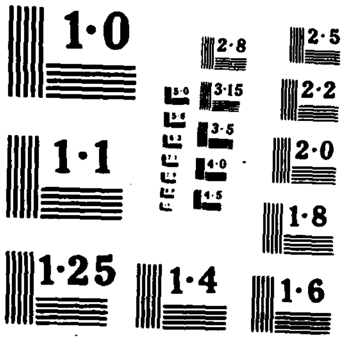
AD-A169 950 A MATHEMATICAL FORMULATION OF A LARGE SPACE STRUCTURE CONTROL PROBLEM(U) CALIFORNIA UNIV LOS ANGELES DEPT OF ELECTRICAL ENGINEERING A V BALAKRISHNAN SEP 85 1/1

UNCLASSIFIED AFOSR-TR-86-0444 AFOSR-83-0318 F/G 22/2 NL


1·0
2·8
2·5
3·15
2·2
3·5
1·1
4·0
2·0
4·5
1·8
1·25
1·4
1·6

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AD-A169 950

DTIC FILE COPY

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

(2)

1. REPORT NUMBER: AFOSR-TR- 86-0444	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): A MATHEMATICAL FORMULATION OF A LARGE SPACE STRUCTURE CONTROL PROBLEM	5. TYPE OF REPORT & PERIOD COVERED: interim
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): A.V. Balakrishnan	8. CONTRACT OR GRANT NUMBER(s): AFOSR 83-0318
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Electrical Engineering Departrment, SEAS, UCLA, Los Angeles, CA 90024	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F, 9769-01
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research (NM), Building 410, Bolling AFB, Washigton, D.C. 20332	12. REPORT DATE: September 1985
	13. NUMBER OF PAGES: 5
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

To be presented at 24th IEEE Conference on Decision and Control, Fort Lauderdale, Florida, December 11-13, 1985.
To be published in the Proceedings.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper presents an abstract-mathematical formulation of a Large Space Structure Control problem. The physical apparatus consists of a softly supported antenna attached to the space shuttle by a flexible beam-like truss. The control objective is to slew the antenna on command within the given accuracy and maintaining stability. The control forces and torques are applied at the shuttle end as well as the antenna end and in addition provision is made for a small number of 2-axis proof-mass actuators along the beam. The beam motion is modelled by partial differential equations. Of the

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

86 7 23 164

variety of Control problems possible we touch only on the time-optimal proble.

''lsac

AFOSR-TR- 86-0444

A MATHEMATICAL FORMULATION OF A LARGE SPACE STRUCTURE CONTROL PROBLEM

A. V. Balakrishnan

24th IEEE Conference on Decision and Control
Fort Lauderdale, Florida
December 11-13, 1985

To be published in the Proceedings

Accesion For	
NTIS CRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and / or Special
A-1	

A MATHEMATICAL FORMULATION OF A LARGE SPACE STRUCTURE CONTROL PROBLEM

A. V. Balakrishnan +

Department of Electrical Engineering
6731 Boelter Hall
UCLA
Los Angeles, CA 90024

1. Introduction

This paper presents an abstract-mathematical formulation of a Large Space Structure Control problem experiment being conducted by the Space Controls Branch (L.W. Taylor[1]) at the NASA Langley Research Center. Briefly, the physical apparatus consists of a softly supported antenna attached to the space shuttle by a flexible beam-like truss. The control objective is to slew the antenna on command within the given accuracy and maintaining stability, based on noisy sensor data and limited control authority; allowance must also be made for random disturbance. The control forces and torques are applied at the shuttle end as well as the antenna end and in addition provision is made for a small number of 2-axis proof-mass actuators along the beam.

The beam motion is modelled by partial differential equations, and we begin in Section 2 with the equations of motion as derived by L.W. Taylor.[1] The abstract formulation as a nonlinear wave-equation in a Hilbert space is given in Section 3. Existence and uniqueness theory is in Section 4. The basic controllability results are in Section 5 and the stabilizability results in Section 6. Of the variety of Control problems possible we touch only on the time-optimal problem, briefly in Section 7.

2. Equations of Motion

We shall need to be brief here -- for necessary elaboration see [1]. The equations of motion, using the continuum model (as opposed to a finite-element model) consist of standard beam bending and torsion partial differential equations with driving end conditions and forces applied at the locations of the proof-mass actuators.

Roll Beam Bending

$$PA \frac{\partial^2 u_\phi}{\partial t^2} + EI_\phi \frac{\partial^4 u_\phi}{\partial s^4} = \sum_{n=1}^{4} [f_{\phi,n}\delta(s-s_n) + g_{\phi,n} \frac{\partial\delta}{\partial s}(s-s_n)]$$

Pitch Beam Bending

$$PA \frac{\partial^2 u_\theta}{\partial t^2} + EI \frac{\partial^4 u_\theta}{\partial s^4} = \sum_{n=1}^{4} [f_{\theta,n}\delta(s-s_n) + g_{\theta,n} \frac{\partial\delta}{\partial s}(s-s_n)]$$

Yaw Beam Torsion

$$PI_\psi \frac{\partial^2 u_\psi}{\partial t^2} - GI_\psi \frac{\partial^2 u_\psi}{\partial s^2} = \sum_{n=1}^{4} g_{\psi,n}\delta(s-s_n)$$

$$\omega_1(t) = \begin{vmatrix} \dot{u}'_\phi(t,0+) \\ \dot{u}'_\theta(t,0+) \\ \dot{u}_\psi(t,0+) \end{vmatrix}$$

$$\omega_4(t) = \begin{vmatrix} \dot{u}'_\phi(t,L-) \\ \dot{u}'_\theta(t,L-) \\ \dot{u}_\psi(t,L-) \end{vmatrix}$$

These are the angular velocity vectors of the shuttle and the antenna respectively. Let

$$g_1(t) = \begin{vmatrix} g_{\phi,1}(t) \\ g_{\theta,1}(t) \\ g_{\psi,1}(t) \end{vmatrix}$$

$$g_4(t) = \begin{vmatrix} g_{\phi,4}(t) \\ g_{\theta,4}(t) \\ g_{\psi,4}(t) \end{vmatrix}$$

and let the force applied at reflector center of mass be

$$F_r = [F_x, F_y, 0]^T .$$

Then

$$g_1(t) = -(\hat{I}_1\dot{\omega}_1 + \omega_1 \otimes I_1\omega_1 - M_1(t) - M_D(t))$$

$$g_4(t) = -(\hat{I}_4\dot{\omega}_4 + \omega_4 \otimes \hat{I}_4\omega_4 - M_4(t) - r \otimes F_r(t)) - m_4 r \otimes \ddot{\xi}_4 .$$

where

$$\hat{I}_4 = I_4 + m_4 \begin{vmatrix} r_y^2 & -r_x r_y & 0 \\ -r_x r_y & r_x^2 & 0 \\ 0 & & r_x^2 + r_y^2 \end{vmatrix} ,$$

$$\xi_4 = \begin{vmatrix} -u_\phi(L-) \\ u_\theta(L-) \\ z(L-) \end{vmatrix} = \text{coordinates of beam tip} ,$$

and

$r = (r_x, r_y, 0)^T$ = coordinates of reflector center of mass with respect to axes through beam tip.

+ Research supported in part under AFOSR Grant 83-0318.

Let

$$f_1(t) = \begin{vmatrix} f_{\phi,1}(t) \\ f_{\theta,1}(t) \end{vmatrix} \qquad f_3(t) = \begin{vmatrix} f_{\phi,3}(t) \\ f_{\theta,3}(t) \end{vmatrix}$$

$$f_2(t) = \begin{vmatrix} f_{\phi,2}(t) \\ f_{\theta,2}(t) \end{vmatrix} \qquad f_4(t) = \begin{vmatrix} f_{\phi,4}(t) \\ f_{\theta,4}(t) \end{vmatrix}$$

Then

$$f_1(t) = - \begin{vmatrix} m_1\ddot{u}_\phi(t,0+) \\ m_1\ddot{u}_\theta(t,0+) \end{vmatrix}$$

$$f_2(t) = - \begin{vmatrix} m_2\ddot{u}_\phi(t,s_2) + m_2\ddot{\Delta}_{\phi,2} \\ m_2\ddot{u}_\theta(t,s_2) + m_2\ddot{\Delta}_{\theta,2} \end{vmatrix}$$

$$f_3(t) = - \begin{vmatrix} m_3\ddot{u}_\phi(t,s_3) + m_3\ddot{\Delta}_{\phi,3} \\ m_3\ddot{u}_\theta(t,s_3) + m_3\ddot{\Delta}_{\theta,3} \end{vmatrix}$$

$$f_4(t) = -m_4 \begin{vmatrix} 1 & 0 & r_x \\ 0 & 1 & r_y \end{vmatrix} \begin{vmatrix} \ddot{u}_\phi(L-) \\ \ddot{u}_\theta(L-) \\ \ddot{u}_\psi(L-) \end{vmatrix} + \begin{vmatrix} F_y \\ -F_x \end{vmatrix} .$$

3. Abstract Formulation

The Hilbert space H is $L_2(0,L)^3 \times R^{14}$, $0 < L < \infty$. The points s_2, s_3, $0 < s_2 < s_3 < L$ are fixed. We define the operator A on the domain $\mathcal{D} \subset H$, consisting of 3×1 functions $u_\phi(\cdot)$, $u_\theta(\cdot)$, $u_\psi(\cdot)$ such that $u_\phi(\cdot)$, $u_\phi'(\cdot)$, $u_\phi''(\cdot)$, $u_\phi'''(\cdot) \in L_2[0,L]$ and $u_\phi'''(\cdot)$ has L_2-derivatives in $[0,s_2]$, $[s_2,s_3]$ and $[s_3,L]$; $u_\theta'(\cdot)$ and $u_\theta''(\cdot) \in L_2[0,L]$; the remaining "scalar" part (in R^{14}) is specified by

$$\begin{aligned} x_4 &= u_\phi(0+) & x_{11} &= u_\phi'(L-) \\ x_5 &= u_\theta(0+) & x_{12} &= u_\theta'(L-) \\ x_6 &= u_\phi(L-) & x_{13} &= u_\psi(L-) \\ x_7 &= u_\theta(L-) & x_{14} &= u_\phi(s_2) \\ x_8 &= u_\phi'(0+) & x_{15} &= u_\theta(s_2) \\ x_9 &= u_\theta'(0+) & x_{16} &= u_\phi(s_3) \\ x_{10} &= u_\psi(0+) & x_{17} &= u_\theta(s_3) \end{aligned}$$

The operator A is defined by $Ax = y$;

$$\begin{aligned} y_1 &= EI_\phi u_\phi''''(\cdot) & y_8 &= -EI_\phi u_\phi''(0+) \\ y_2 &= EI_\theta u_\theta''''(\cdot) & y_9 &= -EI_\theta u_\theta''(0+) \\ y_3 &= -GI_\psi u_\psi''(\cdot) & y_{10} &= -GI_\psi u_\psi'(0+) \\ y_4 &= EI_\phi u_\phi'''(0+) & y_{11} &= EI_\phi u_\phi''(L-) \\ y_5 &= EI_\theta u_\theta'''(0+) & y_{12} &= EI_\theta u_\theta''(L-) \\ y_6 &= -EI_\phi u_\phi'''(L-) & y_{13} &= GI_\psi u_\psi'(L-) \\ y_7 &= -EI_\theta u_\theta'''(L-) & & \end{aligned}$$

$$y_{14} = EI_\phi(u_\phi'''(s_2+) - u_\phi'''(s_2-))$$

$$y_{15} = EI_\theta(u_\theta'''(s_2+) - u_\theta'''(s_2-))$$

$$y_{16} = EI_\phi(u_\phi'''(s_3+) - u_\phi'''(s_3-))$$

$$y_{17} = EI_\theta(u_\theta'''(s_3+) - u_\theta'''(s_3-))$$

It may then be verified that $\mathcal{D}$ is dense and A is self-adjoint and nonnegative definite. The control system dynamics are then formulated as a nonlinear wave equation over H:

$$M\ddot{x}(t) + Ax(t) + Bu(t) + FN(t) + K(\dot{x}(t)) = \ldots$$

where M is the 17×17 matrix specified by

$$M = (m_{i,j})$$

where all $m_{i,j}$ are zero except

$$\begin{aligned} m_{1,1} &= PA & m_{5,5} &= m_1 \\ m_{2,2} &= PA & m_{6,6} &= m_4 \\ m_{3,3} &= PI_\psi & m_{7,7} &= m_4 \\ m_{4,4} &= m_1 & & \end{aligned}$$

$$m_{13,6} = m_{6,13} = m_4 r_x$$

$$m_{13,7} = m_{7,13} = m_4 r_y$$

$$\begin{vmatrix} m_{8,8} & m_{8,9} & m_{8,10} \\ m_{9,8} & m_{9,9} & m_{9,10} \\ m_{10,8} & m_{10,9} & m_{10,10} \end{vmatrix} = I_1$$

$$\begin{vmatrix} m_{11,11} & m_{11,12} & m_{11,13} \\ m_{12,11} & m_{12,13} & m_{12,13} \\ m_{13,11} & m_{13,12} & m_{13,13} \end{vmatrix} = \hat{I}_4$$

$$\begin{aligned} m_{14,14} &= m_2 & m_{16,16} &= m_3 \\ m_{15,15} &= m_2 & m_{17,17} &= m_3 . \end{aligned}$$

We note that M defines a self-adjoint positive definite (nonsingular) linear operator on H onto H. The "control" $u(t)$ is 10×1:

$$u(t) = \begin{vmatrix} M_1(t) \\ M_4(t) \\ m_2\ddot{\Delta}_{\phi,2} \\ m_2\ddot{\Delta}_{\theta,2} \\ m_3\ddot{\Delta}_{\phi,3} \\ m_3\ddot{\Delta}_{\theta,3} \end{vmatrix}$$

and B is correspondingly a 17×10 constant matrix given by

$$B = \begin{vmatrix} 0_{7\times10} \\ I_{10\times10} \end{vmatrix}$$

($0_{7\times10}$ denotes 7×10 zero matrix)

($I_{10\times10}$ denotes 10×10 identity matrix) ,

$N(t)$ is the noise disturbance which is 3×1 so that F is 17×3:

$$F = \begin{vmatrix} 0_{7\times 3} \\ I_{3\times 3} \\ 0_{7\times 3} \end{vmatrix}$$

Finally $K(\dot{x}(t))$ is a nonlinear function of $\dot{x}(t)$ given by

$$K(\dot{x}(t)) = \begin{vmatrix} \omega_1 \otimes I_1\omega_1 \\ \omega_4 \otimes I_4\omega_4 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

$$\omega_1 = \text{col.}\ [\dot{x}_8(t),\ \dot{x}_9(t),\ \dot{x}_{10}(t)]$$

$$\omega_4 = \text{col.}\ [\dot{x}_{11}(t),\ \dot{x}_{12}(t),\ \dot{x}_{13}(t)]$$

$\otimes$ denoting vector cross-product and I_1, I_4 are non-singular self-adjoint 3×3 (moment) matrices.

In going over to the state-space form with

$$Y(t) = \begin{vmatrix} x(t) \\ \dot{x}(t) \end{vmatrix}$$

we have:

$$\dot{Y}(t) = AY(t) + Bu(t) + FN(t) + K(Y(t)) \qquad (3.2)$$

where

$$A = \begin{vmatrix} 0 & I \\ -M^{-1}A & 0 \end{vmatrix}$$

$$Bu = \begin{vmatrix} 0 \\ -M^{-1}Bu \end{vmatrix}$$

and in the notation,

$$Y = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix}$$

we have:

$$K(Y) = \begin{vmatrix} 0 \\ -M^{-1}K(y_2) \end{vmatrix}$$

$$FN(t) = \begin{vmatrix} 0 \\ -M^{-1}FN(t) \end{vmatrix}$$

We now have a choice of inner products. The energy inner product is:

$$[Y,Z]_E = [My_2,z_2] + [\sqrt{A}\,y_1, \sqrt{A}\,z_1] \qquad (3.3)$$

where

$$Y = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix}, \qquad Z = \begin{vmatrix} z_1 \\ z_2 \end{vmatrix}.$$

We also have the M-inner product:

$$[Y,Z]_M = [My_1,z_1] + [My_2,z_2] \quad . \qquad (3.4)$$

We will denote the corresponding completed spaces by $\mathcal{H}_E$ and $\mathcal{H}_M$.

We can show that A generates a dissipative semi-group over $\mathcal{H}_E$ and an unbounded semigroup over $\mathcal{H}_M$. The resolvent is compact and in either inner product we have the representation for the semigroup (see [2]):

$$S(t)Y = \sum S(t)\,P_k\,Y\ , \qquad (3.5)$$

where P_k is a two-dimensional projection for each $k \geq 1$, and P_0 is the projection on the null space of A^2, and

$$P_k S(t)\,P_k = S(t)\,P_k$$

$$P_k P_j = 0 \qquad j \neq k\ .$$

In the energy inner product, P_0 is of course zero.

Proportional Damping

There is reason to believe that we may assume "proportional" damping. [Private communication, L.W. Taylor.] In this case we may modify A to be:

$$A = \begin{vmatrix} 0 & 1 \\ -M^{-1}A & -2\zeta\sqrt{M^{-1}}\,\sqrt{A} \end{vmatrix} \qquad (3.6)$$

where ζ is the fixed damping factor. In this case the semigroup is exponentially damped in the energy norm, although not of course in the M-norm, because of the nonemptiness of the zero-eigen-function space of A (and hence of A), which are not affected by propor-*tional damping*. (*We may characterize* $\sqrt{A}$ without invoking the modes, although we shall not go into this here.) See [3] for more on square roots.

If we omit the nonlinearity for the moment, the linear equation

$$M\ddot{x} + Ax + 2\zeta\sqrt{M}\sqrt{A}\,x + Bu(t) = 0 \qquad (3.7)$$

has the energy-norm solution (modal expansion, zero initial conditions)

$$x(t) = \sum_1^\infty \phi_k \int_0^t a_k(t-\sigma)\,[u(\sigma), \ldots_k]\,d\sigma \qquad (3.8)$$

where Φ_k are the eigen-functions

$$A\phi_k = \gamma_k^2 M \phi_k\ , \qquad \gamma_k > 0$$

$$\phi_k = \begin{vmatrix} \phi_{k,8} \\ \vdots \\ \phi_{k,13} \end{vmatrix}$$

(indices denote components)

$$a_k(t) = e^{-\sigma_k t}\,\frac{\sin \lambda_k t}{\lambda_k}$$

$$\sigma_k = -\zeta\gamma_k$$

$$\lambda_k = \sqrt{1-\zeta^2}\,\gamma_k\ .$$

For the M-norm case we must of course add

$$\int_0^t (t-\sigma)\, P_0(Bu(\sigma))\, d\sigma$$

to (3.8). We note that there exists nonzero x such that:

$$[x, Bu] = 0$$

$$Ax = 0 \; .$$

On the other hand

$$\Omega_k \neq 0 \qquad \text{for any } k \geq 1.$$

4. Existence and Uniqueness of Solutions

[illegible] and uni[illegible] [illegible] the nonlinear deterministic equation:

$$\dot{Y}(t) = AY(t) + Bu(t) + K(Y(t)) \; . \qquad (4.1)$$

The function $K(\cdot)$ is a homogeneous polynomial of degree two and is only locally Lipschitzian. Nevertheless because of the fact that

$$[K(y_2), y_2] = 0 \; ,$$

we can show that there is no explosion and that the integral version:

$$Y(t) = \int_0^t S(t-\sigma) Bu(\sigma)\, d\sigma + \int_0^t S(t-\sigma) K(Y(\sigma))\, d\sigma$$

$$+ \; S(t)\, Y \qquad 0 < t \qquad (4.2)$$

has a unique solution, continuous in t (in either norm) with the bound (under no damping)

$$\|Y(t)\|_E \leq \|Y(0)\|_E + \int_0^t \|u(\sigma)\|\, d\sigma \; .$$

Moreover the usual Picard-iteration

$$Y_n(t) = \int_0^t S(t-\sigma) K(Y_{n-1}(\sigma))\, d\sigma + \int_0^t S(t-\sigma) Bu(\sigma)\, d\sigma$$

converges to the solution ("mild" solution!). The corresponding theory for the input disturbance will appear elsewhere.

5. Controllability

Our main result is that the system is (approximately) controllable in the energy-norm (in some time) and not controllable in the M-norm. First of all we observe that we can define control $u_f(t)$ so that

$$Bu_f(t) = -K(\dot{x}(t)) \; .$$

This feedback control makes the system linear in other words. Controllability in the energy-norm then follows from (3.8), since following [2], if

$$\sum_1^\infty [\phi_k, M\Omega]\, \omega_k a_k(t) = 0$$

for all $t > 0$, the linear independence of the functions $\{a_k(t)\}$ implies that

$$[\phi_k, M\Omega]\, \omega_k = 0$$

and we have $\Omega_k \neq 0$ for any $k \geq 1$.

The existence of a nonzero x (a "tumbling mode") such that

$$[x, Bu] = 0 \; ,$$

$$Ax = 0$$

also implies that

$$[K(y), x] = 0 \qquad \text{for every } y$$

and hence such an x cannot be reached from the origin by using any control $u(\cdot)$ for both the linear and nonlinear equations.

6. Time-optimal Control

Of the variety of control problems, we shall briefly mention time-optimal control. The "rapid slewing" requirement to any given direction within an error cone would translate at the first level to an open-loop deterministic time-optimal control problem to a target set subject to a control constraint

$$\|u(t)\| \leq C \qquad \text{a.e.}$$

If the target set be such that we can find a (minimizing) sequence of controls $u_n(\cdot)$ satisfying the constraint with corresponding times T_n, and we may as well assume that

$$T_0 = \lim_n T_n$$

It is not difficult to prove the existence of an optimal control $u_0(\cdot)$ corresponding to the time T_0. We may then invoke the maximum principle of Fattorini[4] (for an appropriate class of target sets).

7. Stabilizability

The abstract formulation does make the problem of stabilization by feedback control quite accessible. This is because (setting noise-disturbance to zero) it is immediate from (2.2) that setting

$$E(t) = \tfrac{1}{2}\|Y(t)\|_E^2$$

we have

$$\frac{d}{dt} E(t) \leq -[Bu(t), \dot{x}(t)]$$

(see [5] for more on this), since

$$[K(\dot{x}(t)), \dot{x}(t)] = 0 \; .$$

Hence we can also ensure that the energy decreases (does not increase):

$$\frac{d}{dt} E(t) \leq -[P\dot{x}(t), \dot{x}(t)]$$

by taking

$$u(t) = P \begin{vmatrix} \omega_1(t) \\ \omega_4(t) \\ \dot{u}_?(t, s_2) \\ \dot{u}_?(t, s_2) \\ \dot{u}_?(t, s_3) \\ \dot{u}_?(t, s_3) \end{vmatrix}$$

for any matrix P which is positive definite.

References

[1] L.W. Taylor and A.V. Balakrishnan. "A Mathematical Problem and a Spacecraft Control Laboratory Experiment (SCOLE) Used to Evaluate Control Laws for Flexible Spacecraft ... NASA/IEEE Design Challenge." Proceedings of the NASA SCOLE Workshop, Hampton, Virginia, December 6-7, 1984.

[2] A.V. Balakrishnan. Applied Functional Analysis. Springer-Verlag, New York, 1981.

[3] H.O. Fattorini. The Cauchy Problem. Cambridge University Press, 1984.

[4] H.O. Fattorini. "The Maximum Principle for Non-linear Non-convex Systems with Set Targets." 24th IEEE Conference on Decision and Control, December 1985.

[5] A.V. Balakrishnan. "A Mathematical Formulation of the SCOLE Control Problem: Part 1." NASA CR 172581.

END

DTIC

8-86