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EVALUATING EXPERIMENTAL DESIGNS FOR FITTING RESPONSE SURFACES OF DETERMINISTIC MODELS

THESIS

Bryan K. Ishihara Captain, USAF

AFIT/GOR/OS/85D-10

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In addition, the response surface equations are evaluated in terms of their predictive power and their explanatory power. The predictive power addresses the equation's ability to adequately estimate the true surface and to accurately predict a future response for a given set of inputs. The explanatory power addresses the equation's ability to present a response equation which is simple to interpret so that the true surface can be easily evaluated and the results can be easily explained.

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# EVALUATING EXPERIMENTAL DESIGNS FOR FITTING RESPONSE SURFACES OF DETERMINISTIC MODELS

#### THESIS

Presented to the Faculty of the School of Engineering of the Air Force Institute of Technology Air University In Partial Fulfillment of the Requirements for the Degree of Master of Science in Operations Research

Bryan K. Ishihara, M.B.A., B.A., B.A.

Captain, USAF

December 1985

Approved for public release; distribution unlimited

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Bryan K. Ishihara

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#### ABSTRACT

This thesis evaluates several bias minimizing and variance minimizing experimental designs in terms of their effectiveness and efficiency in constructing response equations for a deterministic, nuclear exchnage problem. The criteria which is used to evaluate these designs includes: 1) number of required design points; 2) number of terms in the response equation; 3) accuracy of fit of the response equation; 4) orthogonality of the design; and 5) rotatability of the design.

In addition, the response surface equations are evaluated in terms of their predictive power and their explanatory power. The predictive power addresses the equation's ability to adequately estimate the true surface and to accurately predict a future response for a given set of inputs. The explanatory power addresses the equation's ability to present a response equation which is simple to interpret so that the true surface can be easily evaluated and the results can be easily explained.

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#### I. <u>OVERVIEW</u>

#### BACKGROUND

Response Surface Methodology (RSM) is a method for fitting mathematical models to surfaces generated by experiment. RSM is concerned with the functional relationship

$$V = h(U_1, U_2, \dots, U_N)$$
(1)

between the response variable, V, and the n independent variables  $U_1$ ,  $U_2$ ,...,  $U_N$ . The response relationship is usually expressed as a low order polynomial or some other non-linear function which approximates the true surface. Graphically, the response relationship is represented as a surface in a space whose coordinates are the n+1 variables,  $U_1$ ,  $U_2$ ,...,  $U_N$  and V. For example, the response surface can be expressed as a series of contours, similar to the rise and fall of land masses on a topographic map.

In applying RSM, the analyst selects an experimental design (i.e., a theory which requires the least number of points to accurately fit the model) such that the functional relationship

$$y = f(x_1, x_2, ..., x_N)$$
 (2)

fitted by the method of least squares closely approximates the true function

 $y' = g(x_1, x_2, \dots, x_p)$  (3)

over a specified region. When f inadequately represents

g, there are two errors associated with this problem:

variance error - which is error due to sampling;
 and 2) bias error - which is error due to the lack of

#### fit of f to g

Initially, criteria for evaluating the adequacy of designs have focused primarily on minimizing the variance error while the question of bias error has been given somewhat secondary considerations. However, a bias minimizing design takes on added significance when RSM is applied to a deterministic model because there is no sampling error associated with this model. In this context, a deterministic model is a model in which the response variables are free from stochastic variation.

For example, non-deterministic models, such as computer simulations, yield different results (due to random sampling of distributions) each time the test is conducted. However, deterministic models, such as linear programming problems, yield the same results each time they are applied because sampling is not involved for the fixed output level. Therefore, any error associated with the deterministic model is solely attributable to bias error (lack of fit) and none of it to variance error (sampling).

In addressing the concept of experimental designs for response surfaces, most of the RSM literature has focused on those designs which minimize variance. These designs have received a large share of attention because RSM has been

almost exclusively applied to non-deterministic models. Examples of variance minimizing designs are provided in Box and Behnken's work (1:455) on three level factorial designs.

Thus far, only a limited amount of literature has been published on bias minimizing designs. Only the basic theory of bias minimizing designs exists in the literature with the applications of the theory still to be fully documented. In general, the applications of RSM have focused on generating response surface equations for two purposes: to predict future responses and to determine the input values of the optimum response. In determining the optimum operating conditions, the experimenter employs various search techniques, such as gradient search methods, to solve the problem. However, in examining the response surface of a deterministic model, the experimenter may not be interested in determining the optimum operating conditions. Rather, as in this thesis, the experimenter may be interested in adequately representing the true surface with his predicted surface so that he can explore the relationships between the input variables and accurately predict the response values. As a result, the experimenter is interested in selecting a design which allows him to employ simple analytical techniques to the response equation to examine these relationships.

The basic theory of bias minimizing designs proposes that a design can be constructed which requires fewer number of design points and which achieves a better fit of the

function to the response surface than a variance minimizing design because a bias minimizing design does not consider any variance in its formulation. Manacapilli (17:6.1) has stated that for experiments with four or more variables, a bias minimizing design provides a function with a better fit to a surface than a variance minimizing design. However, the bias minimizing design requires more design points to achieve the fit. Thus, these results indicate that not all bias minimizing designs achieve a better fit with fewer design points.

#### PROBLEM STATEMENT

In developing experimental designs for response surfaces, researchers have focused on variance minimizing designs as the basic criterion in order to capture the response surface. However, deterministic models possess characteristics whereby a bias minimizing design may be a more appropriate choice. Thus, the problem is to find a bias minimizing design which best fits the response surface of a deterministic model and to compare the efficiency and effectiveness of this design to a variance minimizing design.

#### RESEARCH QUESTION

What experimental designs produce effective and efficient response surfaces of deterministic models? SUBSIDIARY QUESTIONS

RSM when applied to a deterministic model is

investigated to find bias minimizing experimental designs. The results of this design are evaluated over various factor levels and its effectiveness is compared to the results of variance minimizing designs. The primary measures of effectiveness (MOE) are: orthogonality of the design, accuracy of fit to the true response surface, and number of required design points.

Some of the questions to be answered from these MOEs are:

1) What are the tradeoffs of using a bias minimizing design instead of a variance minimizing design to fit the response surface of a deterministic model?

2) When is this design more efficient or more effective than a variance minimizing design?

3) Is the analysis of response surfaces made easier with a bias minimizing design?

4) Is the bias minimizing design a practical method (in terms of the required number of design points) to fit a response surface of a deterministic model?

5) Do these designs produce equivalent results?
6) Can an orthogonal bias minimizing design be constructed to facilitate the analysis of the surface?

#### LITERATURE REVIEW

The concepts of Response Surface Methodology were first developed by Box and Wilson in the early 1950's (5:1). This methodology is designed to be a practical and efficient manner in dealing with the problem of determining the

optimum operating conditions for a specific process. For example, RSM is ideally suited for agricultural and chemical experiments in which the experimenter seeks to maximize the yield or purity of the product. An extensive list of the applications of RSM in chemical, agricultural, and other related fields can be found in Hill and Hunter's article (11:591).

More recent applications of RSM have been in the area of animal husbandry. Articles by Toyomizu, Akiba, Horiguchi, and Matsumoto, (24:886) and by Roush, Petersen, and Arscott, (21:1504) have identified the uses of RSM in determining the optimum operating conditions for raising chickens.

With regards to military applications, Smith and Mellichamp (22) used RSM to perform multidimensional parametric analysis to study mathematical programming models. In addition, several recent Air Force Institute of Technology (AFIT) thesis efforts have applied Smith and Mellichamp's approach and techniques to various military related problems. For example, Manacapilli (17) applied RSM along with economic production functions to a nuclear exchange linear programming model. Manacapilli showed that economic production functions could be fitted to the response surface of a deterministic model so that basic economic theory could be used to analyze the surface.

Another application of RSM to a nuclear exchange model is the work by Graney (19). In his thesis, Graney combined

two different response surfaces (representing two different objectives) as a means of evaluating a multiple objective problem.

Further use of Smith and Mellichamp's paper in this research effort is defined in Chapter II under the proposed methodology section.

#### GENERAL METHODOLOGY

In general, the basic approach in answering the research question is to apply various experimental designs to a response surface of a deterministic model and to evaluate the differences in the results.

Specifically, the first step is to identify bias minimizing designs, such as the Box-Draper designs and the Koshal designs.

The second step is to identify variance minimizing designs. Several variance minimizing designs are considered in this study. These designs include, but are not limited to, second order rotatable designs and central composite rotatable designs. These types of designs are selected because of their frequent use in RSM.

Next, both the bias minimizing and variance minimizing designs are applied to a deterministic model. The deterministic model to be used in this research effort is a nuclear exchange model. The nuclear exchange model is a simple linear programming model designed to optimally allocate the number of strategic weapons per weapon type

(input variables) to targets so that maximum damage occurs (response variable). This model was selected because of its basic simplicity and because past AFIT thesis efforts, by Graney (19:51) and by Manacapilli (17:6.1) have used a similar model with favorable results.

Finally, the functions obtained from the two types of experimental designs are evaluated in terms of their effectiveness and efficiency.

In terms of effectiveness, the fit of the postulated surfaces to the actual surface is measured. To measure the fit of a postulated surface, a random set of data points from the actual surface is collected and is applied to the function which is used to generate the postulated surface. The differences between the expected response and the actual response are then computed. The sum of the deviations squared (SDS) is then obtained and a comparison is made between each of the functions estimated by different designs based on their SDS value. The designs which generate models which most closely represent the true surface are the model which minimizes SDS.

In terms of efficiency, the minimum number of design points required to generate the surface are evaluated. As stated earlier, Manacapilli showed that a bias minimizing design required more design points than a variance minimizing design while gaining only a small improvement in the fit of the postulated surface. If this result holds for the generalized bias minimizing design, then the

effectiveness and efficiency trade-offs between bias and variance minimizing designs are evaluated.

One of the measures of this trade-off is an effectiveness-efficiency factor. This factor is given by

E=(error in fit of design)\*(number of design points) (4)

where the error in the fit of the design is the mean of the differences between the predicted and the actual responses divided by the actual responses.

This factor simultaneously takes into consideration the precision with which the design estimates the true surface and the number of required experimental design points (18:135).

The next chapter expands the methodology and discusses the specific concepts involved in applying RSM to a deterministic model.

#### II. METHODOLOGY

#### INTRODUCTION

This chapter details the methodology used in applying Response Surface Methodology (RSM) to a deterministic model. It builds on the general methodology described in Chapter I by explaining the various experimental designs which are used in this study, by detailing the nuclear exchange linear programming (LP) models and by outlining the application of experimental designs in generating the response surfaces.

#### SECOND ORDER EXPERIMENTAL DESIGNS FOR RESPONSE SURFACES

A major goal of Response Surface Methodology is to answer the question of what happens to the response variable when input variables are allowed to vary over a specified range. Instead of utilizing a one variable at a time procedure (which changes the level of one factor while fixing all other input variables at a specified level), the theory of experimental design is utilized. By applying the appropriate experimental designs, a response function is generated to estimate the response value for any given combination of input variables. Two books which are excellent references on the theory of experimental design have been written by Davies (8) and by Hicks (10). An extensive list of applications of experimental design in RSM can be found in Steinberg and Hunter's article (23:71).

Furthermore, a significant portion of RSM literature

has focused on second order experimental designs. That is, designs specifically constructed for fitting a second order polynomial equation to the data. This type of design produces a second order polynomial equation of the form :

 $y = b_0 + \sum b_j x_j + \sum b_{ij} x_i^2 + \sum b_{ij} x_i x_j$  (5) This equation includes the first few terms of a Taylor series expansion. Therefore, a second order design usually produces an adequate response equation in estimating

the actual surface because the response equation depicting the actual surface can usually be represented by a Taylor series expansion. The difference between the actual and predicted surfaces is due to the higher order terms of the Taylor series expansion which are not included in the equation for the predicted surface, but are included in the equation for the actual surface.

<u>ORTHOGONALITY</u>. One of the characteristics of the experimental design which is sought by the experimenter is the mutual orthogonality of coefficients. Orthogonality is that property of an experiment which ensures that the different classes of effects shall be capable of direct and seperate estimation without any entanglement. The sum of squares of all the effects are then independent and additive (8:587).

Orthogonality permits surer assessment of those areas on which process design and control efforts should be concentrated. It also provides means for unambiguous simplification and improvement of response models along with

the potential for uncovering basic mechanisms (20:419). That is, those terms which are most important in predicting the surface can be easily identified.

<u>ROTATABILITY</u>. A second desirable characteristic of an experimental design is that the design be rotatable. An experimental design is said to be rotatatle if the variance of the estimated response 9, at some point  $(x_1, x_2, \dots, x_K)$ , depends on the distance from the point to the design center and not on direction. In other words, as far as the design is concerned, points in the factor space which are the same distance from the origin are treated as being equally important (18:165).

Box and Draper (3:339) indicate that a rotatable design will minimize bias provided that certain conditions are met. These conditions are: the region of interest is spherical and the fitted model is an mth order polynomial and the true model is an nth order polynomial, such that if m+n=2r, then the appropriate design is an rth order rotatable design; if m+n=2r+1, then the appropriate design is a rth order rotatable design with moments of order 2r+1 all zero. Thus, in fitting a second order polynomial to a true surface of order three, the fifth moments of the design must all be zero for the design to be rotatable.

<u>NUMBER OF DESIGN POINTS</u>. A third desirable characteristic of an experimental design is that the fewest possible number of design points be used to estimate the response surface. The fewest number of design points is a

function of the number of factors and the type of design being used.

The minimum possible number of design points that can be used to estimate any surface is defined by the expression (k+1)\*(k+2)/2, where k defines the number of factors in the experiment. This quantity is in stark contrast to the number of data runs which are required if the experimenter chose to utilize a one variable at a time procedure to estimate the surface. For example, in a simple two variable problem in which the first variable is allowed to vary between 0 and 450 and the second variable is allowed to vary between 0 and 750, Manacapilli calculated that 338,701 (451 \* 751) runs are required to produce the exact surface of a two factor experiment if the variables are varied one at a time (17:3.8). On the otherhand, certain experimental designs are able to estimate this surface with only six data runs. The savings in time and cost are readily apparent.

#### BIAS MINIMIZING DESIGNS

Since the only error associated with deterministic models is bias error, bias minimizing designs are very effective in fitting response surfaces to these models. The bias minimizing designs which are utilized in this project are: Box-Draper designs and Koshal designs.

BOX-DRAPER BIAS MINIMIZING DESIGNS. The Box-Draper bias minimizing design is constructed in a similar manner as a central composite design. That is, the design is composed of a fractional factorial portion, an axial point portion

and center points. Like the central composite design, the bias minimizing design is nearly orthogonal and nearly rotatable.

For the case of three independent variables the design matrix is given by

		X 1	X2	X3	
		T <u>+</u> P	±p	±p	
		±9	Ø	Ø	
D	=	0	±٩	20 I	
		0	Ø	±q	(4)
		2	Ø	<u>ø</u> :	(6)

The specific values of p and q are dependent on the second and fourth moments of the design matrix. The specific values which minimize bias and a detailed derivation of the design matrix can be found in Box and Draper's article (2:622) and Myers' book (18:196).

KOSHAL DESIGNS. The Koshal design is a simple design used by Koshal in fitting frequency distributions by the method of maximum likelihood (12:577). The Koshal design requires the minimum number of design points to fit a quadratic response surface, and the major advantage of using this design is its simplicity and economy.

For the case of three independent variables the design matrix is given by

		X1	X2	X3	
		5	5	5	
		.5	5	5	
		5	.5	5	
		5	5	.5	
		1.5	5	5	
D	=	5	1.5	~.5	
		5	5	1.5	
		5	5	.5 ;	(7)
		5	.5	.5	
		.5	.5	5	

A detailed derivation of the design matrix can be found in Kanemasu's article (12:578).

#### VARIANCE MINIMIZING DESIGNS

As stated earlier, there are two errors associated with the problem of fitting the predicted surface to the actual surface: variance error and bias error. As their names imply, variance minimizing designs are constructed to minimize variance error and bias minimizing designs are constructed to minimize bias error.

Although deterministic models do not generate any variance error, variance minimizing designs have been shown to produce adequate results in fitting response surfaces to these models. In fact, Box and Draper hypothesized that designs which minimize variance do a very good job at minimizing bias also (2:622). The variance minimizing

designs which are utilized in this project are: Central Composite Rotatable Designs, Box-Behnken Designs, Hybrid Designs, and Minimum Point Designs.

<u>CENTRAL COMPOSITE ROTATABLE DESIGNS</u>. The central composite rotatable design was first developed by Box and Wilson as a practical alternative to the 3 <sup>K</sup> factorial design for estimating a second order response equation. This design is probably the most widely known and recommended design for estimating quadratic response surfaces (16:412).

The central composite design is the 2<sup>K</sup> factorial or fractional factorial, augmented by axial points and center points. For the case of three independent variables the design matrix is given by

		X 1	X2	ХЗ	
		±1	±1	±1 ]	
		±q	Ø	0	
D	*	0	±q	0	
		2	Ø	±q	(9)
		0	Ø	0	(8)

With the proper selection of q, the central composite design is orthogonal and rotatable. A detailed derivation of the q-values can be found in Box and Hunter's article (4:195).

<u>BOX-BEHNKEN DESIGNS</u>. In 1960, Box and Behnken developed a new class of three level factorial designs which were useful for estimating the coefficients in a second degree graduating polynomial (1:455). These designs are

nearly rotatable and nearly orthogonal (i.e., a small correlation exists between the constant term and the squared terms and between the squared terms themselves). A practical advantage of the Box-Behnken design over the full central composite design is that when evaluating problems with large number of factors (i.e., greater than five), the Box-Behnken design requires fewer design points to estimate the surface.

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The designs are formed by combining factorial designs with incomplete block designs (1:457). For the case of three independent variables, the design matrix is given by

$$D = \begin{bmatrix} x_1 & x_2 & x_3 \\ \pm 1 & \pm 1 & 0 \\ \pm 1 & 0 & \pm 1 \\ 0 & \pm 1 & \pm 1 \\ 0 & 0 & 0 \end{bmatrix}$$
(9)

A detailed derivation of the design matrix can be found in Box and Behnken's article (1:455).

<u>HYBRID DESIGNS</u>. Hybrid designs were created to achieve the same degree of orthogonality as central composite designs or regular polyhedral designs, to be nearly minimum point, to be nearly rotatable, and to possess some ease in coding (20:419).

The hybrid designs are constructed in a similar manner as central composite designs. That is, the design is composed of a fractional factorial portion, an axial point portion, and center points. The major difference between

2 ~ 8

the hybrid and central composite designs is that hybrid designs are augmented with an extra variable column which resembles a cross polytope design. For the case of three independent variables the design matrix is given by



A detailed derivation of the design matrix can be found in Roquemore's article (20:419).

MINIMUM POINT SECOND ORDER DESIGNS. The purpose of second order designs with minimum number of points is to provide a low cost, practical design to estimate a quadratic response surface. For a k-factor design, the minimum number of points is (k+1)\*(k+2)/2. The design should give rise to least squares estimates with minimum generalized variance (6:613).

For the case of three independent variables, the design matrix is given by

		X 1	X2	ХЗ	
		-1	-1	-1	
		1	-1	-1	
		-1	1	-1	
		; -1	-1	1	
	2	i p	Р	-1	
D		P	-1	Р	
		1 -1	P	P	
		9	1	1	
		; 1	đ	1	(11)
		1	1	٩	

The specific values of p and q, along with a detailed derivation of the design matrix can be found in Box and Draper's article (6:613).

#### MISCELLANEOUS

Several articles have been published which have offered an alternate approach to minimizing bias error in response surfaces. The articles by: 1) Karson, Manson, and Hader(14:461), 2) Karson (13:1565), 3) Karson and Spruill (15:329), and 4) Cote, Manson and Hader (7:633), have applied similar techniques to address the problem of minimizing bias. That is, their methods consist of developing an estimator for a given design which minimizes bias.

Although these works will not be used in this project, their availability is noted here for any future work on this topic.

#### NUCLEAR EXCHANGE MODEL

The deterministic model used to generate the response values for this study is a nuclear exchange linear programming model. In this rather simplistic model of a nuclear force structure, the single objective of the model is to maximize the damage expectancy of the force structure. The damage expectancy is maximized given the effectiveness of each weapon type against each target type.

For example, consider the hypothetical strategic force of three weapon systems (ICBM, Bomber, SLBM) and two target types. Table 1 shows the amount of damage expected (D1, D2, D3,..., D6) given a particular weapon type and target type.

#### TABLE 1

SINGLE SHOT DAMAGE EXPECTANCY TABLE

#### WEAPON IYPE (FACIORS)

TGT TYPE	ICBM (M1)	BOMBER (B1)	SLBM (S1)
TARGET 1	D1	D2	D3
TARGET 2	D4	D5	D6

The damage expectancy values in Table 1 represent single shot damage expectancy, that is, the damage expectancy from a single warhead of a particular weapon against a single target. In this model, an additional warhead could be launched at the same target. Therefore, it is possible to increase the damage expectancy against a single target by

launching two warheads (of the same or different weapon type) at the target. In those cases where two warheads are launched at the same target, the new damage expectancy value for that target can be computed in the following manner

$$DE_{MEW} = 1 - [(1 - DE_1) * (1 - DE_2)]$$
(12)

Thus, there are six additional combinations of damage expectancy which must be considered for each target type (e.g., M1\*M1, M1\*B1, M1\*S1, B1\*B1, B1\*S1, S1\*S1). After considering all possible combinations, the LP model now maximizes the total damage expectancy subject to the limitations on the number of weapons, number of targets, and other targeting constraints. Further details of the specific model used to generate the data along with the LP formulation are available in Appendix A.

#### APPLICATION OF THE EXPERIMENTAL DESIGN

As mentioned earlier, there is a practical advantage in employing the theory of experimental design in estimating a response surface. For example, in the simple three factor problem used in this study, if each factor were allowed to vary one at a time, the total number of runs required to generate the surface is 14,601,339,651 (1351\*1801\*6001). By constructing the appropriate experimental design, the surface could be estimated with as few as ten runs.

The experimental designs which are used in this study are second order designs. The purpose of a second order design is to fit a second order response equation to the

data. The second order response equation for the three factor case is of the form

$$DE = b_0 + b_1 (M1) + b_2 (B1) + b_3 (S1) + b_{11} (M1*M1) + b_{22} (B1*B1) + b_{33} (S1*S1) + b_{12} (M1*B1) + b_{13} (M1*S1) + b_{23} (B1*S1) (13)$$

where, M1 = number of ICBM warheads, B1 = number of bomber warheads, and S1 = number of SLBM warheads.

One of the requirements to produce a second order design is that at least three levels of each factor must be selected so that the coefficients of the response equation can be estimated. The three levels which are usually selected are the values at the upper end (having a coded value of +1) and lower end (having a coded value of -1) of the range for each input parameter plus the mid-point (having a coded value of 0) between the extreme points. These points are selected and coded in this manner so that the coefficients of the response equation are not highly correlated.

Table 2 indicates the uncoded and coded values of the three factor problem used in this project.
#### TABLE 2

#### FACTOR LEVELS

WEAPON	IYPES	(FACTORS)	LEVELS	(CODED VE	<u>ALUES)</u>
	M1		1650(-1)	2325(0)	3000(+1)
	Bi		1200(-1)	2100(0)	3000 (+1)
	51		6000(-1)	9000 (0)	12000(+1)

In many of the central composite designs which are used in this study, the coded values are greater than +1 and less than -1. The appearance of these values can best be explained by examining the central composite design for the three factor case. In this situation, the central composite design selects its initial design points (factorial portion) at the corners of a cube centered at the origin. Because the region of interest is taken to be spherical, the design positions additional points along the axes so that representative points can be sampled over the entire region. These axial design points lie near the boundary of the sphere and along one of the axes. Therefore, the axial points have values greater than +1 and less than -1 due to the radius of the sphere.

There is a possibility of significant error in the estimated response values because the axial points are positioned outside the original feasible region. That is, if the true surface is radically different outside the feasible region than it is within the feasible region, a significant error occurs due to the positioning of the

axial points. In that case, a different means of coding or a different design (which does not utilize points outside this region) should be employed. However, if the true surface is not radically different outside the feasible region than it is within the feasible region, then the axial design points do not contribute a significant error in estimating the predicted surface.

Furthermore, many of the designs which have been selected in this study have been originally constructed with multiple center points. Often times, multiple center points are used to minimize variance, to achieve rotatability or to achieve orthogonality. However, the replication of the center points is not required while analyzing deterministic models because there is no sampling error associated with this type of model. Although pure rotatability and orthogonality may be sacrificed in these cases, the designs still maintain a high degree of rotatability and orthogonality. Therefore, only one center point is included in any design which has been originally constructed with multiple center points.

The designs which are used in this study for the three factor case have already been outlined. The designs which are used in the three, four, five, and six factor problems are provided in Appendix B.

After selecting the appropriate experimental design and after defining the LP, the next step in the methodology is to execute the appropriate number of force allocation runs

using the LP model. One run is accomplished for each of the identified combinations in order to obtain the optimum damage expectancy for each set of input factor values.

The final steps in obtaining a response equation is to use the results of a particular design as input to a multiple linear regression program. The coded levels of each of the factors are input to the program as the independent variables, and the damage expectancy is input as the dependent variable. Using multiple stepwise linear regression, the coefficients of the response equation are determined. The coded values are then uncoded in order to produce a relationship in which the actual number of warheads of the weapon systems can be used. The response equation for uncoded values portrays the relationship of each factor to the response variable. A sample of the input and the output for the stepwise linear regression is found in Appendix C.

The next chapter portrays the results and details the analysis of the effectiveness and efficiency of the various designs used in this study.

## III. RESULTS AND ANALYSIS

## INTRODUCTION

This chapter presents the analysis of the response equations for the various designs which are used in this study. The purpose of this analysis is not to evaluate the LP but rather, to evaluate the designs and the response equations.

The analysis is divided into two parts: the predictive power of the response equations and the explanatory power of the response equations. The predictive power of the response equation addresses the equation's ability to adequately estimate the true surface and to accurately predict a response for a given set of inputs. The explanatory power addresses the equation's ability to present a response equation which is simple to interpret so that the true surface is easily evaluated and the results are easily explained.

## PREDICTIVE POWER OF RESPONSE SURFACE EQUATIONS

As stated in Chapter I, the basis for evaluating the predictive capabilites of an equation of a response surface design is eq(4), the effectiveness - efficiency factor.

E=(error in fit of design)\*(number of design points) (4) This factor generates a value so that various designs are compared in terms of their precision in estimating the

true surface and the number of required experimental design points.

In order to obtain the error in fit of the design, a random set of points is used as inputs into the response surface equation and the differences between the actual and predicted responses are recorded. In comparing these response equations, the random data set is not used to estimate any response equation, but rather to compare the predicted values of the estimated equations against each other. The benefit of the random data set is that all equations are compared against a common data set so that prediction bias is avoided. Prediction bias arises when the choice of a regression equation is uniquely related to the observations from which it was created.

The response equations which are used to evaluate the error in fit of the design are modified versions of the full regression equations. That is, selected terms from the full regression equations have been omitted from the modified regression equaitons because these terms account for less than two percent of the total sum of squares.

The primary reason for excluding these terms is to obtain a relatively simple equation (i.e., an equation with as few terms as possible) which is easy to interpret, but still produces a good fit. If the full regression equation is used instead of the modified equation to estimate the true surface, the ability of the response equation to predict future response values is improved; however, the ability to interpret the equation is significantly

degraded. That is, the interpretation of the equation is very difficult due to the complexity of the equation. Thus, by selecting two percent of the total sum of squares as the criteria for determining what terms to include in the response equation, a reasonably good fit is obtained while still preserving the interpretative value of the response equation.

The response surface equations along with the random data sets which are used in the analysis of the predictive power can be found in Appendix D.

Tables 3 through 6 depict the results of the predictive power of the various designs which are used in this study.

#### TABLE 3.

PREDICTIVE POWER OF THREE FACTOR DESIGNS

PERCENT ERROR IN FIT NUMBER OF DESIGN IO RANDOM DATA SEI DESIGN POINTS E - YALVE VARIANCE DESIGNS

	BOX-BEHNKEN	*	1.08	13	14.04
	HYBRID 310+CP	*	1.28	11	14.08
	HYBRID 311A	*	1.63	11	17.93
	CENTRAL COMP	*	1.56	15	23.40
	MIN POINT		6.12	10	61.20
B	IAS DESIGNS				
	BOX-DRAPER	*	1.38	15	20.70
	KOSHAL		2.74	10	27.40

\* indicates nearly rotatable design

# TABLE 4.

# PREDICTIVE POWER OF FOUR FACTOR DESIGNS

PERCEN	T ERRO	R IN FIT	. NUMBER OF	
DESIGN TO RA	NDOM D	ATA SET	DESIGN POINTS	<u>e - Value</u>
VARIANCE DESIG	NS			
HYBRID 416A+	CP *	1.06	17	18.02
HYBRID 416C	*	1.80	16	28 <b>.80</b>
BOX-BEHNKEN	*	1.18	25	29 <b>.50</b>
CENTRAL COMP	*	1.31	25	32.75
MIN POINT		4.37	15	65.55
BIAS DESIGNS				
BOX-DRAPER	*	0.96	25	24,00
KOSHAL		1.80	15	27.00

\* indicates nearly rotatable design

## TABLE 5.

## PREDICTIVE POWER OF FIVE FACTOR DESIGNS

PERCENT ERR DESIGN IO RANDOM	OR IN FIT DATA SET	NUMBER OF DESIGN POINTS	E - YALUE
VARIANCE DESIGNS			
CENTRAL COMP *	1.73	27	46.71
(HALF REPLICATE) BOX-BEHNKEN *	1.41	41	57.81
MIN POINT	5.88	21	123.48
BIAS DESIGNS			
KOSHAL	1.15	21	24.15
BOX-DRAPER *	1.37	27	36.99
(HALF-REPLICATE) BOX-DRAPER *	1.31	43	56.33
VE ULL/			

\* indicates nearly rotatable design

## TABLE 6.

#### PREDICTIVE POWER OF SIX FACTOR DESIGNS

DESIGN	PERCEN	r erf VDOM	ROR IN FIT DATA SET	NUMBER OF DESIGN POINTS	E <u>- Value</u>
VARIANCE	DESIGN	3			
HYBRID	628A	*	1.33	28	37.24
HYBRID	628B	*	1.35	28	37 <b>.80</b>
MIN PO	INT		1.76	28	49.28
BOX-BEI	HNKEN	*	1.15	49	56.35
CENTRAI (HALF-I	L COMP REPLICA	* TE)	1.33	45	59.85
BIAS DES	IGNS				
KOSHAL			1.72	28	48.16
BOX-DRI	APER REPLICA	* (E)	1.39	45	62.55

\* indicates nearly rotatable design

<u>ACCURACY</u>. Tables 3 through 7 indicate that the accuracy of a response surface equation is dependent on the number of design points and on the degree of rotatability of the design. That is, the accuracy of a response surface equation is highly correlated to the number of design points, but this relationship is not a perfect correlation and their differences in fit may be explained by the degree of the rotatability of the designs.

For example, it may be hypothesized that the design

with the most points produces the most accurate fit. That is, each design selects its design points under the assumption that the design points are representative of their surrounding area. In theory, by selecting more design points, the area for which each point is an estimate is decreased. Thus, the total error in the estimate of the true surface is reduced. But as Tables 3 through 7 indicate, more design points does not guarantee a more accurate fit. This result is rather surprising, but it does indicate that other contributing factors influence the degree of accuracy of the response equation other than the number of design points.

A major contributing factor which influences the accuracy of an equation of a response surface design may be the degree of rotatability of the design. In general, the variance and bias minimizing designs are able to generate similar degrees of fit in estimating a response surface equation to the true surface, provided that the variance minimizing designs are nearly rotatable. Rotatability is an important characteristic for variance minimizing designs when estimating deterministic models because the selection of the design points is not strictly based on minimizing bias. But, as stated in Chapter I, if the region of interest is spherical, rotatable designs minimize bias. The importance of rotatability in minimizing bias is highlighted by the fact that the only variance minimizing design which fails to consistently produce a relatively accurate fit is

the minimum point design, which is a non-rotatable design.

However, none of the designs mentioned thus far is perfectly rotatable; thus, bias is not totally minimized for these designs. In fact, all of the designs, except the minimum point and the Koshal designs, are nearly rotatable. That is, these nearly rotatable designs have varying degrees of rotatability and thus, have varying degrees of effectiveness in minimzing bias. Therefore, it might be hypothesized that the differences in accuracy between the designs which are not accounted for by the number of design points, are accounted for by the degrees of rotatability of the designs.

For example, the response surface equation of the hybrid design (416A+CP) achieves a more accurate fit to the true surface than the response equation of the Box-Behnken design for the four factor problem. In this example, the hybrid design requires eight fewer points than the Box-Behnken design, but is able to achieve a higher degree of accuracy. By the hypothesis, this indicates that the hybrid design (416A+CP) has a higher degree of rotatability than the Box-Behnken design for four factors. This hypothesis is confirmed by the fact that the Box-Behnken design is not highly rotatable for four factors, while the hybrid design maintains a high degree of rotatability.

A similar argument can be made to explain why no single design is able to achieve the most accurate fit for all factors. That is, for the different number of factors,

these designs have different number of design points and different degrees of rotatability. Thus, the degrees of accuracy for the response equations are affected.

EFECTIVENESS-EFFICIENCY FACTOR. Since the accuracy of fit of the design is highly correlated to the number of design points, the results which are obtained concerning the effectivenss-efficiency factor are not surprising. That is, a design which provides the most accurate fit with the fewest number of design points does not exist. Even a perfectly rotatable design does not provide the most accurate fit, if a sufficient number of design points are not specified.

Thus, in attempting to minimize the effectivenessefficiency factor, it remains for the experimenter to evaluate the problem to determine what degree of accuracy is required and how many design points are available. In evaluating the trade-off between accuracy and the number of design points, one major consideration is the cost of performing additional data runs. In those problems in which cost of executing additional data runs is minimal, the experimenter may concentrate on achieving a higher degree of accuracy. On the other hand, in those cases in which set-up costs and run time are prohibitive, the experimenter may select the most economical design (in terms of the fewest design points) which achieves an adequate level of accuracy.

The trade-offs between accuracy (level of fit) and cost (number of design points) are further complicated when the

experimenter desires a response equation which has superior explanatory powers.

## EXPLANATORY POWER OF RESPONSE SURFACE EQUATION

The basis for evaluating the explanatory capabilities of an equation of a response surface design is the interpretation of the coefficients of the response equation. The coefficients provide answers to such questions as:

"What is the overall effect to the dependent variable,

if one independent variable is increased by an amount  $\boldsymbol{X}$ 

and all other independent variables are held constant?" By applying the analysis techniques of multiple regression, similar "what...if..." questions can be answered quickly and accurately. The answers to these questions are quick and easy because the basic interpretation of the coefficients in multiple regression analysis is that the coefficients of the main effect terms represent the marginal effects of change of the dependent variable when the values of the independent variables are altered.

EFFECT OF MULTICOLLINEARITY ON REGRESSION

<u>COEFFICIENTS</u>. As stated earlier, Box and Hunter have identified a small correlation that exists between the constant term and the squared term and between the squared terms themselves (4:201). This correlation is commonly referred to as multicollinearity in statistical literature. Since Box and Hunter concentrated on the predictive power of the response equation, multicollinearity did not affect their results because of the fact that when some or all independent variables are correlated among themselves, the ability to obtain a good fit is not generally inhibited nor does it tend to affect prediction of new observations (19:384). That is, the predictive power of the response equation is not generally affected when multicollinearity exists.

However, the common interpretation of regression coefficients as measuring the change in the expected value of the dependent variable when the corresponding independent variable is increased by one unit while all independent variables are held constant is not fully applicable when a high degree of multicollinearity exists. While it may be conceptually possible to vary one independent variable and hold the others constant, it may not be possible in practice to do so for independent variables that are highly correlated (19:385). Because a high degree of correlation among the independent variables exists in non-orthogonal designs, these designs do not possess a high degree of explanatory power by the fact that the regression coefficients of the response equations do not accurately measure the marginal effects of change of the dependent variable when the values of the independent variables are altered. Thus, only those designs which have a high degree of orthogonality will be considered in the explanatory power section of this study. The designs which are to be evaluated are: the Central Composite Rotatable designs, the

Box-Behnken designs, the Hybrid designs, and the Box-Draper Bias Minimizing designs.

<u>THREE FACTOR RESULTS</u>. Equation (14) is the basic response equation for the three factor problem.

$$DE = b_0 + b_1 (M1) + b_2 (B1) + b_3 (S1)$$
  
-  $b_{33} (S1*S1) - b_{23} (B1*S1)$  (14)

The terms which have been omitted from the equation account for less than two percent of the total sum of squares, and therefore are dropped from the final equation.

Table 7 lists the coefficients of the response equation for the various designs for the three factor problem.

#### TABLE 7.

## COEFFICIENTS OF THE RESPONSE EQUATION FOR THREE FACTOR DESIGNS

DESIGN

	b <sub>o</sub>	Þ <sub>1</sub>	<sup>b</sup> 2	ь <b>з</b>	b 33	<sup>b</sup> 23
(1) Central Comp	124.9285	. 4848	1.153	1.004	.0000276	.000062
(2) Box- Behnken	-648.3319	. 4965	1.225	1.180	.0000375	. 0000665
(3) Hybrid (311A)	68.4118	.5243	1.195	.961	.0000251	.0000602
(4) Hybrid (310+CP)	-626.1627	. 5002	1.327	1.163	.0000352	.0000774
(5) Box- Draper	-285.6179	. 4798	1.223	1.085	.0000318	. 0000667

FOUR FACTOR RESULTS. Equation (15) is the basic response equation for the four factor problem.

 $DE = b_0 + b_1(M1) + b_2(M2) + b_3(B1) + b_4(S1)$ - b\_4 (S1\*S1) - b\_2 (M2\*S1) - b\_3 (B1\*S1) (15)

The terms which have been omitted from the equation account for less than two percent of the total sum of squares, and therefore are dropped from the final equation.

Table 8 lists the coefficients of the response equation for the various designs for the four factor problem.

## TABLE 8.

## COEFFICIENTS OF THE RESPONSE EQUATION FOR FOUR FACTOR DESIGNS

DESIGN

		b <sub>o</sub>	b 1	b 2	b3	b <sub>4</sub>	b <sub>4.4</sub>	b 24	Ъ <sub>34</sub>
(1)	Central Comp	939.3	. 416	.961	1.02	. 907	<b>. 0000</b> 23	.000054	.000051 ·
(2)	Box- Behnken	1320.0	.383	.946	.987	.877	.000023	.000054	.000051
(3)	Hybrid (416C)	1129.1	. 434	1.12	1.08	.793	.000015	. 000068	.0000645
(4)	Hybrid (416A+CF	1060.1 ')	. 439	1.09	1.11	.823	.000017	.000064	.0000601
(5)	Box- Draper	1332.3	.395	.944	. 988	.887	.000024	.000053	.0000503

<u>FIVE FACTOR RESULTS</u>. Equation (16) is the basic response equation for the five factor problem.

$$DE = b_0 + b_1(M1) + b_2(M2) + b_3(B1)$$
$$+ b_4(B2) + b_5(S1) - b_{55}(S1*S1)$$
(16)

The terms which have been omitted from the equation account for less than two percent of the total sum of squares, and therefore are dropped from the final equation.

Table 9 lists the coefficients of the response equation for the various designs for the five factor problem.

## TABLE 9.

## COEFFICIENTS OF THE RESPONSE EQUATION FOR FIVE FACTOR DESIGNS

#### DESIGN

and and a subserve a station

		<u> </u>	<sup>0</sup> 1	2	03	<sup>D</sup> 4	5	<sup>D</sup> 55	
(1)	Central Comp	1615.7	.501	. 630	.712	.755	.951	.0000261	
(2)	Box- Behnken	844.0	. 603	. 680	.729	.769	1.04	.0000295	
(3)	Box~ Draper(H)	`536 <b>.</b> 6	.604	.683	.739	.785	1.09	.0000329	
(4)	Box~ Draper(F)	388.0	.629	.678	.756	. 798	1.09	.0000327	

<u>SIX FACTOR RESULTS</u>. Equation (17) is the basic response equation for the six factor problem.

$$DE = b_0 + b_1(M1) + b_2(M2) + b_3(M3)$$
  
+ b\_4(B1) + b\_5(B2) + b\_6(S1)  
- b\_{66}(S1\*S1) - b\_{56}(B2\*S1) (17)

The terms which have been omitted from the equation account for less than two percent of the total sum of squares, and therefore are dropped from the final equation.

Table 10 lists the coefficients of the response equation for the various designs for the six factor problem.

#### TABLE 10.

## COEFFICIENTS OF THE RESPONSE EQUATION FOR SIX FACTOR DESIGNS

DESIGN

		ь <b>о</b>	ь <sub>1</sub>	<sup>ь</sup> 2	ь <sup>3</sup>	ь <sub>4</sub>	<sup>b</sup> 5	<sup>ь</sup> 6	b 66	56
(1)	Central Composit	620.1 e	.562	.606	.673	. 685	1.24	1.05	.0000235	.000057
(2)	Box- Behnken	-642.1	.536	.628	. 654	.692	1.39	1.28	. 0000331	<b>. 00007</b> 3
(3)	Hybrid (628A)	715.9	.563	.598	.626	. 684	1.23	1.04	.0000228	. 000057
(4)	Hybrid (628B)	695.9	.554	. 606	.614	.683	1.23	1.04	<b>. 00002</b> 32	.000057
(5)	Box- Draper	-101.0	.568	.645	.678	.695	1.22	1.21	<b>. 0000</b> 328	. 000054

<u>INTERPRETATION OF COEFFICIENTS</u>. The parameter  $b_0$  is the y-intercept of the regression surface. This term does not have any particular meaning as a separate term in any of the regression models because the range of the variables in the model does not include the point where M1=0,...,S1=0. If this point had been in the relevant range of the variables, the  $b_0$  term would indicate the response level when all of the independent variables are set to zero.

When the regression model does not include cross product or squared terms (i.e., a first order equation), the parameters  $b_K \ (k \neq 0)$  indicate the change in the mean response per unit increase in  $x_K$  when all other variables included in the model are held constant. That is,  $b_1$ indicates the change in the expected value of the response with a unit increase in  $x_1$  when all other variables in the model are held constant. Since the experimental designs which are being considered in this section are nearly orthogonal, the independent variables are almost uncorrelated and the variables have an additive effect.

When the regression model includes cross product or squared terms, such as Eq (14) of the three factor model, the coefficients of the main effect terms no longer indicate the change in the mean response per unit increase in  $x_K$  for any given level of the remaining variables, if that variable is also included in the cross product or squared terms. For example, in Eq(14), b, no longer indicates the change in the

mean response for a unit increase in B1 because of the interaction of B1 and S1. Similarly,  $b_3$  no longer indicates the change in the mean response for a unit increase in S1 because of the interaction of S1 and B1 and because of the interaction of S1 with itself. Because of the complexity of a regression model which includes several cross product and squared terms, the most desirable regression equation which explains the response surface is one in which the main effects dominate and the cross product and squared terms take minor, secondary roles. In this manner, the parameters  $b_K$  would indicate the change in the mean response per unit increase in  $x_K$  when all other variables included in the model are held constant.

Differences between Coefficients. Tables 7 through 10 indicate that no two equations have the same coefficients to estimate the same surface. That is, each design produces a unique response equation to estimate the true surface. These results are not unexpected and does not disqualify the use of multiple regression analysis techniques as an analytical tool when evaluating response surfaces of deterministic models.

For example, small differences between the coefficients are to be expected because the predicted surface is not a perfect representation of the actual surface. The errors in the predicted surface are due to two factors: first, each design selects different quantities of sample points to estimate the true surface and each design selects its design

points at different locations in the region of interest; and second, a small amount of multicollinearity is still present between some of the variables in these equations.

The selection of the number of design points and the placement of the design points have a tremendous impact on the fitted regression function because the response equation is influenced by whatever design points are selected. Because each of the designs selects points from different areas of the true surface, the response equation must necessarily be different for each design. Any change in the selection of the design points influences the coefficients of the response equation to accomodate this change.

For example, the response equation of the hybrid design (311A) is

y = .5243 (M1) + 1.195 (B1) + .961 (S1)

- .00000251 (S1\*S1) - .0000602 (B1\*S1) + 68.4118 (18)

If the design is altered so that the first and third variables in the design are switched, then the new design matrix is coded as follows: the S1 variable is coded as variable 1; the B1 variable is still coded as variable 2; and the M1 variable is coded as variable 3. As a result, different response values are obtained from the same design matrix because different uncoded values have been used in the MPOS input files. The resulting response equation for

the altered hybrid design is:

y = .4678 (M1) + 1.11095 (B1) + .98925 (S1)

- .0000275 (S1\*S1) - .0000512 (B1\*S1) + 209.2299 (19)

The predictive power for both the original and the altered designs are equivalent. That is, the degrees of accuracy for both equations are the same. However, a totally different set of coefficients has been generated from the altered design to estimate the true surface.

A second reason to account for the small differences between the coefficients is the small amount of multicollinearity which exists between the intercept term and the squared terms and between the squared terms themselves. Variables which possess even a small amount of multicollinearity, produce regression coefficients which tend to vary from one design to the next.

For example, a regression equation is developed from a representative sample of points (design 1) from a large population. If multicollinearity exists between some or all of the variables, a second sampling (design 2) from the population will necessarily produce a different regression equation because of the multicollinearity between the variables. The degree of variability between the coefficients of the two equations may be directly related to the degree of multicollinearity between the variables.

Multicollinearity can be eliminated between the variables of an experimental design, if the design is made

perfectly orthogonal. A full factorial design is perfectly orthogonal, if the required transformations are performed on the squared terms of the design. (The required transformation is:  $X_i^2 - \overline{X}_i^2$ .) The full factorial design is not commonly used in RSM because it requires a large number of design points. However, this design serves as an excellent standard to which other designs can be compared.

By performing the necessary transformations, the full factorial three factor design for the deterministic model produces the following response equation:

$$y = .4979 (M1) + 1.216 (B1) + 1.1978 (S1)$$
  
- .00003865 (S1\*S1) - .0000649 (B1\*S1)  
- 964.8424 (20)

The response equation of the full factorial design achieves the same degree of accuracy as the other nearly orthogonal designs in estimating the true surface. Thus, the primary difference between the full factorial design and the other nearly orthogonal designs is that multicollinearity does not exists between the variables in the full factorial design so that all of the variables are uncorrelated.

In that case, the full factorial design provides the most representative estimate of the true surface. That is, the regression equation which is developed from the full factorial design is strictly determined by the number and placement of its design points. Whereas, the response equations of the other designs are also determined by the

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number and placement of the design points, but, the effects of multicollinearity also influence the coefficients of some of the variables of the response equations.

Thus, the degree of variability which exists between the coefficients of the full factorial response equation and the coefficients of the other response equations may be a measure of the degree of multicollinearity which exists in those designs. For example, the coefficients of the response equation for the Box-Behnken design compare more favorably to the full factorial design than some of the other three factor designs. This may indicate that the influence of multicollinearity for three factor designs, is not as significant on the variables of the Box-Behnken design as on the variables of the other designs. However, this statement is just a hypothesis because there is no measure of the degree of orthogonality for the various designs.

In short, the effects of multicollinearity on response surface equations is of primary importance when evaluating deterministic models because the analyst is attempting to draw direct inferences from the coefficients of the response equation. If the degree of multicollinearity is too great between the variables in the equation, any information obtained from the coefficients may be misleading.

As stated earlier, a small amount of multicollinearity exists in all of the equations (except the full factorial), therefore, no two designs produce the same set of

coefficients when estimating a surface. Although the differences between the coefficients are small, their effects may be quite significant.

To further address the subject matter of the effects of the differences between the coefficients, the designs and the response equations of the five factor model are evaluated. The five factor model is selected because of the simplicity of the response equations. These equations include five main effect terms and one squared term in its modified form, eq(16). In this manner, the coefficients of the M1, M2, B1 and B2, terms should represent the marginal effects of change of the dependent variable when the independent variables are altered. The coefficient of the S1 term is excluded from the list because the effects of change caused by S1 are influenced by the S1-squared term. Therefore, the marginal effects of S1 vary for different levels of S1. Tables 11 through 14 compare the predicted change to the actual change of the dependent variable when one independent variable is altered while all other independent variables are held constant.

## TABLE 11.

12.4.23

ACTUAL AND PREDICTED CHANGES WHEN M1 IS DECREASED BY 100

# CHANGE IN RESPONSE

DE	SIGN	ACTUAL	PREDICTED	PERCENT ERROR
(1)	Central Composite	-57.0	-50.1	12.1
(2)	Box- Behnken	-57.0	-60.3	5.8
(3)	Box- Draper (H)	-57.0	-60.4	6.0
(4)	Box- Draper (F)	-57.0	-62.9	10.4

## TABLE 12.

ACTUAL AND PREDICTED CHANGES WHEN M2 IS INCREASED BY 400

# CHANGE IN RESPONSE

DE	BIGN	ACTUAL	PREDICTED	PERCENT ERROR
(1)	Central Composite	260.0	251.9	3.1
(2)	Box- Behnken	260.0	271.8	4.5
(3)	Box- Draper (H)	260.0	273.0	5.0
(4)	Box- Draper (F)	260.0	271.0	4.2

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## TABLE 13.

ACTUAL AND PREDICTED CHANGES WHEN B1 IS INCREASED BY 200

CHANGE IN RESPONSE

DES	SIGN	ACTUAL	PREDICTED	PERCENT ERROR
(1)	Central Composite	140.0	142.5	1.8
(2)	Box- Behnken	140.0	145.9	4.2
(3)	Box- Draper (H)	140.0	147.9	5.6
(4)	Box- Draper (F)	140.0	151.1	7.9

## TABLE 14.

ACTUAL AND PREDICTED CHANGES WHEN B2 IS DECREASED BY 500

		CHANGE IN RESPONSE	
DESIGN	ACTUAL	PREDICTED	PERCENT ERROR
(1) Central Composite	-37 <b>0.0</b>	-377.25	2.0
(2) Box- Behnken	-37 <b>0.0</b>	-384.5	3.9
(3) Box- Draper (H)	-370.0	-392.3	6 <b>.0</b>
(4) Box- Draper (F)	-370.0	-398.8	7.8

There are two primary reasons to explain the discrepancies between the actual and the predicted changes of the various response surface equations.

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The first reason for these discrepancies is due to multicollinearity. As was stated earlier, the effects of multicollinearity influence the coefficients of the correlated variables, and thereby influence their ability to accurately measure the marginal effects of change.

The second reason for these discrepancies is due to the fact that several terms have been omitted from the full regression model. In this five factor example, ten interaction terms and four squared terms have been removed. If these variables are inserted into eq(16), the differences between the predicted change and the actual change of the dependent variable is reduced. The full regression equation for the five factor model is

 $y = b_{0} + b_{1} (M1) + b_{2} (M2) + b_{3} (B1) + b_{4} (B2) + b_{5} (S1)$ +  $b_{12} (M1*M2) + b_{13} (M1*B1) + b_{14} (M1*B2) + b_{15} (M1*S2)$ +  $b_{23} (M2*B1) + b_{24} (M2*B2) + b_{25} (M2*S1) + b_{34} (B1*B2)$ +  $b_{35} (B1*S1) + b_{45} (B2*S1) + b_{11} (M1*M1) + b_{22} (M2*M2)$ +  $b_{33} (B1*B1) + b_{44} (B2*B2) + b_{55} (S1*S1)$ (21)

Tables 15 through 18 contain the same information as Tables 11 through 14, except for the fact that the full regression model (5 main effects, 10 interaction terms, 5 squared terms, and 1 intercept term) is used to determine

the predicted change of the response values in Tables 15 through 18.

## TABLE 15.

## ACTUAL AND PREDICTED CHANGES WHEN M1 IS DECREASED BY 100 (FULL MODEL)

# CHANGE IN RESPONSE

DE	BIGN	ACTUAL	PREDICTED	PERCENT ERROR
(1)	Central Composite	-57.0	-57.5	0.9
(2)	Box- Behnken	-57.0	-59.9	5.1
(3)	Box- Draper (H)	-57.0	-60.2	5.6
(4)	Box- Draper (F)	-57.0	-60.3	5.8

## TABLE 16.

ACTUAL AND PREDICTED CHANGES WHEN M2 IS INCREASED BY 400 (FULL MODEL)

			CHANGE IN RESPONSE		
DE	SIGN	ACTUAL	PREDICTED	PERCENT ERRO	JR
(1)	Central Composite	269.0	247.6	4.8	
(2)	Box- Behnken	260.0	267.5	2.9	
(3)	Box- Drap <del>er</del> (H)	26 <b>0.0</b>	272.5	4.8	
(4)	Box- Draper (F)	260.0	269.8	3.8	

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## TABLE 17.

## ACTUAL AND PREDICTED CHANGES WHEN B1 IS INCREASED BY 200 (FULL MODEL)

			CHANGE IN RESPONSE	
D	ESIGN	ACTUAL	PREDICTED	PERCENT ERROR
(1)	Central Composite	140.0	140.1	0.1
(2)	Box- Behnken	140.0	145.1	3.6
(3)	Box- Draper (H)	140.0	146.9	4.9
(4)	Box~ Draper (F)	140.0	148.2	5.8

## TABLE 18.

## ACTUAL AND PREDICTED CHANGES WHEN B2 IS DECREASED BY 500 (FULL MODEL)

## CHANGE IN RESPONSE

DESIGN		ACTUAL	PREDICTED	PERCENT ERROR	
(1)	Central Composite	-370.0	-364.1	1.6	
(2)	Box- Behnken	-37 <b>0.0</b>	-377.9	2.1	
(3)	Box- Draper (H)	-370.0	-387.1	4.6	
(4)	Box- Draper (E)	-370.0	-389.5	5.3	

From the full regression models, the analyst is able to obtain very effective measures of the marginal effects of the independent variables. The procedure to obtain this measure consists of calculating the response value from the response equation twice. During the first calculation, all variables are set to a base level; during the second calculation, all variables remain fixed, except for the variable whose marginal effect is to be measured. Subsequently, the difference in the response values of the two calculations represents the marginal effect of that variable.

If the full regression model is used to measure the marginal effects of an independent variable, then the requirement for low multicollinearity between the variables is eliminated. The analyst does not obtain the marginal effects information from the coefficients, but from the difference between the estimated response values. Since multicollinearity does not affect a design's ability to produce a response equation that predicts effectively, the response equation of a non-orthogonal design is able to measure the marginal effects of an independent variable as well as the equation of an orthogonal design.

The drawback to using the full regression model is the complexity of eq(21). The ability to measure the marginal effects is no longer a simple proposal and the marginal effects of an independent variable is dependent on the level at which it is evaluated. Essentially, the analyst performs a one variable at a time procedure to obtain his information. Therefore, the method involving the modified

regression model may be a more attractive alternative because of its simplicity, provided that the decision maker is able to accept some limitations in the magnitudes of the estimates of the predicted marginal effects.

<u>Analytical Value of the Modified Response</u> <u>Equations</u>. The question of whether or not any error level is too great is dependent on the degree of accuracy which the decision maker requires. An important point which the decision maker must consider, is that the information pertaining to the marginal effects of the independent variables is readily available from the simplified response equation, but not easily obtainable from other sources.

In examiming the predicted marginal effects of the modified equations, Tables 11 through 14 indicate that there are discrepancies between the predicted and the actual changes in the response values. The range of the error between the predicted and actual change is 1.8% to 12.1%. In general, an error of less than 2% is likely to be accepted in just about any circumstance. However, an error of 12% may not be acceptable in certain situations.

A simple test to check the magnitude of the error of the predicted change for the coefficients of the modified response equation, is to compare the predicted change of the modified response equation to the predicted change of the full regression equation. If the level of error is acceptable to the decision maker, the modified response equation is used and is a very useful analytical tool.

However, if the level of error is not acceptable, the analyst may choose to use the full regression equation to measure the marginal effects of change for the independent variables. But as was noted earlier, this procedure may be rather lengthy, in that the marginal effects of change of any variable is not constant throughout the entire surface. Therefore, the variables must be evaluated at several different levels in order to determine the marginal effect.

An alternative to using the full regression model when the level of error of the modified equation is unacceptable, is to evaluate the ratios of the coefficients of the modified equations. In regression analysis, the ratio of the coefficients indicates the contribution of one factor as compared to another factor. Therefore, although the exact magnitude of their impact on the response value can not be determined, the relative importance of the various independent variables can be assessed by rank ordering the ratios of the coefficients.

For example, the modified response equation for the five factor Box-Behnken design is

y = .6029 (M1) + .6795 (M2) + .7294 (B1) + .7689 (B2) + 1.0363 (S1) - .0000295 (S1\*S1) + 843.9903 (22)

The mean contribution of B2 to the response value, y, is estimated to be 1.27 times as great as the mean contribution

of M1 to the response value. This amount is obtained by calculating the ratio of the coefficients for the two independent variables (.7689/.6029).

In a similar manner, eq(23) is the response equation for the five factor Central Composite Rotatable design. According to this response equation, the mean contribution of B2 is 1.5 times as great as the mean contribution of M1 to the response value (.7545/.5011).

$$y' = .5011 (M1) + .6289 (M2) + .7123 (B1)$$
  
+ .7545 (B2) + .9511 (S1)  
- .0000261 (S1\*S1) + 1615.6741 (23)

Since the various designs which are used in this study produce different coefficients for each design, the ratios of coefficients are different. Hence, the exact magnitude of the contribution of each factor as compared to other factors can not be determined with certainty. Nevertheless, the relative contribution of the factors can still be obtained and these contributions can be rank ordered.

Therefore, an accurate, but less definitive statement about the relative contribution of the two independent variables, is that the mean contribution of B2 is greater than the mean contribution of M1 to the response value. This conclusion is permissible because the relative magnitudes of the coefficients are maintained for the various designs even though the values of the coefficients

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themselves change from one design to another. That is, no matter what orthogonal design is selected, the regression coefficient for B2 always exceeds the coefficient for M1. In this manner, the following conclusion is drawn for the five factor problem: the independent variable which causes the greatest change is S1, provided the values for S1 remain within the relevant range. This variable is followed in order of importance by: B2, B1, M2, and M1. Although this conclusion does not provide an exact measure of the difference between the factors, the relative ranking of the factors is still an important piece of information which the decision maker needs in order to evaluate his alternatives.

## OVERVIEW

This research identifies several experimental designs which are effective and efficient in estimating response surface equations for deterministic models. The specific deterministic model which is used in this study is a nuclear exchange linear programming model, but the results which are obtained in this study apply to any deterministic model.

The purpose of Response Surface Methodlogy in its application to any type of model is to obtain a response surface equation which closely approximates the true surface. A second order polynomial has been shown to be an excellent choice for estimating response surfaces of determininstic, nuclear exchange models. Ideally, the experimental design should use as few points as possible to obtain an accurate fit and the response equation should contain as few terms as possible. That is, the equation should contain as few cross product and squared terms as possible so that direct inferences can be drawn from the coefficients of the main effects.

## SPECIFIC RESULTS

In keeping with the idea of simplifying the response equation, the response surface equations which are used to evaluate error in fit of the design are modified versions of the full regression equations. That is, selected terms from
the full regression equations have been omitted from the modified regression equations because these terms account for less than two percent of the total sum of squares.

In evaluating the response equations of these designs for their effectiveness, the results indicate that the accuracy of a modified response equation is dependent on the number of design points and on the degree of rotatability of the design. The accuracy of a response surface equation is highly correlated to the number of design points, but this relationship is not a perfect correlation, and their differences can be explained by the degree of rotatability of the designs.

In short, more design points improves accuracy, provided the points are spread throughout the entire surface. Similarly, if the region of interest is spherical, a rotatable design improves the accuracy of the response equation.

In evaluating these designs for their efficiency as well as effectiveness, the results indicate that a design which provides the most accurate fit with the fewest number of design points for all factors does not exist. In fact, minimizing the number of design points and maximizing accuracy are dichotomous goals. This trade-off between accuracy and required number of design points is dependent on the problem at hand and needs to be defined seperately for each situation. Therefore, it remains for the decision maker to evaluate the problem to determine which is more

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important: a more accurate fit or fewer design points.

In evaluating the explanatory power of the response equations, the coefficients of the modified response equation represent marginal effects of change of the dependent variable when the independent variables are altered provided specific conditions are met. These conditions are: a high degree of multicollinearity does not exist between the independent variables; the main effect terms of the response equation dominate the squared and the interaction terms; and the modified response surface is able to accurately represent the true surface.

The results of this project indicate that there are limitations in the ability of the coefficients of the modified response equation to predict the marginal effects of change. The accuracy of these models to predict the marginal change can be evaluated by comparing the predicted changes of the coefficients of the modified response equations to the coefficients of the full regression equations. If this difference is unacceptable, then the full regression equation can be used to measure the marginal effects or the ratios of the coefficients of the modified equation can be used to determine the relative importance of the independent variables.

### RECOMMENDATIONS

This study originally intended to examine the differences between bias minimizing and variance

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minimizing designs. As this study evolved, it became clear that there were no significant differences between the two types of experimental designs, provided that the designs maintained certain characteristics, such as, rotatability and orthogonality.

However, very few designs possess pure rotatability or orthogonality. Most of the designs are classified in a gray area called, nearly rotatable and nearly orthogonal. As such, further research is required in specifying nearly rotatable and nearly orthogonal. This research may find what levels of rotatability are necessary to achieve a desired level of predictive power. Also, this research may find what levels of orthogonality are necessary to achieve a desired level of explanatory power.

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## Appendix A: MPOS Input Files

This appendix describes the specific problems which were analyzed for the three, four, five, and six factor cases. In addition, this appendix lists a sample of the Multi-Purpose Optimization System (MPOS) input files which were used to obtain the response value.

### THREE FACTOR PROBLEM

There are three weapon types and five target classes. The three weapon types are:

VALUE	HIGH	VALUE	LOW	IYPE	WEAPON
3000		1650			M1
3000		1200			B1
12000	:	6000			S1

Assumption: Each system's reliability, probability of launch survivability, probability of arrival, and warhead reliability are all 1.0. The single shot probability of kill (which is damage expectancy on a target because of the previous assumptions of reliability, etc.) for each warhead type is provided in the following table:

### TABLE A.1

### SINGLE SHOT PROBABILITY OF KILL FOR THREE FACTOR PROBLEM

TARGET IYPE		WEAPON	WEAPON IYPE				
	M1	B1	51				
TGT CLASS 1	.84	. 96	.73				
TGT CLASS 2	.75	. 88	. 64				
TGT CLASS 3	.55	.70	. 32				
TGT CLASS 4	. 88	. 98	.78				
TGT CLASS 5	.25	. 48	.15				

Target class 4 represents time urgent targets and only ICBMs (M1) and SLBMs (S1) are allowed to hit them. In addition, the maximum allowable damage on target class 2 targets is .95 of the total target value and at least .60 of the total target value of target class 5 targets must be destroyed. Furthermore, a maximum of two warheads can targeted at any individual target.

The number of targets in each class is 6000, 4000, 2000, 1000, and 200, respectively. The measure of effectiveness for this problem is the total damage expectancy, i.e., the sum of the product of the damage expectancy on a target times the number of targets.

EOUR FACTOR PROBLEM

There are four weapon types and five target classes. The four weapon types are:

WEAPON	IYPE	L <u>D₩</u>	YALUE	HIGH	YALUE
M1			1650		3000
M2			500		2000
B1			1200		3000
<b>S</b> 1			6000	1	2000

Assumption: Each system's reliability, probability of launch survivability, probability of arrival, and warhead reliability are all 1.0. The single shot probability of kill (which is damage expectancy on a target because of the previous assumptions of reliability, etc.) for each warhead type is provided in the following table:

### TABLE A.2

### SINGLE SHOT FROBABILITY OF KILL FOR FOUR FACTOR PROBLEM

]	TARGET	IYPE		WEAPON	IEAPON TYPE					
			M1	M2	B1	<b>S1</b>				
TGT	CLASS	1	.84	. 90	.96	.73				
TGT	CLASS	2	.75	. 83	.88	. 64				
TGT	CLASS	3	.55	.62	.70	. 32				
TGT	CLASS	4	.88	.94	. 98	.78				
TGT	CLASS	5	.25	.36	. 48	.15				

Target class 4 represents time urgent targets and only ICBMs (M1, M2) and SLBMs (S1) are allowed to hit them. In addition, the maximum allowable damage on target class 2 targets is .95 of the total target value and at least .60 of the total target value of target class 5 targets must be destroyed. Furthermore, a maximum of two warheads can targeted at any individual target.

The number of targets in each class is 6000, 4000, 2000, 1000, and 200, respectively. The measure of effectiveness for this problem is the total damage expectancy, i.e., the sum of the product of the damage expectancy on a target times the number of targets.

# EIVE EACTOR PROBLEM

There are five weapon types and five target classes. The five weapon types are:

WEAPON	IYPE	LOM	YALUE	HIGH	VALUE
M1			1650		3000
M2			500		2000
B1	,		1200		3000
B2			2000		4000
<b>S1</b>			6000	:	2000

Assumption: Each system's reliability, probability of launch survivability, probability of arrival, and warhead reliability are all 1.0. The single shot probability of kill (which is damage expectancy on a target because of the previous assumptions of reliability, etc.) for each warhead type is provided in the following table: SINGLE SHOT PROBABILITY OF KILL FOR FIVE FACTOR PROBLEM

TARGET TYPE	WEAPON IYPE								
	M1	M2	B1	B2	S1				
TGT CLASS 1	.84	. 90	.96	. 98	.73				
TGT CLASS 2	.75	.83	. 88	.92	.64				
TGT CLASS 3	.55	.62	.70	.74	.32				
TGT CLASS 4	. 88	.94	. 98	.99	.78				
TGT CLASS 5	.25	.36	. 48	.55	.15				

Target class 4 represents time urgent targets and only ICBMs (M1, M2) and SLBMs (S1) are allowed to hit them. In addition, the maximum allowable damage on target class 2 targets is .95 of the total target value and at least .60 of the total target value of target class 5 targets must be destroyed. Furthermore, a maximum of two warheads can targeted at any individual target.

The number of targets in each class is 9000, 6000, 3000, 1500, and 300, respectively. The measure of effectiveness for this problem is the total damage expectancy, i.e., the sum of the product of the damage expectancy on a target times the number of targets. SIX FACTOR PROBLEM

There are six weapon types and five target classes. The six weapon types are:

WEAPON	IYPE	LOW	VALUE	HIGH	VALUE
M1			1650		3000
M2			500		2000
M3			400		1000
B1			1200		3000
B2			2000		4000
<b>S</b> 1			6000	1	2000

Assumption: Each system's reliability, probability of launch survivability, probability of arrival, and warhead reliability are all 1.0. the single shot probability of kill (which is damage expectancy on a target because of the previous assumptions of reliability, etc.) for each warhead type is:

### TABLE A.4

SINGLE SHOT PROBABILITY OF KILL FOR SIX FACTOR PROBLEM

IARGEI IYPE		Ш	EAPON I	YPE		
	M1	M2	M3	B1	B2	S1
TGT CLASS 1	. 84	. 90	.92	.96	. 98	.73
TGT CLASS 2	.75	.83	.87	. 88	.92	.64
TGT CLASS 3	.55	.62	.65	.70	.74	.32
TGT CLASS 4	. 88	.94	. 95	. 98	.99	.78
TGT CLASS 5	. 25	.36	.45	. 48	.55	.15

Target class 4 represents time urgent targets and only ICBMs (M1, M2, M3) and SLBMs (S1) are allowed to hit them. In addition, the maximum allowable damage on target class 2 targets is .95 of the total target value and at least .60 of the total target value of target class 5 targets must be destroyed. Furthermore, a maximum of two warheads can targeted at any individual target.

The number of targets in each class is 9000, 6000, 3000, 1500, and 300, respectively. The measure of effectiveness for this problem is the total damage expectancy, i.e., the sum of the product of the damage expectancy on a target times the number of targets. REGULAR TITLE RESPONSE SURFACE FOR ARSENAL EXCHANGE MODEL

VARI	ABLE	is:	** WE	** WEAPON					
			** 5	/51	ΓEM	**			
X11	ТО	X15	** M1			**			
X21	TO	X25	** B1			**			
X31	TO	X35	** 51			**			
X41	TO	X45	** M1	+	M1	**			
X51	TO	X55	** B1	+	<b>B1</b>	**			
X61	TO	X65	** 51	+	S1	**			
X71	TO	X75	** M1	+	<b>B1</b>	**			
X81	TO	X85	** M1	+	<b>S1</b>	**			
X91	TO	X95	** B1	+	<b>S1</b>	**			

MAXIMIZE

\*\* DE TABLE \*\*

.8400	X11	+	.7500	X12	+	. 5500	X13	+	. 8800	X14	+	. 2500	X15	+
. 9600	X21	+	. 8800	X22	+	.7000	X23	+	.0000	X24	+	. 4800	X25	+
.7300	X31	+	. 6400	X32	+	.3200	X33	+	.7800	X34	+	.1500	X35	+
.9744	X41	+	.9375	X42	+	.7975	X43	+	.9856	X44	+	.4375	X45	+
.9984	X51	+	.9856	X52	+	.9100	X53	+	.0000	X54	+	.7296	X55	+
.9271	X61	+	.8704	X62	+	.5376	X63	+	.9516	X64	+	.2775	X65	+
.9936	X71	+	.9700	X72	+	.8650	X73	+	.0000	X74	+	.6100	X75	+
.9568	X81	+	.9100	X82	+	.6940	X83	+	.9736	X84	+	.3625	X85	+
.9892	X91	+	.9568	X92	+	.7960	X93	+	.0000	X94	+	.5580	X95	

CONSTRAINTS \*\* (1) 95 % DE CRITERIA \*\* .7500 X12 + .8800 X22 + .6400 X32 + .9375 X42 + .9856 X52 + .8704 X62 + .9700 X72 + .9100 X82 + .9568 X92 .LE. 3800

\*\* (2) 60 % DE CRITERIA \*\* .2500 X15 + .4800 X25 + .1500 X35 + .4375 X45 + .7296 X55 + .2775 X65 + .6100 X75 + .3625 X85 + .5580 X95 .LE. 120

\*\* (3) TARGET 1 CONSTRAINT \*\* X11 + X21 + X31 + X41 + X51 + X61 + X71 + X81 + X91 .LE. 6000

\*\* (4) TARGET 2 CONSTRAINT \*\* X12 + X22 + X32 + X42 + X52 + X62 + X72 + X82 + X92 .LE. 4000

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\*\* (5) TARGET 3 CONSTRAINT \*\* X13 + X23 + X33 + X43 + X53 + X63 + X73 + X83 + X93 .LE. 2000

\*\* (6) TARGET 4 CONSTRAINT \*\* X14 + X24 + X34 + X44 + X54 + X64 + X74 + X84 + X94 .LE. 1000

\*\* (7) TARGET 5 CONSTRAINTS \*\* X15 + X25 + X35 + X45 + X55 + X65 + X75 + X85 + X95 .LE. 200

\*\* (8) WEAPON SYSTEM 1 (M1) CONSTRAINT \*\*
X11 + X12 + X13 + X14 + X15 +
2 X41 + 2 X42 + 2 X43 + 2 X44 + 2 X45 +
X71 + X72 + X73 + X74 + X75 +
X81 + X82 + X83 + X84 + X85 .LE. 3000

\*\* (9) WEAPON SYSTEM 2 (B1) CONSTRAINT
X21 + X22 + X23 + X24 + X25 +
2 X51 + 2 X52 + 2 X53 + 2 X54 + 2 X55 +
X71 + X72 + X73 + X74 + X75 +
X91 + X92 + X93 + X94 + X95 .LE. 3000

\*\* (10) WEAPON SYSTEM 3 (S1) CONSTRAINT \*\*
X31 + X32 + X33 + X34 + X35 +
2 X61 + 2 X62 + 2 X63 + 2 X64 + 2 X65 +
X81 + X82 + X83 + X84 + X85 +
X91 + X92 + X93 + X94 + X95 .LE. 9000

PRINT OPTIMIZE

REGULAR TITLE RESPONSE SURFACE FOR ARSENAL EXCHANGE MODEL

VARIABLES	** WEAPUN	**
	** SYSTEM	**
X11 TO X15	** M1	**
X21 TO X25	** B1	**
X31 TO X35	** S1	**
X41 TO X45	** M1 + M1	**
X51 TO X55	** B1 + B1	**
X61 TO X65	** S1 + S1	**
X71 TO X75	** M1 + B1	**
X81 TO X85	** M1 + S1	**
X91 TO X95	** B1 + S1	**
X101 TO X105	** M2	**
X111 TO X115	** M2 + M2	**
X121 TO X125	** M1 + M2	**
X131 TO X135	** M2 + B1	**
X141 TO X145	** M2 + 51	**

MAXIMIZE

\*\* DE TABLE \*\*

\_ \_ \_ \_ . .

.8400	X11	+	.7500	X12	+	.5500	X13	+	. 8800	X14	+	. 2500	X15	+
.9600	X21	+	.8800	X22	+	.7000	X23	+	.0000	X24	+	. 4800	X25	+
.7300	X31	+	.6400	X32	+	.3200	X33	+	.7800	X34	+	.1500	X35	+
.9744	X41	+	.9375	X42	+	.7975	X43	+	.9856	X44	+	.4375	X45	+
. 9984	X51	+	.9856	X52	+	.9100	X53	+	.0000	X54	+	.7296	X55	+
.9271	X61	+	.8704	X62	+	.5376	X63	+	.9516	X64	+	.2775	X65	+
.9936	X71	+	.9700	X72	+	.8650	X73	+	.0000	X74	+	.6100	X75	+
.9568	X81	+	.9100	X82	+	.6940	X83	+	.9736	X84	+	.3625	X85	+
.9892	X91	+	.9568	X92	+	.7960	X93	+	.0000	X94	+	.5580	X95	+
. 9000	X1Ø1	+	.8300	X1Ø2	+	.6200	X1Ø3	+	.9400	X104	+	.3600	X105	+
. 9900	X111	+	.9711	X112	+	.8556	X113	+	.9964	X114	+	. 5904	X115	+
.9840	X121	+	.9575	X122	+	.8290	X123	+	.9928	X124	+	. 5200	X125	+
.9960	X131	+	.9796	X132	+	.8860	X133	+	.0000	X134	+	.6672	X135	+
.9730	X141	+	.9388	X142	+	.7416	X143	+	.9868	X144	+	.4560	X145	

CONSTRAINTS

\*\* (1) 95 % DE CRITERIA \*\*
.7500 X12 + .8800 X22 + .6400 X32 + .9375 X42 + .9856 X52 +
.8704 X62 + .9700 X72 + .9100 X82 + .9568 X92 + .8300 X102 +
.9711 X112 + .9575 X122 + .9796 X132 + .9388 X142 .LE. 3800

\*\* (2) 60 % DE CRITERIA \*\* .2500 X15 + .4800 X25 + .1500 X35 + .4375 X45 + .7296 X55 + .2775 X65 + .6100 X75 + .3625 X85 + .5580 X95 + .3600 X105 + .5904 X115 + .5200 X125 + .6672 X135 + .4560 X145 .GE. 120 \*\* (3) TARGET 1 CONSTRAINT \*\* X11 + X21 + X31 + X41 + X51 + X61 + X71 + X81 X91 + X101 + X111 + X121 + X131 + X141 .LE. 6000 **\*\* (4) TARGET 2 CONSTRAINT \*\*** X12 + X22 + X32 + X42 + X52 + X62 + X72 + X82 X92 + X102 + X112 + X122 + X132 + X142 .LE. 4000 \*\* (5) TARGET 3 CONSTRAINT \*\* **X13 + X23 + X33 + X43 + X53 + X63 + X73 + X83** X93 + X103 + X113 + X123 + X133 + X143 .LE. 2000 **\*\* (6) TARGET 4 CONSTRAINT \*\*** X14 + X24 + X34 + X44 + X54 + X64 + X74 + X84 X94 + X104 + X114 + X124 + X134 + X144 .LE. 1000 \*\* (7) TARGET 5 CONSTRAINTS \*\* X15 + X25 + X35 + X45 + X55 + X65 + X75 + X85 X95 + X105 + X115 + X125 + X135 + X145 .LE. 200 \*\* (8) WEAPON SYSTEM 1 (M1) CONSTRAINT \*\* + X12 + X13 + X14 + X15X11 2 X41 + 2 X42 + 2 X43 + 2 X44 + 2 X45 + X71 + X72 + X73 + X74 + X75 + X81 + X82 + X83 + X84 + X85 + X121 + X122 + X123 + X124 + X125 .LE. 3000 \*\* (9) WEAPON SYSTEM 2 (M2) CONSTRAINT \*\* X101 + X102 + X103 + X104 + X105 +2 X111 + 2 X112 + 2 X113 + 2 X114 + 2 X115 + X121 + X122 + X123 + X124 + X125 +X131 + X132 + X133 + X134 + X135 + X141 + X142 + X143 + X144 + X145 .LE. 2000 \*\* (10) WEAPON SYSTEM 3 (B1) CONSTRAINT X21 + X22 + X23 + X24 + X25 + 2 X51 + 2 X52 + 2 X53 + 2 X54 + 2 X55 + X71 + X72 + X73 + X74 + X75 + X91 + X92 + X93 + X94 + X95 + X131 + X132 + X133 + X134 + X135 .LE. 3000

\*\* (11) WEAPON SYSTEM 4 (S1) CONSTRAINT \*\*
X31 + X32 + X33 + X34 + X35 +
2 X61 + 2 X62 + 2 X63 + 2 X64 + 2 X65 +
X81 + X82 + X83 + X84 + X85 +
X91 + X92 + X93 + X94 + X95 +
X141 + X142 + X143 + X144 + X145 .LE. 9000

PRINT OPTIMIZE

2022/2023

0.5

REGULAR TITLE RESPONSE SURFACE FOR ARSENAL EXCHANGE MODEL

VARIA	ABLE	IS	**	WE	EAF	20N	**	
			**	S١	<b>/</b> 51	TEM	**	
X11	TO	X15	**	M1			**	
X21	ТО	X25	**	<b>B1</b>			**	
X31	ТО	X35	**	<b>S1</b>			**	
X41	то	X45	**	M1	+	M1	**	
X51	TO	X55	**	<b>B1</b>	+	<b>B1</b>	**	
X61	ТО	X65	**	<b>S1</b>	+	<b>S1</b>	**	
X71	то	X75	**	M1	+	<b>B1</b>	**	
X81	то	X85	**	M1	+	<b>S1</b>	**	
X91	TO	X95	**	<b>B1</b>	+	<b>S1</b>	**	
X1Ø1	TO	X105	**	M2			**	
X111	TO	X115	**	M2	+	M2	**	
X121	TO	X125	**	M1	+	M2	**	
X131	ТО	X135	**	M2	+	<b>B1</b>	**	
X141	то	X145	**	M2	+	S1	**	
X151	то	X155	**	<b>B2</b>			**	
X161	ТО	X165	**	<b>B2</b>	+	<b>B2</b>	**	
X171	тο	X175	**	M1	+	<b>B2</b>	**	
X181	TO	X185	**	M2	+	<b>B2</b>	**	
X191	TO	X195	**	B1	+	B2	**	
X201	TO	X205	**	<b>B2</b>	+	<b>S1</b>	**	

MAXIMIZE

\*\* DE TABLE \*\*

.8400	X11	+	.7500	X12	+	.5500	X13	+	. 8800	X14	+	.2500	X15 +	•
.9600	X21	+	. 8800	X22	+	.7000	X23	+	. 0000	X24	+	.4800	X25 +	
.7300	X31	+	.6400	X32	+	.3200	X33	+	.7800	X34	+	.1500	X35 +	
.9744	X41	+	.9375	X42	+	.7975	X43	+	.9856	X44	+	.4375	X45 +	
.9984	X51	+	.9856	X52	+	.9100	X53	+	.0000	X54	+	.7296	X55 +	
.9271	X61	+	.8704	X62	+	.5376	X63	+	.9516	X64	+	.2775	X65 +	•
.9936	X71	+	. 9700	X72	+	.8650	X73	+	.0000	X74	+	.6100	X75 +	•
.9568	X81	+	.9100	X82	+	.6940	X83	+	.9736	X84	+	.3625	X85 +	
.9892	X91	+	.9568	X92	+	.7960	X93	+	.0000	X94	+	.5580	X95 +	•
.9000	X1Ø1	+	.8300	X102	+	.6200	X103	+	.9400	X1Ø4	+	.3600	X105	+
.9900	X111	+	.9711	X112	+	.8556	X113	+	.9964	X114	+	. 5904	X115	+
.9840	X121	+	.9575	X122	+	.8290	X123	+	.9928	X124	+	.5200	X125	+
.9960	X131	+	.9796	X132	+	.8860	X133	+	. 0000	X134	+	.6672	X135	+
.9730	X141	+	.9388	X142	+	.7416	X143	+	.9868	X144	+	. 4560	X145	+
.9800	X151	+	.9200	X152	+	.7400	X153	+	.0000	X154	+	. 5500	X155	+
.9996	X161	+	.9936	X162	+	.9324	X163	+	. 0000	X164	+	.7975	X165	+
.9968	X171	+	.9936	X172	+	.8830	X173	+	.0000	X174	+	.6625	X175	+
.9980	X181	+	.9864	X182	+	.9012	X183	+	.0000	X184	+	.7120	X185	+
.9992	X191	+	.9904	X192	+	.9220	X193	+	. 0000	X194	+	.7660	X195	+
.9946	X201	+	.9712	X202	+	.8232	X203	+	.0000	X204	+	.6175	X205	

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CONSTRAINTS \*\* (1) 95 % DE CRITERIA \*\* .7500 X12 + .8800 X22 + .6400 X32 + .9375 X42 .9856 X52 + .8704 X62 + .9700 X72 + .9100 X82 .9568 X92 + .8300 X102 + .9711 X112 + .9575 X122 + .9796 X132 + .9388 X142 + .9200 X152 + .9936 X162 + .9936 X172 + .9864 X182 + .9904 X192 + .9712 X202 .LE. 5700 \*\* (2) 60 % DE CRITERIA \*\* .2500 X15 + .4800 X25 + .1500 X35 + .4375 X45 .7296 X55 + .2775 X65 + .6100 X75 + .3625 X85 -.5580 X95 + .3600 X105 + .5904 X115 + .5200 X125 + .6672 X135 + .4560 X145 + .5500 X155 + .7975 X165 + .6625 X175 + .7120 X185 + .7660 X195 + .6175 X205 .GE. 180 \*\* (3) TARGET 1 CONSTRAINT \*\* X11 + X21 + X31 + X41 + X51 + X61 + X71 + X81 + X91 X101 + X111 + X121 + X131 + X141 + X151 + X161 + X171 + X181 +X191 + X201 .LE. 9000 \*\* (4) TARGET 2 CONSTRAINT \*\* X12 + X22 + X32 + X42 + X52 + X62 + X72 + X82 + X92 X102 + X112 + X122 + X132 + X142 + X152 + X162 + X172 + X182 +X192 + X202 .LE. 6000 \*\* (5) TARGET 3 CONSTRAINT \*\* X13 + X23 + X33 + X43 + X53 + X63 + X73 + X83 + X93 + X103 + X113 + X123 + X133 + X143 + X153 + X163 + X173 + X183 + X193 + X203 .LE. 3000 \*\* (6) TARGET 4 CONSTRAINT \*\* X14 + X24 + X34 + X44 + X54 + X64 + X74 + X84 + X94 X104 + X114 + X124 + X134 + X144 + X154 + X164 + X174 + X184 + X194 + X204 .LE. 1500 \*\* (7) TARGET 5 CONSTRAINTS \*\* X15 + X25 + X35 + X45 + X55 + X65 + X75 + X85 + X95 X105 + X115 + X125 + X135 + X145 + X155 + X165 + X175 + X185 + X195 + X205 .LE. 300 \*\* (8) WEAPON SYSTEM 1 (M1) CONSTRAINT \*\* X11 + X12 + X13 + X14 + X15 +2 X41 + 2 X42 + 2 X43 + 2 X44 + 2 X45 + X71 + X72 + X73 + X74 + X75 +X81 + X82 + X83 + X84 + X85 + X121 + X122 + X123 + X124 + X125 + X171 + X172 + X173 + X174 + X175 .LE. 2325

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\*\* (9) WEAPON SYSTEM 2 (M2) CONSTRAINT \*\*
X101 + X102 + X103 + X104 + X105 +
2 X111 + 2 X112 + 2 X113 + 2 X114 + 2 X115 +
X121 + X122 + X123 + X124 + X125 +
X131 + X132 + X133 + X134 + X135 +
X141 + X142 + X143 + X144 + X145 +
X181 + X182 + X183 + X184 + X185 . LE. 1250

\*\* (10) WEAPON SYSTEM 3 (B1) CONSTRAINT
X21 + X22 + X23 + X24 + X25 +
2 X51 + 2 X52 + 2 X53 + 2 X54 + 2 X55 +
X71 + X72 + X73 + X74 + X75 +
X91 + X92 + X93 + X94 + X95 +
X131 + X132 + X133 + X134 + X135 +
X191 + X192 + X193 + X194 + X195 .LE. 2100

\*\* (11) WEAPON SYSTEM 4 (B2) CONSTRAINT \*\*
X151 + X152 + X153 + X154 + X155 +
2 X161 + 2 X162 + 2 X163 + 2 X164 + 2 X165 +
X171 + X172 + X173 + X174 + X175 +
X181 + X182 + X183 + X184 + X185 +
X191 + X192 + X193 + X194 + X195 +
X201 + X202 + X203 + X204 + X205 .LE. 3000

\*\* (12) WEAPON SYSTEM 5 (S1) CONSTRAINT \*\*
X31 + X32 + X33 + X34 + X35 +
2 X61 + 2 X62 + 2 X63 + 2 X64 + 2 X65 +
X81 + X82 + X83 + X84 + X85 +
X91 + X92 + X93 + X94 + X95 +
X141 + X142 + X143 + X144 + X145 +
X201 + X202 + X203 + X204 + X205 .LE. 9000

PRINT OPTIMIZE

والمخاط والمرجاني وكروان وترجر وأورا

6 FACTORS \*\*\*\* \*\*\*\*\* 

RESPONSE SURFACE FOR ARSENAL EXCHANGE MODEL

REGULAR TITLE

**       SYSTEM **         X11       TO X15       ** M1       **         X21       TO X25       ** B1       **         X31       TO X35       ** S1       **         X41       TO X45       ** M1 + M1 **       **         X51       TO X55       ** B1 + B1 **       **         X61       TO X65       ** S1 + S1 **       **	
X11       TO       X15       **       M1       **         X21       TO       X25       **       B1       **         X31       TO       X35       **       S1       **         X41       TO       X45       **       M1       +       **         X51       TO       X55       **       B1       +       **         X61       TO       X65       **       S1       +       **	
X21       TO       X25       **       B1       **         X31       TO       X35       **       S1       **         X41       TO       X45       **       M1       +       M1       **         X51       TO       X55       **       B1       **       **       Y </td <td></td>	
X31       TO       X35       ** S1       **         X41       TO       X45       ** M1 + M1 **         X51       TO       X55       ** B1 + B1 **         X61       TO       X65       ** S1 + S1 **	
X41     TO     X45     ** M1     + M1     **       X51     TO     X55     ** B1     + B1     **       X61     TO     X65     ** S1     + S1     **	
X51         TO         X55         **         B1         + B1         **           X61         TO         X65         **         S1         + S1         **	
X61 TO X65 ** S1 + S1 **	i
X71 TO X75 ** M1 + B1 **	
X81 TO X85 ** M1 + S1 **	
X91 TO X95 ** B1 + S1 **	
X101 TO X105 ** M2 **	
X111 TO X115 ** M2 + M2 **	
X121 TO X125 ** M1 + M2 **	
X131 TO X135 ** M2 + B1 **	
$x_1 x_1 x_1 x_1 x_1 x_2 x_1 x_2 x_2 x_3 x_4 x_5 x_4 x_5 x_5 x_5 x_5 x_5 x_5 x_5 x_5 x_5 x_5$	
$\frac{1}{10}$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
$\begin{cases} x & x^{2}x^{2} & x^{2}x^{2} \\ x^{2}x^{2}x^{2}x^{2}x^{2}x^{2}x^{2}x^{2}$	
$X_{221} = X_{223} = X_{233} = X_{2$	
$x_{201} = 10 x_{200}$	
$X_{251} = 10$ X255 $** B_1 + M_3 **$	
X261 TO X265 ** B2 + M3 **	
X271 TO X275 ** M3 + S1 **	
Ră	
$\Delta = 17$	
	8523

MAXIMIZE

6

KANNER DESERTE

\*\* DE TABLE \*\*

.8400	X11	+	.7500	X12	+	.5500	X13	+	.8800	X14	+	. 2500	X15 ·	+
.9600	X21	+	.8800	X22	+	.7000	X23	+	.0000	X24	+	. 4800	X25 ·	+
.7300	X31	+	.6400	X32	+	.3200	X33	+	.7800	X34	+	.1500	X35 ·	÷
.9744	X41	+	.9375	X42	+	.7975	X43	+	.9856	X44	+	.4375	X45 ·	+
.9984	X51	+	.9856	X52	+	.9100	X53	+	.0000	X54	+	.7296	X55 ·	ŧ .
.9271	X61	+	.8704	X62	+	.5376	X63	+	.9516	X64	+	.2775	X65 ·	÷
.9936	X71	+	.9700	X72	+	.8650	X73	+	.0000	X74	+	.6100	X75 ·	t
.9568	X81	+	.9100	X82	+	.6940	X83	+	.9736	X84	+	.3625	X85 ·	+
.9892	X91	+	.9568	X92	+	.7960	X93	+	.0000	X94	+	. 5580	X95 ·	+
. 9000	X1Ø1	+	.8300	X1Ø2	+	. 6200	X103	+	.9400	X1Ø4	+	.3600	X105	+
.9900	X111	+	.9711	X112	+	.8556	X113	+	.9964	X114	+	.5904	X115	+
.9840	X121	+	.9575	X122	+	.8290	X123	+	.9928	X124	+	.5200	X125	+
.9960	X131	+	.9796	X132	+	.8860	X133	+	.0000	X134	+	.6672	X135	+
.9730	X141	+	.9388	X142	+	.7416	X143	+	.9868	X144	+	.4560	X145	+
. 7800	X151	+	.9200	X152	+	.7400	X153	+	.0000	X154	+	.5500	X155	+
.9996	X161	+	.9936	X162	+	.9324	X163	+	.0000	X164	+	.7975	X165	+
.9968	X171	+	.9936	X172	+	.8830	X173	+	.0000	X174	+	.6625	X175	+
.9980	X181	+	. 9864	X182	+	.9012	X183	+	.0000	X184	+	.7120	X185	+
.9992	X191	+	. 9904	X192	+	.9220	X193	+	.0000	X194	+	.7660	X195	+
.9946	X201	+	.9712	X202	+	.8232	X203	+	.0000	X204	+	.6175	X205	+
.9200	X211	+	.8700	X212	+	. 6500	X213	+	.9500	X214	+	. 4500	X215	+
.9936	X221	+	.9831	X222	+	.8775	X223	+	.9975	X224	+	. 6975	X225	+
.9872	X231	+	.9675	X232	+	.8425	X233	+	. 9940	X234	+	.5875	X235	+
.9920	X241	+	.9779	X242	+	.8670	X243	+	.9970	X244	+	. 6480	X245	+
.9968	X251	+	. 9844	X252	+	.8750	X253	+	.0000	X254	+	.7140	X255	+
.9984	X261	+	. 7876	X262	+	. 9090	X263	+	.0000	X264	+	.7525	X265	+
.9784	X271	+	.9532	X272	+	.7620	X273	+	. 7890	X274	+	.5325	X275	
CONSTR	RAINTS	3												
** (1)	95 7	/ I	E CRIT	TERIA	÷	f								
.7500	X12	+	. 8800	X22	+	.6400	X32	+	.9375	X42	+	.9856	X52	+
.8704	X62	+	.9700	X72	+	.9100	X82	+	.9568	X92	+	.8300	X102	+
.9711	X112	+	.9575	X122	+	.9796	X132	+	.9388	X142	+	.9200	X152	+
.9936	X162	+	.9936	X172	+	.9864	X182	+	.9904	X192	+	.9712	X202	+
.8700	X212	+	.9831	X222	+	.9675	X232	+	.9779	X242	+	.9844	X252	+
.9896	X262	+	.9532	X272		.E. 570	00							
** (2)	60 7	. Γ	E CRI	TERIA	¥1	f								
.2500	X15	+	. 4800	X25	+	.1500	X35	+	.4375	X45	+	.7296	X55	+
.2775	X65	+	.6100	X75	+	.3625	X85	+	.5580	X95	+	.3600	X105	+
. 5904	X115	+	. 5200	X125	+	.6672	X135	+	. 4560	X145	+	. 5500	X155	+
.7975	X165	+	.6625	X175	+	.7120	X185	+	.7660	X195	+	.6175	X205	+
. 4500	X215	+	. 6975	X225	+	.5875	X235	+	. 6480	X245	+	.7140	X255	+
.7525	X265	+	.5325	X275	. 0	E. 180	2							
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\*\* (3) TARGET 1 CONSTRAINT \*\* X11 + X21 + X31 + X41 + X51 + X61 + X71 + X81X91 + X101 + X111 + X121 + X131 + X141 + X151 + X161 +X171 + X181 + X191 + X201 + X211 + X221 + X231 + X241 + X251 + X261 + X271 .LE. 9000 \*\* (4) TARGET 2 CONSTRAINT \*\* X12 + X22 + X32 + X42 + X52 + X62 + X72 + X82+ **X92** + X102 + X112 + X122 + X132 + X142 + X152 + X162 + X172 + X182 + X192 + X202 + X212 + X222 + X232 + X242 + X252 + X262 + X272 .LE. 6000 \*\* (5) TARGET 3 CONSTRAINT \*\* X13 + X23 + X33 + X43 + X53 + X63 + X73 + X83 + X93 + X103 + X113 + X123 + X133 + X143 + X153 + X163 + X173 + X183 + X193 + X203 + X213 + X223 + X233 + X243 + X253 + X263 + X273 .LE. 3000 \*\* (6) TARGET 4 CONSTRAINT \*\* X14 + X24 + X34 + X44 + X54 + X64 + X74 + X84 X94 + X104 + X114 + X124 + X134 + X144 + X154 + X164 + X174 + X184 + X194 + X204 + X214 + X224 + X234 + X244 +X254 + X264 + X274 .LE. 1500 **\*\* (7) TARGET 5 CONSTRAINTS \*\*** X15 + X25 + X35 + X45 + X55 + X65 + X75 + X85 X95 + X105 + X115 + X125 + X135 + X145 + X155 + X165 + X175 + X185 + X195 + X205 + X215 + X225 + X235 + X245 + X255 + X265 + X275 .LE. 300 \*\* (8) WEAPON SYSTEM 1 (M1) CONSTRAINT \*\* X11 + X12 + X13 + X14 + X15 + 2 X41 + 2 X42 + 2 X43 + 2 X44 + 2 X45 + X71 + X72 + X73 + X74 + X75 + X81 + X82 + X83 + X84 + X85 + X121 + X122 + X123 + X124 + X125 + X171 + X172 + X173 + X174 + X175 + X231 + X232 + X233 + X234 + X235 .LE. 2325 \*\* (9) WEAPON SYSTEM 2 (M2) CONSTRAINT \*\* X101 + X102 + X103 + X104 + X105 +  $2 \times 111 + 2 \times 112 + 2 \times 113 + 2 \times 114 + 2 \times 115 +$ X121 + X122 + X123 + X124 + X125 +X131 + X132 + X133 + X134 + X135 + X141 + X142 + X143 + X144 + X145 +X181 + X182 + X183 + X184 + X185 + X241 + X242 + X243 + X244 + X245 .LE. 1250

\*\* (10) WEAPON SYSTEM 3 (M3) CONSTRAINT \*\* X211 + X212 + X213 + X214 + X215 +2 X221 + 2 X222 + 2 X223 + 2 X224 + 2 X225 + X231 + X232 + X233 + X234 + X235 +X241 + X242 + X243 + X244 + X245 + X251 + X252 + X253 + X254 + X255 + X261 + X262 + X263 + X264 + X265 + X271 + X272 + X273 + X274 + X275 .LE. 700 \*\* (11) WEAPON SYSTEM 4 (B1) CONSTRAINT X21 + X22 + X23 + X24 + X25 + 2 X51 + 2 X52 + 2 X53 + 2 X54 + 2 X55 + X71 + X72 + X73 + X74 + X75 + + X92 + X93 + X94 + X95 X91 + X131 + X132 + X133 + X134 + X135 + X191 + X192 + X193 + X194 + X195 + X251 + X252 + X253 + X254 + X255 .LE. 2100 \*\* (12) WEAPON SYSTEM 5 (B2) CONSTRAINT \*\* X151 + X152 + X153 + X154 + X155 + 2 X161 + 2 X162 + 2 X163 + 2 X164 + 2 X165 + X171 + X172 + X173 + X174 + X175 + X181 + X182 + X183 + X184 + X185 + X191 + X192 + X193 + X194 + X195 + X201 + X202 + X203 + X204 + X205 + X261 + X262 + X263 + X264 + X265 .LE. 3000 \*\* (13) WEAPON SYSTEM 6 (S1) CONSTRAINT \*\* X31 + X32 + X33 + X34 + X35 +  $2 \times 61 + 2 \times 62 + 2 \times 63 + 2 \times 64 + 2 \times 65 +$ + X82 + X83 X81 + X84 + X85 + + X92 + X93 + X94 + X95 X91 X141 + X142 + X143 + X144 + X145 +X201 + X202 + X203 + X204 + X205 + X271 + X272 + X273 + X274 + X275 .LE. 9000

PRINT OPTIMIZE

# Appendix B: Experimental Designs

### TABLE B.1

### BOX-BEHNKEN EXPERIMENTAL DESIGN FOR THREE FACTORS

NON-CODED CODED

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RUN#	M1	B1	SI	M1	B1	51	RESPONSE
1	3000	3000	9000	1	1	0	10131.4892
2	3000	1200	7 <b>000</b>	1	-1	0	9242.6923
3	1650	3000	7000	-1	1	0	9671.9492
4	1650	1200	7000	-1	-1	Ø	8493.1923
5	3000	2100	12000	1	0	1	10395.4062
6	3000	2100	6000	1	0	-1	8477.3846
7	1650	2100	12000	-1	0	1	9935.8662
8	1650	2100	6000	-1	Ø	-1	7464.8846
9	2325	3000	12000	Ø	1	1	10584.9202
10	2325	3000	6000	Ø	1	-1	8757.9423
11	2325	1200	12000	0	-1	1	9724.2762
12	2325	1200	6000	Ø	-1	-1	7179.1346
13	2325	2100	9 <b>000</b>	Ø	Ø	0	9474.4362

	<u>N(</u>		D		CODED		
RUN#	M1	B1	51	M1	B1	<b>S1</b>	RESPONSE
1	2914	2886	11619	0.873	0.873	0.873	10642.0176
2	2914	2886	6381	Ø.873	0.873	-0.873	9189.1323
3	2914	1314	11619	0.873	-0.873	0.873	9892 <b>.</b> 8949
4	2914	1314	6381	0.873	-0.873	-0.873	7965.0446
5	1736	2886	11619	-0.873	0.873	0.873	10251.0156
6	1736	2886	6381	-0.873	0.873	-0.873	8464 <b>.904</b> 6
7	1736	1314	11619	-0.873	-0.873	0.873	9491 <b>.</b> 9 <b>0</b> 37
8	1736	1314	6381	-0.873	-0.873	-0.873	7081.5446
9	1334	2100	7 <b>000</b>	-1.468	Ø	0	8943.0723
10	3316	2100	9000	1.468	Ø	. Ø	9811.7725
11	2325	779	9000	Ø	-1.468	Ø	8576.7423
12	2325	3421	7000	Ø	1.468	Ø	10099.7576
13	2325	2100	4596	Ø	Ø	-1.468	7071.7846
14	2325	2100	13404	Ø	Ø	1.468	10471.3739
15	2325	2100	7000	Ø	Ø	0	9474.4362

# TABLE B.2

2332223

BOX-DRAPER EXPERIMENTAL DESIGN FOR THREE FACTORS

	NO	N-CODEI	<u>0</u>	1	CODED		
RUN#	M1	B1	<b>S</b> 1	M1	B1	51	RESPONSE
1	1650	1200	6000	-1	-1	-1	6672.8846
2	3000	1200	6000	1	-1	-1	7685.3846
3	1650	3000	6000	-1	1	-1	8256.8846
4	165 <b>0</b>	1200	12000	-1	-1	1	9494.5061
5	2457	228 <b>0</b>	6000	0.1925	0.1925	-1	8228.5346
6	2457	1200	76 <b>00</b>	0.1925	-1	0.1925	9208.0423
7	1650	2280	76 <b>00</b>	-1	0.1925	0.1925	9471.1782
8	2127	3000	12000	-0.2912	1	1	10520.1731
9	3000	1836	12000	1	-0.2912	1	10265.9406
10	3000	3000	8100	1	1	-0.2912	9924.1292

# TABLE B.3

(17)

MINIMUM POINT EXPERIMENTAL DESIGN FOR THREE FACTORS

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KOSHAL EXPERIMENTAL DESIGN FOR THREE FACTORS

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	N	ON-CODE	D		CODED		
RUN#	M1	B1	51	M1	B1	51	RESPONSE
1	1988	1650	7500	-0.5	-0.5	~0.5	8282.3846
2	2662	1650	7500	0.5	-0.5	~0.5	8695.0323
3	1988	2550	7500	-0.5	0.5	~0.5	<sup>.</sup> 8940.8523
4	1988	1650	10500	-0.5	-0.5	0.5	9484.6413
5	3337	1650	7500	1.5	-0.5	-0.5	9079.7823
6	1988	3 <b>450</b>	7500	-0.5	1.5	-0.5	957 <b>0.8</b> 523
7	1988	1650	13500	-0.5	-0.5	1.5	10175.8413
8	2662	2 <b>550</b>	7500	0.5	0.5	-0.5	9325.0323
9	2662	1650	10500	0.5	-0.5	0.5	9714.0709
10	1988	2550	10500	-0.5	0.5	0.5	9920.9244

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n		EV.	I			11		11
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				TABLE B.	5		
CEN	ITRAL C	OMPOSIT	E EXPER	IMENTAL	DESIGN	FOR THREE	FACTORS
	N	<u>ON-CODE</u>	D		CODED		
RUN#	M1	Bt	<b>S</b> 1	M1	B1	51	RESPONSE
1	3000	3000	12000	1	1	1	10792.2127
2	3000	3000	6000	1	1	-1	9142.692
3	3000	1200	12000	1	-1	1	9954 <b>.0</b> 46
4	3000	1200	6000	1	-1	-1	7685.384
5	1650	3000	12000	-1	1	1	10362.572
6	1650	3000	6000	-1	1	-1	8256.884
7	1650	1200	12000	-1	-1	1	9494.506
8	1650	1200	6000	-1	-1	-1	6672.884
9	3460	2100	7000	1.682	Ø	0	9860.790
10	1190	2100	9000	-1.682	0	0	8860.992
11	2325	3614	9000	0	1.682	. <b>0</b>	9978.864
12	2325	586	9 <b>000</b>	0	-1.682	0	8441.642
13	2325	2100	14046	0	Ø	1.682	10597.912
14	2325	2100	3954	Ø	0	-1.682	6654.484
15	2325	2100	9000	Ø	0	Ø	9474.4362
			В	- 5			

	HYBRID	(311A)	EXPERI	MENTAL	DESIGN FOR	THREE	FACTORS
	V		Ð				
RUN	(# M1	<b>B1</b>	51	M1	B1	<b>S1</b>	RESPONSE
1	2325	2100	15000	Ø	Ø	2	10785.9456
2	2325	2100	3000	Ø	Ø	-2	6034.3846
3	1370	827	12000	-1.414	-1.414	1	9204.3765
4	3280	827	12000	1.414	-1.414	1	9866 <b>. 4</b> 389
5	1370	3373	12000	-1.414	1.414	1	10438.0595
6	3280	3373	12000	1.414	1.414	1	11037.5089
7	3675	2100	6000	2	Ø	-1	8897.4423
8	975	2100	6000	-2	Ø	-1	6958.3846
9	2325	3900	6000	Ø	2	-1	9387.9423
10	2325	300	6000	Ø	-2	-1	6387.1346
11	2325	2100	9000	Ø	Ø	Ø	9474.4362

# TABLE B.6

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				TABLE B	.7		
н	YBRID (	(310+CP)	EXPERI	MENTAL 1	DESIGN FO	DR THREE	FACTORS
	Ņ	ION-CODE	D		ED		
RUN	# M1	B1	51	M1	B1	51	RESPONSE
1	2325	2100	12872	Ø	Ø	1.2906	10366.5168
2	2325	2100	8592	Ø	Ø	-0.136	932 <b>0.</b> 2623
3	1650	1200	10916	-1	-1	0.6386	9244.7525
4	3000	1200	10916	1	-1	0.6386	9704.2925
5	1650	3000	10916	-1	1	0.6386	10113.3956
6	3000	3000	10916	1	1	0.6386	10569.6987
7	3117	2100	6218	1.1736	Ø	-0.9273	8679.6623
8	1533	2100	6218	-1.1736	Ø	- <b>0.92</b> 73	7516.6546
9	2325	3156	6218	Ø	1.1736	-0.9273	8967.4223
10	2325	1044	6218	Ø	-1.1736	-0.9273	7181.3746
11	2325	2100	9000	Ø	Ø	Ø	9474.4361
			<b>F</b> .	_ 7			
			В	- /			

## TABLE B.8

### FULL FACTORIAL EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON				CODED			
RUN#	M1	B1	<b>S</b> 1	M1	B1	<b>S</b> 1	RESPONSE	
1	3000	3000	12000	1	1	1	10792.2129	
2	3000	3000	7000	1	1	Ø	10131.4893	
3	3000	3000	6000	1	1	-1	9142.6924	
4	3000	2100	12000	1	Ø	1	10395.4063	
5	3000	2100	9 <b>000</b>	1	Ø	Ø	9704.2061	
6	3000	2100	6000	1	Ø	-1	8477.3848	
7	3000	1200	12000	1	-1	1	9954.0459	
8	3000	1200	9 <b>000</b>	1	-1	Ø	9242.6924	
9	3000	1200	6000	1	-1	-1	7685.3848	
10	232 <b>5</b>	3000	12000	Ø	1	1	10534.9199	
11	2325	3000	9000	Ø	1	Ø	99 <b>01.</b> 7188	
12	2325	3000	6000	Ø	1	-1	8757.9424	
13	2325	2100	12000	Ø	Ø	1	10165.6357	
14	2325	2100	9000	Ø	Ø	Ø	9474.4365	
15	2325	2100	6000	Ø	Ø	-1	7971.1348	
16	2325	1200	12000	Ø	-1	1	9724.2764	
17	2325	1200	9 <b>000</b>	Ø	-1	Ø	8871.4424	
18	2325	1200	6000	Ø	-1	-1	7179.1348	

## TABLE B.8 (continued)

### FULL FACTORIAL EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON	<u>-CODED</u>			CODED			
RUN#	M1	B1	<b>S</b> 1	M1	B1	S1	RESPONSE	
19	1650	3000	12000	-1	1	1	10362.5723	
20	1650	3000	9000	-1	1	Ø	9671.9492	
21	1650	3000	6000	-1	1	-1	8256.8848	
22	1650	2100	12000	-1	Ø	1	9935.8662	
23	1650	2100	9000	-1	Ø	0	9123.1924	
24	1650	2100	6000	-1	Ø	-1	7464.8848	
25	1650	1200	12000	-1	-1	1	9494,5059	
26	1650	1200	9000	-1	-1	Ø	8493.1924	
27	1650	1200	6000	-1	-1	-1	6672.8848	

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# TABLE B.9

# BOX-BEHNKEN EXPERIMENTAL DESIGN FOR FOUR FACTORS

NON-CODED

# CODED

RUN#	F M1	M2	B1	<b>S1</b>	M1	M2	B1	51	RESPONSE
1	3000	2000	2100	9000	1	1	Ø	Ø	10530.8908
2	3000	500	2100	7000	1	-1	Ø	0	9914.4062
3	1650	2000	2100	9000	-1	1	Ø	0	10085.4662
4	1650	500	2100	9000	-1	-1	Ø	Ø	9445.1923
5	2325	1250	3000	12000	Ø	Ø	1	1	11043.7952
6	2325	1250	3000	6000	Ø	Ø	1	-1	9 <b>570.4</b> 423
7	2325	1250	1200	12000	Ø	Ø	-1	1	10242.4877
8	2325	1250	1200	6000	Ø	0	-1	-1	8216.6346
9	3000	1250	2100	12000	1	Ø	Ø	1	10862.7746
10	3000	1250	2100	6000	1	0	0	-1	9325.1923
11	1650	1250	2100	12000	-1	Ø	Ø	1	10444.2508
12	1650	1250	2100	6000	-1	Ø	Ø	-1	8502.3846
13	2325	2000	3000	9000	Ø	1	1	0	10714.9038
14	2325	2000	1200	7000	0	1	-1	0	9873.8762
15	2325	500	3000	7000	Ø	-1	1	0	10111.9192
16	2325	500	1200	9000	Ø	-1	-1	Ø	9186.4423
17	3000	1250	3000	9000	1	Ø	1	0	10645.8738
18	3000	125 <b>0</b>	1200	7000	1	Ø	-1	0	9788.3462
19	1650	1250	3000	7000	-1	0	1	0	10197.4492
20	1650	1250	1200	9000	-1	Ø	-1	Ø	9287.6923
21	2325	2000	2100	12000	Ø	1	Ø	1	10930.8071
22	2325	2000	2100	6000	0	1	0	-1	9427.9423
23	2325	500	2100	12000	Ø	-1	Ø	1	10374.5477
24	2325	500	2100	6000	Ø	-1	Ø	-1	8386.1346
25	2325	1250	2100	9000	Ø	Ø	0	Ø	9999.9361

## TABLE B.10

## BOX-DRAPER EXPERIMENTAL DESIGN FOR FOUR FACTORS

# NON-CODED

# CODED

RUN#	M1	M2	B1	51	M1	M2	B1	<b>S1</b>	RESPONSE
1	2781	1756	2707	11025	.675	.675	.675	.675	11052.2726
2	2781	1756	2707	6975	.675	.675	.675	675	10190.7768
3	2781	1756	1493	11025	.675	.675	675	.675	10514.5027
4	2781	1756	1493	6975	.675	.675	675	675	9542.6223
5	2781	744	2707	11025	.675	675	.675	.675	10680.2466
6	2781	744	2707	6975	.675	675	.675	675	9744.8623
7	2781	744	1493	11025	.675	675	675	.675	10111.3233
8	2781	744	1493	6975	.675	675	675	675	8895.0623
9	1869	1756	2707	11025	675	.675	.675	.675	10772.1974
10	1869	1756	2707	6975	675	.675	.675	675	9880.3320
11	1869	1756	1493	11025	675	.675	675	.675	10220.2949
12	1869	1756	1493	6975	675	.675	675	675	9033.0223
13	1869	744	2707	11025	675	675	.675	.675	10385.9703
14	1869	744	2707	6975	675	675	.675	675	9225.0223
15	1869	744	1493	11025	675	675	675	.675	9 <b>800.</b> 8786
16	1869	744	1493	6975	675	675	675	675	8336.4946
17	1414	1250	2100	9 <b>000</b>	-1.349	7 Ø	Ø	Ø	9689.8318
18	3236	1250	2100	7000	1.349	7 0	Ø	0	10310.0405
19	2325	238	2100	7000	Ø	-1.34	49 Ø	Ø	9574.4913
20	2325	2262	2100	9000	0	1.34	49 0	Ø	10414.2856
21	2325	1250	886	7000	Ø	0	-1.349	7 Ø	9404.5905
22	2325	1250	3314	9000	0	Ø	1.349	7 Ø	10566.3179
23	2325	1250	2100	4953	Ø	0	0-	-1.349	8338.5546
24	2325	1250	2100	13047	Ø	Ø	0	1.349	10861.8458
25	2325	1250	2100	9000	Ø	Ø	Ø	Ø	9999.9361

E CARLES CONTRACTOR CONTRACTOR CONTRACTOR CONTRACTOR CONTRACTOR CONTRACTOR CONTRACTOR CONTRACTOR CONTRACTOR CON
### MINIMUM POINT EXPERIMENTAL DESIGN FOR FOUR FACTORS

NON-CODED

CODED

RUN#	M1	M2	B1	<b>S1</b>	M1	M2	B1	<b>S1</b>	RESPONSE
1	1650	500	1200	6000	-1	-1	-1	-1	7087.8846
2	3000	500	1200	6000	1	-1	-1	-1	81 <b>00.</b> 3846
3	1650	2000	1200	6000	-1	1	-1	-1	8332 <b>.884</b> 6
4	1650	500	3000	6000	-1	-1	1	-1	8671.8846
5	1650	500	1200	12000	-1	-1	-1	1	9704.7061
6	2604	1560	1200	6000	.4114	.4114	-1	-1	869 <b>0.</b> 9723
7	2604	500	2472	6000	.4114	-1	.4114	-1	8872.3723
8	2604	500	1200	10500	.4114	-1	-1	.4114	9683.8477
9	1650	1560	2472	6000	-1	.4114	.4114	-1	9017.5923
10	1650	1560	1200	10500	-1	.4114	-1	.4114	9804.7301
11	1650	500	2472	10500	-1	-1	.4114	.4114	9979.3780
12	1886	2000	3000	12000	650	Z 1	1	1	11181.9322
13	3000	76Ø	3000	12000	1 ·	6502	1	1	11071.2087
14	3000	2000	1512	12000	1	1	6502	1	10875.2048
15	3000	2000	3000	69 <b>00</b>	1	1	1	6502	10485.3723

- 3

KOSHAL EXPERIMENTAL DESIGN FOR FOUR FACTORS

NON-CODED

CODED

RUN#	M1	M2	B1	<b>S1</b>	M1	M2	B1	<b>S</b> 1	RESPONSE
1	2055	95 <b>0</b>	1740	7800	4	4	4	4	9167.5423
2	<b>2730</b>	950	17 <b>40</b>	7800	.6	4	4	4	9542.6923
3	2055	1700	1740	7800	4	.6	4	4	9643.9423
4	2055	<b>950</b>	2640	7800	4	4	.6	4	9763.3672
5	2055	95 <b>0</b>	1740	10800	4	4	4	.6	10020.0841
6	3405	95 <b>0</b>	1740	7800	1.6	4	4	4	9788.4241
7	2055	2450	1740	7800	4	1.6	4	4	9959.4841
8	2055	950	3540	78 <b>00</b>	4	4	1.6	4	10186.7272
9	2055	95 <b>0</b>	1740	13800	4	4	4	1.6	10656.2591
10	273Ø	1700	1740	78 <b>00</b>	.6	.6	4	4	9873.9541
11	273 <b>0</b>	95 <b>0</b>	2640	7800	.6	4	.6	4	9993.1372
12	273 <b>0</b>	950	1740	10800	.6	4	4	.6	10249.8541
13	2055	1700	2640	78 <b>00</b>	4	.6	.6	4	10078.6672
14	2055	1700	1740	10800	4	.6	4	.6	10326.9688
15	2055	95 <b>0</b>	2640	10800	4	4	.6	.6	10449.1518

## CENTRAL COMPOSITE EXPERIMENTAL DESIGN FOR FOUR FACTORS

# NON-CODED

# CODED

RUN	# M1	M2	B1	S1	M1	M2	B1	51	RESPONSE
1	3000	2000	3000	12000	1	1	1	1	11453.3481
2	3000	2000	3000	6000	1	1	1	-1	10281.0892
3	3000	2000	1200	12000	1	-	-1	1	10735.7096
4	3000	2000	1200	6000	1	1	-1	-1	9182.6923
5	3000	500	3000	12000	1	-1	1	1	10975.7627
6	3000	500	3000	6000	1	-1	1	-1	9467.6923
7	3000	500	1200	12000	1	-1	-1	1	10164.2461
8	3000	500	1200	6000	1	-1	-1	-1	8100.3846
9	1650	2000	3000	12000	-1	1	1	1	11111.8277
10	1650	2000	3000	6000	-1	1	1	-1	9673.1923
11	1650	2000	1200	12000	-1	1	-1	1	10315.1908
12	1650	2000	1200	6000	-1	1	-1	-1	8332.8846
13	1650	500	3000	12000	-1	-1	1	1	10559.2338
14	1650	500	3000	6000	-1	-1	1	-1	8671.8846
15	1650	500	1200	12000	~1	-1	-1	1	9704.7061
16	1650	500	1200	6000	-1	-1	-1	-1	7087.8846
17	975	1250	2100	9000	-2	Ø	Ø	Ø	9 <b>540.</b> 3961
18	3675	1250	2100	9000	2	Ø	Ø	Ø	10459.4762
19	2325	Ø	2100	9000	Ø	-2	Ø	0	9474.4362
20	2325	2750	2100	7000	Ø	2	Ø	Ø	10599.9208
21	2325	1250	300	9000	Ø	Ø	-2	Ø	9028.9423
22	2325	1250	3900	7000	0	Ø	2	0	10825.9638
23	2325	1250	2100	3000	Ø	Ø	Ø	-2	7071.8846
24	2325	1250	2100	15000	Ø	Ø	Ø	2	11239.2052
25	2325	1250	2100	7000	Ø	Ø	Ø	Ø	9999.9361

# HYBRID (416A+CP) EXPERIMENTAL DESIGN FOR FOUR FACTORS

NON-CODED

Research

CODED

RUN#	M1	M2	B1	51	M1	M2	B1	51	RESPONSE
1	2325	1250	2100	14353	Ø	Ø	Ø	1.7844	11118.1515
2	2325	1250	2100	4517	Ø	Ø	0 -	-1.4945	8057.9946
3	1650	500	1200	10932	-1	-1	-1	.644	9458.6389
4	3000	500	1200	10932	1	-1	-1	.644	9918.1789
5	1650	2000	1200	10932	-1	1	-1	.644	10084.3805
6	3000	2000	1200	10932	1	1	-1	.644	10523.0236
7	1650	500	3000	10932	-1	-1	1	.644	10327.2820
8	3000	500	3000	10932	1	-1	1	.644	10765.2599
9	1650	2000	3000	10932	-1	at • -	1	.644	10901.3249
1Ø	3000	2000	3000	10932	1	1	1	.644	11290.6917
11	3463	1250	2100	6278	1.6853	Ø	0	9075	9716.9823
12	1187	1250	2100	6278	-1.6853	Ø	Ø	9075	8333.0546
13	2325	2514	2100	6278	Ø	1.6853	Ø	9075	9887.5823
14	2325	Ø	2100	6278	0 -	1.6853	Ø	9075	8149.0546
15	2325	1250	3617	6278	Ø	0 1.	6853	9075	10090.3072
16	2325	1250	583	6278	Ø	0-1.0	6853	9075	7851.5946
17	2325	1250	2100	9000	Ø	Ø	Ø	Ø	9999.9361

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	1		1060				2		
RUN#	M1	M2	B1	<b>S</b> 1	M1	M2	B1	<b>S1</b>	RESPONSE
1	2325	1250	2100	14296	Ø	Ø	0	1.7654	11107.4868
2	2325	1250	2100	9000	0	Ø	0	0	9999.9361
3	1650	500	1200	10702	-1	-1	-1	.5675	9405.6469
4	3000	500	1200	10702	1	-1	-1	.5675	9865.1869
5	1650	2000	1200	10702	-1	1	-1	.5675	10033.6885
6	3000	2000	1200	10702	1	1	-1	.5675	10474.6316
7	1650	500	3000	10702	-1	-1	1	.5675	10274.2900
8	3000	500	3000	10702	1	-1	1	.5675	10718.6746
9	1650	2000	3000	10702	-1	1	1	.5675	10855.9919
10	3000	2000	3000	10702	1	1	1	.5675	11255.6627
11	3317	1250	2100	5847	1.4697	0	0	-1.0509	943 <b>5.502</b> 3
12	1383	1250	2100	5847	-1.4697	0	0	-1.0509	8166.7146
13	2325	2352	2100	5847	<b>Ø</b> 1	. 4697	Ø	-1.0509	9586.3623
14	2325	148	2100	5847	0 -1	. 4697	Ø	-1.0509	7996.0546
15	2325	1250	3423	5847	Ø	Ø 1.	4697	-1.0504	9796.1623
16	2325	1250	777	5847	0	0 -1.	4697	-1.0504	7746.4746
				В	- 16				
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# BOX-BEHNKEN EXPERIMENTAL DESIGN FOR FIVE FACTORS

			NON-CI	ODED				CODED			
RUN 🛊	HI	Ħ2	B1	B2	<b>S</b> 1	H1	H2	<b>B</b> 1	B2	<b>S</b> 1	RESPONSE
1	3888	2 <b>990</b>	2100	3 <b>966</b>	9 <b>000</b>	1	1	8	8		14743.1514
2	3068	500	2188	3880	9 <b>888</b>	1	-1	8	•		13768.1514
2	1658	2 <b>986</b>	2100	3000	9 <b>000</b>	-i	i	8	8	8	13973.6514
4	1650	580	21	3860	9 <b>888</b>	-1	-1	0	8	8	12888.7422
5	2325	1258	3808	4086	9 <b>888</b>	0	8	1	1	8	15248.9814
6	2325	1258	3600	2888	9 <b>898</b>		8	1	-1	8	13768.9814
7	2325	1258	1285	4886	9 <b>865</b>	8	8	-1	1	8	13988.9814
8	2325	1259	1299	2000	9 <b>990</b>	0	•	-1	-1	8	12385.4922
9	2325	2000	2188	3000	12000		1	0	ŧ	1	15379.2168
18	2325	2006	2198	3886	6 <b>995</b>		1	8	8	-i	12697.4922
11	2325	588	2188	3888	12888	0	-1	8	8	1	14748.6172
12	2325	500	2188	3888	6 <b>868</b>	8	-1	9	8	-i	11452.4922
13	3020	1250	3 <b>888</b>	3888	9 <b>888</b>	1	0	1	8	0	14885.6514
14	3000	1250	1298	3888	9 <b>888</b>	1		-1	8	8	13625.6514
15	1659	1258	3680	3860	9 <b>888</b>	-1	8	1	8	0	14116.1514
16	1658	1250	1290	3000	9 <b>866</b>	-1	0	-1	6	1	12719.2422
17	2325	1259	2188	4880	12680		8	. 0	1	1	15574.3164
18	2325	1250	2180	4888	6 <b>888</b>	9	8	9	1	-1	12994.9922

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# TABLE B.16 (continued)

			NON-CI	DED				CODED			
RUN #	M1	H2	B1	B2	<b>S</b> 1	MI	H2	<b>B1</b>	B2	<b>S</b> 1	RESPONSE
19	2325	1250	2168	2000	12886	9	8	0	-1	1	14518.9814
28	2325	1258	2188	2888	6 <b>888</b>	ŧ	8	8	-1	-1	11154.9922
21	2325	2886	3888	3 <b>888</b>	9 <b>868</b>	9	1	1	8	8	14988.4814
22	2325	2 <b>886</b>	1286	3 <b>886</b>	9 <b>988</b>	0	1	-1	6		13728.4014
23	2325	586	3008	38 <b>88</b>	9 <b>868</b>	•	-1	1	8	•	14813.4814
24	2325	588	1288	3868	9 <b>860</b>	8	-i	-1	8		12682.9922
25	3 <b>888</b>	1250	2188	4 <b>988</b>	9 <b>000</b>	1	9	9	1	8	14995.6514
26	3000	1259	2100	2999	9 <b>888</b>	1	8	8	-1	8	13515.6514
27	1658	1258	2100	4886	9 <b>880</b>	-1		8	1	8	14226.1514
28	1658	1258	2188	2995	9 <b>960</b>	-1	U	8	-1	•	12591.2422
29	2325	1258	3888	3 <b>888</b>	1 <b>2000</b>	1	0	1	•	1	15487.2764
39	2325	1259	3888	3888	6 <b>888</b>	8	8	1	8	-1	12867.7422
31	2325	1250	1298	3860	12888	8	0	-1	8	1	14628.9814
32	2325	1258	1285	3680	6888	0	8	-1	0	-i	11273.9922
33	3996	1250	2198	3888	12868	1	8	8	ß	1	15293.6865
34	3899	1258	2188	3868	6 <b>886</b>	. 1	8	8	8	-1	12588.7422
35	1658	1250	2188	3888	12808	-1	Û	0	8	1	14834.1465
36	1658	1258	2198	3888	6 <b>888</b>	-1	8	9	8	-1	11555.2422
37	2325	2000	2100	4586	9 <b>888</b>	8	1	ð	1	0	15898.4814
38	2 <b>325</b>	2986	2100	2990	9 <b>888</b>	Û	1	0	-1	0	13618.4014
39	2325	588	2198	4898	9 <b>888</b>	9	-1	8	1	0	14123.4014
40	2325	586	2189	2006	9 <b>000</b>		-1	•	-1	0	12474.9922
41	2325	1250	2188	3888	9 <b>068</b>	•	0	1			13878.9014

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# BOX-DRAPER EXPERIMENTAL DESIGN FOR FIVE FACTORS

			NON-CI	DDED				CODED			
RUN #	M1	Ħ2	B1	<b>B2</b>	<b>S1</b>	HI	M2	<b>B</b> 1	<b>B</b> 2	<b>S1</b>	RESPONSE
1	2688	1564	2477	3419	18257	.419	.419	.419	.419	. 419	15281.8667
2	2688	1564	2477	3419	7743	.419	.419	.419	.419	419	1423 <b>8.</b> 5615
2	2688	1564	2477	2581	18257	.419	.419	. 419	419	.419	14768.3715
4	2688	1564	2477	2581	7743	.419	.419	.419	419	419	1361 <b>8. 44</b> 15
5	2688	1564	1723	3419	10257	.419	.419	419	.419	.419	14868.6915
6	2688	1564	1723	3419	7743	.419	.419	419	.419	419	13782.7615
7	2688	1564	1723	2581	18257	.419	.419	419	419	.419	14248.5715
8	2688	1564	1723	2581	7743	.419	. 419	419	419	419	12967.1524
9	26 <b>8</b> 8	936	2477	3419	18257	.419	419	.419	.419	. 419	14988.2915
18	2688	936	2477	3419	7743	.419	419	.419	. 419	419	13822.3615
11	2688	936	2477	2581	10257	.419	419	.419	419	.419	14368.1715
12	2609	936	2477	2581	7743	.419	419	.419	419	419	13189.4324
13	2688	936	1723	3419	10257	.419	419	419	.419	.419	14452.4915
14	2688	936	1723	3419	7743	. 419	419	419	.419	419	13216.8724
15	2688	936	1723	2581	18257	. 419	-,419	419	-,419	.419	13832.3715
16	2698	936	1723	2581	7743	. 419	419	419	-,419	419	12445.9124
17	2842	1564	2477	3419	19257	-,419	.419	.419	.419	.419	15865.8715
18	2842	1564	2477	3419	7743	419	.419	.419	.419	419	13982.2815
19	2842	1564	2477	2581	18257	419	. 419	. 419	-, 419	.419	14445.7515
28	7847	1544	7477	25R1	7743	- 410	410	410	- 419	- 419	13288.5124

# TABLE B.17 (continued)

			NON-CO	DED				CODED			
RUN #	MI	H2	B1	<b>B2</b>	51	H1	H2	B1	B2	<b>S</b> 1	RESPONSE
21	2842	1564	1723	3419	10257	419	.419	419	.419	.419	14538 <b>.8</b> 715
22	2842	1564	1723	3419	7743	419	.419	419	.419	419	13387.9524
23	2842	1564	1723	2581	18257	419	.419	419	419	.419	13917.9515
24	2842	1564	1723	2581	7743	419	.419	419	419	419	12356.9924
25	2842	936	2477	3419	10257	419	419	. 419	.419	.419	14657.6715
26	2842	936	2477	3419	7743	419	419	. 419	.419	419	13458.2324
27	2842	936	2477	2581	18257	419	419	. 419	419	.419	14037.5515
28	2842	936	2477	2581	7743	419	419	. 419	419	419	12679.2724
29	2842	936	1723	3419	18257	419	419	419	.419	.419	14129.8715
30	2842	936	1723	3419	7743	419	419	-,419	.419	419	12786.7124
31	2842	936	1723	2581	18257	-,419	419	419	419	.419	13589.7515
32	2842	936	1723	2581	7743	-,419	-,419	-,419	419	419	12015.7524
33	2998	1259	2188	3888	9 <b>880</b>	. 997	8	0	ŧ	8	14254.5115
34	1652	1258	2188	3888	9 <b>888</b>	-,997	8	Û	I	8	13487.2915
35	2325	1998	2188	3000	9 <b>988</b>	8	.997	9	8	ł	14357.1015
36	2325	592	2100	3000	9 <b>886</b>	9	997	9	0	8	13384.7015
37	23 <b>25</b>	1250	2997	3 <b>888</b>	9 <b>886</b>	8	•	.997	0	1	14498.8015
38	23 <b>25</b>	1250	1283	3888	7 <b>888</b>	8		997	0	•	13228.1324
39	2325	1 <b>259</b>	2199	3997	9 <b>888</b>	8	6	0	.997	0	14688.6815
40	2325	1258	2198	2003	9 <b>888</b>	0	9	0	997	Î	131 <b>68.</b> 2524
41	2325	1 <b>250</b>	2100	3 <b>896</b>	11991	•	9	0		.997	15861.8431
42	2325	1250	2188	3880	6 <b>88</b> 9	8	•			997	12880.9324
43	2325	1258	2100	3888	9988				ŧ		13878.9015

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# BOX-DRAPER (HALF-REPLICATE) EXPERIMENTAL DESIGN FOR FIVE FACTORS

			NON-C	ODED				CODED			
RUN #	H1	H2	<b>B</b> 1	<b>B2</b>	<b>S</b> 1	H1	Ħ2	<b>B</b> 1	<b>B</b> 2	S1	RESPONSE
1	2 <b>8</b> 59	955	1745	2686	7818	394	394	394	394	394	12135.5524
2	2591	1545	1745	2696	7818	. 394	. 394	394	394	394	13829.5724
3	2591	955	2455	26 <b>8</b> 6	7818	. 394	394	, 394	394	394	13164.6724
4	2859	1545	2455	2686	7818	394	. 394	. 394	394	394	13258.8524
5	2591	955	1745	3394	7818	. 394	394	-, 394	. 394	394	13264.8324
6	2859	1545	1745	3394	7818	394	. 394	-, 394	.394	-, 394	13350.2124
7	2859	955	2455	3394	7818	394	394	. 394	. 394	-, 394	13485.3124
8	2591	1545	2455	3394	7818	. 394	. 394	. 394	.394	394	14289.7815
9	2591	955	1745	2686	10182	. 394	394	394	394	. 394	13834.4315
18	2859	1545	1745	2686	10182	-, 394	. 394	-, 394	394	. 394	13914.6915
11	2 <b>85</b> 9	955	2455	2686	19182	394	394	. 394	394	. 394	14828.1915
12	2591	1545	2455	26 <b>86</b>	18182	. 394	.394	. 394	-, 394	. 394	14714.9315
13	2957	955	1745	3394	18182	394	394	394	. 394	. 394	14114.3115
14	2591	1545	1745	3394	10182	. 394	. 394	394	.394	. 394	14801.0515
15	2591	955	2455	3394	10182	. 394	394	. 394	. 394	. 394	14914.5515
16	2859	1545	2455	3394	10182	394	. 394	. 394	. 394	. 394	14994.8115

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			NON-CO	DED		CODED					
RUN #	H1	N2	B1	B2	<b>S1</b>	H1	H2	B1	B2	Si	RESPONSE
17	2 <b>95</b> 7	1258	2188	3000	9 <b>865</b>	.937	0	8	8	8	14231.1415
18	1693	1258	2188	3 <b>880</b>	9 <b>868</b>	937	9	0	0	8	13518.6615
19	2325	1953	2188	3868	9 <b>885</b>	8	.937		8		14327.8515
20	2325	547	2188	3866	9 <b>866</b>	•	937	0	5	8	13413.9515
21	2325	1258	2943	3888	9 <b>880</b>	8	8	.937	8	8	14461.0015
22	2325	1258	1257	3000	9 <b>869</b>	8		937		0	13275.6524
23	2325	1258	2190	3937	9 <b>888</b>	8	8	8	.937	8	14564.2815
24	2325	1258	2188	2863	7 <b>888</b>	0	8	0	937	8	13155.4524
25	2325	1258	2188	3 <b>880</b>	11811		8	•	8	.937	15828.3718
26	2325	1250	2188	3888	6189			8	8	937	12198.5924
27	2325	1258	2188	3 <b>900</b>	9 <b>888</b>				I		13878.9815

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#### TABLE B.18 (continued)

# MINIMUM POINT EXPERIMENTAL DESIGN FOR FIVE FACTORS

			NON-C	ODED				CODED			
RUN #	<b>H1</b>	H2	<b>B</b> 1	B2	<b>S</b> 1	Mi	Ħ2	<b>B</b> 1	<b>B2</b>	SI	RESPONSE
1	1658	588	1209	2000	6 <b>888</b>	-1	-1	-1	-1	-1	92 <b>98.</b> 2424
2	3 <b>860</b>	500	1288	2888	6 <b>868</b>	1	-1	-1	-1	-1	18251.2424
3	1650	2868	1280	2000	6 <b>888</b>	-1	1	-1	-1	-1	18456.7424
4	1650	500	3000	2006	6 <b>988</b>	-1	-1	1	-1	-1	10812.2424
5	1650	500	1288	4088	6000	-1	-1	-i	1	-1	11648.2424
6	1658	500	1288	2000	12868	-1	-1	-1	-i	1	13888.6515
7	1778	642	1288	2886	6 <b>996</b>	8188	- <b>.</b> 81 <b>8</b> 8	-i	-1	-1	9419.3624
8	1778	500	1378	2000	6888	8188	-1	8108	-1	-1	9453 <b>.88</b> 24
9	1778	588	1200	21 <b>89</b>	6 <b>888</b>	8188	-1	-1	8188	-1	9473.9624
19	1778	586	1280	2006	6568	8188	-1	-1	-1	81 <b>8</b> 8	9688.6424
11	1658	642	1370	2899	6 <b>888</b>	-1	8108	8108	-1	-1	9472.5224
12	1658	642	1288	2189	6 <b>890</b>	-1	8108	-1	8188	-1	9493.4824
13	1658	642	1286	2000	6568	-1	8108	-1	-1	8108	97 <b>89. 8</b> 824
14	1658	566	1378	2189	6 <b>899</b>	-1	-1	8108	8108	-i	9527.1224
15	1658	500	1378	2888	6568	-1	-1	8108	-1	8108	9733.8 <b>824</b>
16	1658	500	1288	2189	6568	-1	-1	-1	81 <b>08</b>	8188	9754.6824
17	2686	2000	3 <b>860</b>	4886	12000	. 5355	1	1	1	1	16396.5653
18	3008	1652	3 <b>880</b>	4886	12 <b>888</b>	1	. 5355	1	i	1	16364.7917
19	3888	2000	2582	4868	12988	1	1	.5355	1	1	16313.8837
28	3060	2989	3080	3535	12888	1	1	1	. 5355	1	16276.9228
21	3800	2868	3866	4886	18686	1	1	1	1	.5355	16197.7212

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# KOSHAL EXPERIMENTAL DESIGN FOR FIVE FACTORS

			NON-C	ODED				CODED			
RUN 🛊	M1	H2	<b>B1</b>	<b>B</b> 2	<b>S</b> 1	MI	H2	<b>B</b> 1	B2	<b>S</b> 1	RESPONSE
1	2188	1000	1886	2667	8 <b>866</b>	33	33	33	33	33	12426.8824
2	2775	1888	1800	2667	8000	.67	33	33	33	33	12937.1324
3	2189	1750	1888	2667	8888	33	.67	33	33	33	13849.3824
4	2180	1869	2708	2667	8868	33	33	.67	33	33	13218.8824
5	2188	1980	1888	3667	8888	33	33	33	.67	33	13346.8824
6	2108	1980	1800	2667	11886	33	33	33	33	.67	14843.7315
7	3 <b>458</b>	1966	1896	2667	8008	1.67	33	33	33	33	13433.2315
8	2166	2588	1888	2667	8888	33	1.67	33	33	33	13634.7315
9	2188	1886	3686	2667	8888	33	33	1.67	33	33	13919.7315
18	2190	1888	1999	4667	8 <b>868</b>	33	33	33	1.67	33	14139.7315
11	2188	1966	1888	2667	14000	33	33	33	33	1.67	15831.9435
12	2775	1750	1886	2667	8888	.67	.67	33	33	33	13535.9815
13	2775	1886	2799	2667	8000	.67	33	.67	33	33	13678.4815
14	2775	1888	1800	3667	8000	.67	33	33	.67	33	13788.4815
15	2775	1888	1890	2667	11000	.67	33	33	33	.67	14428.4815
16	2100	1750	2788	2667	8 <b>868</b>	33	.67	.67	33	33	13777.2315
17	2198	1750	1888	3667	8 <b>980</b>	33	. 67	33	.67	33	13887.2315
19	2188	1750	1899	2667	11898	33	.67	33	33	. 67	14531.2315
19	2188	1966	2766	3667	8 <b>986</b>	33	33	. 67	.67	33	14829.7315
20	2188	1969	2788	2667	11000	33	33	.67	33	.67	14673.7315
21	2188	1999	1886	3667	11000	33	33	33	.67	.67	14783.7315

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			NON-C	DDED				CODED			
RUN #	H1	H2	<b>B1</b>	B2	<b>S</b> 1	M1	<b>H2</b>	B1	B2	. <b>S1</b>	RESPONSE
1	1659	580	1288	2000	6 <b>885</b>	-1	-1	-1	-1	-i	92 <b>88.</b> 2424
2	3 <b>888</b>	2000	1286	2888	6999	1	1	-1	-1	-1	11496.2424
3	3 <b>660</b>	588	3888	2000	6898	1	-1	1	-1	-1	11838.2424
4	1650	2000	3888	2000	6 <b>890</b>	-1	i	1	-1	-1	12857.2424
5	3 <b>999</b>	588	1299	4000	6 <b>890</b>	1	-1	-1	1	-1	12091.2424
6	1658	2000	1288	4000	6 <b>888</b>	-1	1	-1	1	-1	12296.7424
7	1658	588	3968	4988	6888	-1	-1	1	1	-1	12652.2424
8	3 <b>968</b>	2886	3888	4888	6 <b>888</b>	1	1	1	1	-1	14718.1515
9	3000	588	1288	2000	12000	1	-1	-1	-1	1	13778.1515
10	1658	2000	1288	2888	12000	-1	1	-1	-1	1	13983.6515
11	1659	500	3888	2980	12888	-1	-1	1	-1	1	14268.6515
12	3888	2000	3 <b>886</b>	2000	12888	1	1	1	-1	1	15521.9467
13	1650	500	1286	4986	12999	-1	-1	-i	1	1	14488.6515
14	3000	2999	1288	4980	12889	1	1	-1	1	1	15694.2388
15	3 <b>884</b>	500	3888	4888	12 <b>000</b>	1	-1	1	1	1	15984.3588
14	1488	7888	7888	1088	17088	_1					14050 0100

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TABLE B.21

# CENTRAL COMPOSITE ROTATABLE (HALF-REPLICATE) EXPERIMENTAL DESIGN FOR FIVE FACTORS

TABLE B.21 (continued)

			NON-CO	DED				CODED			
RUN #	H1	M2	<b>B</b> 1	B2	<b>S</b> 1	N1	M2	B1	B2	<b>S</b> 1	RESPONSE
17	3675	1250	2186	3 <b>888</b>	9 <b>886</b>	2		9	8	8	14648.4815
18	975	1258	2188	38 <b>86</b>	9 <b>868</b>	-2	8		8		12999.7424
19	23 <b>25</b>	2758	2198	3886	9000		2	0	•	8	14845.5J15
28	2325		2198	3860	9 <b>888</b>	8	-2	8	8	8	12979.9924
21	2325	1258	3980	3000	9 <b>888</b>	8	8	2	8	8	1513 <b>8.901</b> 5
22	23 <b>25</b>	1250	388	3000	9 <b>866</b>	8	8	-2	0	0	12433.4924
23	2325	1258	2190	5686	9 <b>000</b>	8	0	0	2	8	15350.9015
24	2325	1258	2188	1888	9 <b>866</b>	8	Ĵ	2	-2		12177.4924
25	2325	1258	2196	3886	15886		8	0	8	2	15747.8788
26	2325	1250	2196	3000	3000	8	0	8	8	-2	18876.7424
27	2325	1259	2198	3988	9 <b>986</b>		8	0	8	8	13878.9815

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# BOX-BEHNKEN EXPERIMENTAL DESIGN FOR SIX FACTORS

			NON-CO	DED					CO	DED			
RUN #	M1	N2	N3	<b>B</b> 1	<b>B</b> 2	Si	M1	H2	M3	B1	B2	<b>S</b> 1	RESPONSE
1	2325	2 <b>888</b>	788	2188	4888	12888	8	1	8	8	1	1	16271.1880
2	2325	2000	720	2106	4688	6868	8	1	8	8	1	-1	14283.5682
3	2325	2000	788	2166	2200	12888	1	1		8	-1	1	15246.5586
4	2325	2 <b>268</b>	788	2108	2998	6888	1	1		9	-1	-1	12417.1591
5	2325	508	788	2100	4885	12888	0	-1	8	9	1	1	15644.7588
6	2325	586	720	2100	4900	6 <b>885</b>	t	-1		8	1	-1	11172.1591
7	2325	588	786	2166	2998	12888	8	-1		8	-1	1	14531.0682
8	2325	588	766	2100	2988	6888	0	-1	8	9	-1	-1	13812.1591
9	3888	1258	1882	2198	3088	12908	1	0	1	0	8	1	15817.5486
18	3866	1258	1888	2188	3085	6 <b>880</b>	1	8	1	0	8	-1	13489.4891
11	3888	1250	488	2100	3099	12888	1	8	-1	8	8	1	15541.3000
12	3000	1250	488	2180	3888	6888	1	8	-1	8	8	-1	12967.4891
13	1658	1250	1888	2168	3086	12888	-1	8	1			1	15358.008
14	1650	1250	1866	2166	3025	6 <b>888</b>	-1		1		1	-1	12455.9091
15	1650	1258	488	2166	3888	12000	-1		-i	8		1	11933.9891
16	1658	1258	488	2188	3880	6868	-1	8	-1	8	0	-1	15 <b>8</b> 58 <b>.886</b>
17	3888	2888	488	3888	3880	9 <b>888</b>	1	t	8	1	8	8	15715.6000
18	3860	2888	488	1200	3888	<b>7888</b>	1	1	8	-1	8	0	14628.8182
19	3000	598	488	3888	3886	9 <b>888</b>	1	-1	8	1	8	8	14995.8182
28	3866	580	488	1200	3000	9 <b>866</b>	1	-1	8	-1	8	8	13645.8182
21	1658	2900	488	3888	3088	7 <b>886</b>	-1	1	8	1	8	8	15111.3182
22	1658	2996	488	1290	3080	9 <b>880</b>	-1	1	8	-1	8	8	13851.3182
23	1650	508	488	3888	3888	9 <b>888</b>	-1	-1	8	1	8	8	14136.3182
24	1650	500	488	1200	3866	9 <b>868</b>	-1	-1	0	-1	8	0	12736.4091
25	2325	2000	1888	2166	4886	9 <b>868</b>		i	1	0	1	0	15699.5000
26	2325	2888	1860	2100	2969	9 <b>888</b>	8	1	1	8	-1	8	14333.8682
27	2325	2888	488	2100	4968	9 <b>880</b>	0	1	-1	8	1		15399.8682
28	2325	2000	488	2188	2000	9 <b>880</b>	0	1	-1	8	-1	0	13919.6682
29	2325	588	1669	2188	4966	9 <b>628</b>	0	-1	1	9	1	8	14838.8682
38	2325	588	1686	2100	2000	9 <b>888</b>		-1	1	8	-1		13358.8682
31	2325	508	488	2188	4060	9 <b>880</b>	0	-1	-1	8	1		14424.8682
32	2325	588	198	2188	2988	9 <b>888</b>	8	-1	-1	8	-1		12853.6591

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# TABLE 8.22 (continued)

			NON-CO	DED					CO	DED			
RUN 🛊	M1	M2	M3	B1	B2	<b>S</b> 1	M1	M2	H3	Bi	B2	Si	RESPONSE
22	2325	1250	1888	3000	3900	12000	8	8	1	- <b>1</b>	8	1	16011.1300
34	2325	1258	1988	3000	3960	6 <b>888</b>	8	8	1	1	8	-1	13768.4891
35	2325	1258	1868	1208	3889	12000	8	8	1	-1	8	1	15154.0000
36	2325	1258	1886	1298	3899	6 <b>888</b>	8	0	1	-1	8	-1	12174.6591
37	2325	1258	408	3868	3888	12898	8	8	-1	1	8	1	15734.8988
38	2325	1258	488	3866	3888	6888	8	8	-1	1	8	-1	13246.4891
39	2325	1250	488	1288	3866	12888	8	8	-1	-1	8	1	14855.5888
48	2325	1250	488	1288	3966	6888	0	8	-1	-1	0	-1	11652.6591
41	3000	1258	7 <b>88</b>	3808	4299	9 <b>868</b>	1	8	8	1	1	8	1592 <b>8.6889</b>
42	3888	1250	788	3899	2988	9 <b>888</b>	1	8	8	1	-1	8	14653.3182
43	3888	1258	786	1200	4888	9 <b>888</b>	1	8	8	-1	1	8	14873.3182
44	3888	1250	788	1299	2988	9 <b>888</b>	1	8	8	-1	-i	8	13393.3182
45	1650	1250	788	3868	4986	9 <b>888</b>	-1	8	8	1	1	0	15363.8182
46	1650	1258	788	3868	2889	9 <b>868</b>	-1	8	8	1	-1	8	13883.8182
47	1658	1258	788	1288	4888	9 <b>888</b>	-1	8	8	-1	1	8	14103.8182
48	1650	1250	788	1200	2988	9 <b>888</b>	-1	8	9	-1	-1	0	12438.9891
49	2325	1258	788	2166	3 <b>886</b>	7 <b>888</b>	8	8	8	8	6	8	14378.5682

# BOX-DRAPER (HALF-REPLICATE) EXPERIMENTAL DESIGN FOR SIX FACTORS

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RUN	<b>≇ 1</b> 13	1	H2	M3	B1	<b>B</b> 2	Si	M1	H2	M3	B1	<b>B</b> 2	Si	RESPONSE
1	284	42	936	574	1723	2581	18257	419	419	419	419	419	.419	1393 <b>8.</b> 5553
2	284	42	936	574	1723	3419	7743	-,419	419	419	419	.419	419	13316.8614
3	284	42	936	574	2477	2581	7743	419	419	419	.419	419	419	13289.5974
4	284	49	936	574	2477	3419	10257	419	419	419	.419	.419	.419	15878.6153
5	284	42	936	826	1723	2581	7743	419	419	. 419	419	419	419	12764.6194
6	204	42	936	826	1723	3419	10257	419	419	.419	419	.419	. 419	14724.1413
7	204	42	936	826	2477	2581	10257	419	419	.419	.419	419	. 419	14631.9613
8	284	42	936	826	2477	3419	7743	419	419	.419	.419	.419	419	14888.4931
9	28	42	1564	574	1723	2581	7743	419	.419	419	419	419	419	13867.5564
10	28	42	1564	574	1723	3419	19257	419	.419	419	419	.419	.419	14959.2003
11	28	42	1564	574	2477	2581	19257	419	.419	419	.419	419	.419	14867.0203
12	204	42	1564	574	2477	3419	7743	419	.419	419	.419	.419	419	14323.5521
13	204	42	1564	826	1723	2581	18257	419	.419	.419	419	419	.419	14512.5463
14	20	42	1564	826	1723	3419	7743	419	.419	.419	419	.419	419	13969.0781
15	204	42	1564	826	2477	2581	7743	419	.419	.419	.419	419	419	13876.8981
16	204	42	1564	826	2477	3419	19257	419	.419	.419	.419	.419	.419	15516.5944
17	26	88	936	574	1723	2581	7743	.419	419	419	419	419	419	12975.7954
18	26	8	936	574	1723	3419	19257	. 419	419	419	419	.419	.419	14873.0958
19	26	88	936	574	2477	2581	10257	. 419	419	419	.419	419	. 419	14780.9158
20	26	88	936	574	2477	3419	7743	.419	419	419	.419	.419	419	14243.1041
21	26	88	936	826	1723	2581	10257	.419	-,419	.419	419	419	.419	14426.4418
22	26	88	936	826	1723	3419	7743	.419	419	.419	419	. 419	419	13888.6301
23	26	88	936	826	2477	2581	7743	.419	419	.419	.419	419	419	13796 <b>. 458</b> 1
24	26	88	936	826	2477	3419	10257	.419	419	.419	.419	.419	.419	15442.4314
25	261	88	1564	574	1723	2581	10257	.419	.419	419	419	419	.419	14661.5008
26	26	88	1564	574	1723	3419	7743	.419	.419	419	419	.419	419	14123.6891
27	26	88	1564	574	2477	2581	7743	.419	. 419	419	. 419	419	419	14031.5091
28	26	88	1564	574	2477	3419	10257	.419	. 419	419	.419	. 419	.419	15685.8414
29	26	8	1564	826	1723	2581	7743	_419	.419	. 419	419	419	419	13677.0351
30	26	88	1564	826	1723	3419	10257	.419	.419	.419	-,419	.419	.419	15346.8994
31	26	88	1564	826	2477	2581	10257	.419	.419	.419	.419	419	.419	15278.6414
32	26	88	1564	826	2477	3419	7743	.419	. 419	.419	.419	.419	419	14825.8951

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# TABLE B.23 (continued)

			NON-CO	IDED					CC	IDED			
RUN 8	#1	H2	H3	<b>B</b> 1	B2	SI	M1	M2	M3	Bi	B2	<b>S</b> 1	RESPONSE
33	2998	1250	788	2199	3888	9 <b>886</b>	.997	8	6	8	8	8	14762.1668
34	1652	1250	788	2100	3888	9 <b>888</b>	997	0	8	8	8	8	13994.9753
35	2325	1998	788	2188	3886	9 <b>866</b>		.997		8	8		14864.6857
36	2325	582	788	2188	3888	7 <b>886</b>		997	8	8	8		13892.5387
37	2325	1258	999	2199	3888	9 <b>888</b>	•	8	.997	0		8	14584.9472
38	2325	1258	481	2100	38 <b>86</b>	9 <b>888</b>	9	8	997	9	8	0	14172.1892
39	2325	1250	788	2997	3886	<b>7890</b>	8	9	8	.997	0	8	15886.6782
48	2325	1250	788	1283	3888	9 <b>868</b>	0	8	8	997	8	8	13758.4582
41	2325	1258	788	2168	3997	9 <b>888</b>	1	8	8	8	.997	8	15116.3482
42	2325	1258	788	2188	2883	9 <b>888</b>		8	8	0	997	8	13648.7882
43	2325	1258	788	2100	3888	11991	9	8	0	0	8	.997	15447.5764
44	2325	1250	788	2100	3866	6889	0	8	9	0	0	997	12728.5991
45	2325	1250	788	2188	3088	9 <b>668</b>	8	0	8	8	8	8	14378.5682

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# HYBRID (628A) EXPERIMENTAL DESIGN FOR SIX FACTORS

			NON-CO	DED					C	ODED			
RUN #	M1	H2	N3	B1	B2	<b>S</b> 1	MI	H2	H3	<b>B</b> 1	B2	<b>S</b> 1	RESPONSE
1	2325	1 <b>258</b>	788	2198	3888	16103	•	8		0		2.3677	163 <b>88.4422</b>
2	2325	1250	788	2100	3888	3567	8			8		-1.8110	11896.2991
2	1658	500	488	1298	2000	18829	-1	-1	-1	-i	-1	. 6996	12725.8411
4	3888	2000	488	1288	2980	18829	1	1	-1	-1	-1	. 6896	14515 <b>.8</b> 662
5	3000	508	1000	1266	2859	18829	1	-1	1	-1	-1	. 6896	13 <b>954.0662</b>
6	1658	2000	1686	1298	2906	10829	-1	1	1	-1	-1	. 6896	14159.5662
7	3000	500	488	3000	2888	18829	1	-1	-1	1	-1	. 6896	148 <b>88.8</b> 662
8	1658	2888	488	3666	2008	19829	-1	1	-1	1	-1	. 6896	14995.7768
9	1650	1 <b>258</b>	1888	3000	2000	18829	-1	-1	1	1	-1	. 6896	14444.5662
18	3999	2990	1888	3088	2888	18829	1	1	1	1	-1	. 6896	15775.9555
11	3000	500	468	1200	4989	16827	1	-1	-1	-1	1	. 6896	15888.7768
12	165 <b>8</b>	2888	488	1280	4858	10829	-1	1	-1	-1	1	. 6896	15177.7768
13	165 <b>8</b>	588	1838	1200	4888	10829	-1	-1	1	-i	1	. 6096	14664.5662
14	3 <b>688</b>	2000	1608	1288	4966	16829	1	1	1	-1	1	. 6896	1595 <b>8.8</b> 355
15	1658	500	468	3000	4220	18829	-1	-1	-1	1	1	. 5896	15485.7768
16	3888	2 <b>888</b>	488	3888	4888	18829	1	1	-1	1	1	. 6896	16589.2275
17	3 <b>008</b>	588	1999	3000	4000	18829	i	-1	1	1	1	. 6896	16166.1555
18	1658	2680	1880	3888	4986	16829	-1	1	1	1	1	. 6896	16327.4275

				NON-	CODED					CO	IDED			
F	IUN (	• N	L H	2 M3	B1	<b>B2</b>	<b>S1</b>	N1	M2	<b>H</b> 3	B1	B2	<b>S</b> 1	RESPONSE
1	9	379	5 12	50 70	8 2188	3000	5907	2.1749	7 8	9	8	8	-1.031	13778.6847
2	10	85	7 12	58 78	<b>5</b> 21 <b>80</b>	38 <b>98</b>	5 <b>98</b> 7	-2.1749	) ()	8	8	0	-1.831	11519.8122
2	1	232	5 28	31 78	2198	3866	5 <b>987</b>		2.1749	9 B	8		-1.831	13991.6952
2	2	232	5	8 78	2108	3000	5 <b>98</b> 7		-2.1749	98	8		-1.031	11614.8991
2	3	232	5 12	5 <b>8</b> 135	2 2188	3888	5987	8	8	2.1749		9	-1.031	13228.9288
2	24	232	5 12	50 4	B 21 <b>88</b>	3888	5 <b>987</b>	0	9	-2.1749		1	-1.831	12885.6382
2	5	232	5 12	59 79	<b>a 40</b> 57	3880	5907	8	0	8	2.1749	0	-1.031	14383.2952
2	26	232	5 12	50 70	<b>B</b> 143	3886	5 <b>987</b>	6	8	8	-2.1749	8	-1.031	1 <b>8982.438</b> 1
2	27	232	5 12	50 78	2180	5175	59 <b>87</b>		8	8	8	2.1749	7 -1.831	1454 <b>8.</b> 8542
2	28	232	5 12	58 78	2186	825	5987	8	8	8	0 -	-2.1749	-1.031	18652.3711

#### TABLE B.24 (continued)

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### HYBRID (628B) EXPERIMENTAL DESIGN FOR SIX FACTORS

			NON-CO	DED					C	ODED			
RUN #	<b>H1</b>	M2	M3	<b>B</b> 1	B2	<b>S</b> 1	M1	H2	M3	B1	B2	<b>S</b> 1	RESPONSE
1	2325	1250	760	2188	3888	15928	8		8		8	2.3894	16343.6433
2	2325	1258	788	2168	3000	9 <b>008</b>	8	ê	8	8	0	8	14378.5682
2	1658	580	480	1298	2000	18732	-1	-1	-1	-1	-1	.5773	12663.8251
4	3608	2000	488	1288	2886	19732	1	1	-1	-1	-1	.5773	1447 <b>8.</b> 4922
5	3 <b>998</b>	508	1889	1288	2866	10732	1	-1	1	-1	-1	.5773	13989.4922
6	1658	2000	1988	1200	2900	1 <b>8732</b>	-1	1	1	-1	-1	.5773	14114.9922
7	3888	588	488	3886	2005	18732	1	-1	-1	1	-1	.5773	14755.4922
8	1658	2888	488	3888	2000	18732	-1	1	-1	i	-1	.5773	14968.9922
9	1658	1258	1696	3888	2000	18732	-1	-1	1	1	-1	.5773	14399.9922
10	3080	2008	1688	3000	2900	10732	1	1	1	1	-i	.5773	15753.6298
11	3 <b>800</b>	588	480	1288	4000	10732	1	-1	-1	-1	1	.5773	14974.6138
12	1658	2000	488	1200	4888	18732	-1	1	-1	-1	1	.5773	15151.6130
13	1650	508	1000	1200	4888	18732	-1	-1	1	-1	1	.5773	14619.9922
14	3888	2000	1000	1208	4898	10732	1	1	1	-1	1	.5773	15927.7898
15	1658	500	488	3999	4888	10732	-1	-1	-1	1	1	.5773	15379.6138
16	3000	2008	480	3888	4005	18732	1	1	-1	1	1	.5773	16487.8788
17	3999	588	1680	3888	4866	10732	1	-1	1	1	1	.5773	16143.8298
18	1658	2800	1988	3868	4888	19732	-1	1	1	1	1	.5773	163 <b>86.070</b> 8

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			NON-CO	DED					C	DED			
RUN #	M1	<b>M2</b>	M3	<b>B</b> 1	B2	SI	MI	H2	H3	B1	B2	<b>S1</b>	RESPONSE
19	3675	1250	786	2100	3866	5536	2.9		8	8		-1.1547	13439.7441
28	975	1258	788	2188	3888	5536	-2.8	•	0	8		-1.1547	11362.4621
21	2325	275	788	2188	3880	5536		2.8	8			-1.1547	13653.3531
22	2325		786	2166	3868	5536	•	-2.8	8	8		-1.1547	11361.5091
23	2325	1258	1388	2198	3806	5536	ŧ	•	2.8	8		-1.1547	12930.3531
24	2325	1258	188	2190	3 <b>888</b>	5536		8	-2.8		1	-1.1547	11886.3531
25	23 <b>25</b>	1258	780	3988	3866	5536		8	8	2.6	0	-1.1547	13997.7441
26	2325	1 <b>258</b>	788	388	3860	5536	8	8	0	-2.8		-1.1547	18795.4621
27	2325	1258	788	2199	5000	5536	6	9	8	8	2.8	-1.1547	14233.3882
28	2325	1258	780	2199	1988	5536		ł		0	-2.8	-1.1547	10568.3531

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# TABLE B.25 (continued)

# MINIMUM POINT EXPERIMENTAL DESIGN FOR SIX FACTORS

			NON-CO	DED			CODED									
RUN #	NI	M2	M3	<b>B</b> 1	B2	51	M1	N2	M3	81	<b>B</b> 2	<b>S</b> 1	RESPONSE			
1	2742	1714	486	1288	2500	6 <b>968</b>	.6183	.6183	-1	-1	-i	-1	11438.9145			
2	2742	500	885	1200	2888	6 <b>900</b>	. 6183	-1	. 6183	-1	-1	-1	18853.3184			
2	2742	580	488	2656	2000	6 <b>866</b>	. 6183	-1	-1	. 6183	-1	-1	11718.7887			
4	2742	589	486	1288	3618	6 <b>898</b>	.6183	-1	-1	-1	. 6183	-1	11919.7781			
5	2742	588	488	1288	2000	19855	.6183	-1	-1	-1	-1	.6183	13485.2117			
6	1650	1714	885	1200	2888	6996	-1	.6183	. 6183	-1	-1	-1	11 <b>020.</b> 1813			
7	1658	1714	488	2656	2000	6 <b>860</b>	-1	. 6183	-1	.6183	-1	-1	11894.8633			
8	1658	1714	480	1290	3618	6 <b>868</b>	-1	.6183	-1	-1	.6183	-1	12986.6419			
9	1650	1714	400	1288	2996	18855	-1	. 6183	-1	-1	-1	. 6183	13571.4967			
10	1650	500	885	265 <b>6</b>	2886	6988	-1	-1	. 6183	. 6183	-1	-1	11389.8437			
11	1650	500	885	1266	3618	6 <b>880</b>	-1	-1	. 6183	-1	.6183	-1	11498.1214			
12	1650	500	885	1200	2000	18885	-1	-1	. 6183	-1	-1	. 6183	• 13117.5683			
13	1650	500	488	2656	3618	6 <b>886</b>	-1	-1	-1	. 6183	.6183	-1	12375.5834			
14	1650	580	486	2656	2005	18885	-1	-1	-1	.6183	-1	.6183	13882.1612			
15	1659	580	488	1200	3618	18885	-1	-1	-1	-1	. 6183	. 6183	13 <b>988.</b> 1142			
16	1727	2 <b>890</b>	1000	3888	4966	12998	8854	1	1	1	1	1	16611.1197			
17	3888	586	1000	3888	4096	12 <b>890</b>	1	8854	1	1	1	1	16465.4144			
18	3000	2660	434	3888	4000	12006	1	1	8854	1	1	1	16782.5889			
19	3000	2000	1998	1303	4900	12006	1	1	1	8854	1	1	16267.3343			
28	3888	2000	1888	3888	2115	12888	1	1	1	1	8854	1	16184.2918			

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							TABLE B.2	26 (conti	nued)					
				NON-CO	DED					CO	DED			
RL	in #	H1	N2	H3	B1	B2	<b>S</b> 1	M1	N2	M3	Bi	B2	51	RESPONSE
21	i ;	3888	2060	1865	3006	4000	6344	1	1	1	1	1	6854	1 <b>5594.484</b> 2
22	2 :	3886	588	488	1200	2006	6868	1	-1	-1	-1	-1	-1	18629.9891
23	5	1650	2868	468	1288	2996	6 <b>888</b>	-1	1	-1	-1	-1	-1	18835.4891
24	•	1650	588	1890	1288	2999	6 <b>868</b>	-i	-1	1	-1	-1	-1	1 <b>8188. 797</b> 1
2	<b>i</b> :	1658	588	489	3886	2986	6 <b>999</b>	-1	-1	-1	1	-1	-1	11198.9891
26		1658	588	488	1290	4888	6 <b>888</b>	-1	-1	-i	-1	1	-1	11418.9891
27	1	1650	588	488	1299	2800	12000	-1	-1	-1	-1	-1	1	13389.3182
28	3	1658	580	488	1280	2886	6998	-1	-1	-1	-i	-1	-1	9578.9 <b>8</b> 91
							B - 3	56						

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### KOSHAL EXPERIMENTAL DESIGN FOR SIX FACTORS

NON-CODED

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RUN #	Ħ1	H2	N3	<b>B1</b>	82	51	H1	<b>H2</b>	M3	<b>B</b> 1	<b>B</b> 2	<b>S</b> 1	RESPONSE
1	2807	1786	614	1843	2714	8143	.7143	.7143	2857	2857	2857	2857	14156.8833
2	2887	1836	914	1843	2714	8143	.7143	2857	.7143	2857	2857	2857	13876.3833
2	2897	1836	614	2743	2714	8143	.7143	2857	2857	.7143	2857	2857	14299.3833
4	2887	1836	614	1843	3714	8143	.7143	2857	2857	2857	.7143	2857	14489.3833
5	2807	1836	614	1843	2714	11143	.7143	2857	2857	2857	2857	.7143	14986.6895
6	2132	1786	914	<sup>-</sup> 1843	2714	8143	2857	.7143	.7143	2857	2857	2857	13976.8838
7	2132	1786	614	2743	2714	8143	2857	.7143	2857	.7143	2857	2857	143 <b>99.88</b> 38
8	2132	1786	614	1843	3714	8143	2857	.7143	~.2857	2857	.7143	2857	14589.8838
9	2132	1786	614	1843	2714	11143	2857	.7143	2857	-, 2857	2857	.7143	15875.1895
18	2132	1036	914	2743	2714	8143	2857	2857	.7143	.7143	2857	2857	14119.3038
11	2132	1836	914	1843	3714	8143	2857	2857	.7143	2857	.7143	2857	14229.3838
12	2132	1836	914	1843	2714	11143	-, 2857	2857	.7143	2857	2857	.7143	14871.5533
13	2132	1836	614	2743	3714	8143	2857	2857	2857	.7143	.7143	2857	14652.3838
14	2132	1836	614	2743	2714	11143	2857	-,2857	2857	.7143	2857	.7143	15189.1895
15	2132	1836	614	2743	3714	11143	2857	2857	2857	2857	.7143	.7143	15288.1895
16	3482	1836	614	1843	2714	8143	1.7143	2857	2857	2857	2857	2857	14054.0533
17	2132	2536	614	1843	2714	8143	2857	1.7143	~.2857	2857	2857	-,2857	14257.3838
18	2132	1836	1214	1843	2714	8143	2857	2857	1.7143	2857	2857	2857	13696.3838
19	2132	1836	614	3643	2714	8143	2857	2857	2857	1.7143	2857	2857	14542.3838
28	2132	1836	614	1843	4714	8143	2857	2857	2857	2857	1.7143	2857	14762.3838

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			NON-CO	DED			CODED								
RUN #	M1	H2	H3	B1	B2	<b>S</b> 1	M1	H2	M3	Bi	B2	51	RESPONSE		
21	2132	1836	614	1843	2714	14143	2857	2857	2857	2857	2857	1.7143	15481.4128		
22	2887	1836	614	1843	2714	8143	.7143	2857	2857	2857	2857	2857	13669.3833		
23	2132	1786	614	1843	2714	8143	2857	.7143	2857	2857	2857	2857	13769.8838		
24	2132	1836	914	1843	2714	8143	2857	2857	.7143	2857	2857	2857	13481.1979		
25	21 <b>32</b>	1836	614	2743	2714	8143	2857	2857	2857	.7143	2857	2857	13912.3 <b>8</b> 38		
26	2132	1836	614	1843	3714	8143	2857	2857	2857	2857	.7143	2857	14022.303B		
27	2132	1836	614	1843	2714	11143	2857	2857	-,2857	2857	2857	.7143	14664.5533		
28	2132	1036	614	1843	2714	8143	2857	2857	2857	2857	2857	2857	13228.1979		

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#### CENTRAL COMPOSITE ROTATABLE (HALF-REPLICATE) EXPERIMENTAL DESIGN FOR SIX FACTORS

CODED

M3

-1

-1

B1

-1

-1

**B**2

-1

1

**S1** 

1

-1

RESPONSE

13389.3182

11418.9891

		NUN-CU	VEV					
H1	H2	M3	B1	B2	51	M1	H2	
1658	588	488	1260	2888	12000	-1	-1	
1658	586	488	1288	4868	6868	-1	-1	
1658	588	480	3888	2000	6888	-1	-1	
1650	588	488	3888	4868	12886	-1	-1	
1650	586	1005	1288	2888	6888	-1	-1	
1650	586	1886	1208	4888	12888	-1	-1	
1650	588	1588	3688	2969	12888	-1	-1	
1658	588	1685	3888	4889	6660	-1	-1	
1659	2888	486	1200	2000	6000	-1	1	
1650	2888	488	1288	4886	12888	-1	1	

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RUN \$

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3	1658	500	488	3 <b>860</b>	2000	6888	-1	-1	-1	1	-1	-1	11198.9891
4	1658	588	488	3888	4868	12896	-1	-1	-1	1	i	1	15788.2288
5	1650	588	1885	1288	2888	6 <b>888</b>	-1	-1	1	-1	-1	-1	10100.9071
6	1650	586	1885	1288	4888	12886	-1	-1	1	-1	1	1	15184.8688
7	1650	588	1588	3666	2965	12888	-1	-1	1	1	-1	1	14922.0008
8	1650	588	1685	3888	4889	6 <b>689</b>	-1	-1	1	1	1	-1	13552.9891
9	1650	2888	488	1286	2888	6 <b>885</b>	-1	1	-1	-1	-1	-1	10835.4091
10	1650	2888	488	1288	4986	12000	-1	1	-1	-i	1	1	15484.1988
11	1658	2688	488	3000	2969	12888	-1	1	-1	1	-1	1	15310.0200
12	1658	2908	488	3000	4969	6888	-1	1	-1	1	1	-1	14235.8182
13	1658	2000	1000	1290	2889	12000	-1	1	1	-1	-1	1	14691.0000
14	1658	2968	1090	1288	4886	6 <b>888</b>	-1	1	1	-1	1	-1	13197.4891
15	1658	2888	1000	3868	2868	6 <b>866</b>	-1	1	1	1	-1	-1	12957.9891
16	1658	2008	1000	3 <b>866</b>	4950	12000	-1	1	1	. 1	1	1	16585.5688
17	3000	588	488	1200	2000	6888	1	-1	-1	-1	-1	-1	18629.9891
18	3888	586	488	1200	4888	12000	1	-1	-1	-1	1	1	15313 <b>.8488</b>
19	3000	588	488	3998	2999	12000	1	-1	-1	1	-i	1	15135 <b>.0688</b>
28	3888	500	408	3888	4869	6 <b>888</b>	1	-1	-1	1	1	-1	14043.8182
21	3886	588	1900	1200	2 <b>960</b>	12000	1	-1	1	-1	-1	1	14492.8182
22	3888	580	1999	1288	4936	6 <b>888</b>	1	-1	1	-1	1	-1	12991.9891
23	3888	588	1689	3888	2986	6 <b>888</b>	1	-1	1	1	-1	-1	12738.9891
24	3888	568	1665	3888	4888	12896	1	-1	1	1	1	1	16431 <b>.0000</b>
25	3888	2000	468	1280	2000	12999	1	1	-1	-1	-1	1	14887 <b>.0008</b>
26	3 <b>886</b>	2000	488	1288	4988	6 <b>888</b>	1	1	-1	-1	1	-1	13714.9891
27	3888	2888	488	3866	2000	6 <b>886</b>	1	1	-1	1	-1	-1	13461.9891
28	3866	2900	488	3000	4968	12000	1	1	-1	1	1	1	16767.3688
29	3886	2800	1628	1299	2860	6 <b>898</b>	1	1	1	-1	-1	-1	12396 <b>. 989</b> 1
38	3886	2999	1000	1268	4080	12000	1	1	1	-1	1	1	16219.8888
31	3000	2000	1969	3868	2000	12000	1	1	1	1	-1	1	16045.8000
32	3000	2200	1968	3888	4888	6 <b>888</b>	ĩ	1	1	1	1	-1	15432.8182

TABLE B.28 (continued)

	NON-CODED							CODED										
RUN #	N1	Ħ2	M3	Bi	<b>B</b> 2	<b>S</b> 1	H1	N2	M3	Bi	B2	S1	RESPONSE					
33	3938	1258	700	2100	3808	7 <b>868</b>	2.378	0			8	0	15264.4578					
34	728	1258	788	2160	3886	9 <b>888</b>	-2.378					8	13445.4951					
35	2325	3834	788	2198	3860	9 <b>868</b>		2.378				8	15474.9188					
36	2325		788	2188	3886	9 <b>888</b>		-2.378	8	0		8	13566.8682					
37	2325	1258	1413	2168	3886	9 <b>69</b> 8			2.378	8	1		14878.8142					
38	2325	1250		2188	3888	9 <b>880</b>	8	1	-2.378	8	ŧ	8	13895.5682					
39	2325	1250	788	4248	3886	7 <b>820</b>	8	8	0	2.378	8	0	15746.0020					
48	2325	1258	788		3888	7 <b>890</b>	0	8		-2.378	8	0	12889.1591					
41	2325	1250	766	2100	5378	7 <b>88</b> 8	8	1		8	2.378	0	15962 <b>. 4996</b>					
42	2325	1258	788	2166	622	9 <b>628</b>	9	8		8	-2.378	8	12469.3991					
43	2325	1250	788	2199	3888	16134	8	0	8	0	0	2.378	16386.9436					
44	2325	1250	788	2188	3888	1866	8	0		8	0	-2.378	9953.7664					
45	2325	1258	786	2198	3000	9 <b>860</b>	8	0	8	8	8	8	14378.5682					

Appendix C: Linear Regression Results

This section contains the results of the stepwise linear regression run of a sample input for the three factor problem. The experimental design which was used to complete this sample run is the Box-Draper bias minimizing design for three factors.

BNDP2R - STEPWISE REGRESSION BMDP STATISTICAL SOFTWARE, INC. 1964 WESTWOOD BLVD. SUITE 202 LOS ANGELES, CA, USA 90025 (213) 475-5700 PROGRAM REVISED OCTOBER 1903 MANUAL REVISED -- 1903 COPYRIGHT (C) 1903 REGENTS OF UNIVERSITY OF CALIFORNIA PROGRAM CONTROL INFORMATION /problem title is 'sample for three factor problem'.

variables are 4. /input format is free. file is bd3a. names are M1,B1,S1,DE, /variable Misqd, Bisqd, Sisqd, MIBI,MISI,BISI. add = 6. /transform Hisqd = H1 + H1. Bisqd = Bi + Bi. Sisqd = Si + Si. M1B1 = M1 + B1. M1S1 = M1 + S1. B1S1 = B1 + S1. depend is DE. /regress independent are M1,B1,S1, Misqd,Bisqd,Sisqd, M1B1,M1S1,B1S1. tol = .001. /print covariance. correlation.

/end.

rreg.

# PROBLEM TITLE IS sample input for 3 factor problem

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NUMBER OF VARIABLES	S TO READ IN		, 4	
NUMBER OF VARIABLES	6 ADDED BY TRA	ISFORMATIONS.	. 6	
TOTAL NUMBER OF VAR	RIABLES		. 19	
NUMBER OF CASES TO	READ IN		, TO END	
CASE LABELING VAR	(ABLES			
MISSING VALUES CHEC	KED BEFORE OR	AFTER TRANS.	NEITHER	
BLANKS ARE			MISSING	
INPUT FILE		UNIT 7.	bd3a	Ì
REWIND INPUT UNIT F	RIOR TO READIN	16 DATA	YES	
NUMBER OF WORDS OF	DYNAMIC STORAG	Æ	, 25598	
VARIABLES TO BE US	ED			
1 M1	2 B1	3 51	4 DE	5 Mlsqd
6 B1sqđ	7 S1sqd	8 M1B1	9 M1S1	18 B1S1
INPUT FORMAT IS Free				

MAXIMUM LENGTH DATA RECORD IS 80 CHARACTERS.

REGRESSION INTERCEPT	RO
WEIGHT VARIABLE	
PRINT COVARIANCE MATRIX	ES
PRINT CORRELATION MATRIX	ES
PRINT ANOVA AT EACH STEP	ES
PRINT STEP OUTPUT	ES
PRINT REGRESSION COEFFICIENT SUMMARY TABLE	ES
PRINT PARTIAL CORRELATION SUMMARY TABLE	NO
PRINT F-RATIO SUMMARY TABLE	NO N
PRINT SUMMARY TABLE	ES
PRINT RESIDUALS AND DATA	NO
PRINT CORRELATION OF REGRESSION COEFFICIENTS	ES
PRINT NORMAL PROBABILITY PLOT	NO
PRINT DETRENDED NORMAL PROBABILITY PLOT	NO
PRINT DIAGNOSTIC PLOTS	NO
PRINT PLOTS FOR XVAR AND YVAR	NO
PRINT PLOTS AT EACH STEP	NO
PRINT PLOTS AND DATA	NO
PRINT PLOTS WITH STATISTICS	NO
NUMBER OF CASES READ.	15

COVARIANCE MATRIX

		M1	B1	S1	Misqd	B1sqd	Sisqd	H1B1	M151	B1S1	
		1	2		3	5	6	7 8		7	18
MI	1.	7434									
Bi	2.	4257e-88	.7434								
<b>S1</b>	3.	88 <del>888</del> 8888	.8888e+88	.7434							
Misqd	5	3692e-88	. 8888e+88	.8515e-88	. 4796						
Bisqd	6	3362e-88	. <b>200</b> 0e+00	. 8888e+88	1838	. 4796					
Sisqd	7	3362e-88	4257e-08	. 8888e+88	1838	1838	. 4796				
M1B1	8.	8515e-88	.2554e-97	. 8988e+84	.2989e-89	.2989e-89	.2989e-8	9.3319			
M151	9.	4257e-88	. 8668e+88	.1277e-87	. 8888e+88	. 8888e+96	. 8588e+8	01277e-87	.3319		
<b>B1S1</b>	10 .	4257e-08	.8515e-88	.1277e-87	. 8888e+68	. 8869e+86	. 8000e+0	<b>1277e-87</b>	. 8968e+66	.3319	

CORRELATION MATRIX

		M1	B1	Si	Nisqd	<b>B1sqd</b>	Sisqd	M1B1	MISI	B1S1
			1	2	3	5	6	7	8	9 10
MI	1	1.0966								
Bi	2	. 8688	1.0000							
S1	3	. 2008	. 2888	1.8888						
Misqd	5	. 8688	. 8888	. 8988	1.8005					
Bisqd	6	. 8988	. 9668	. 9999	3833	1.9999				
Sisqd	7	. 6888	. 8868	. 8888	3833	3833	1.8888			
MIBI	8	. 8986	. 6685	. 8688	. 8888	. 2999	. 8066	1.0000		
MISI	9	. 6999	. 8696	. 8686	. 9966	. 2008	. 8839	. 2889	1.0000	
<b>B1S1</b>	10	. 6888	. 2699	. 8888	. 0000	. 2000	. 8998	. 8869	. 2000	1.0006

S	TEPPIN	6 Alg(	DRITHM	l							,	•		•			.F		
M	AXIMUM	NUMBE	ER OF	STEPS	5.										28				
D	EPENDE	NT VAF	RIABLE													4	DE		
H	INIMUM	ACCEP	PTABLE	FT	) EI	ITER					•				4,988	;	4.888		
H	AXIMUM	ACCEF	PTABLE	FT	) RI	EHOV	Ε.		,				•		3.988	,	3.988		
H	INIMUM	ACCEI	PTABLE	TOLI	RAI	ICE.					•			•	. 98198				
S	UBSCRI	pts of	F THE	INDE	PENI	)ENT	۷	AR:	IAI	BL.I	ES				1	2	3	5	6
															7	8	9	18	

STEP NO.

STD. ERROR OF EST. 1138.3025

ANALYSIS OF VARIANCE

	sum of squares	DF	Mean Square
RESIDUAL	18149256.	14	1295733.

VARIABLES IN EQUATION FOR DE

		STD. ERROR	STD REG		F	
VARIABLE (Y-INTERCEPT	COEFFICIENT 9161.82715 )	of coeff	COEFF	TOLERANCE	to remove	LEVEL

#### VARIABLES NOT IN EQUATION

		PARTIAL		F	
VARI	ABLE	CORR.	TOLERANCE	TO ENTER	LEVEL
H1	1	.24528	1.39668	.83	1
B1	2	.42422	1.89899	2.85	1
S1	3	.84465	1.89566	32.37	1
Misqd	5	.86236	1.00000	.85	i
Bisqd	6	.84787	1.88888	.83	1
Sisqd	7	17424	1.00000	.41	1
MIBI	8	01485	1.89999	. 80	t
M151	9	06771	1.00000	.86	1
<b>B</b> 1S1	10	89125	1.00000	.11	1

#### STEP NO. 1

VARIABLE ENTERED 3 S1

MULTIPLE R	.8447
NULTIPLE R-SQUARE	.7134
ADJUSTED R-SQUARE	.6914
STD. ERROR OF EST.	632.3545

# ANALYSIS OF VARIANCE

	sum of squares	DF	MEAN SQUARE	F RATIO
REGRESSION	12941919.	1	.1294192e+88	32.37
RESIDUAL	5198338.5	13	399872.2	

#### VARIABLES IN EQUATION FOR DE

			STD. ERROR	STD REG		F	
VARI	ABLE	COEFFICIENT	OF COEFF	COEFF	TOLERANCE	TO REMOVE	LEVEL
(Y-INTER	CEPT	9161.82715 )					
S1	3	1115.15485	196.0181	.845	1.00000	32.37	1

#### VARIABLES NOT IN EQUATION

			PARTIAL		F	
•	VARIABLE		VARIABLE CORR.		TO ENTER	LEVEL
•	81	1	45821	1.50000	7.19	1
•	B1	2	.79247	1.00000	28.26	1
•	Misqd	5	.11650	1.90000	.17	t
,	Bisqd	6	. 88792	1.80000	. 89	1
•	Sisqd	7	32548	1.00000	1.42	1
	M1B1	8	02625	1.00000	.01	1
	M1S1	9	12649	1.00000	. 28	1
	B1S1	18	17845	1.00000	.36	1

STEP NO. 2

VARIABLE ENTERED 2 B1

MULTIPLE R	.9452
NULTIPLE R-SQUARE	.8934
ADJUSTED R-SQUARE	.8756
STD. ERROR OF EST.	401.4315

# ANALYSIS OF VARIANCE

1.1.

		sum of squares	DF	Mean Square	F RATIO
	REGRESSION	16286489.	2	8183245.	58.28
•	RESIDUAL	1933766.9	12	161147.2	

#### VARIABLES IN EQUATION FOR DE

			STD. ERROR	STD REG		F	
VA	RIABLE	COEFFICIENT	OF COEFF	COEFF	TOLERANCE	to remove	LEVEL
(Y-INT	ERCEPT	9161.82715 )					
B1	2	56 <b>8.87</b> 813	124.4363	. 424	1.00000	28.26	1
S1	3	1115.15485	124.4363	.845	1.00000	88.31	1

#### VARIABLES NOT IN EQUATION

			PARTIAL		F	
•	VARIABLE		CORR.	TOLERANCE	TO ENTER	LEVEL
	H1	1	.75126	1.00000	14.25	1
	Misqd	5	.19181	1.80608	.42	1
	Bisqd	6	.14416	1.80000	.23	1
	Sisqd	7	53365	1.89898	4.38	1
	MIBI	8	84383	1 <b>. 86666</b>	.82	1
,	MISI	9	20740	1.00000	.49	1
	B1S1	18	27947	1.00000	.93	1
STEP NO. 3

VARIABLE ENTERED 1 M1

MULTIPLE R	.9765
MULTIPLE R-SQUARE	. 9536
ADJUSTED R-SQUARE	. 9489
STD. ERROR OF EST.	276.7283

## ANALYSIS OF VARIANCE

stradu kroktel skutski svanna seconda i harana seconda seconda starada starada i manara useesen arda M

	SUM OF SQUARES	DF	HEAN SQUARE	F RATIO
REGRESSION	17297892.	3	5765964.	75.29
RESIDUAL	842364.86	11	76578.55	

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TOWN.

### VARIABLES IN EQUATION FOR DE

			STD. ERROR	STD REG	6 F			
V	ARIABLE	COEFFICIENT	OF COEFF	COEFF	TOLERANCE	to remove	LEVEL	
(Y-IN	TERCEPT	9161.82715	)					
Mi	1	323.83828	85.7866	.245	1.00066	14.25	i	
B1	2	560.07B13	85.7886	.424	1.88968	42.63	1	
<b>S1</b>	3	1115.15485	85.7886	.845	1.00000	167.88	1	

### VARIABLES NOT IN EQUATION

			PARTIAL		F		
•	VARIABLE		CORR.	TOLERANCE	ERANCE TO ENTER		
•	Misqd	5	.28941	1.88888	.91	1	
•	Bisqd	6	.21841	1.00800	.59	1	
	Sisqd	7	88855	1.00000	18.88	1	
	M181	8	86520	1.00000	.84	1	
	N1S1	9	31423	1.86866	1.18	1	
	B1S1	18	42344	1.80000	2.18	1	

MULTIPLE R	.9919
MULTIPLE R-SQUARE	. 9839
ADJUSTED R-SQUARE	.9775
STD. ERROR OF EST.	170,7814

	SUM OF SQUARES	DF	Mean Square	F RATIO
REGRESSION	17848594.	4	4462149.	152.99
RESIDUAL	291663.80	18	29166.38	

1											 • • •
1											
	STEP NO.	4									
			-	<b>-</b>							
	VARIABLE	ENTER	ED 7	S1 <b>sqd</b>							
	MULTIPLE	R		.9919							
	MULTIPLE	R-SQU	ARE	.9839							
	ADJUSTED	R-590/	<b>NRE</b>	.9775							
	STD. ERR	ior of e	EST.	70.7814							
	ANALYSIS	i of var	RIANCE								
				SUM OF S	QUARES	DF	HEA	n square	F RATIO		
		REGRES	SSION	17848594	•	4	4462	149.	152.99		
		RESID	JAL	291663.		10	2916	6.30			
				VARIA	BLES IN EG	NATIO	IN FOR	DE			
					STD. ERROR	ST	) REG				
	VARI	ABLE	COEFFI	ICIENT	of coeff	COE	FF	TOLERANCE	to remove	LEVEL	
	(Y-INTER	CEPT	9368.	52344 )							
	M1	1	323.	83828	52.9391		245	1.00000	37.42	1	
	B1	2	568.	7813	52.9391		424	1.0000	111.93	1	
	S1	3	1115.3	15485	52.9391		845	1.00000	443.73	1	
	Sisod	7	-286.	38565	65.9874		174	1.00000	18.88	1	
	•										
			VARIAB	LES NOT 1	N EQUATION	1					
			PARTI	AL.	F						
	. VAR	IABLE	CORR	TOLERA	NCE TO E	INTER	LEV	EL			
	. Misod	5	037	76 .853		01		1			
	, Bisad	6	- 168	39 .853	17	26		1			
	. MIBI	R	119	B1 1.000		11		1			
	. M1S1	9	- 534	3 1.844	<b>1 3</b> .	59		1			
	BISI	1	- 719	51 1.884		67		-			
			• / • /					•			
								• <del>•</del>			
								C - 9			

•	VARIA	RL E	PARTIAL CORR.	TOLERANCE	F To enter	LEVEL
	*****	***	001111	. AFEW WAF		ta to T to to
:	Misqd	5	83776	.85387	.01	1
	Bisqd	6	16839	.85387	.26	1
	MIBI	8	11981	1.90000	.11	1
	M151	9	53403	1.88888	3.59	1
	B151	10	71961	1.00000	9.67	1

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STEP NO. 5

VARIABLE ENTERED 10 BIS1

NULTIPLE R	. 9961
NULTIPLE R-SQUARE	.9922
ADJUSTED R-SQUARE	.9879
STD. ERROR OF EST.	125.0028

ANALYSIS OF VARIANCE

anta "ramma" receive "normal" second (receive second second second receive second "rampra 'receive "receive B

	sum of squares	DF	MEAN SQUARE	F RATIO
REGRESSION	17999626.	5	3599925.	238.39
RESIDUAL	148629.45	9	15625.58	

VARIABLES IN EQUATION FOR DE

			STD. ERROR	STD REG		F	
VAR	IABLE	COEFFICIENT	OF COEFF	COEFF	TOLERANCE	to remove	LEVEL
(Y-INTE	RCEPT	9368.52344					
M1	1	323.83829	38.7483	. 245	1.2000	<b>37.85</b>	1
B1	2	568.07813	38.7483	. 424	1. <b>00000</b>	288.93	1
S1	3	1115.15485	38.7483	.845	1.00000	828.26	1
Sisqd	7	-286.38565	48.2483	174	1.00000	35.24	1
BISI	10	-188.28658	57.9887	891	1.20688	9.67	1

VARIABLES NOT IN EQUATION

			PARTIAL		F		
•	VARIABLE		CORR. TOLERANCE		TO ENTER	LEVEL	
•	MIsqd	5	85438	. 85387	.82	1	
	Bisqd	6	24250	.85387	.58	1	
	MIBL	8	15958	1.00000	.21	1	
	MISI	9	76987	1.90080	11.58	1	

1.1

STEP NO. 6

VARIABLE ENTERED 9 MIS1

MULTIPLE R	. 9984
NULTIPLE R-SQUARE	. 9968
ADJUSTED R-SQUARE	. 9945
STD. ERROR OF EST.	84.7437

### ANALYSIS OF VARIANCE

	sum of squares	DF	hean square	F RATIO
REGRESSION	18882884.	6	3813881.	419.66
RESIDUAL	57451.914	8	7181.489	

### VARIABLES IN EQUATION FOR DE

			STD. ERROR	STD REG			
VAR	IABLE	COEFFICIENT	OF COEFF	COEFF	TOLERANCE	TO RENOVE	LEVEL
(Y-INTE	RCEPT	9368.52344	)				
H1	1	323.8382	26.2678	.245	1	151.97	1
B1	2	568.87813	26.2678	. 424	1.00000	454.58	1
S1	3	1115.15485	26.2698	.845	1.00000	1892.12	1
S1sqd	7	-286.38565	32.7848	174	1	76.68	1
N1S1	9	-133.79181	39.3128	668	1. Direct	11.58	1
B1S1	10	-188.28658	39.3128	891	1.00000	21.83	1

### VARIABLES NOT IN EQUATION

			PARTIAL		F	
	VARIA	BLE	CORR.	TOLERANCE	TO ENTER	LEVEL
•	Mfand	F		06787	86	
٠	ntedo	J	69369	. 2008/	. 53	1
	Bisqd	6	3794	.85387	1.18	1
	M1B1	8	24967	1.68686	.47	1
•						
•						

\*\*\*\*\* F LEVELS( 4.800, 3.900) OR TOLERANCE INSUFFICIENT FOR FURTHER STEPPING

# CORRELATION MATRIX OF REGRESSION COEFFICIENTS

		H1	B1	<b>S1</b>	Sisqd	N151	BISI
		1	2	2	7	9	10
HI	1	1.0000					
B1	2	. 8868	1.0000				
S1	3	. 2000	. 1999	1.8000			
Sisqd	7	. 8966	. 2020	. 8868	1.0000		
MISI	9	. 8868		. 8866	. 2008	1.3888	
BISI	10	. 2008	. 8699	. 8880	. 2000	. 2008	1.0000

TABLE D.1 (continued)

# RESPONSE SURFACE EQUATIONS FOR THREE FACTORS

	DESIGN	EQUATION
6)	CENTRAL COMP	Y = 0.4848(M1) + 1.153(B1) + 1.004(S1)
	RUTHTHBLE	0000276(S1*S1)0000618(B1*S1)
		+ 124.9285
7)	KOSHAL	Y = 0.5453(M1) + 0.6747(B1) + 0.6047(S1)
		0000142(S1*S1) + 2396.592

D - 2

## TABLE D.2

# RESPONSE SURFACE EQUATIONS FOR FOUR FACTORS

	DESIGN	EQUATION
1)	BOX - BEHNKEN	Y = 0.3833(M1) + 0.9461(M2) + 0.9871(B1)
		+ 0.8775(S1)0000229(S1*S1)
		0000539(M2*S1)0000512(B1*S1)
		+ 1319.9931
2)	BOX - DRAPER	Y = 0.3950(M1) + 0.9441(M2) + 0.9879(B1)
		+ 0.8868(S1)000024(S1*S1)
		0000527(M2*S1)0000503(B1*S1)
		+ 1332.3008
3)	HYBRID	Y = 0.434(M1) + 1.1172(M2) + 1.0793(B1)
	(4186)	+ 0.7926(S1)0000149(S1*S1)
		00000677(M2*S1)00000645(B1*S1)
		+ 1129.0738
4)	HYBRID	Y = 0.4394(M1) + 1.0915(M2) + 1.1122(B1)
	(4104+08)	+ 0.823(S1)0000174(S1*S1)
		0000643(M2*S1)0000601(B1*S1)
		+ 1060.9086
5)	MIN POINT	Y = 0.4088(M1) + 1.0122(M2) + 1.055(B1)
		+ 0.9707(S1)0000265(S1*S1)
		0000582(M2*S1)000055(B1*S1)
		+ 810 7629

D - 3

TABLE D.2 (continued)

## RESPONSE SURFACE EQUATIONS FOR FOUR FACTORS

	DESIGN	EQUATION
6)	CENTRAL COMP	Y = 0.416(M1) + 0.9609(M2) + 1.0236(B1)
		+ 0.907(S1)0000233(S1*S1)
		0000536(M2*S1)0000512(B1*S1)
		+ 939.3
7)	KOSHAL	Y = 0.4087(M1) + 0.4833(M2) + 0.5322(B1)
	<b>د</b> آر	<sup>4</sup> + 0.2406(S1) +5142.2911

D - 4

## TABLE D.3

## RESPONSE SURFACE EQUATIONS FOR FIVE FACTORS

	DESIGN	EQUATION
1)	BOX - BEHNKEN	Y = 0.6029(M1) + 0.6795(M2) 0.7294(B1)
		+ 0.7689(B2) + 1.0363(S1)
		0000295(S1*S1) + 843.9903
2)	BOX - DRAPER	Y = 0.6289(M1) + 0.6776(M2) + 0.7556(B1)
		+ 0.7976(B2) + 1.0942(S1)
		0000329(S1*S1) + 387.9629
3)	BOX - DRAPER (HALE-REPLICATE)	Y = 0.6037(M1) + 0.6825(M2) + 0.7393(B1)
		+ 0.7846(B2) + 1.0926(S1)
		0000329(S1*S1) + 536.6394
4)	MIN POINT	Y = 1.2691(M1) + 2.3605(M2) + 0.9176(B1)
		+ 0.938(B2) + 0.6281(S1)
		001015 (M1*M2) - 640.3131
5)	CENTRAL COMP	Y = 0.5011(M1) + 0.6289(M2) + 0.7123(B1)
		+ 0.7545(B2) + 0.9511(S1)
		0000261(S1*S1) + 1615.6741
6)	KOSHAL	Y = .7010(M1) + 0.7668(M2) + 0.8011(B1)
		+ 0.8332(B2) + 0.9541(S1)
		- .000024(S1*S1) + 465.7382

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	TABLE D.4
RESPONSE SU	RFACE EQUATIONS FOR SIX FACTORS
DESIGN	EQUATION
1) BOX – BEHNKEN	Y = 0.5358(M1) + 0.6281(M2) + 0.6543(M3)
	+ 0.6921(B1) + 1.3939(B2) + 1.2779(S1)
	0000331(S1*S1)0000726(B2*S1)
	- 642.153
2) BOX - DRAPER (HALE-REPLICATE)	Y = 0.5678(M1) + 0.6446(M2) + 0.6260(M3)
	+ 0.6954(B1) + 1.2237(B2) + 1.2081(S1)
	00003276(S1*S1)00005422(B2*S1)
	- 101.0283
3) HYBRID	Y = 0.5625(M1) + 0.5781(M2) + 0.6260(M3)
(6284)	+ 0.6840(B1) + 1.2294(B2) + 1.0361(S1)
	0000228(S1*S1)0000568(B2*S1)
	+ 715.9275
4) HYBRID	Y = 0.5544(M1) + 0.6061(M2) + 0.6138(M3)
(6288)	+ 0.6833(B1) + 1.2290(B2) + 1.0444(S1)
	0000232(S1*S1)0000565(B2*S1)
	+ 695.8883
5) MIN POINT	Y = 0.5825(M1) + 0.6558(M2) + 0.5929(M3)
	+ 0.7219(B1) + 1.2001(B2) + 2.3404(S1)
	0000961(S1*S1)0000558(B2*S1)
	- 4816.2332
	D - 6

## TABLE D.4 (continued)

## RESPONSE SURFACE EQUATIONS FOR SIX FACTORS

DESIGN

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# EQUATION

6)	CENTRAL COMP	Y	=	0.5622(M1)	+	0.6062(M2)	+	0.6728(M3)
	RUIHINDLE		+	Ø.6852(B1)	+	1.2372(82)	+	1.0462(S1)
			_	.0000235(S1	*	31) — .0000	569	7(B2*S1)
			+	620.122				
7)	KOSHAL	Y	=	0.5837(M1)	+	Ø.6582(M2)	+	0.7360(M3)
			+	0.7033(B1)	+	0.9630(B2)	+	1.0337(S1)
			-	.0000262(S1	*9	51) — .0000	262	2(B2*S1)
			+	956 1621				

# RANDOMLY SELECTED CHECK POINTS FOR THREE FACTORS

		<u>M1</u>	<u>B1</u>	<u>51</u>
	1	1691	1747	6014
	2	1660	1332	10359
	3	2031	2624	9151
	4	2527	1435	7752
	5	2888	2594	8166
	6	1831	1787	6474
	7	1985	2011	7846
	8	2955	1415	7076
	9	2411	1959	6152
	10	1757	2614	8822
	11	2724	2224	9792
	12	2326	2643	11871
	13	2980	1743	11841
	14	1750	2523	9492
	15	1729	2243	9347
	16	2768	2812	8627
	17	2666	1462	8818
	18	2157	2223	8 <b>0</b> 82
	19	2292	2201	9033
	20	1771	2802	7756
*	21	1650	1200	6000
*	22	3000	3000	12000

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CONSTRUCTION CONSTRUCTION

# RANDOMLY SELECTED CHECK POINTS FOR FOUR FACTORS

		<u>M1</u>	<u>M2</u>	<u>B1</u>	<u>51</u>
	1	2257	643	1739	6585
	2	1665	1443	1400	7407
	3	2063	1087	1242	8236
	4	2645	1954	2986	9142
	5	1881	540	2441	11017
	6	2521	1306	1352	8755
	7	2496	973	1283	7598
	8	2247	1943	1613	7274
	9	2779	553	2957	11808
	10	2550	1120	1741	11147
	11	1739	1056	2371	11175
	12	2242	770	1539	11044
	13	2608	759	2499	8595
	14	2515	628	1278	10883
	15	2134	1876	1921	8122
	16	2401	1174	1837	11724
	17	1800	1819	1373	11811
	18	2996	1972	1949	6088
	19	1707	1972	1958	8375
	20	1712	1870	2027	11200
**	21	1650	500	1200	6000
**	22	3000	2000	3000	12000
			D - 9		

			TABLE I	0.7		
	RANDOMLY	SELECTED	CHECK PO	INTS FOR	FIVE FACTORS	
	<u>M1</u>	<u>M2</u>	<u>B1</u>	<u>82</u>	<u>51</u>	
1	1691	645	1747	3911	6014	
. 2	1660	535	1332	2462	10359	
3	2031	1832	2624	2970	9151	
4	2527	562	1435	2822	7752	
5	2888	1824	2574	2866	8166	
6	1831	1133	1787	2050	6474	
7	1985	1670	2011	2832	7846	
8	2955	557	1415	2750	7076	
9	2411	1656	1959	2652	6152	
10	1757	875	2614	2936	8822	
11	2295	1252	2235	3613	92 <b>08</b>	
12	2568	705	1973	2700	6598	
13	2073	1977	1368	2588	11543	
14	2868	1753	2326	3932	6215	
15	1802	1031	1402	2705	6648	
16	2108	598	1572	3301	6262	
17	1835	1147	1840	2236	8226	
18	1873	1281	2344	3994	6801	
19	2216	976	2997	2275	8600	
20	2138	704	1972	2694	655 <b>0</b>	
** 21	1650	500	1200	2000	6000	
** 22	3000	2000	3000	4000	12000	
			D - 101			

## RANDOMLY SELECTED CHECK POINTS FOR SIX FACTORS

\$			<u>M1</u>	<u>M2</u>	MB	<u>B1</u>	<u>B2</u>	<u>51</u>
		1	2295	1249	561	2371	2244	11147
		2	1782	1833	674	2733	2698	11940
42		3	2316	768	845	2553	2149	11165
		4	2944	1912	443	1428	2237	8188
		5	1830	1017	912	2320	2333	7243
<b>3</b>		6	2171	1034	608	1272	2532	8582
		7	1671	1720	692	2082	2292	69 <b>0</b> 7
8		8	1678	953	765	27 <b>0</b> 8	3442	7633
		9	1812	1700	514	1234	2135	9257
		10	1691	1857	958	2399	2551	11751
		11	1660	963	416	2626	3353	6585
8		12	2031	1323	765	2213	2724	7407
		13	2527	517	621	2925	2798	8236
ж.		14	2888	1128	704	1359	3910	9142
8		15	1831	819	548	1415	2253	11017
		16	1985	645	554	2201	3079	8755
		17	2955	669	943	29 <b>07</b>	3265	7598
		18	2411	1066	798	2125	2122	7274
		19	1757	1944	613	2888	3076	11808
		20	2568	1360	863	1539	2291	9235
	**	21	1650	500	400	1200	2000	6000
	**	22	3000	2000	1000	3000	4000	12000
					D - 11			
					M . TT			
	(A) (A)	<u>ja k</u> i		STATES &		N N N N N N N N N N N N N N N N N N N		

## BIBLIOGRAPHY

1. Box, G.E.P. and D.W. Behnken. "Some New Three Level Designs for the Study of Quantitative Variables." <u>Technometrics, Vol.</u> 2: 455 (1960).

2. Box, G.E.P. and N.R. Draper. "A Basis for the Selection of a Response Surface Design." <u>Journal of American</u> <u>Statistical Association, Vol. 54</u>: 622 (1959).

3. Box, G.E.P. and N.R. Draper. "The Choice of a Second Order Rotatable Design." <u>Biometrika, Vol.</u> 50: 335 (1963).

4. Box, G.E.P. and J.S. Hunter. "Multifactor Experimental Designs for Exploring Response Surfaces." <u>Annals of</u> <u>Mathematical Statistics, Vol. 28</u>: 195 (1957).

5. Box, G.E.P. and K.B. Wilson. "On the Experimental Attainment of Optimum Conditions." <u>Journal of Royal</u> <u>Statistical Society, Series B, Yol. 13</u>: 1 (1951).

6. Box, M.J. and N.R. Draper. "On Minimum Point Second Order Designs." <u>Technometrics</u>, <u>Vol.</u> <u>16</u>: 613 (1974).

7. Cote, R., A.R. Manson, and R.J. Hader. "Minimum Bias Approximation of a General Regression Model." <u>Journal of</u> <u>American Statistical Association, Vol. 68</u>: 633 (1973).

8. Davies, O.L. <u>Design and Analysis of Industrial</u> <u>Experiments (2nd Edition).</u> New York: Hafner Publishing Co., Inc. (1956).

9. Graney, R. "An Optimization Methodology for Multicriteria Comparison Using Response Surface Methods and Mathematical Programming." Unpublished MS Thesis. AFIT (AU), WPAFB, OH, Mar 1984.

10. Hicks, C.R. <u>Fundamental Concepts in the Design of</u> <u>Experiments (2nd Edition)</u>. New York: Holt, Rinehart and Winston (1973).

11. Hill, W.J. and W.G. Hunter. "A Review of Response Surface Methodology: A Literature Survey." <u>Technometrics</u>, <u>Vol.</u> 8: 571 (1966).

12. Kanemasu, H. "A Statistical Approach to Efficient Use and Analysis of Simulation Models," Proceedings of the 42nd Session of the International Statistical Institute, 1979, Manila, Phillipines, 573-601.

13. Karson, M.J. "Design Criterion for Minimum Bias Estimation and Experimental Design for Response Surface." Journal of American Statistical Association, Vol. 65: 1565 (1970).

14. Karson, M.J., A.R. Manson and R.J. Hader. "Minimum Bias Estimation and Experimental Design for Response Surfaces." <u>Technometrics, Vol. 11</u>: 461 (1969).

15. Karson, M.J. and M.L. Spruill. "Design Criteria and Minimum Bias Estimation." <u>Communications in Statistics</u>, <u>Vol.</u> 4: 339 (1975).

16. Lucas J.M. "Which Response Surface Design is Best." <u>Technometrics, Vol. 18</u>: 411 (1976).

17. Manacapilli, T. "A Methodology for Identifying Cost Effective Strategic Force Mixes." Unpublished MS Thesis. AFIT (AU), WPAFB, OH, Dec 1984.

18. Myers, R.H. <u>Response Surface Methodology</u>. Ann Arbor: Edwards Brothers (1971).

19. Neter, J., W. Wasserman, and M.H. Kutner. <u>Applied Linear</u> <u>Statistical Models (Second Edition)</u>. Homewood, Illinois: Richard D. Irwin, Inc. (1985).

20. Roquemore, K.G. "Hybrid Designs for Quadratic Response Surfaces." <u>Technometrics, Vol. 18</u>: 419 (1976).

21. Roush, W.B., R.G. Petersen, and G.H. Arscott. "An Application of Response Surface Methodology to Research in Poultry Nutrition." <u>Poultry Science, Vol. 58</u>: 1504 (1979).

22. Smith, P.W. and J.M. Mellichamp. "Multidimensional Parametric Analysis Using Response Surface Methodology and Mathematical Programming as Applied to Military Problems," Proceedings of the Pacific Conference on Operations Research, April 23-28, 1979, Seoul, Korea, Vol. I, 592-615.

23. Steinberg, D.M. and W.G. Hunter. "Experimental Design: Review and Comment." <u>Technometrics.</u> <u>Vol. 26</u>: 71 (1984).

24. Toyomizu, M., Y. Akiba, M. Horiguchi and T. Matsumoto. "Multiple Regression and Response Surface Analysis of the Effects of Dietary Protein, Fat, and Carbohydrate on the Body Protein and Fat in Growing Chicks." <u>Journal of</u> <u>Nutrition, Yol. 112</u>: 886 (1982).

BIB - 2

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