaces" after Laurent Schwartz. To appear in Zeit. Wahrsch. verw. Gebiete.

[18] Ustunel, A.S. (1982). Some applications of stochastic integration on infinite dimensions. Stochastics, 7, p. 255-288.

[19] Ustunel, A.S. (1983). Analytic semimartingales and their boundary values. J. Functional Analysis, Vol. 51, no. 2, p. 142-158.

[20] Ustunel, A.S. (1983). Distributions-valued semimartingales and applications to control and filtering. Preprint, to appear in Lecture Notes in Math.

[21] Ustunel, A.S. (1982). Stochastic integration on nuclear spaces and its applications. Annales de l'Institut Henri Poincaré, Vol. XVIII, no. 2, p. 165-200.

[22] Yosida, K. (1974). Functional Analysis. 3rd Edition, Springer-Verlag, Berlin-Heidelberg-New York.

END

DTIC

9—86