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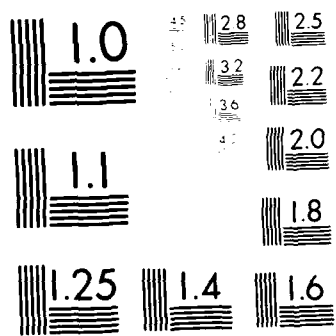
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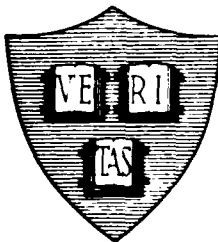
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**DUGDALE MODEL FOR CIRCUMFERENTIAL THROUGH-CRACKS IN
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DUGDALE MODEL FOR CIRCUMFERENTIAL THROUGH-CRACKS IN PIPES LOADED BY BENDING

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Abstract

The paper contains an analytical solution to the title problem by means of the semi-membrane theory of cylindrical shells. Results include the crack opening displacements as a function of circumferential angle, the fracture parameter J , and the effects of the crack on the compliance of the pipe.

INTRODUCTION

Ideally the failure load of a cracked pipe could be calculated from theory and the results of a small number of tests on simple specimens. Unfortunately this is not the case nor is it likely to be any time in the near future. A good idea of the "state of the art" can be gained from reading recent reports on research projects conducted by General Electric [1], Battelle [2], and the Proceedings of an A.S.M.E. Symposium [3]. The fundamental theory of nonlinear fracture mechanics is still being developed. How it applies to real structures, pipes in particular, is not a routine matter. Engineering procedures based on available knowledge plus a considerable amount of expensive testing must be used to get information for maintenance or design purposes.

Frequently the practice is to apply experimentally determined factors to results for cracks in flat plates to correct for curvature effects or to make adjustments to results from elastic solutions for cracks in cylindrical shells to account for plasticity. Such procedures are not unreasonable but



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leave unanswered such questions as: can tests on small diameter pipes be trusted to predict what will happen to large diameter pipes? It seems that more effective use could be made of cylindrical shell theory without resort to finite element methods. The present paper is directed toward that end in the case of circumferential through-cracks.

The analysis to be presented has some limitations of its own, however. It applies when the pipe material is such that the Dugdale model for plasticity effects is appropriate. It applies only when the crack is sufficiently long for the semi-membrane theory of cylindrical shells to be valid. For short cracks with a Dugdale zone shallow shell theory should be used, and that, strangely enough, is a more difficult problem. It could be solved but no one has yet done it. Most interest seems to be in cracks which extend at least an eighth of the circumference and that is long enough for the present analysis to be valid. For some further discussion see [4]. The analysis here is limited to the bending case because that seems to be most important technically. Tension, torsion, or combined loading cases can be done by the same methods.

Results of the sort obtained here could be obtained by finite element methods and without need for the Dugdale model but, of course, at the expense of much more computing. The principal advantage of the present approach is that the results come out in closed form in terms of elementary functions. Some numerical root-finding is required, but programming to do parameter studies would be trivial. The analysis includes linear elastic fracture and plastic collapse as limiting cases. A theory of tearing based on the present analysis and an additional physical hypothesis has been constructed and will be the subject of a subsequent paper.

FUNDAMENTAL EQUATIONS

Dimensionless displacements, stress functions, and stress measures are related to corresponding dimensional quantities (with an overbar) by the relations

$$\begin{aligned}
 (\bar{u}, \bar{v}, \bar{w}) &= \frac{\sigma_F R}{E} (u, v, \epsilon^{-2} w) \\
 (\bar{\chi}_z, \bar{\chi}_\theta, \bar{\psi}) &= \sigma_F R^2 h \epsilon^2 (R \chi_z, R \chi_\theta, \epsilon^{-2} \psi) \\
 (\bar{N}_z, \bar{N}_\theta, \bar{N}_{z\theta}) &= \sigma_F h (N_z, N_\theta, N_{z\theta}) \\
 (\bar{M}_z, \bar{M}_\theta, \bar{M}_{z\theta}) &= \sigma_F R h \epsilon^2 (M_z, M_\theta, M_{z\theta}) \\
 M &= \pi R^2 h \sigma_F \sigma
 \end{aligned} \tag{1}$$

Here σ_F is the flow stress of the material, h is the pipe thickness, R is the pipe radius, and E is Young's modulus, M is the applied bending moment on the pipe, σ is a dimensionless load parameter and ϵ is a small parameter given by

$$\epsilon^2 = (h/R)[12(1-\nu^2)]^{-1/2} \tag{2}$$

The analysis in the present paper is similar to that in [4] where it was found that the most significant results are obtained by assuming a semi-membrane state of stress in the cylinder and neglecting the small contribution of edge effects. The solution thus obtained is correct except in the immediate vicinity of singularities such as the crack tips. Nevertheless quantities such as stress intensity factors, overall compliance, and so forth, important to fracture mechanics, turn out to be correct. A semi-membrane state is derived from two complex-valued functions Φ and φ each of which satisfy the characteristic equation

$$(\ddot{\Phi} + \Phi)'' - i\epsilon^{-2}\Phi'' = 0 \tag{3}$$

Here primes and dots refer to differentiation with respect to z or θ respectively where zR is the axial distance measured from the cracked section and θ is the circumferential angle measured from the middle of the crack. The functions Φ and φ are related by

$$\Phi'' = \epsilon^2 \varphi \tag{4}$$

Dimensionless displacements and stress functions are given in terms of Φ by the formulas

$$\begin{aligned}
 \epsilon^2 u &= \ddot{\Phi}' & \epsilon^2 \chi_z &= -i\ddot{\Phi}' \\
 \epsilon^2 v &= -\ddot{\Phi} & \epsilon^2 \chi_\theta &= i\ddot{\Phi} \\
 w &= -\ddot{\Phi} + i\varphi & \psi &= i\ddot{\Phi} + \varphi
 \end{aligned} \tag{5}$$

Dimensionless membrane and bending stress measures are given in terms of φ by

$$\begin{aligned} N_z &= \ddot{\varphi} & M_z &= -i(\varphi'' + \nu \ddot{\varphi}) \\ N_\theta &= \varphi'' & M_\theta &= -i(\ddot{\varphi} + \nu \varphi'') \\ N_{z\theta} &= -\dot{\varphi}' & M_{z\theta} &= -i(1-\nu)\dot{\varphi}' \end{aligned} \quad (6)$$

In these formulas the physical variables, displacements, etc., are to be understood as the real part of the expressions on the right-hand-side of the equation.

ANALYSIS

Because of symmetry boundary conditions can be applied on the section $z = 0$. In physical terms the crack edges are free of stress. A Dugdale zone is assumed to exist beyond the crack tips for an as yet undetermined distance. Within the Dugdale zone the axial membrane stress equals the flow stress. The axial bending stress is ignored on the basis of the assumption that the axial stress equals the flow stress throughout the thickness of the pipe. For a sufficiently large applied bending moment compressive stresses on the side of the pipe far from the crack could exceed the flow stress and so a compressive Dugdale zone (of unknown extent) is admitted to prevent it. All remaining boundary conditions on $z = 0$ follow directly from the symmetry of the problem. In the analysis the pipe is assumed to be infinitely long and subjected to a bending moment at infinity in such a way that the crack is symmetrically located on the tension side. According to the semi-membrane theory only two boundary conditions apply on any segment of the section $z = 0$. In other words two of the possible four conditions are redundant. Boundary conditions on stress measures are more conveniently expressed in terms of conditions on stress functions obtained in accordance to procedures in [5]. Because of symmetry the boundary conditions need only be stated in the range $0 < \theta < \pi$. In terms of dimensionless quantities they are

$$\chi_z = 0 \quad \text{all } \theta \quad (7)$$

$$\begin{aligned}\epsilon^2 \chi_\theta &= 0 & (0 \leq \theta < \alpha) \\ &= \sin(\theta - \alpha) - (\theta - \alpha) & (\alpha < \theta < \beta) \end{aligned} \quad (8)$$

$$\begin{aligned}&= \sin \theta - (\pi - \theta) + \frac{\pi}{2} \sigma \cos \theta - G_1 \sin \theta & (\gamma < \theta < \pi) \\ u &= 0 & (\beta < \theta < \gamma) \end{aligned} \quad (9)$$

G_1 is a constant of integration. For positive values of θ the crack extends from $\theta = 0$ to $\theta = \alpha$, the tension Dugdale zone lies in $\alpha < \theta < \beta$, the compressive Dugdale zone (if any) lies in $\gamma < \theta < \pi$, and similarly for negative values of θ . By means of equation (5) the boundary conditions (on $z = 0$) can be restated in terms of $\ddot{\Phi}$

$$\begin{aligned}R \{i\ddot{\Phi}_c'\} &= 0 & \text{all } \theta \\ \ddot{\Phi}_c' &= 0 & \beta < \theta < \gamma \\ R \{i\ddot{\Phi}_c\} &= 0 & 0 \leq \theta < \alpha \\ &= 1 - \frac{1}{2}(\theta - \alpha)^2 - \cos(\theta - \alpha) & \alpha < \theta < \beta \\ &= - (1 + \cos \theta) + \frac{1}{2}(\pi - \theta)^2 + \frac{\pi}{2} \sigma \sin \theta + G_1 \cos \theta + G_2 & \gamma < \theta \leq \pi\end{aligned} \quad (10)$$

G_2 is a constant of integration. The subscript c has been introduced to indicate that these conditions apply to the complete solution. The complete solution $\ddot{\Phi}_c$ is a superposition of a function $\ddot{\Phi}$ which can be assumed to vanish at infinity, a beam solution (which accounts for bending by moments at infinity) given by

$$\ddot{\Phi}_B = \frac{1}{2} [\epsilon^2 z^2 \cos \theta + i(2 \cos \theta - \theta \sin \theta) \sigma] \quad (11)$$

and a collection of four elementary solutions given by

$$\ddot{\Phi}_E = i a - \epsilon b z + i c \cos \theta + \epsilon d z \cos \theta \quad (12)$$

which in physical terms mean a null solution (does nothing), a real axial translation, an imaginary lateral translation, and a real rotation. The constants a , b , c , and d are real, and as yet undetermined. In case there is no compressive Dugdale zone then $\gamma = \pi$ and the third of equations (8) becomes inoperative. The boundary conditions for the reduced problem read

$$R \{i\ddot{\Phi}'\} = 0 \quad \text{all } \theta \quad (13)$$

$$\ddot{\Phi}' = \epsilon(b - d \cos \theta) \quad \beta < \theta < \gamma \quad (14)$$

$$\begin{aligned} R \{i\ddot{\Phi}\} &= \frac{1}{2}(2 \cos \theta - \theta \sin \theta) \sigma + a + c \cos \theta & 0 < \theta < \alpha \\ &= \frac{1}{2}(2 \cos \theta - \theta \sin \theta) \sigma + a + c \cos \theta \\ &\quad + 1 - \frac{1}{2}(\theta - \alpha)^2 - \cos(\theta - \alpha) & \alpha < \theta < \beta \\ &= \frac{1}{2}[2 \cos \theta - (\theta - \pi) \sin \theta] \sigma + a + c \cos \theta \\ &\quad - (1 + \cos \theta) + \frac{1}{2}(\pi - \theta)^2 + G_1 \cos \theta + G_2 & \gamma < \theta < \pi \end{aligned} \quad (15)$$

According to [6] the necessary and sufficient conditions for $\ddot{\Phi}$ to vanish at infinity are

$$\int_0^\pi \ddot{\Phi} d\theta = 0, \quad \int_0^\pi \ddot{\Phi} \cos \theta d\theta = 0 \quad (16)$$

and in such a case $\ddot{\Phi}'$ is accurately approximated by

$$\ddot{\Phi}' = -i^{3/2} \epsilon(\ddot{\Phi} + \frac{1}{2}\Phi)'' \quad (17)$$

Now put

$$\ddot{\Phi} = i^{1/2} H(\theta), \quad \text{on } z = 0 \quad (18)$$

then by (17)

$$\ddot{\Phi}' = \epsilon(\ddot{H} + \frac{1}{2}H) \quad \text{on } z = 0 \quad (19)$$

Equation (13) will be satisfied if H is real. Equation (14) becomes a simple differential equation for H in the range $\beta < \theta < \gamma$, while (15) gives H explicitly over the rest of the range of θ .

Complete results for H are as follows

$$H = -\sqrt{2}(a + c \cos \theta) - \sqrt{2}(\cos \theta - \frac{1}{2}\theta \sin \theta)\sigma \quad 0 \leq \theta \leq \alpha \quad (20)$$

$$\begin{aligned} H &= -\sqrt{2}(a + c \cos \theta) - \sqrt{2}(\cos \theta - \frac{1}{2}\theta \sin \theta)\sigma \\ &\quad - \sqrt{2}[1 - \frac{1}{2}(\theta - \alpha)^2 - \cos(\theta - \alpha)] & \alpha \leq \theta \leq \beta \end{aligned} \quad (21)$$

$$H = 2(b + d \cos \theta) + 4P \cos x + 4Q \sin x \quad \beta \leq \theta \leq \gamma \quad (22)$$

$$\begin{aligned}
 H = & -\sqrt{2} [a + G_2 + (c + G_1) \cos \theta] \\
 & -\sqrt{2} [\cos \theta + \frac{1}{2} (\pi - \theta) \sin \theta] \sigma \\
 & + \sqrt{2} [1 + \cos \theta - \frac{1}{2} (\pi - \theta)^2] \quad \gamma \leq \theta \leq \pi \quad (23)
 \end{aligned}$$

On physical grounds the displacements and stress functions must be continuous at α , β , and γ . This means that H , $\dot{H}$, and $\ddot{H}$ must be continuous at β and γ . Continuity at α is automatic. The extent of the Dugdale zones is determined by the condition that $\dot{\chi}_z$ (or $\ddot{H}$) be continuous at β and γ . Recall that in plane stress applications of the idea an analogous condition must be imposed to avoid infinite stresses. In the present context a discontinuity in $\ddot{H}$ means (mathematically) the presence of a concentrated load but it should be understood that semi-membrane theory is an asymptotic approximation in which a large stress acting over a short distance gets replaced by a concentrated load.

To simplify the discussion a bit consider the case in which there is no compressive Dugdale zone. Then $\gamma = \pi$ and $Q = 0$. There are six unknown constants namely, a , b , c , d , P , and β . There are six equations namely, the four continuity conditions on H and its first three derivatives and the two conditions (16) which can be written in the equivalent form

$$\int_0^\pi (\ddot{H} + H) d\theta = 0 \quad , \quad \int_0^\pi \ddot{H} \cos \theta d\theta = 0 \quad (24)$$

As it turns out the continuity conditions involve a , b , c , and d only in the combinations

$$A = a + \sqrt{2} b \quad , \quad C = c + \sqrt{2} d \quad (25)$$

As an algebraic system the four continuity conditions are four linear equations for the three constants A , C , and P . The condition that the equations be compatible is

$$\begin{aligned}
 \sigma = & \{ [\sin \beta - \sin \alpha + (\beta - \alpha) \cos \beta] \sin y + \sqrt{2} (\beta - \alpha) \sin \beta \cos y \} \\
 & + \left[\frac{1}{2} (\beta + 3 \sin \beta \cos \beta) \sin y + \sqrt{2} \sin^2 \beta \cos y \right] \quad (26)
 \end{aligned}$$

where

$$y = (\pi - \beta)/\sqrt{2}$$

This is a transcendental equation for the determination of β given the load parameter σ and the crack length α . Only A , C , and P are required to determine the crack opening displacements $u_c(0, \theta)$, and (according to the Dugdale model) the fracture parameter $J = 2\sigma_F \bar{u}_c(0, \theta)$. However, b and d are needed to calculate the extension and rotation of the pipe (over and above beam theory) at sections remote from the crack. These deformations relate to the increased flexibility of the pipe due to the presence of the crack. The constants in question are determined by use of (24).

Results for A , C , P , Q , and σ are listed here for the case $\gamma < \pi$. If no compressive Dugdale zone is present replace γ by π .

$$Q = \sigma \sin \gamma - (\pi - \gamma) \quad (27)$$

$$P \sin x = Q \cos x - \sigma \sin \beta + \beta - \alpha \quad (28)$$

$$x = (\gamma - \beta)/\sqrt{2} \quad (29)$$

$$A = \sigma \cos \beta + \frac{1}{2}(\beta - \alpha)^2 - \sqrt{2}(P \cos x + Q \sin x) \quad (30)$$

$$C \sin \beta = -\frac{1}{2}(7 \sin \beta + \beta \cos \beta) \sigma + \beta - \alpha + \sin(\beta - \alpha) \quad (31)$$

$$N_1 = [\sin \beta - \sin \alpha + (\beta - \alpha) \cos \beta] \sin x - \sqrt{2}[\pi - \gamma - (\beta - \alpha) \cos x] \sin \beta \quad (32)$$

$$D_1 = \frac{1}{2}(\beta + 3 \sin \beta \cos \beta) \sin x - \sqrt{2}(\sin \gamma - \sin \beta \cos x) \sin \beta \quad (33)$$

$$N_2 = [\sin \gamma - (\pi - \gamma) \cos \gamma] \sin x + \sqrt{2}[(\pi - \gamma) \cos x - (\beta - \alpha)] \sin \gamma \quad (34)$$

$$D_2 = \sqrt{2}(\sin \gamma \cos x - \sin \beta) \sin \gamma - \frac{1}{2}[3 \sin \gamma \cos \gamma - (\pi - \gamma)] \sin x \quad (35)$$

$$\sigma = N_1/D_1 = N_2/D_2 \quad (36)$$

In case $\gamma = \pi$ the formula $\sigma = N_1/D_1$ reduces to (26). The value of σ for which the axial membrane stress at $\theta = \pi$ first reaches the flow stress is given by

$$\sigma = (\sqrt{2} \sin y - \beta + \alpha)/(\sqrt{2} \sin y - \sin \beta) \quad (37)$$

where $y = (\pi - \beta)/\sqrt{2}$.

At the plastic collapse load

$$\gamma = \beta = (\alpha + \pi)/2 \quad (38)$$

and the collapse value of the load parameter is given by

$$\sigma_L = 4 \left(\cos \frac{\alpha}{2} - \frac{1}{2} \sin \alpha \right) / \pi \quad (39)$$

For $\gamma < \pi$ the expressions in (36) must be used simultaneously to determine β and γ given σ and α by some numerical root-finding process. Small values of $\pi - \gamma$ are especially troublesome because the second expression for σ in (36) becomes indeterminate at $\gamma = \pi$. A polynomial approximation for $(\sin z)/z$ given in [7] may be found helpful.

Results for a, b, c, d , and G_1 follow from

$$\begin{aligned} \pi a = & (\pi - \beta)A + [\sin \beta - \sin \gamma - (\pi - \gamma)\cos \gamma]\sigma + \frac{1}{8}[(\beta - \alpha)^3 - (\pi - \gamma)^3 + 3(\pi - \gamma)^4] \\ & + [2 \sin x + \sqrt{2}(\pi - \gamma)]P + 2(1 - \cos x)Q \end{aligned} \quad (40)$$

$$b = \sqrt{2}(A - a)/2 \quad (41)$$

$$G_1 \cos \gamma = 1 + \cos \gamma - C \cos \gamma - \sqrt{2}P - [2 \cos \gamma + \frac{1}{2}(\pi - \gamma) \sin \gamma]\sigma \quad (42)$$

$$\begin{aligned} \pi c = & -(\beta - \gamma + \sin \beta \cos \beta - \sin \gamma \cos \gamma)C - (\pi - \gamma - \sin \gamma \cos \gamma)G_1 \\ & - 2\sigma[\beta + \frac{7}{8}\sin \beta \cos \beta + \frac{1}{8}\beta \cos 2\beta + \pi - \gamma - \frac{7}{8}\sin \gamma \cos \gamma + \frac{1}{8}(\pi - \gamma) \cos 2\gamma] \\ & - 2[\sin \beta - \sin \alpha - \frac{1}{2}(\beta - \alpha)\cos \alpha - \frac{1}{2}\cos \beta \sin(\beta - \alpha) \\ & + \sin \gamma - \frac{1}{2}(\pi - \gamma) + \frac{1}{2}\sin \gamma \cos \gamma] + 2\sqrt{2}P[2 \sin \gamma - 2 \sin \beta \cos x - \sqrt{2} \cos \beta \sin x] \\ & - 2\sqrt{2}Q[\sqrt{2} \cos \gamma + 2 \sin \beta \sin x - \sqrt{2} \cos \beta \cos x] \end{aligned} \quad (43)$$

$$d = \sqrt{2}(C - c)/2 \quad (44)$$

A general expression for the complete axial displacement (dimensionless) on the cracked section is given by

$$\epsilon u_c(0, \theta) = \ddot{H} + \frac{1}{2}H - b + d \cos \theta \quad (45)$$

Displacements on the crack are obtained from (45) by using expressions for H valid in the range $0 < \theta < \alpha$. The result is

$$\epsilon u_c(0, \theta) = \frac{1}{2} \sqrt{2} [(3 \cos \theta - \frac{1}{2} \theta \sin \theta) \sigma - A + C \cos \theta] \quad (46)$$

The dimensionless crack opening displacement $\delta = 2u_c(0, \alpha)$ is given by

$$\epsilon \delta = \sqrt{2} [(3 \cos \alpha - \frac{1}{2} \alpha \sin \alpha) \sigma - A + C \cos \alpha] \quad (47)$$

According to the Dugdale model the fracture parameter J equals the flow stress times the crack opening displacement. In dimensional form the result is

$$J = \sigma_F^2 R \delta / E \quad (48)$$

Values of the field variables (stress measures, displacements, etc.) could be calculated for values of z other than $z = 0$ by solving the characteristic equation (3) for Φ with boundary conditions according to (18) and (20) to (23), possibly by means of Fourier series. At present it seems there would not be much interest in such calculations. Disturbances in the stress distribution due to the crack decay exponentially as z increases, otherwise the pipe behaves as a beam. How rapidly these effects decay depends mostly on the value of ϵ . The smaller ϵ is the slower the decay. For $\epsilon = .1$ the effects have mostly died out three or four diameters away from the cracked section. The crack does, however, cause a rotation and an axial displacement which persist. Dimensionless axial displacements approach

$$\epsilon u_c = \epsilon \sigma z \cos \theta - b + d \cos \theta \quad \text{as } z \rightarrow \infty \quad (49)$$

The term with z corresponds to the pipe bending as a beam fixed at $z = 0$. The additional physical rotation and axial displacement of cross sections are given by

$$\Omega = \sigma_F d / E \epsilon \quad (50)$$

$$U = - \sigma_F R b / E \epsilon \quad (51)$$

Results from the present analysis should reduce to those from linear elastic fracture mechanics in the limit $\sigma_F \rightarrow \infty$. Since J must remain finite (48) shows that $\delta \rightarrow 0$ in the same limit, as expected. The extent of the Dugdale zone $\beta - \alpha$ must also become small. Somewhat lengthy calculations show that for small $\beta - \alpha$

$$\epsilon \delta = \sqrt{2} (\beta - \alpha)^2 \quad (52)$$

$$\sigma = (\beta - \alpha) / \mu \sin \alpha \quad (53)$$

where

$$\mu = 1 + \frac{\sqrt{2}}{4} \frac{\alpha + \alpha \cot^2 \alpha - \cot \alpha}{\cot [(\pi - \alpha) / \sqrt{2}] + \sqrt{2} \cot \alpha} \quad (54)$$

and

$$\left(\frac{\partial u_c}{\partial \theta} \right)_{\theta=\alpha} = -\epsilon^{-1} \sqrt{2} (\beta - \alpha) \quad (z = 0) \quad (55)$$

These equations imply

$$\epsilon \delta = \sqrt{2} \sigma^2 \mu^2 \sin^2 \alpha \quad (56)$$

It follows then from (48) that

$$J = \frac{\sqrt{2} R}{E \epsilon} \sigma_F^2 \sigma^2 \mu^2 \sin^2 \alpha \quad (57)$$

but from (1) repeated here

$$M = \pi R^2 h \sigma_F \sigma \quad (58)$$

it follows that

$$J = \frac{\sqrt{2}}{\pi^2 E \epsilon R^3 h^2} M^2 \mu^2 \sin^2 \alpha \quad (59)$$

This agrees with the result in [8] if account is taken of the relation

$$I = 2h R J \quad (60)$$

between the I in [8] and J . Equations (48), (52), and (55) lead to

$$J = \frac{1}{2} \sqrt{2} E \epsilon R \left(\frac{1}{R} \frac{\partial \bar{u}_c}{\partial \theta} \right)_{\theta=\alpha}^2 \quad (61)$$

which with $J = K^2/E$ yields

$$K = 2^{-1/4} \epsilon^{1/2} R^{1/2} E \left(\frac{1}{R} \frac{\partial \bar{u}_c}{\partial \theta} \right)_{\theta=\alpha} \quad (62)$$

This last equation shows that according to semi-membrane theory in the elastic case the opening angle at the crack tip is a measure of the stress intensity factor. Why this should be so must remain

somewhat obscure for the present because an analytical solution to the full shell equations valid in the neighborhood of the crack tip is not available. The angle here must be the limit of some quantity obtained from such a solution.

CONCLUDING REMARKS

Rather complete results have been obtained for the problem of through circumferential cracks in pipes with use of the Dugdale model and according to thin shell theory. One would expect the theory to be more accurate for low strain hardening materials and for thin pipes. The thinness requirement is not too severe as far as shell theory is concerned. Shell theory is reasonably good for values of R/h as small as ten and not grossly in error for R/h as small as five or six. See the paper by LeFort et al. in [3]. Errors involved in the use of the Dugdale model as applied in the present paper are more difficult to assess. The shape of the plastic zone near the tip of a circumferential crack is probably quite different from what it is in a flat plate. Stresses tend to decay more slowly in the axial direction and equalize more rapidly in the circumferential direction than they do in a plate. One might be tempted to argue either way on the matter but perhaps it would be better to await the results of more refined analyses and/or the results of experiment. A sort of adaptation of the line-spring model might allow the theory to deal with strain hardening in a more realistic fashion but there seems not to be much point in pushing the model too far at present.

With the introduction of an additional hypothesis governing crack growth the present model can be used as the basis for a theory of tearing. That something about the shape of the crack tip remains invariant as the crack grows has been suggested as such an hypothesis [9, 10]. An adaptation of that hypothesis to the circumferential crack problem will be the subject of a subsequent paper.

ACKNOWLEDGEMENT

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