AD-A169 289 INFLUENCE OF SHEAR MODULUS ON THE BEHAVIOR OF ACOUSTICALLY TRANSPARENT MATERIALS(U) NAVAL RESEARCH LAB WASHINGTON DC P S DUBBELDAY 30 APR 86 NRL-MR-5745 1/1

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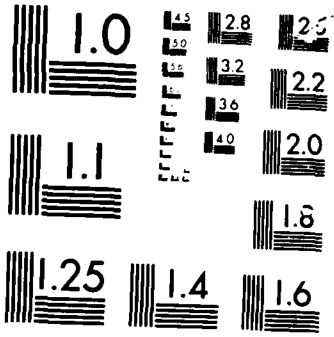

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Naval Research Laboratory

Washington, DC 20375-5000 NRL Memorandum Report 5745 April 30, 1986

Influence of Shear Modulus on the Behavior of Acoustically Transparent Materials

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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED

1b. RESTRICTIVE MARKINGS: None

2a. SECURITY CLASSIFICATION AUTHORITY: None

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE: None

3. DISTRIBUTION / AVAILABILITY OF REPORT: Approved for public release; distribution is unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S): NRL Memorandum Report 5745

5. MONITORING ORGANIZATION REPORT NUMBER(S): None

6a. NAME OF PERFORMING ORGANIZATION: Underwater Sound Ref. Det. Naval Research Laboratory

6b. OFFICE SYMBOL *(If applicable)*: Code 5975

7a. NAME OF MONITORING ORGANIZATION: None

6c. ADDRESS *(City, State, and ZIP Code)*: P.O. Box 568337, Orlando, FL 32856-8337

7b. ADDRESS *(City, State, and ZIP Code)*: None

8a. NAME OF FUNDING / SPONSORING ORGANIZATION: ONR/NRL Non-Special Focus Prgm.

8b. OFFICE SYMBOL *(If applicable)*: Code 5900

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: Various

8c. ADDRESS *(City, State, and ZIP Code)*: P.O. Box 568337, Orlando, FL 32856-8337

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61153N		RR011-08-42	1472-0

11. TITLE *(Include Security Classification)*: Influence of Shear Modulus on the Behavior of Acoustically Transparent Materials

12. PERSONAL AUTHOR(S): Dubbelday, Pieter S.

13a. TYPE OF REPORT: Interim

13b. TIME COVERED: FROM 9/1/84 TO 12/1/85

14. DATE OF REPORT *(Year, Month, Day)*: 1986 April 30

15. PAGE COUNT: 16

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP
20	01	X

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*: rho-c material, reflection of acoustic waves, acoustic transparency

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

Materials are under construction that match the density rho and dilatational sound speed c as closely as possible to the values of seawater, while maintaining sufficient rigidity to serve for structural purposes. The demand for rigidity implies a larger shear modulus than is typical for the usual rho-c elastomers. Matching of density and sound speed results in transparency for fluids only; the finite shear modulus in a solid admits the presence of a shear wave, which causes deviation from ideal rho-c behavior. In this report the effect is analyzed of a finite shear modulus on the reflection of plane waves by an infinite plate of the rho-c material. Examples are given of the reflection coefficient as a function of incidence angle for various combinations of density and sound speed close to ideal, and various ratios of plate thickness to dilatational wavelength. The effect of a finite loss factor in the shear modulus is shown.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL: Dr. Pieter S. Dubbelday

22b. TELEPHONE *(Include Area Code)*: 305, 857-5197

22c. OFFICE SYMBOL: Code 5975

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

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CONTENTS

INFLUENCE OF SHEAR MODULUS ON THE BEHAVIOR OF ACOUSTICALLY TRANSPARENT MATERIALS

INTRODUCTION

Several publications [1-4] have appeared in recent years that report on the development of an acoustically transparent material based on a fluoroepoxy filled with microballoons. Such a material can be designed to have a density and dilatational sound speed that closely match those of the acoustic medium, usually seawater, but with sufficient rigidity to serve as a material for structural members or backing plates of acoustical systems. The matching of density and dilitational sound speed leads to perfect acoustic transparency for reflection of sound at the interface of fluids only. A solid has a finite shear modulus; and for other than perpendicular incidence, a shear wave will be present in addition to the dilatational wave. This shear wave causes a deviation from ideal transparency.

In this study an analysis is given of the influence of the shear modulus on the reflection and transmission of a plane sound wave by an infinite plate with strictly or approximately matching density and sound speed. It shows the limits on plate thickness and frequency imposed by the requirement of negligible reflection. The effect of a finite loss factor in the shear modulus is also investigated.

ANALYSIS

The analysis of the reflection and transmission of a plane sound wave by a plate is given in Ref. 5. The main points of the derivation are repeated here. In the general case (Fig. 1) a fluid with density ρ_o is present at one side of the plate (thickness h = 2d) where the incident wave arrives, with

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Manuscript approved 7 April 1986

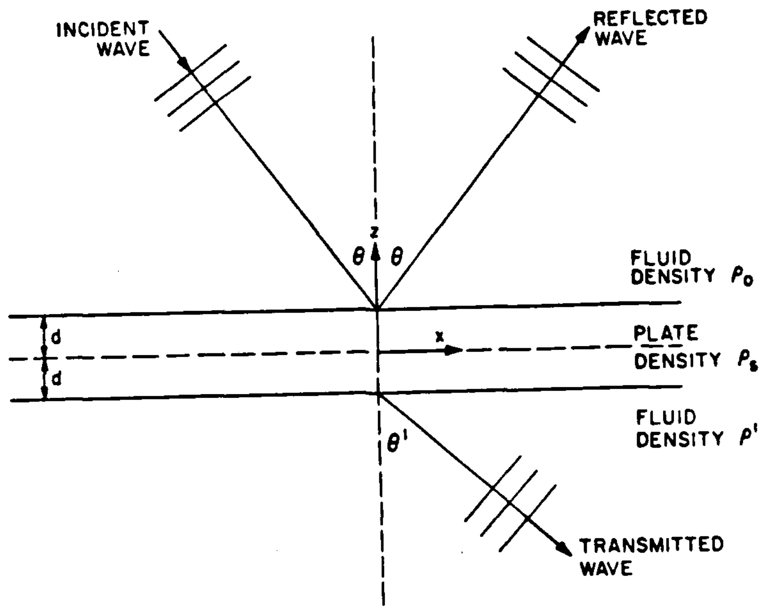

Fig. 1 - Geometry of fluid-loaded plate

angle θ to the normal and wavenumber k_o. Another fluid with density ρ' and sound speed c' is at the other boundary of the plate. The transmitted wave forms an angle θ' with the normal.

The propagation of waves in the solid, excited by the incoming wave, is treated by exact elasticity theory. The plane-crested wave in the solid with wavenumber k is a combination of a dilatational wave, represented by a scalar potential ϕ with amplitudes A_s and B_a for the symmetric and antisymmetric contributions, respectively, and a shear wave represented by the z-component of a vector potential ψ with amplitudes D_s and C_a. Other field variables are the pressure amplitudes of the incoming wave P_i, of the transmitted wave P', and of the reflected wave P_r. At the boundaries of the plate with the two fluids, one imposes conditions of continuous normal stress and velocity and zero tangential stress. The matrix of the coefficients of the seven field variables in the six boundary conditions (after manipulation of the rows) is shown in Table 1. Here $P_o = P_i \exp(ik_o d \cos\theta)$, $P = P_r \exp(-ik_o d \cos\theta)$,

Table 1 - Matrix of Coefficients of Equations Describing Reflection and Transmission by Fluid-Loaded Plate

Field variables---->

$i\,B_a/d^2$	C_a/d^2	A_s/d^2	$i\,D_s/d^2$	$P_0/(\rho_0 c_s^2)$	$P/(\rho_0 c_s^2)$	$P'/(\rho' c_s^2)$
$(k^2-s'^2)d^2\sin q'd$	$2iks'd^2\sin s'd$	0	0	$\frac{1}{2}(\rho_0/\rho_s)$	$\frac{1}{2}(\rho_0/\rho_s)$	$-\frac{1}{2}(\rho'/\rho_s)$
$-2ikq'd^2\cos q'd$	$-(k^2-s'^2)d^2\cos s'd$	0	0	0	0	0
0	0	$(k^2-s'^2)d^2\cos q'd$	$-2iks'd^2\cos s'd$	$\frac{1}{2}(\rho_0/\rho_s)$	$\frac{1}{2}(\rho_0/\rho_s)$	$\frac{1}{2}(\rho'/\rho_s)$
0	0	$2ikq'd^2\sin q'd$	$-(k^2-s'^2)d^2\sin s'd$	0	0	0
$q'd\cos q'd$	$-ikd\cos s'd$	0	0	$-\frac{1}{2}\frac{ik_0 d\cos\theta}{(k_s d)^2}$	$\frac{1}{2}\frac{ik_0 d\cos\theta}{(k_s d)^2}$	$-\frac{1}{2}\frac{ik_0 d\cos\theta'}{(k_s d)^2}$
0	0	$q'd\sin q'd$	$ikd\sin s'd$	$\frac{1}{2}\frac{ik_0 d\cos\theta}{(k_s d)^2}$	$-\frac{1}{2}\frac{ik_0 d\cos\theta}{(k_s d)^2}$	$-\frac{1}{2}\frac{ik_0 d\cos\theta'}{(k_s d)^2}$

and $P' = P_t \exp(-ik'd \cos\theta')$. The wavenumbers k_o, k, and k' are related through the coincidence condition $k = k_o \sin\theta = k' \sin\theta'$. Also $q'^2 = k_d^2 - k^2$, and $s'^2 = k_s^2 - k^2$, where k_d and k_s are the wavenumbers of the dilatational wave and shear wave in the solid.

The ratios of the field variables may be found from the matrix as ratios of the corresponding subdeterminants. One finds for the case where the same fluid is present on both faces of the plate that the reflection and transmission coefficients R and T are given by

$$R = \frac{p_r}{p_i} = \frac{p}{p_o} \exp(2ik_o d \cos\theta) \tag{1}$$

$$T = \frac{p_t}{p_i} = \frac{p'}{p_o} \exp(2ik_o d \cos\theta), \tag{2}$$

where

$$\frac{p}{p_o} = \frac{-A + D}{A + B - C + D}, \tag{3}$$

$$\frac{p'}{p_o} = \frac{B + C}{A + B - C + D}, \tag{4}$$

$$A = \frac{1}{2}\left(\frac{\rho_o}{\rho_s}\right)^2 (q'd)^2 (k_s d)^4 \cos q'd \sin q'd \cos s'd \sin s'd, \tag{5}$$

$$B = \frac{1}{2}\, i\left(\frac{\rho_o}{\rho_s}\right) k_o d\, q'd\, \Delta_s \cos q'd \cos s'd \cos\theta, \tag{6}$$

$$C = \frac{1}{2}\, i \left(\frac{\rho_o}{\rho_s}\right) k_o d \; q'd \; \Delta_a \sin q'd \sin s'd \cos \theta, \tag{7}$$

$$D = \frac{1}{2}\, (k_o d)^2 \, \Delta_a \, \Delta_s \, (\cos^2 \theta)/(k_s d)^4, \tag{8}$$

and

$$\Delta_a = m_{11}m_{22} - m_{12}m_{21}, \tag{9}$$

$$\Delta_s = m_{33}m_{44} - m_{34}m_{43} , \tag{10}$$

in terms of the elements m_{ij} of the matrix in Table 1.

It is of interest to formulate the condition for zero transmission through the plate, which is equivalent to total reflection if there are no losses in the plate. From Eq. (4) one sees that this is equivalent to B + C = 0. Inserting the expressions for Δ_a and Δ_s from Eqs. (9) and (10), one may reduce this to

$$\begin{aligned}[(kd)^2 - (s'd)^2]^2 &\cos s'd \sin s'd \\ &+ 4(kd)^2 \, q'd \, s'd \cos q'd \sin q'd = 0.\end{aligned} \tag{11}$$

A computer program was written to evaluate the reflection coefficient R and transmission coefficient T according to Eqs. (1) and (2). Input to the program is the ratio of density of the fluid to that of solid ρ_o/ρ_s, sound speed in the fluid relative to the dilatational speed in the solid c_o/c_d, the ratio of shear to bulk modulus in the solid G/K, the loss tangent in the shear modulus η_G, and the thickness of the plate relative to the wavelength of dilatational waves in the plate material h/λ. The bulk modulus K is assumed lossless.

If one introduces the following symbols

$$a = (\rho_o/\rho_s)\ q'd\ (k_s d)^2 \cos q'd \cos s'd$$

$$a' = (\rho_o/\rho_s)\ q'd\ (k_s d)^2 \sin q'd \sin s'd$$

$$d = \Delta_a\ k_o d/(k_s d)^2$$

$$d' = \Delta_s\ k_o d/(k_s d)^2\ , \tag{12}$$

one may write for the expressions A, B, C, D,

$$\begin{aligned} A &= 1/2\ aa' \\ B &= 1/2\ i\ ad' \cos\theta \\ C &= 1/2\ i\ a'd \cos\theta \\ D &= 1/2\ dd' \cos^2\theta\ . \end{aligned} \tag{13}$$

The reflection factor P/P_o may be expressed as

$$\frac{P}{P_o} = \frac{(d'/a')(d/a)\cos^2\theta - 1}{(1 - i\frac{d}{a}\cos\theta)(1 + i\frac{d'}{a'}\cos\theta)}\ . \tag{14}$$

A "structural response function" Ω was introduced in Ref. 6 by setting

$$i\Omega \cos\theta = \frac{P_o + P}{P_o - P}\ . \tag{15}$$

Here one finds a similar structural response function from

$$\frac{P_o + P}{P_o - P} = \frac{B - C + 2D}{2A + B - C}$$

as

$$\Omega = \frac{(d'/a') - (d/a) - 2i(d/a)(d'/a')\cos\theta}{2 + i(d'/a')\cos\theta - i(d/a)\cos\theta} . \tag{16}$$

The structural response function is related to the effective impedance Z of the plate surface, defined as the ratio of the total pressure $P + P_o$ at the side of the incoming wave to the surface particle speed perpendicular to the plate. One finds

$$Z = -\frac{\rho_o c_o}{\cos\theta} \frac{P_o + P}{P_o - P} = -i\Omega\rho_o c_o . \tag{17}$$

One may define effective surface impedances for antisymmetric and symmetric waves separately. First the pressure at the side of the incoming wave is split in a part contributing to antisymmetric waves and a part contributing to symmetric waves, namely

$$P + P_o = \frac{1}{2}(P + P_o - P') + \frac{1}{2}(P + P_o + P'). \tag{18}$$

Similarly the displacement perpendicular to the plate, w(d), is written as

$$w(d) = \frac{1}{2}[w(d) + w(-d)] + \frac{1}{2}[w(d) - w(-d)]. \tag{19}$$

The two impedances are then defined by

$$Z_a = \frac{P_o + P - P'}{w(d) + w(-d)} \tag{20}$$

and

$$Z_s = \frac{P_o + P + P'}{w(d) - w(-d)}. \tag{21}$$

In terms of the symbols introduced before, one finds $Z_a = -id/a$ and $Z_s = id'/a'$.

It would be instructive to express Z in terms of Z_a and Z_s in the form of an equivalent circuit diagram. Thus far this has not been successfully accomplished.

RESULTS AND DISCUSSION

Figure 2 shows the reflection coefficient as a function of incidence angle θ for various values of relative density and sound speed. The three figures arranged along a column have equal values of ρ_o/ρ_s, namely 0.995, 1.000, and 1.005. The three figures arranged in a row have equal value of c_o/c_d: 0.995, 1.000, and 1.005. The four curves marked 1, 2, 3, 4 vary in value of the thickness h (=2d) relative to the wave length λ of dilatational waves in the material. The four curves correspond to h/λ = 0.50, 0.25, 0.10, 0.05. The value of the shear modulus G relative to the bulk modulus K was found from measurements [7] to be 0.13. In Fig. 2 the loss tangent in G is assumed zero.

A striking feature in Fig. 2 is the appearance of a spike in the h/λ = 0.5 curve indicating 100% reflection. This may be explained by considering Fig. 3, which is the locus of points for which the relative wave speed $\gamma = c/c_s$ is such that zero transmission occurs, as a function of the dimensionless frequency $k_s d$. The expression for this was given in Eq. (11). According to the coincidence relation $c_o = c \sin \theta$, the points for γ when the angle θ is varied lie on a vertical line starting at the point with abscissa $k_s d$ and ordinate c_o/c_s. If this line intersects the characteristic curve of Fig. 3, total reflection will occur. In the example depicted in Fig. 3 this occurs for $\gamma = 18.9°$ and $\theta = 9.16°$. This is relatively rare for near-ρc materials. A small amount of loss in the shear modulus reduces the height of the spike to a negligible size.

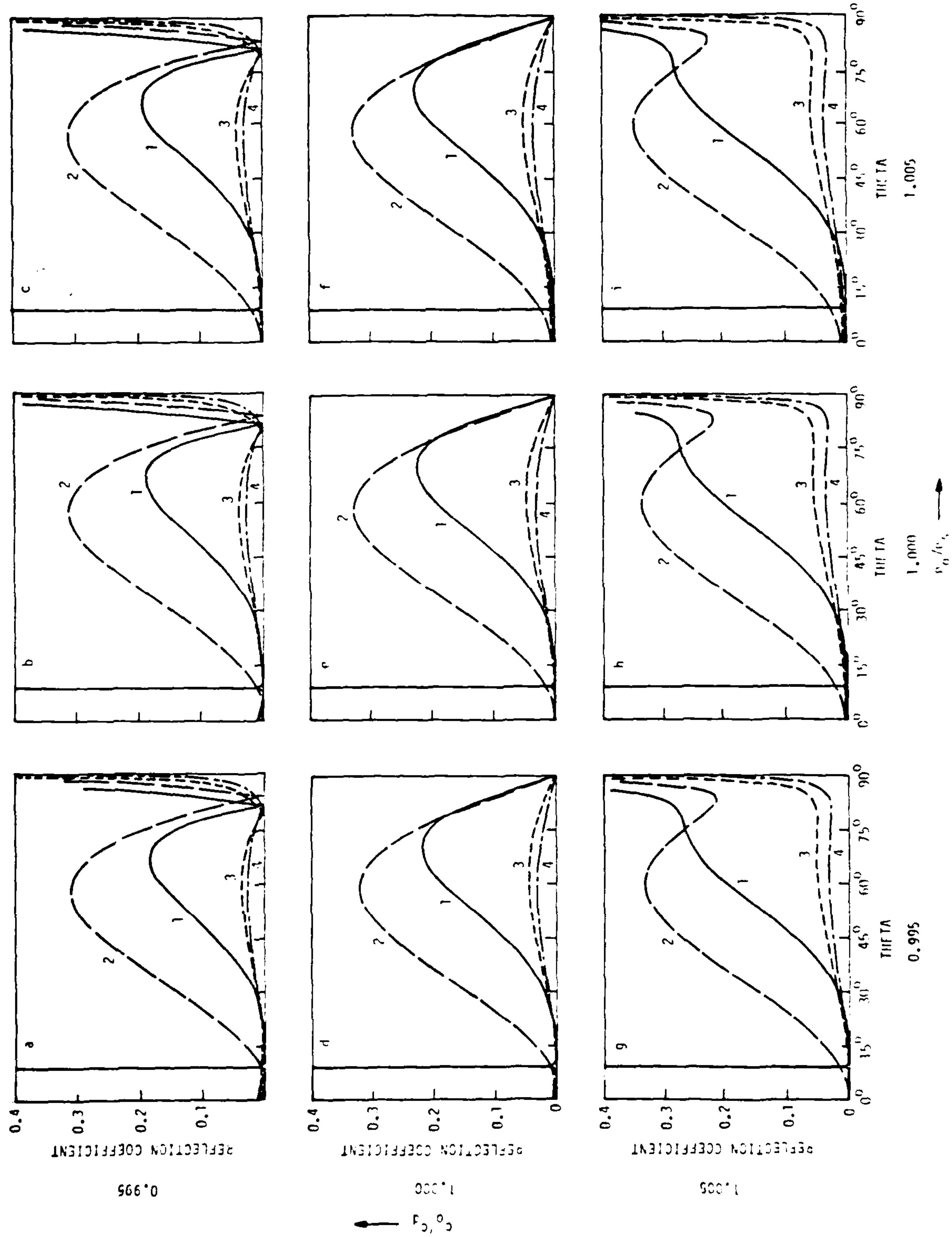

Fig. 2 - Reflection coefficient as a function of incidence angle θ. h/λ = 0.5 (curve 1), 0.25 (curve 2), 0.1 (curve 3), 0.05 (curve 4)

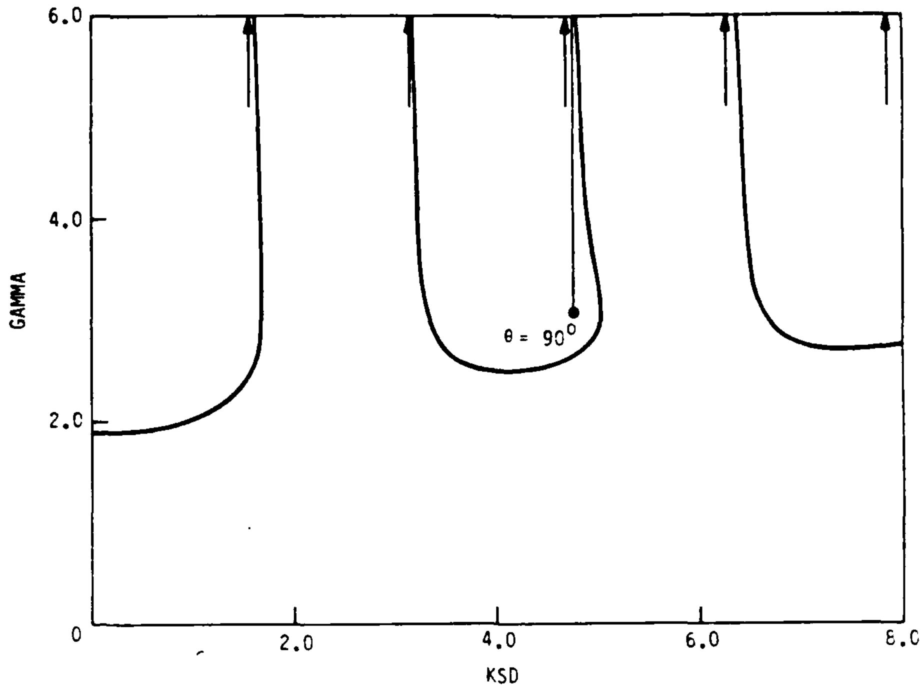

Fig. 3 - Wavespeed $\gamma = c/c_s$, as a function of dimensionless frequency $k_s d$ for which zero transmission occurs. Arrows indicate asymptotes. The vertical line is the locus of points with $h/\lambda = 0.5$ ($k_s d = 4.719$) and θ varying from 90 to 0°.

A global viewing of the nine sets of curves in Fig. 2 shows that at the intermediate range of θ, say from 30 to 60°, there is little difference in the appearance of the curves for varying density and sound speed. As to the small angle range, it appears that the relevant parameter is the product ρc; the groups that have similar appearance are a; b,d; c,e,g; f,h; and i. This corresponds to the fact that for fluids the behavior near $\theta = 0°$ is determined by the product ρc. Near $\theta = 90°$ one sees that there are three groups distinguished by the presence or absence of zero reflection. For c_o/c_d = 0.995 there is an angle where the reflection is zero; for c_o/c_d = 1.000 the zero reflection occurs at 90°; and for c_o/c_d = 1.005 there is no angle near 90° at which the reflection coefficient is zero.

In Figure 4 examples are given where both the density and the sound speed differ by +5% or -5% compared with ideal ρc. It appears that too small a density and sound speed are more deleterious than too large a density and sound speed, judging by the reflection coefficient.

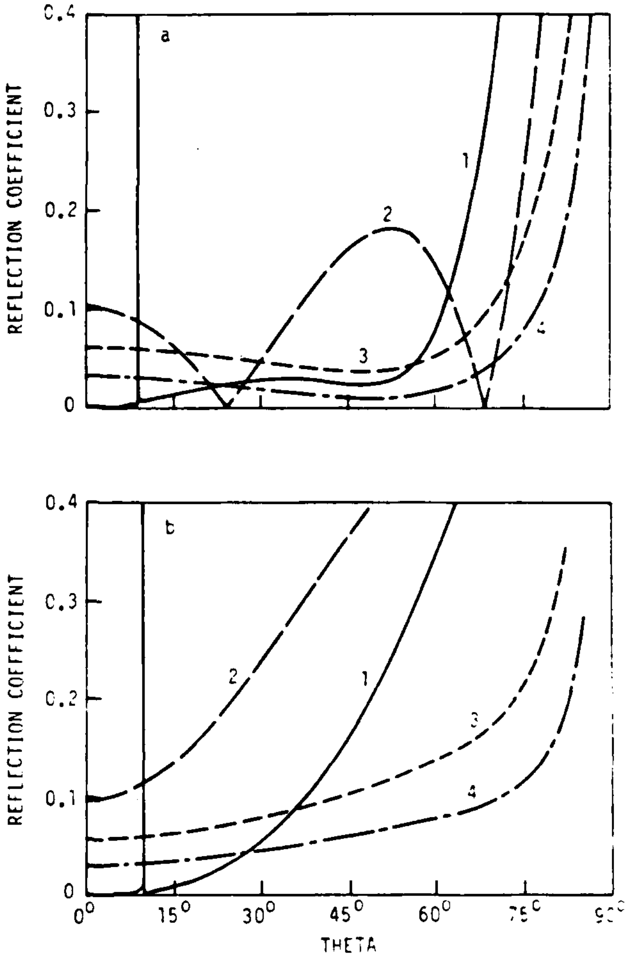

Fig. 4 - Reflection coefficient as a function of incidence angle θ.

a) $\rho_o/\rho_s = c_o/c_d = 0.95$
b) $\rho_o/\rho_s = c_o/c_d = 1.05$

curve 1 $h/\lambda = 0.5$
curve 2 $h/\lambda = 0.25$
curve 3 $h/\lambda = 0.1$
curve 4 $h/\lambda = 0.05$

In Figure 5 the effect of a finite loss tangent in G is shown, namely $\eta_G = 0.01$, which is about the value measured for the shear modulus of the rigid ρc material. Three combinations of density and sound speed were chosen. One sees that the effect is quite small.

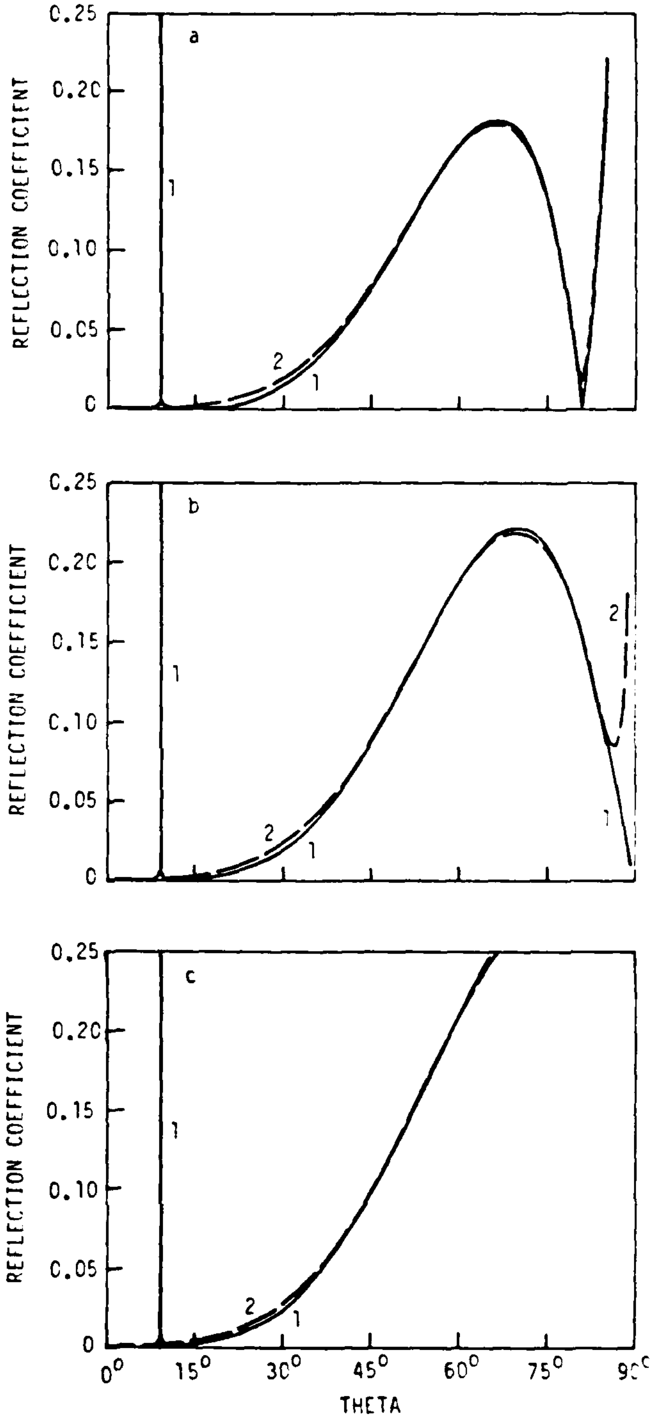

Fig. 5 - Reflection coefficient as a function of incidence angle θ.

$h/\lambda = 0.5$

a) $\rho_o/\rho_s = c_o/c_d = 0.995$
b) $\rho_o/\rho_s = c_o/c_d = 1.000$
c) $\rho_o/\rho_s = c_o/c_d = 1.005$

curve 1 - $\eta_G = 0.00$
curve 2 - $\eta_G = 0.01$

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